
Lecture Set 8: Induction Machines

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Spring 2021

About this Lecture Set

- Reading
 - *Electromechanical Motion Devices, 2nd Edition,*
Sections 6.1-6.9, 6.12
- Goal
 - Understand the theory, operation, and modeling of
induction machines

Lecture 68

Preliminaries

Sample Applications

- Low Power:
 - Shaded pole machines (small fans)
 - Permanent Split Capacitor Machines (Residential HVAC)
- Medium Power:
 - Industrial HVAC
 - Pumps
 - Vehicle Propulsion
 - Drive Applications
- High Power:
 - Ship, Train Propulsion
 - Wind Power Generation

Characteristics

- The Good
- The Bad
- The Ugly

Types of IM Machines

- Wound Rotor
- Squirrel Cage
- Solid Rotor

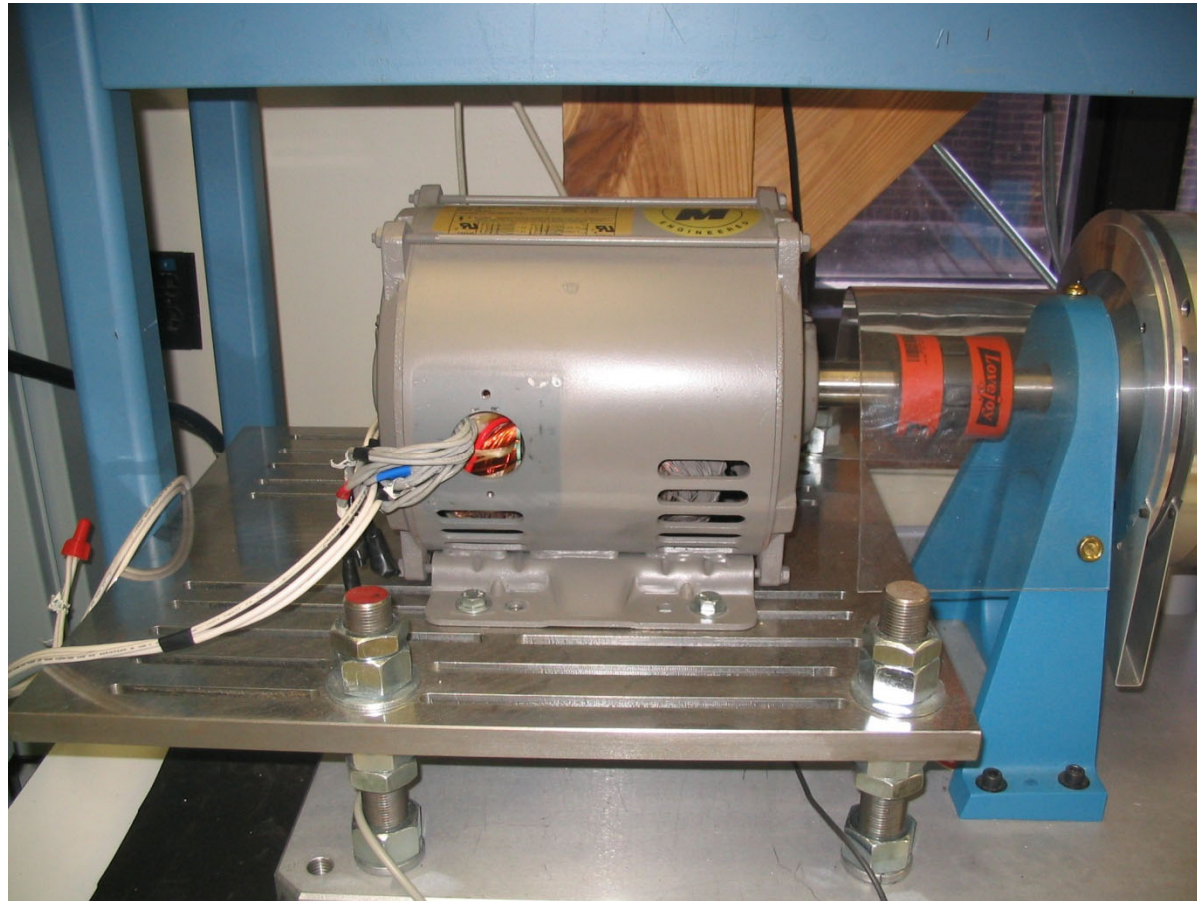
1.5 MW Induction Generator

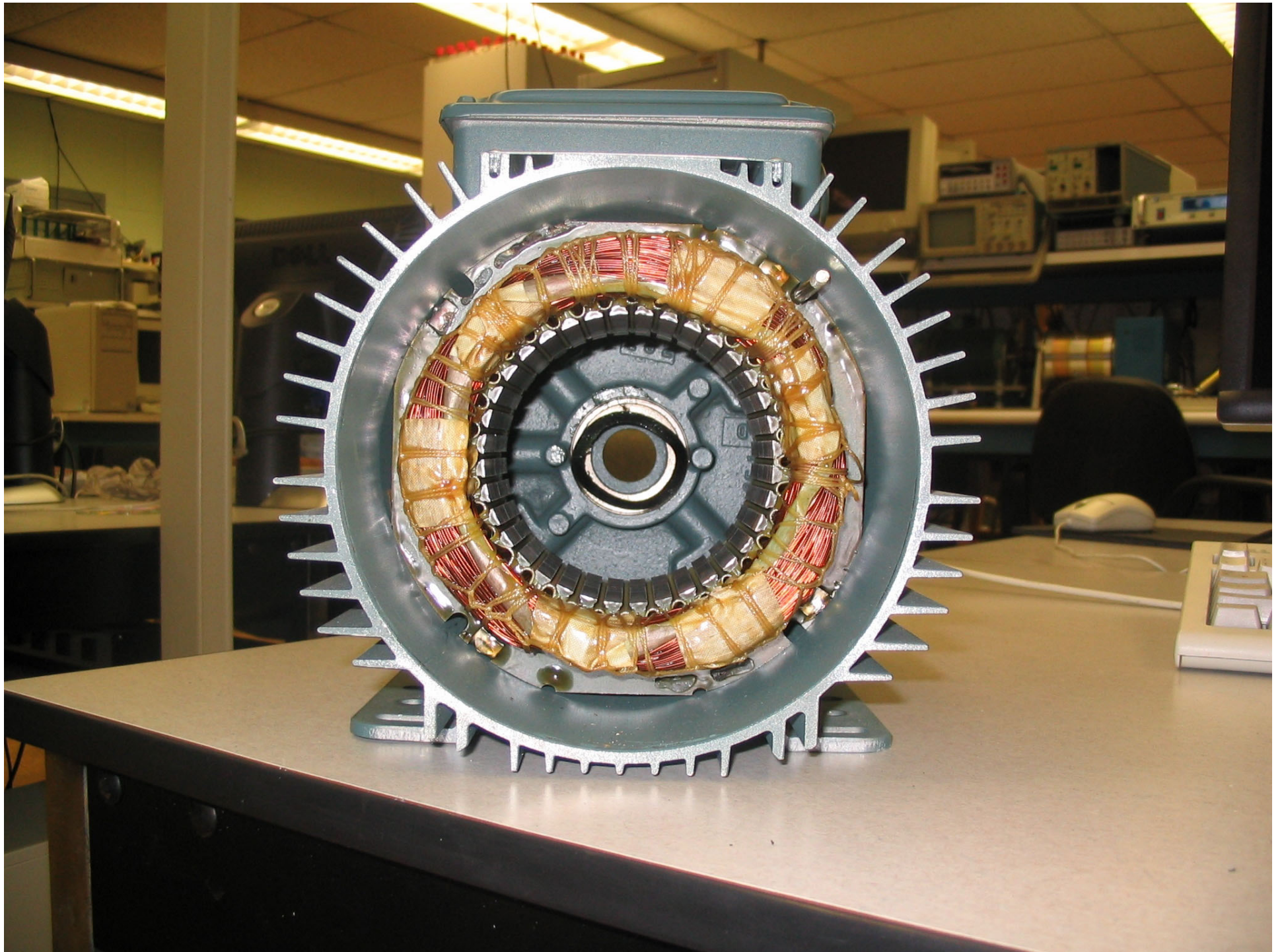


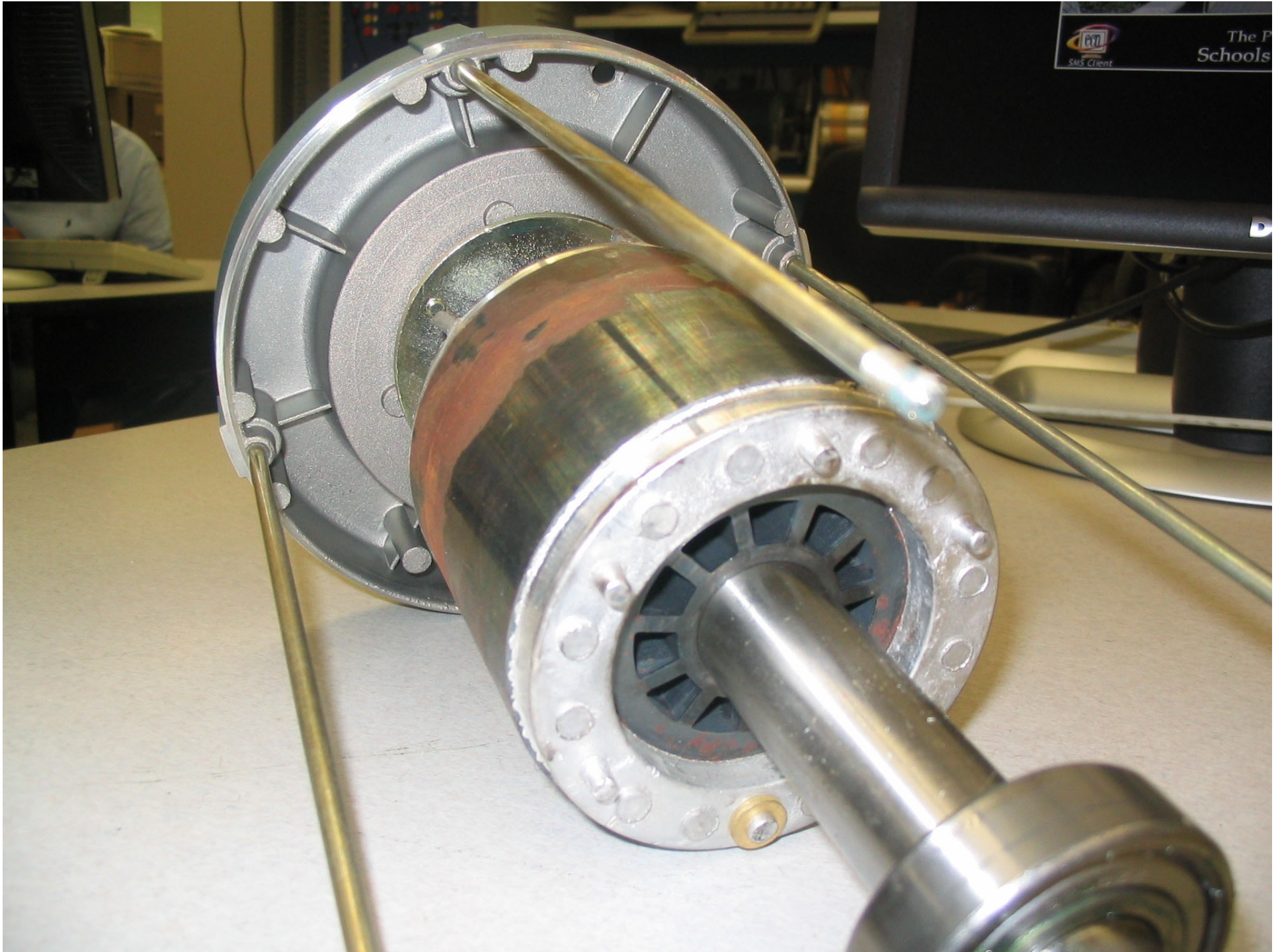
150 Hp @ 1800 RPM



5 Hp @ 1800 RPM



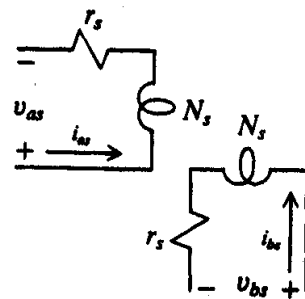
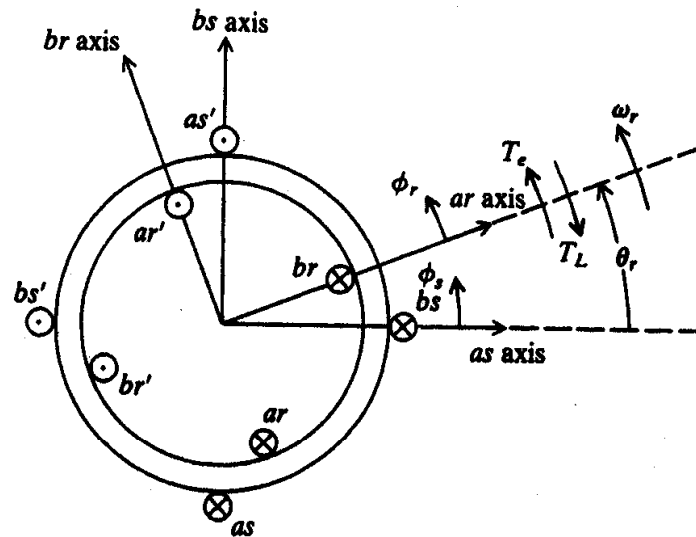




Lecture 69

Theory of Operation: Stator MMF

Two-Phase IM



Stator MMF

Stator MMF

Stator MMF

Stator MMF

Stator MMF

Stator MMF

Lecture 70

Theory of Operation: Rotor MMF

Rotor MMF

Rotor MMF

Rotor MMF

Rotor MMF

Rotor MMF

Rotor MMF

Comparison of MMFs

- As viewed from stator

- As viewed from rotor

IM Torque Production

Lecture 71

Overview of Machine Models

Our Analysis

- Machine Variable Model
- Referred Machine Variable Model
- QD Model
- Steady-State Model

Machine Variable Model

- Voltage equations

$$\mathbf{v}_{abs} = \mathbf{r}_s \mathbf{i}_{abs} + p \boldsymbol{\lambda}_{abs}$$

