Lecture Set 7:
Phasors and Transformers

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About this Lecture Set

- Reading
  - *Power Magnetic Devices: A Multi-Objective Design Approach, Appendix E,* (Available through IEEE Xplore or blackboard)
  - *Electromechanical Motion Devices, 2nd Edition*, Section 1.5
- Goal
  - Review phasors and analyze a single-phase transformer in order to set the stage for the induction machine

Lecture 64

Review of Phasor Analysis

Review of Phasor Analysis

- Basis of phasor analysis:

$$\frac{d}{du}e^{au} = ae^{au}$$

$$e^{j\phi} = \cos\phi + j\sin\phi$$

Review of Phasor Analysis

- Consider the differential equation

$$a_0 y + a_1 \frac{dy}{dt} + a_2 \frac{d^2 y}{dt^2} + \ldots = b_0 x + b_1 \frac{dx}{dt} + b_2 \frac{d^2 x}{dt^2} + \ldots$$

- This may also be written as

$$\sum_{n=0}^{N} a_n \frac{d^n y}{dt^n} = \sum_{m=0}^{M} b_m \frac{d^m x}{dt^m}$$

- Form of input (and output)

$$f = \sqrt{2} A_f \cos(\omega t + \phi_f)$$

Review of Phasor Analysis

- Phase-Shifted Problem

$$\sum_{n=0}^{N} a_n \frac{d^n y_p}{dt^n} = \sum_{m=0}^{M} b_m \frac{d^m x_p}{dt^m}$$

- Form of input (and output)

$$f_p = \sqrt{2} A_f \sin(\omega t + \phi_f)$$

Review of Phasor Analysis

- Composite variables

$$\underline{f} = f + jf_p$$

- Clearly

$$f = \text{real}(\underline{f})$$

$$f_p = \text{imag}(\underline{f})$$

Review of Phasor Analysis

- We can show

$$\underline{f} = \sqrt{2}\,\hat{f}e^{j\omega t}$$

- where

$$\hat{f} = A_f e^{j\phi_f} \qquad \hat{f} = A_f \angle \phi_f$$

$$\phi_f = \text{angle}(\hat{f}) \qquad A_f = \left|\hat{f}\right|$$

Review of Phasor Analysis

Review of Phasor Analysis

- We can show

$$\hat{y} = \hat{x}\frac{\sum_{m=1}^{M} b_m \left(j\omega \right)^m}{\sum_{n=1}^{N} a_n (j\omega)^n}$$