$$\mathbf{v}_{abr} = \mathbf{r}_r \mathbf{i}_{abr} + p \boldsymbol{\lambda}_{abr}$$

- Flux Linkage Equations

$$\begin{bmatrix} \boldsymbol{\lambda}_{abs} \\ \boldsymbol{\lambda}_{abr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}_{sr} \\ (\mathbf{L}_{sr})^T & \mathbf{L}_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abs} \\ \mathbf{i}_{abr} \end{bmatrix}$$

- Inductance Matrices

$$\mathbf{L}_s = \begin{bmatrix} L_{ls} + L_{ms} & 0 \\ 0 & L_{ls} + L_{ms} \end{bmatrix}$$

$$\mathbf{L}_r = \begin{bmatrix} L_{lr} + L_{mr} & 0 \\ 0 & L_{lr} + L_{mr} \end{bmatrix}$$

$$\mathbf{L}_{sr} = L_{sr} \begin{bmatrix} \cos \theta_r & -\sin \theta_r \\ \sin \theta_r & \cos \theta_r \end{bmatrix}$$

- Torque equation

$$T_e = -\frac{P}{2} L_{sr} [(i_{as} i_{ar} + i_{bs} i_{br}) \sin \theta_r + (i_{as} i_{br} - i_{bs} i_{ar}) \cos \theta_r]$$

Referred Machine Variable Model

- Referral of variables

$$\mathbf{i}'_{abr} = \frac{N_r}{N_s} \mathbf{i}_{abr}$$

$$\mathbf{v}'_{abr} = \frac{N_s}{N_r} \mathbf{v}_{abr}$$

$$\boldsymbol{\lambda}'_{abr} = \frac{N_s}{N_r} \boldsymbol{\lambda}_{abr}$$

- Voltage equations

$$\mathbf{v}_{abs} = \mathbf{r}_s \mathbf{i}_{abs} + p \boldsymbol{\lambda}_{abs}$$

$$\mathbf{v}'_{abr} = \mathbf{r}'_r \mathbf{i}'_{abr} + p \boldsymbol{\lambda}'_{abr}$$

$$\mathbf{r}'_r = \left(\frac{N_s}{N_r} \right)^2 \mathbf{r}_r$$

- Flux linkage equations

$$\begin{bmatrix} \boldsymbol{\lambda}_{abs} \\ \boldsymbol{\lambda}'_{abr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}'_{sr} \\ (\mathbf{L}'_{sr})^T & \mathbf{L}'_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abs} \\ \mathbf{i}'_{abr} \end{bmatrix}$$

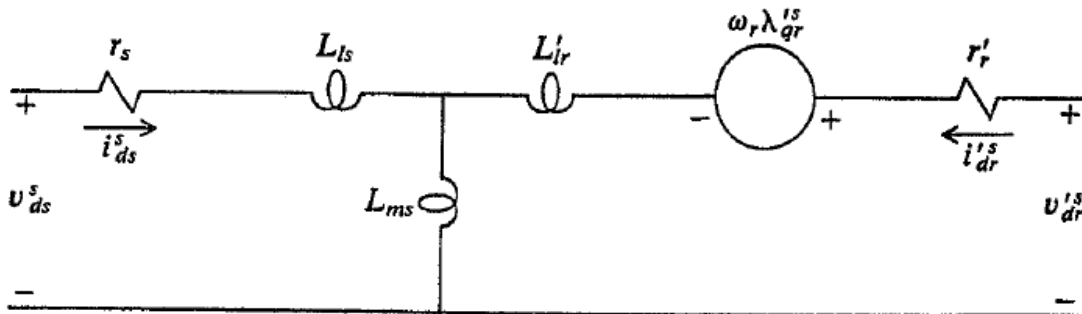
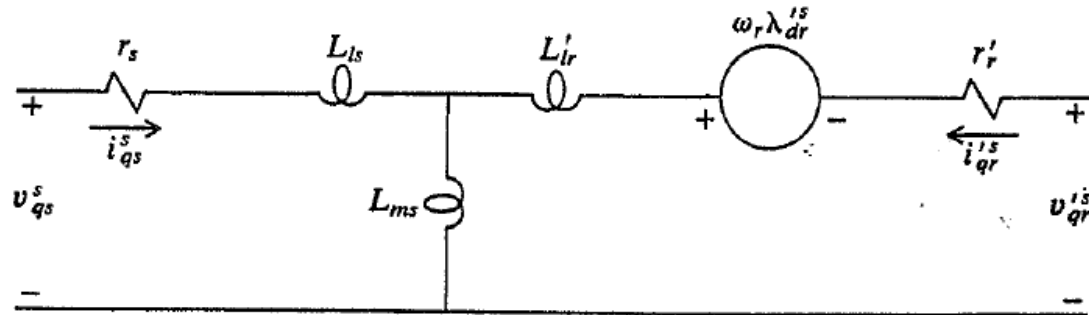
$$\mathbf{L}'_r = \begin{bmatrix} L'_{lr} + L_{ms} & 0 \\ 0 & L'_{lr} + L_{ms} \end{bmatrix}$$

$$\mathbf{L}'_{sr} = \frac{N_s}{N_r} \mathbf{L}_{sr} = L_{ms} \begin{bmatrix} \cos \theta_r & -\sin \theta_r \\ \sin \theta_r & \cos \theta_r \end{bmatrix}$$

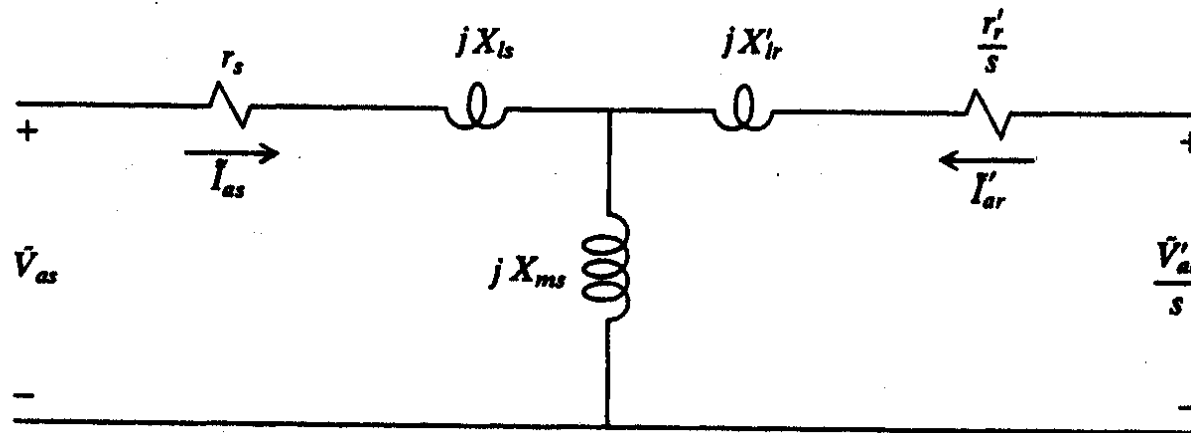
- Torque equation

$$T_e = -\frac{P}{2} L_{ms} [(i_{as} i'_{ar} + i_{bs} i'_{br}) \sin \theta_r + (i_{as} i'_{br} - i_{bs} i'_{ar}) \cos \theta_r]$$

QD Model



Phasor Model



- Torque:
$$T_e = 2 \left(\frac{P}{2} \right) L_{ms} \operatorname{Re} [j \tilde{I}_{as}^* \tilde{I}'_{ar}]$$

Lecture 72

Machine Variable Model: Voltage and Flux Linkage Equations

Voltage Equations

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

$$v_{ar} = r_r i_{ar} + \frac{d\lambda_{ar}}{dt}$$

$$v_{br} = r_r i_{br} + \frac{d\lambda_{br}}{dt}$$

$$\mathbf{v}_{abs} = \mathbf{r}_s \mathbf{i}_{abs} + p \boldsymbol{\lambda}_{abs}$$

$$\mathbf{v}_{abr} = \mathbf{r}_r \mathbf{i}_{abr} + p \boldsymbol{\lambda}_{abr}$$

$$(\mathbf{f}_{abs})^T = [f_{as} \quad f_{bs}]$$

$$(\mathbf{f}_{abr})^T = [f_{ar} \quad f_{br}]$$

Flux Linkage Equations – Scalar Form

$$\lambda_{as} = L_{asas}i_{as} + L_{asbs}i_{bs} + L_{asar}i_{ar} + L_{asbr}i_{br}$$

$$\lambda_{bs} = L_{bsas}i_{as} + L_{bsbs}i_{bs} + L_{bsar}i_{ar} + L_{bsbr}i_{br}$$

$$\lambda_{ar} = L_{aras}i_{as} + L_{arbs}i_{bs} + L_{arar}i_{ar} + L_{arbr}i_{br}$$

$$\lambda_{br} = L_{bras}i_{as} + L_{brbs}i_{bs} + L_{brar}i_{ar} + L_{brbr}i_{br}$$

Flux Linkage Equations – Matrix Form

$$\boldsymbol{\lambda}_{abs} = \mathbf{L}_s \mathbf{i}_{abs} + \mathbf{L}_{sr} \mathbf{i}_{abr}$$

$$\boldsymbol{\lambda}_{abr} = (\mathbf{L}_{sr})^T \mathbf{i}_{abs} + \mathbf{L}_r \mathbf{i}_{abr}$$

$$\begin{bmatrix} \boldsymbol{\lambda}_{abs} \\ \boldsymbol{\lambda}_{abr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}_{sr} \\ (\mathbf{L}_{sr})^T & \mathbf{L}_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abs} \\ \mathbf{i}_{abr} \end{bmatrix}$$

Derivation of Magnetizing Inductances

Derivation of Magnetizing Inductances

Derivation of Magnetizing Inductances

Derivation of Magnetizing Inductances

Derivation of Magnetizing Inductances

Flux Linkage Equation Summary

$$\mathbf{L}_s = \begin{bmatrix} L_{ss} & 0 \\ 0 & L_{ss} \end{bmatrix} = L_{ss} \mathbf{I}$$

$$L_{ss} = L_{ls} + L_{ms}$$

$$L_{rr} = L_{lr} + L_{mr}$$

$$\mathbf{L}_r = \begin{bmatrix} L_{rr} & 0 \\ 0 & L_{rr} \end{bmatrix} = L_{rr} \mathbf{I}$$

$$L_{ms} = \frac{N_s^2}{\mathcal{R}_m}$$

$$L_{mr} = \frac{N_r^2}{\mathcal{R}_m}$$

$$\mathbf{L}_{sr} = L_{sr} \begin{bmatrix} \cos \theta_r & -\sin \theta_r \\ \sin \theta_r & \cos \theta_r \end{bmatrix}$$

$$L_{sr} = \frac{N_r N_s}{\mathcal{R}_m}$$

Lecture 73

Machine Variable Model: Torque Equation

Starting Point

$$\begin{bmatrix} \boldsymbol{\lambda}_{abs} \\ \boldsymbol{\lambda}_{abr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}_{sr} \\ (\mathbf{L}_{sr})^T & \mathbf{L}_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abs} \\ \mathbf{i}_{abr} \end{bmatrix}$$

Derivation of Torque Equation

Derivation of Torque Equation

Lecture 74

Referred Machine Variable Model: Flux Linkage Equations

Why

Starting Point

$$\begin{bmatrix} \boldsymbol{\lambda}_{abs} \\ \boldsymbol{\lambda}_{abr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}_{sr} \\ (\mathbf{L}_{sr})^T & \mathbf{L}_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abs} \\ \mathbf{i}_{abr} \end{bmatrix}$$

$$\mathbf{L}_s = \begin{bmatrix} L_{ss} & 0 \\ 0 & L_{ss} \end{bmatrix} = L_{ss} \mathbf{I}$$

$$\mathbf{L}_r = \begin{bmatrix} L_{rr} & 0 \\ 0 & L_{rr} \end{bmatrix} = L_{rr} \mathbf{I}$$

$$\mathbf{L}_{sr} = L_{sr} \begin{bmatrix} \cos \theta_r & -\sin \theta_r \\ \sin \theta_r & \cos \theta_r \end{bmatrix}$$