$$y = \text{real}(\sqrt{2}\,\hat{y}e^{j\omega t})$$

$$y = \sqrt{2}A_y \cos(\omega t + \phi_y)$$

Review of Phasor Analysis

Review of Phasor Analysis

Review of Phasor Analysis

Review of Phasor Analysis

Lecture 65

Phasor Representation of Circuit Elements

Review of Phasor Analysis

- Consider a resistor

Review of Phasor Analysis

- Consider a capacitor

Review of Phasor Analysis

- Consider an inductor

Review of Phasor Analysis

- Consider a series RLC circuit

Review of Phasor Analysis

Review of Phasor Analysis

- Numerical example

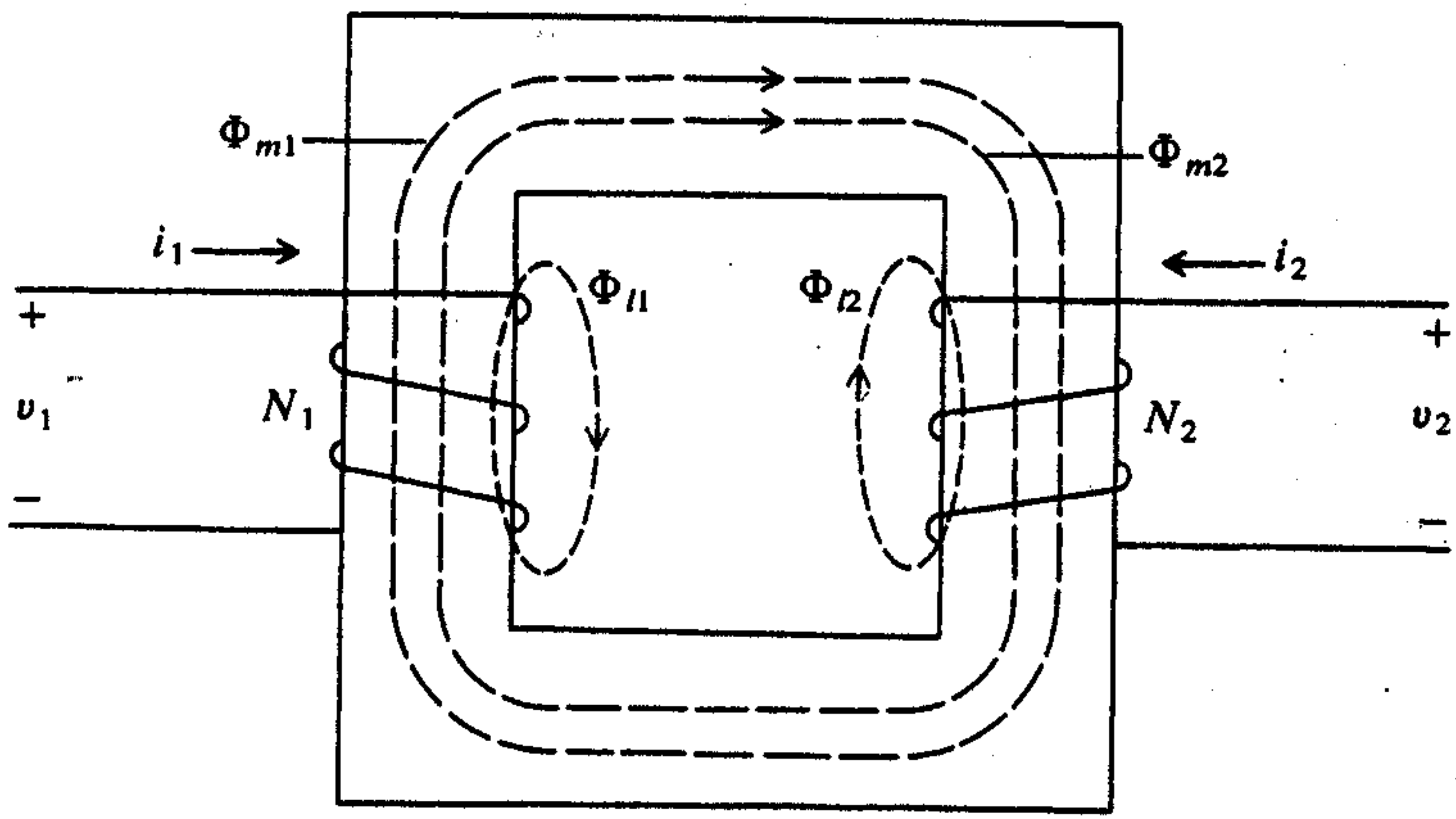
Review of Phasor Analysis

- Numerical example

Lecture 66

Transformers

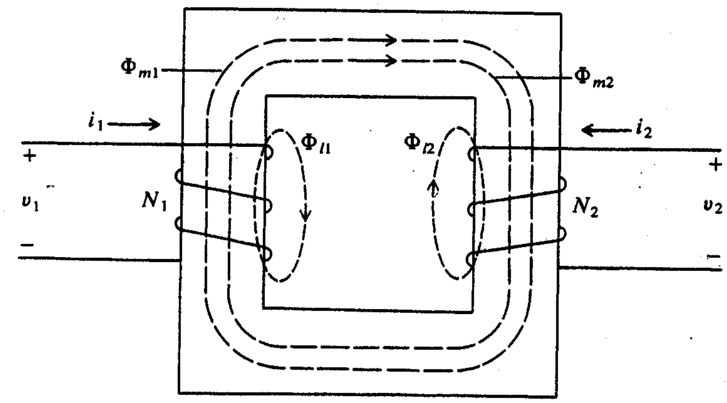
A Single Phase Transformer



Construction

Analysis

- Voltage Equations



$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt}$$

$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

Analysis

- Flux linkage equations

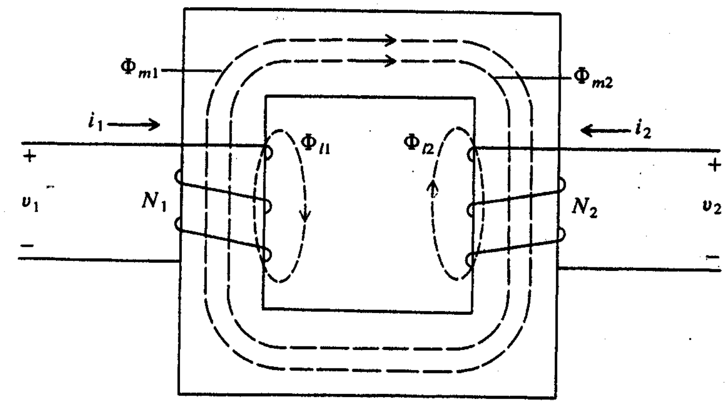
$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2$$

$$L_{11} = \frac{N_1^2}{\mathcal{R}_{l1}} + \frac{N_1^2}{\mathcal{R}_m} = L_{l1} + L_{m1}$$