Referral of Flux Linkage Equations

Referral of Flux Linkage Equations

Summary

$$\begin{bmatrix} \boldsymbol{\lambda}_{abs} \\ \boldsymbol{\lambda}'_{abr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}'_{sr} \\ (\mathbf{L}'_{sr})^T & \mathbf{L}'_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abs} \\ \mathbf{i}'_{abr} \end{bmatrix}$$

$$\mathbf{L}'_r = \left(\frac{N_s}{N_r} \right)^2 \mathbf{L}_r = \begin{bmatrix} L'_{rr} & 0 \\ 0 & L'_{rr} \end{bmatrix}$$

$$L'_{rr} = L'_{lr} + \left(\frac{N_s}{N_r} \right)^2 L_{mr} = L'_{lr} + L_{ms}$$

$$\mathbf{L}'_{sr} = \frac{N_s}{N_r} \mathbf{L}_{sr} = L_{ms} \begin{bmatrix} \cos \theta_r & -\sin \theta_r \\ \sin \theta_r & \cos \theta_r \end{bmatrix}$$

Lecture 75

Referred Machine Variable Model: Voltage and Torque Equations

Referral of Voltage Equations

$$\mathbf{v}_{abs} = \mathbf{r}_s \mathbf{i}_{abs} + p \boldsymbol{\lambda}_{abs}$$

$$\mathbf{v}_{abr} = \mathbf{r}_r \mathbf{i}_{abr} + p \boldsymbol{\lambda}_{abr}$$

Referral of Voltage Equations

- Finally, we get

$$\mathbf{v}_{abs} = \mathbf{r}_s \mathbf{i}_{abs} + p \boldsymbol{\lambda}_{abs}$$

$$\mathbf{v}'_{abr} = \mathbf{r}'_r \mathbf{i}'_{abr} + p \boldsymbol{\lambda}'_{abr}$$

- Where

$$\mathbf{r}'_r = \left(\frac{N_s}{N_r} \right)^2 \mathbf{r}_r$$

Referral of Torque Equation

- It can be shown that

$$T_e = -\frac{P}{2} L_{ms} [(i_{as} i'_{ar} + i_{bs} i'_{br}) \sin \theta_r + (i_{as} i'_{br} - i_{bs} i'_{ar}) \cos \theta_r]$$

Lecture 76

The Stationary Reference Frame

Stationary Reference Frame

- Stator transformation

$$\mathbf{f}_{qds}^S = \mathbf{K}_s^S \mathbf{f}_{abs}$$

$$\mathbf{f}_{abs} = (\mathbf{K}_s^S)^{-1} \mathbf{f}_{qds}^S$$

$$\begin{bmatrix} f_{qs}^S \\ f_{ds}^S \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} f_{as} \\ f_{bs} \end{bmatrix}$$

- Rotor transformation

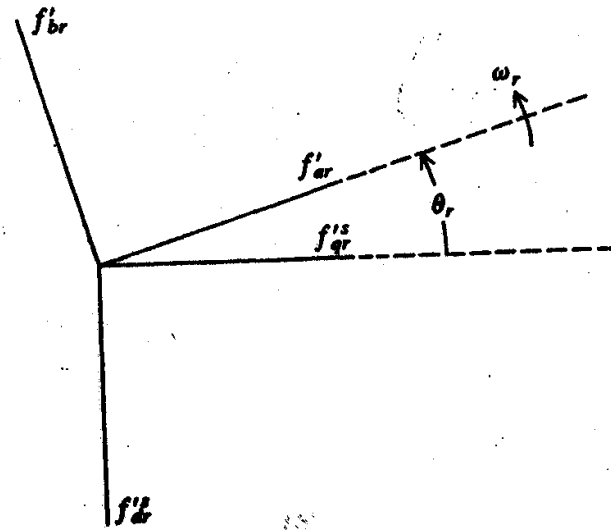
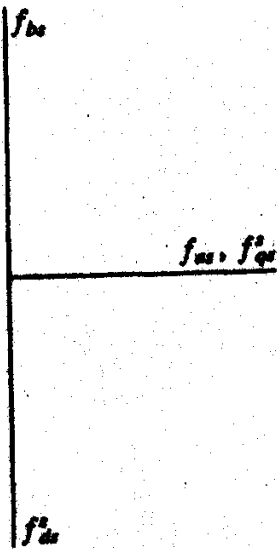
$$\mathbf{f}'_{qdr} = \mathbf{K}_r^S \mathbf{f}'_{abr}$$

$$\mathbf{f}'_{abr} = (\mathbf{K}_r^S)^{-1} \mathbf{f}'_{qdr}$$

$$\begin{bmatrix} f'_{qr} \\ f'_{dr} \end{bmatrix} = \begin{bmatrix} \cos \theta_r & -\sin \theta_r \\ -\sin \theta_r & -\cos \theta_r \end{bmatrix} \begin{bmatrix} f'_{ar} \\ f'_{br} \end{bmatrix}$$

Stationary Reference Frame

Geometrical Interpretation



Lecture 77

Transformation of the Voltage Equations to the Stationary Reference Frame

Transformation of Voltage Equations

Transformation of Voltage Equations

Transformation of Voltage Equations

Transformation of Voltage Equations

Summary

$$\mathbf{v}_{qds}^s = \mathbf{r}_s \mathbf{i}_{qds}^s + p \boldsymbol{\lambda}_{qds}^s$$

$$\mathbf{v}_{qdr}'^s = \mathbf{r}_r' \mathbf{i}_{qdr}'^s - \omega_r \boldsymbol{\lambda}_{dqr}'^s + p \boldsymbol{\lambda}_{qdr}'^s$$

$$\boldsymbol{\lambda}_{dqr}'^s = [\lambda_{dr}'^s \quad -\lambda_{qr}'^s]^T$$

Lecture 78

Transformation of the Flux Linkage Equations to the Stationary Reference Frame

Transformation of Flux Linkage Equations

$$\boldsymbol{\lambda}_{abs} = \mathbf{L}_s \mathbf{i}_{abs} + \mathbf{L}'_{sr} \mathbf{i}'_{abr}$$

$$\boldsymbol{\lambda}_{abr} = \mathbf{L}'_{sr} \mathbf{i}_{abs} + \mathbf{L}'_{rr} \mathbf{i}'_{abr}$$

Transformation of Flux Linkage Equations

Transformation of Flux Linkage Equations

Transformation of Flux Linkage Equations

Transformation of Flux Linkage Equations

$$\boldsymbol{\lambda}_{qds}^s = L_{ss} \mathbf{i}_{qds}^s + L_{ms} \mathbf{i}'_{qdr}$$

$$\boldsymbol{\lambda}'_{qdr} = L_{ms} \mathbf{i}_{qds}^s + L'_{rr} \mathbf{i}'_{qdr}$$

$$L_{ss} = L_{ls} + L_{ms}$$

$$L'_{rr} = L'_{lr} + L_{ms}$$

Lecture 79

Transformation of the Torque Equation to the Stationary Reference Frame

Transformation of the Torque Equations

$$T_e = \frac{P}{2} L_{sr} \mathbf{i}_{abs}^T \begin{bmatrix} -\sin \theta_r & -\cos \theta_r \\ \cos \theta_r & -\sin \theta_r \end{bmatrix} \mathbf{i}_{abr}$$

Transformation of the Torque Equations

Transformation of the Torque Equations

Transformation of the Torque Equations

$$T_e = \frac{P}{2} L_{ms} (i_{qs}^s i_{dr}'^s - i_{ds}^s i_{qr}'^s)$$

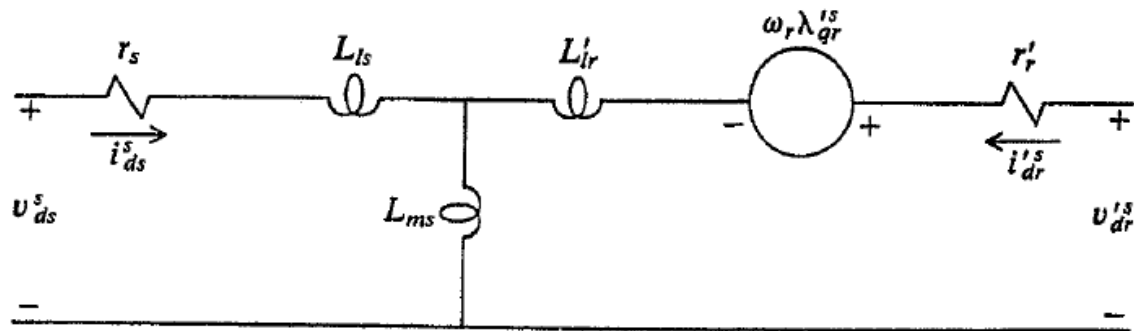
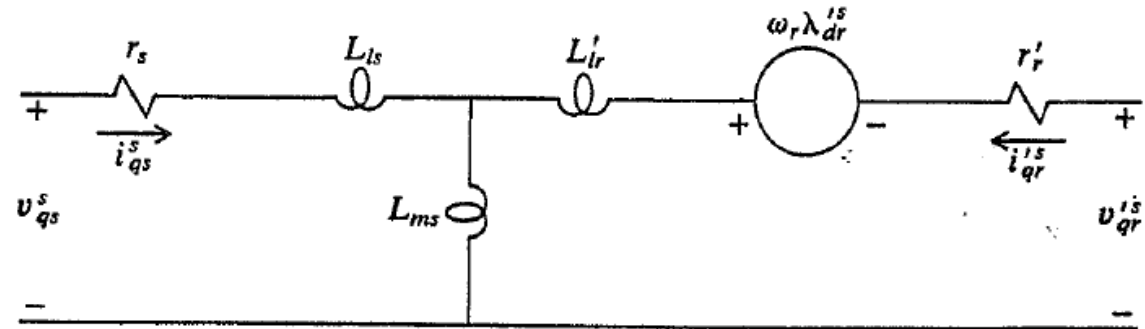
$$T_e = \frac{P}{2} (\lambda_{qr}'^s i_{dr}'^s - \lambda_{dr}'^s i_{qr}'^s)$$

$$T_e = \frac{P}{2} (\lambda_{ds}^s i_{qs}^s - \lambda_{qs}^s i_{ds}^s)$$

Lecture 80

QD Equivalent Circuit

QD Equivalent Circuit



QD Equivalent Circuit

QD Equivalent Circuit

Comments on QD Machine

- When is it valid ?

- What is it used for ?

Lecture 81

Balanced Steady-State Operation and the Phasor Model

Stator Side

- Steady-state forms

$$F_{as} = \sqrt{2}F_s \cos[\omega_e t + \theta_{esf}(0)]$$

$$F_{bs} = \sqrt{2}F_s \sin[\omega_e t + \theta_{esf}(0)]$$

- We can show

$$\tilde{F}_{qs}^s = \tilde{F}_{as} = F_s e^{j\theta_{esf}(0)}$$

$$\tilde{F}_{ds}^s = -\tilde{F}_{bs} = -(-jF_s e^{j\theta_{esf}(0)})$$

$$\tilde{F}_{qs}^s = -j\tilde{F}_{ds}^s$$

Stator Side

Stator Side

Rotor Side

- Rotor relationships

$$F'_{ar} = \sqrt{2}F'_r \cos[(\omega_e - \omega_r)t + \theta_{erf}(0)]$$

$$F'_{br} = \sqrt{2}F'_r \sin[(\omega_e - \omega_r)t + \theta_{erf}(0)]$$

- We can show

$$\tilde{F}'_{qr}{}^s = \tilde{F}'_{ar}$$

$$\tilde{F}'_{dr}{}^s = -\tilde{F}'_{br}$$

$$\tilde{F}'_{qr}{}^s = -j\tilde{F}'_{dr}{}^s$$

Rotor Side

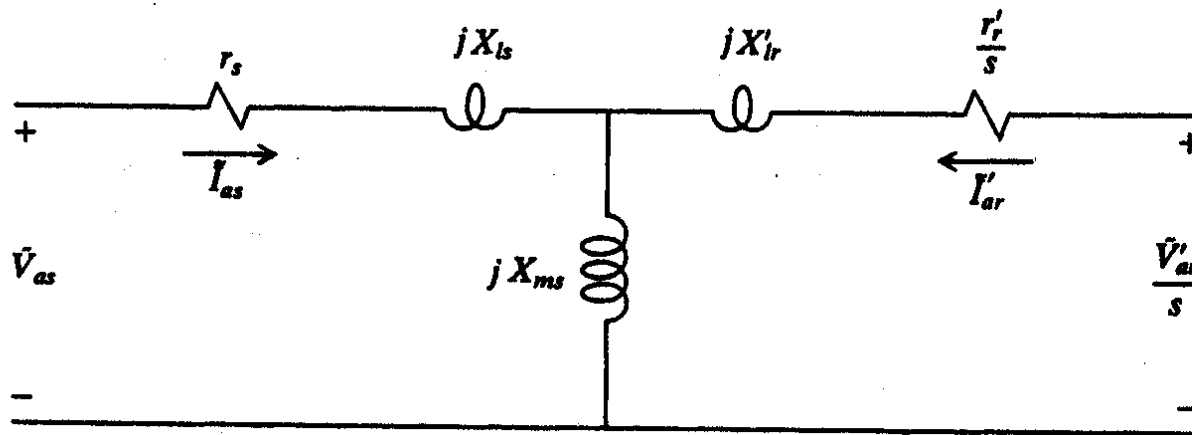
Rotor Side

Rotor Side

Lecture 82

Balanced Steady-State Operation
Phasor Equivalent Circuit

Phasor Equivalent Circuit (2-Phase)



- Torque:
$$T_e = 2 \left(\frac{P}{2} \right) L_{ms} \operatorname{Re} [j \tilde{I}_{as}^* \tilde{I}'_{ar}]$$

Phasor Equivalent Circuit (2-Phase)

- Voltage Equations

$$\tilde{V}_{as} = (r_s + j\omega_e L_{ls})\tilde{I}_{as} + j\omega_e L_{ms}(\tilde{I}_{as} + \tilde{I}'_{ar})$$

$$\frac{\tilde{V}'_{ar}}{s} = \left(\frac{r'_r}{s} + j\omega_e L'_{lr} \right) \tilde{I}'_{ar} + j\omega_e L_{ms}(\tilde{I}_{as} + \tilde{I}'_{ar})$$

- Slip

$$s = \frac{\omega_e - \omega_r}{\omega_e}$$

- Torque

$$T_e = 2 \left(\frac{P}{2} \right) L_{ms} \operatorname{Re}[j\tilde{I}_{as}^* \tilde{I}'_{ar}]$$

Derivation

Derivation

Derivation

Derivation

Derivation

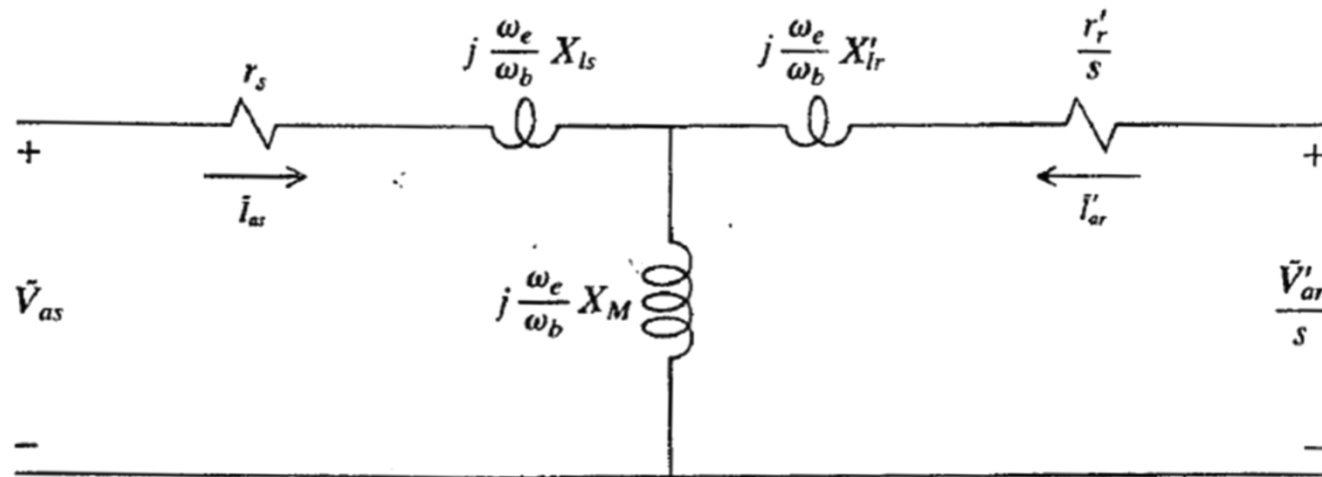
Chief Problems with Model

- Magnetizing Saturation
- Leakage Saturation
- Distributed System Effects
- Thermal Effects

Lecture 83

Balanced Steady-State Operation of 3-Phase Machines

Phasor Equivalent Circuit (3-Phase)



- Where

$$T_e = 3 \left(\frac{P}{2} \right) L_M \operatorname{Re} [j \tilde{I}_{as}^* \tilde{I}'_{ar}] \quad L_M = \frac{3}{2} L_{ms}$$

Delta Connected Machine

Wye Connected Machine

Lecture 84

Analyzing Machine Operation

Typical Operating Situations

- Utility grid
 - Fixed voltage, fixed frequency voltage source
- Inverter (power electronic control)
 - Voltage source based inverter (variable voltage, variable frequency)
 - Volts-per-hertz control
 - Current source based inverter (variable current, variable frequency)
 - Maximum torque per amp control
 - Field-oriented control
 - Direct torque control

Derivation of Rotor Current

Torque for a Given Stator Current

- We can show

$$T_e = \frac{N \left(\frac{P}{2} \right) \omega_s L_M^2 I_s^2 r'_r}{(r'_r)^2 + (\omega_s L'_{rr})^2}$$

Torque for a Given Stator Current

Torque for a Given Stator Current

Torque for a Given Stator Current

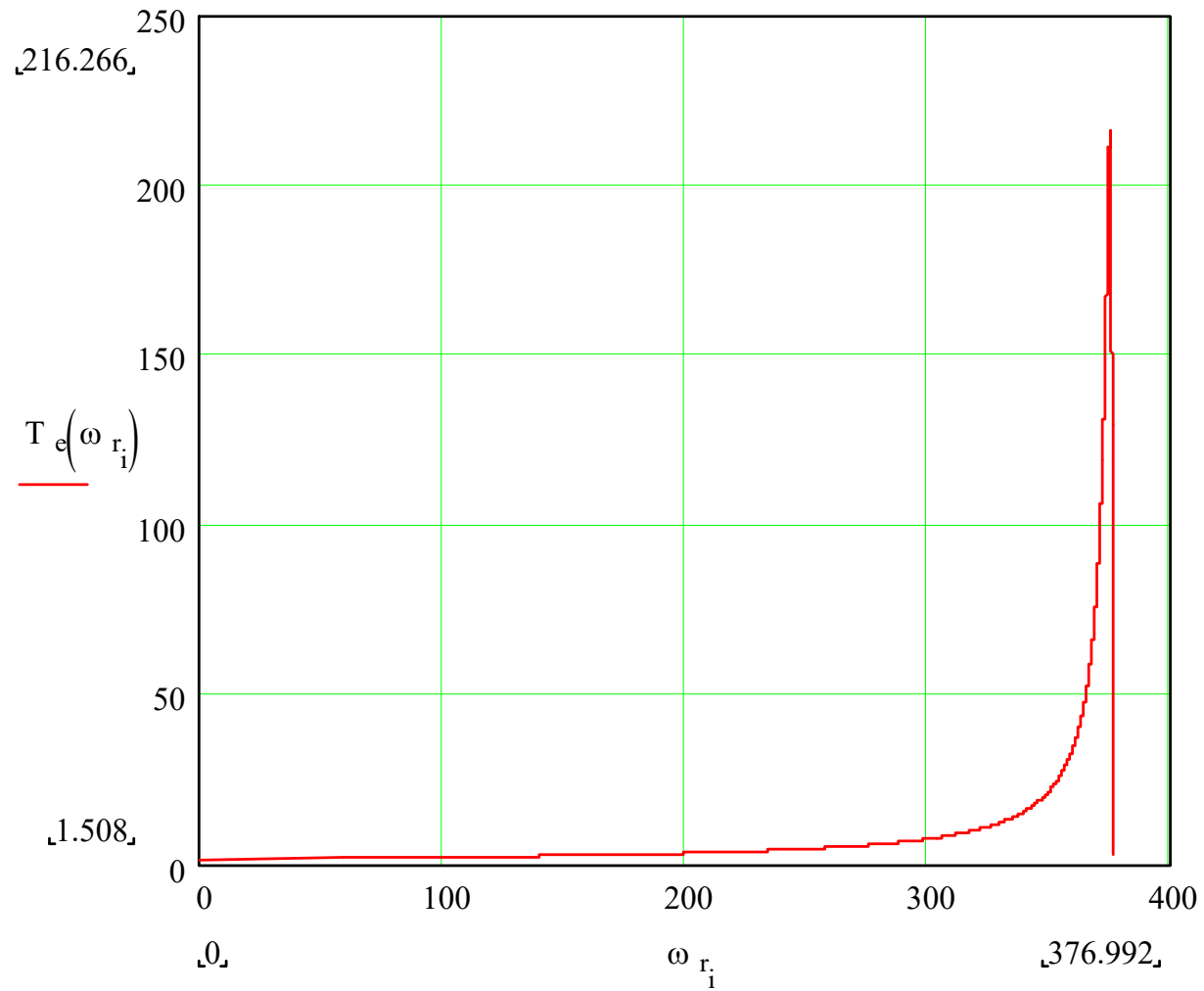
Lecture 85

Operation From A Current Source

Operation from Current Source

- Machine Parameters
- 460 V, 1-1, rms; 50 Hp @ 1800 rpm
 - Stator resistance: 72.5 m Ω
 - Rotor resistance: 41.3 m Ω
 - Stator and referred rotor leakage: 1.32 mH
 - Magnetizing Inductance: 30.1 mH
- Operating Conditions
 - Armature current: 50 A, rms
 - Frequency: 60 Hz

Current Source Torque - Speed



Maximum Torque Per Amp Control

- Control Summary

$$I_s = \sqrt{\frac{2|T_e^*|(r_r'^2 + (\omega_s L_{rr}')^2)}{NP|\omega_s|L_M^2 r_r'}}$$

$$\omega_s = \frac{r_r'}{L_{rr}'}$$

- To show this, start with

$$T_e = \frac{N \left(\frac{P}{2} \right) \omega_s L_M^2 I_s^2 r_r'}{(r_r')^2 + (\omega_s L_{rr}')^2}$$

Maximum Torque Per Amp Control

Maximum Torque Per Amp Control

Maximum Torque Per Amp Control

MTPA Control Example

- Consider the 50 Hp example
- Suppose we want 200 Nm at a speed of 1000 rpm
- Compute the magnitude of the a-phase current, A
- Compute the required voltage
- Compute the efficiency

MTPA Control Example

MTPA Control Example

MTPA Control Example

Lecture 86

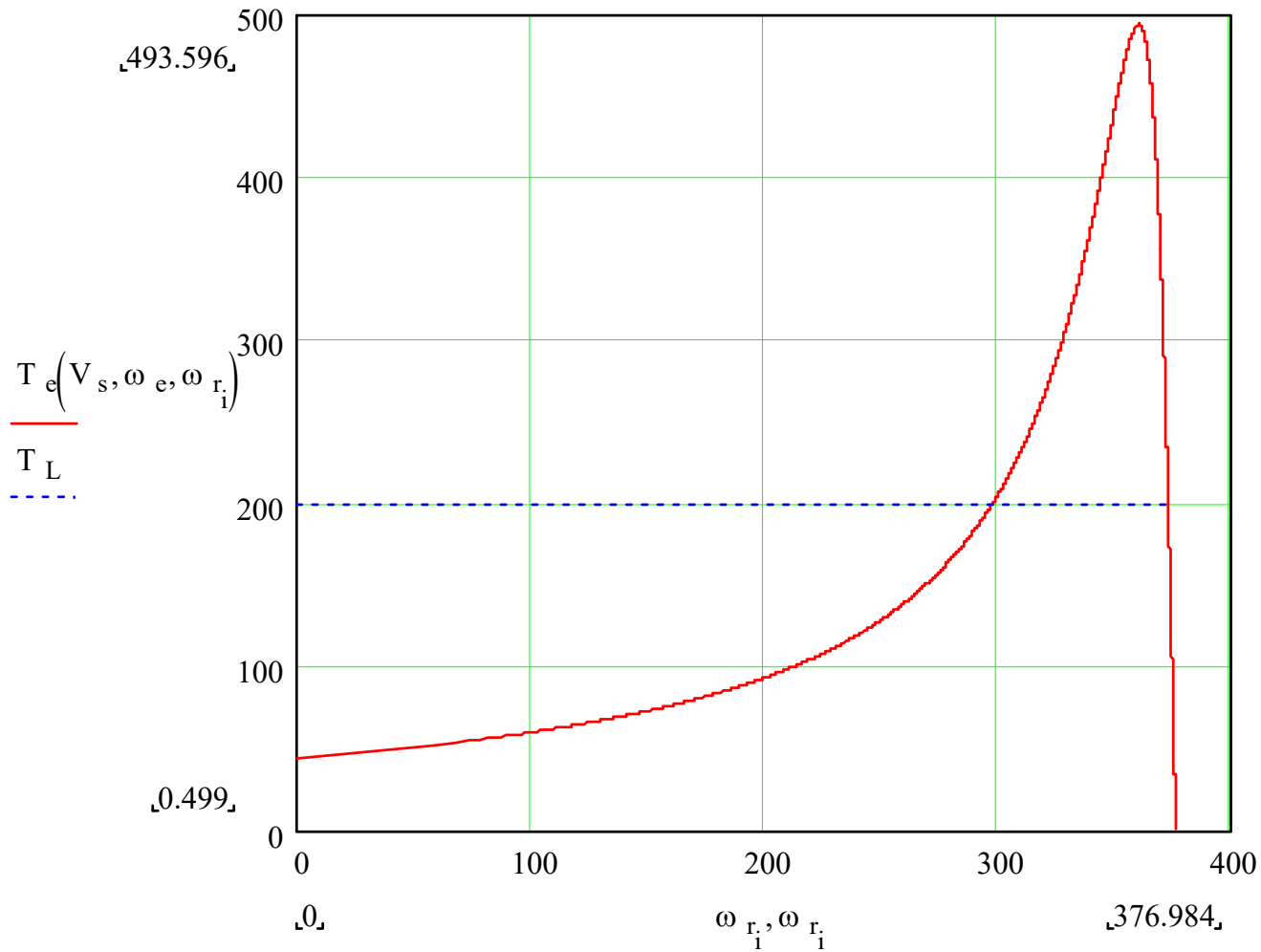
Operation From A Voltage Source

Operation From Voltage Source: Stator Current

Example 1

- Consider our 50 Hp machine
- Fed from 460 V 1- ϕ rms source
- Load torque of 200 Nm
- Objective: Compute speed and current

Steady-State Operating Point

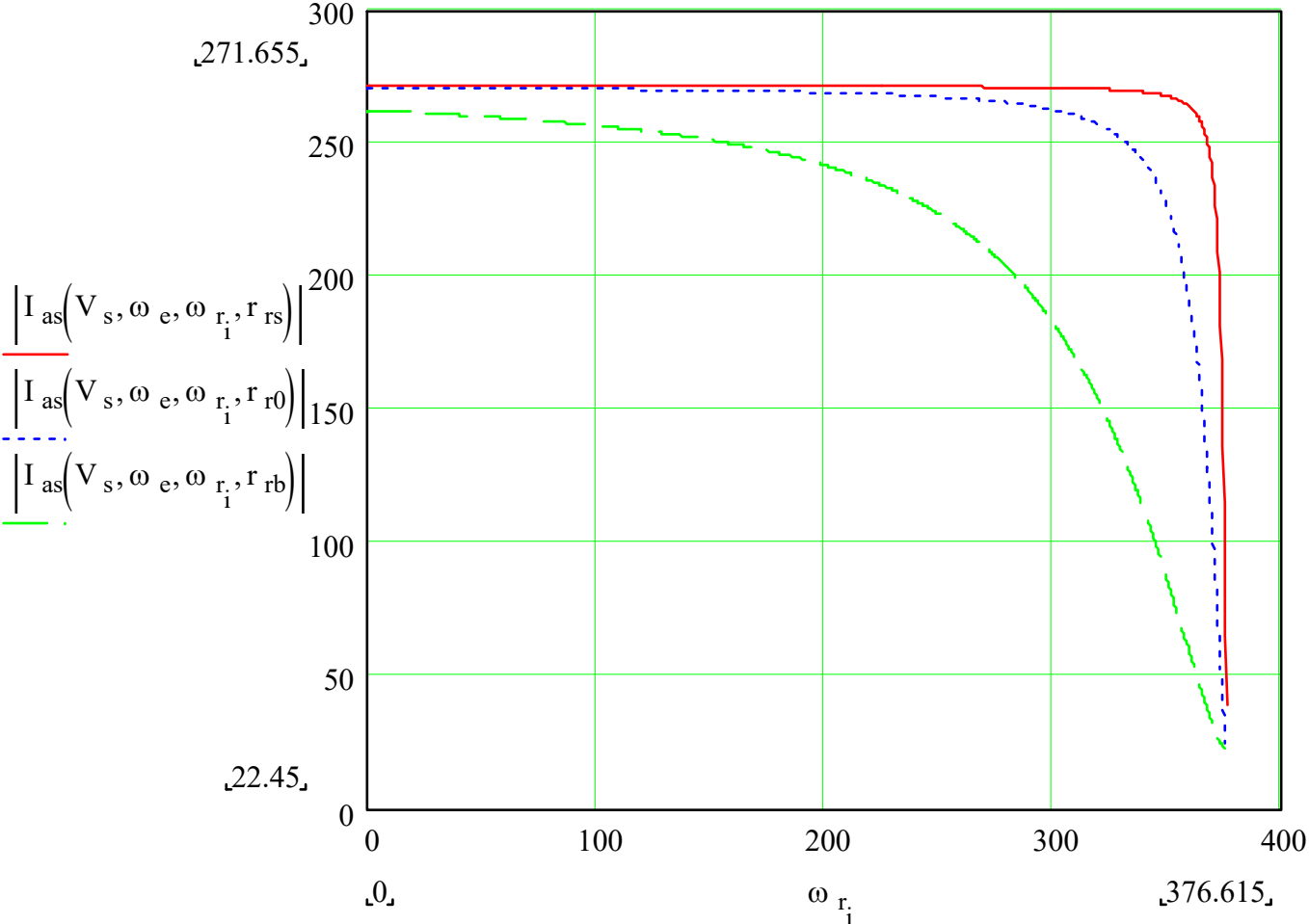


Steady-State Operating Point

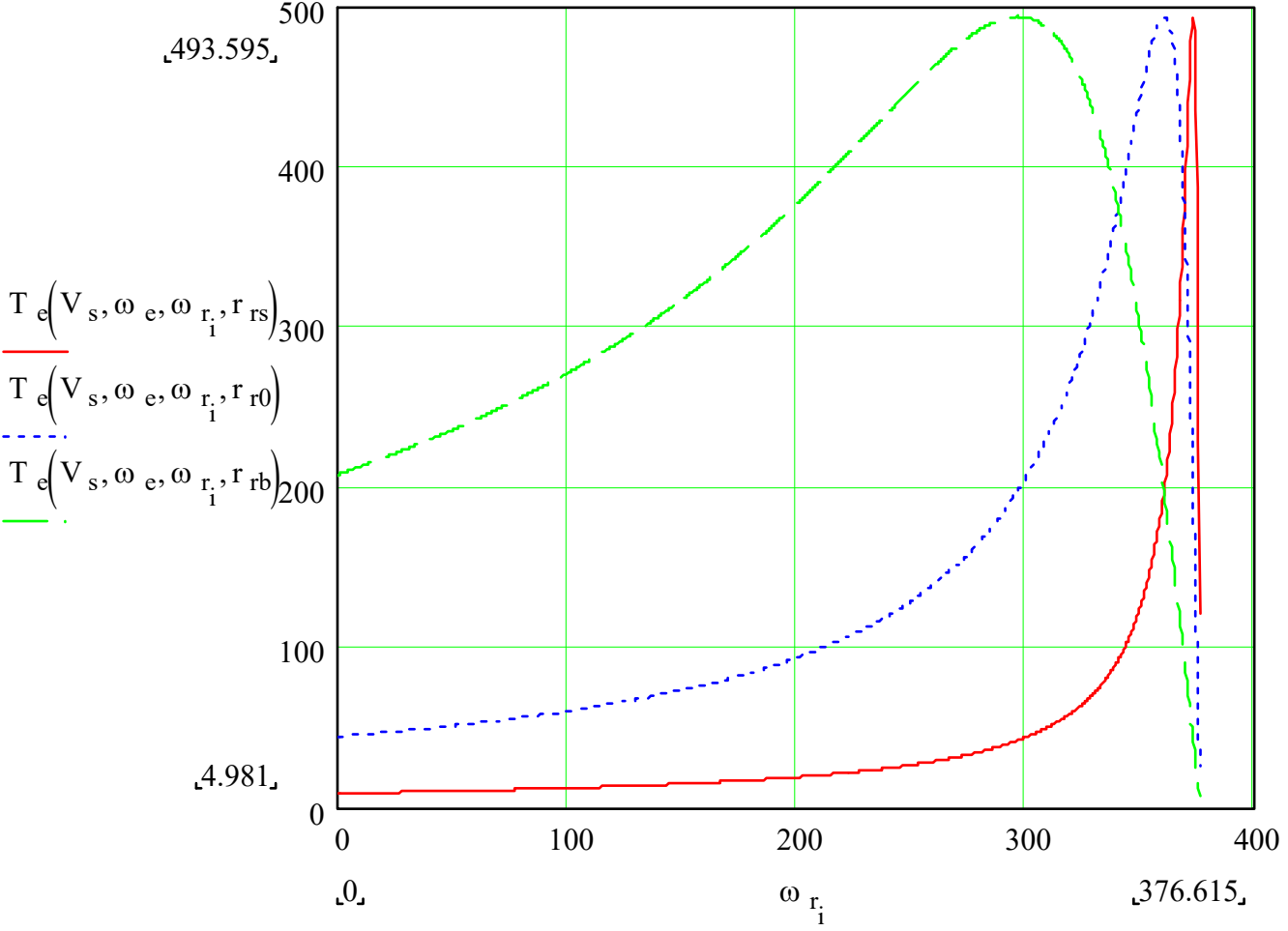
Example 2

- Consider our 50 Hp machine
- Fed from 460 V 1- ϕ rms, 60 Hz source
- Let's look at machine properties versus speed

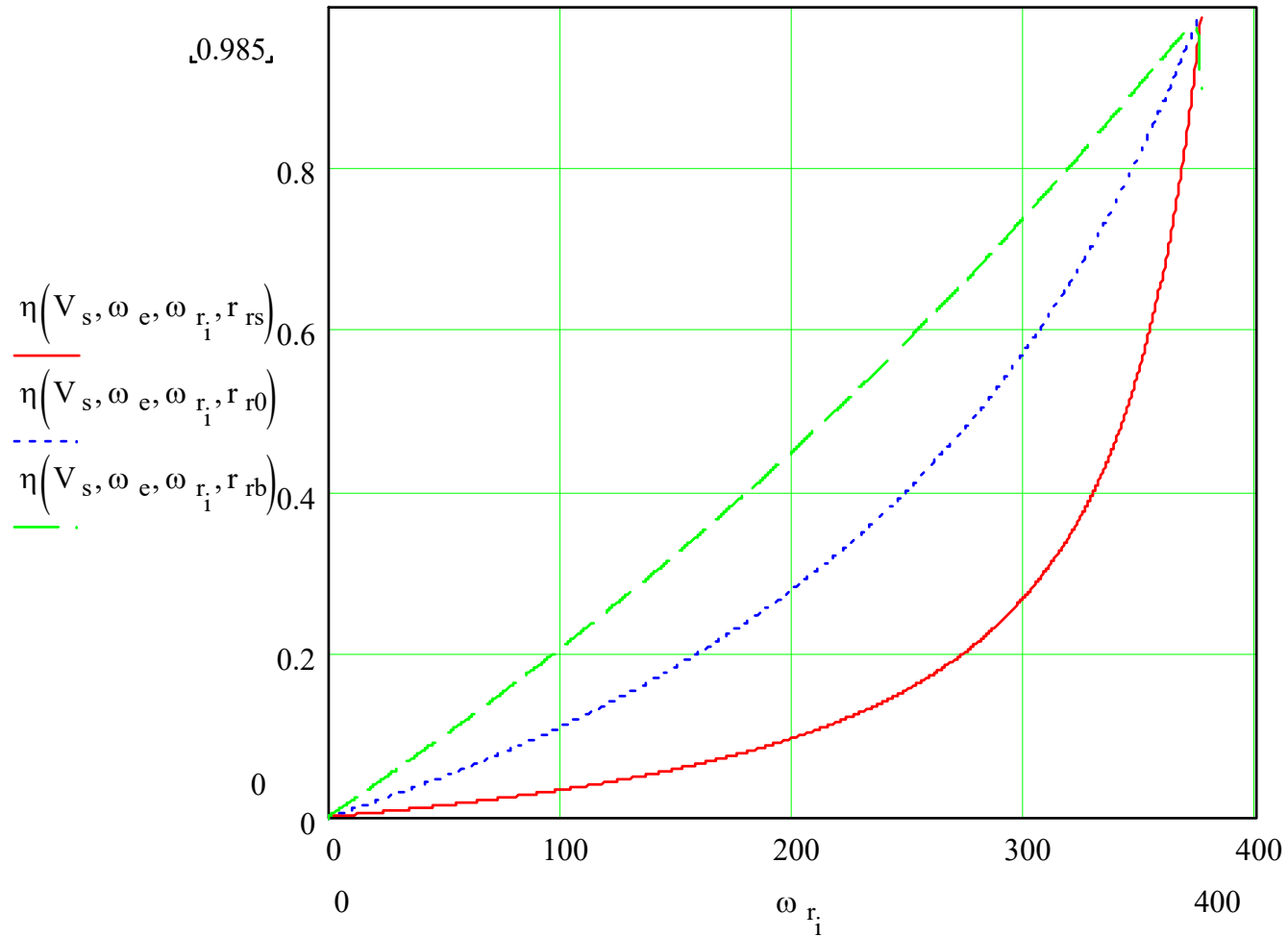
Current vs Speed



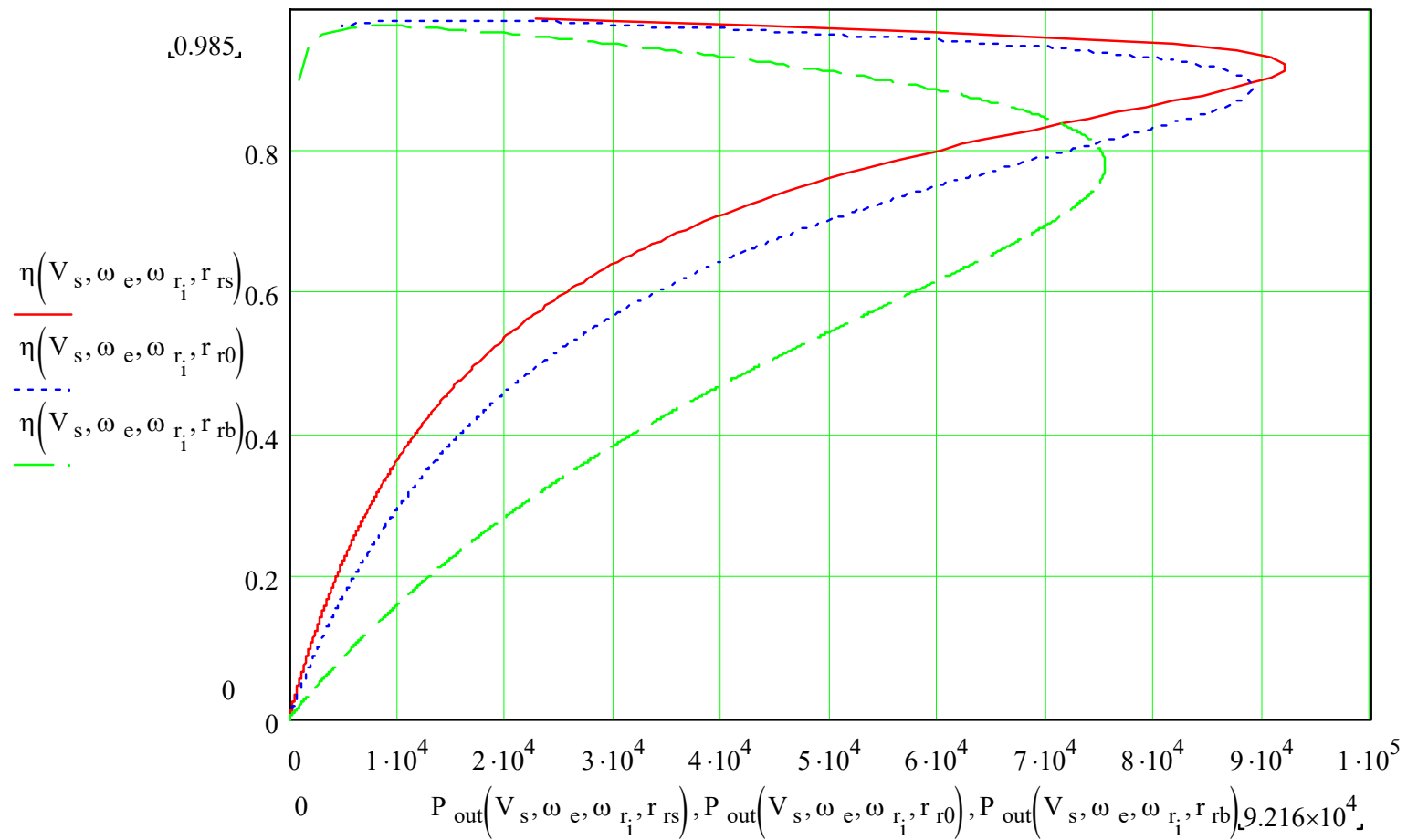
Torque vs Speed



Efficiency vs Speed



Efficiency vs Output Power



Lecture 87

Parameter Identification

Parameter Identification

- DC Test

Parameter Identification

- Blocked Rotor Test

Parameter Identification

- Blocked Rotor Test

Parameter Identification

- No Load Test

Lecture 87

Volts Per Hertz Control

Volts Per Hertz Control

- The idea:

Volts Per Hertz Control

- The idea:

Volts Per Hertz Control