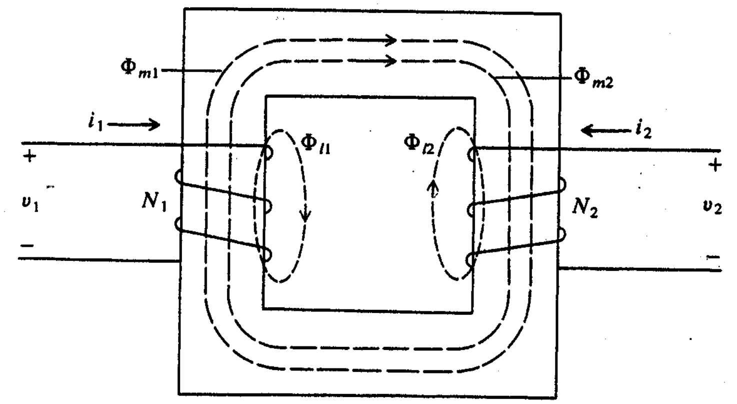
$$L_{22} = \frac{N_2^2}{\mathcal{R}_{l2}} + \frac{N_2^2}{\mathcal{R}_m} = L_{12} + L_{m2}$$

$$L_{12} = L_{21} = \frac{N_1 N_2}{\mathcal{R}_m}$$



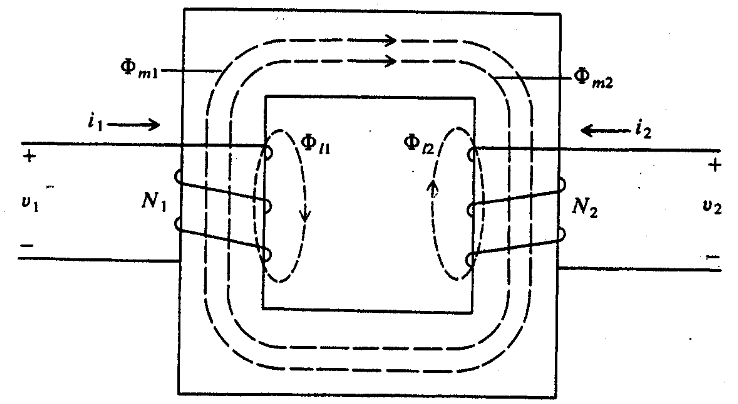
Analysis

- Flux Terms



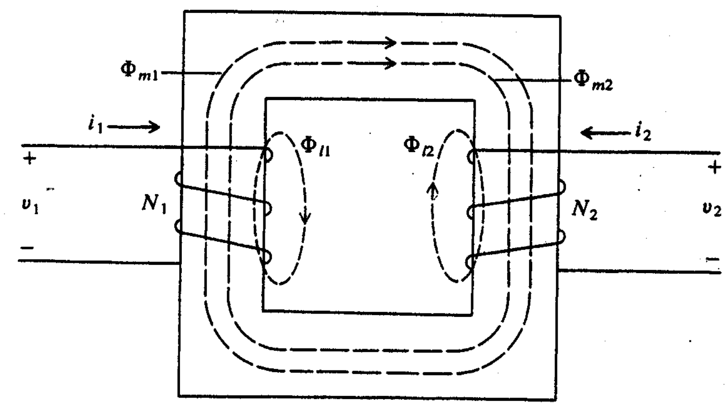
Analysis

- Flux Terms



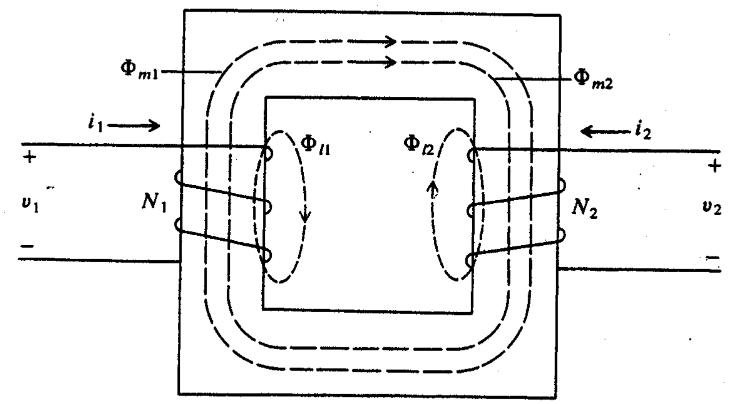
Analysis

- Flux Terms

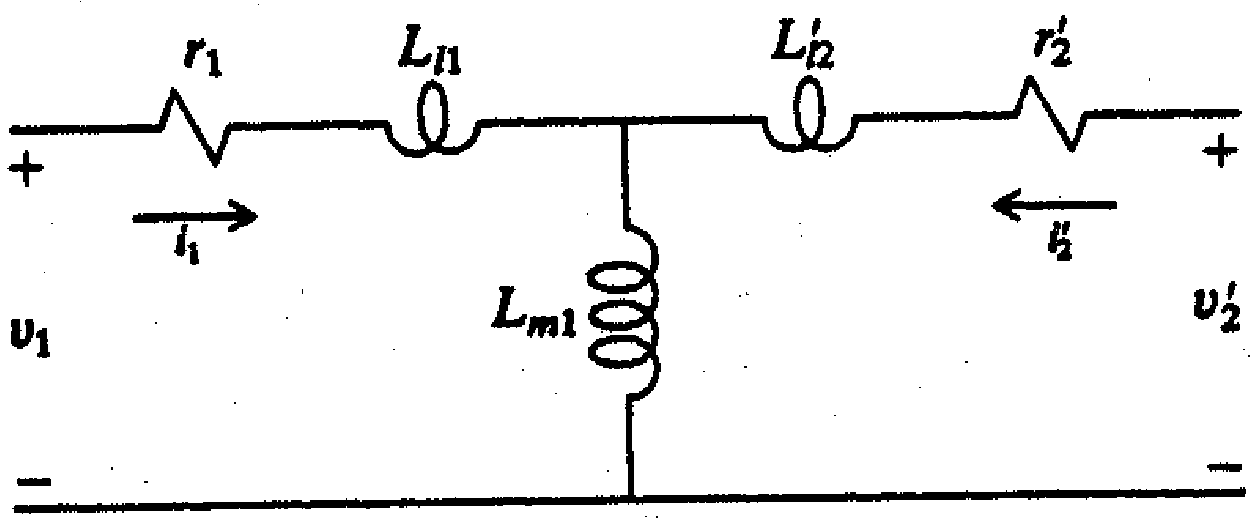


Analysis

- Flux Terms



T-Equivalent Circuit Model



T-Equivalent Model

- Definition of referred variables

$$i_2' = \frac{N_2}{N_1} i_2$$

$$v_2' = \frac{N_1}{N_2} v_2$$

$$\lambda_2' = \frac{N_1}{N_2} \lambda_2$$

- Voltage equations

$$\begin{bmatrix} v_1 \\ v_2' \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2' \end{bmatrix} \begin{bmatrix} i_1 \\ i_2' \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_1 \\ \lambda_2' \end{bmatrix}$$

$$r_2' = \left(\frac{N_1}{N_2} \right)^2 r_2$$

- Flux linkage equations

$$\lambda_1 = L_{l1} i_1 + L_{m1}(i_1 + i_2')$$
$$\lambda_2' = L_{l2}' i_2' + L_{m1}(i_1 + i_2')$$

$$L_{l2}' = \left(\frac{N_1}{N_2} \right)^2 L_{l2}$$

Derivation of T-Equivalent Circuit

Derivation of T-Equivalent Circuit

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Derivation of T-Equivalent Circuit

An Example

An Example

An Example

Lecture 67

Transformer Examples

Parameter ID Example

- DC Test:
  - $Z_{1dc} = 0.2\ \Omega$

- Short Circuit Test:
  - $Z_{1sc} = 2.63 \text{ at } 80.4^o\ \Omega$

Parameter ID Example

- Short Circuit Test (continued)

Parameter ID Example

- Open Circuit Test
  - $Z_{1oc} = 46.2 \text{ at } 84.7^o \, \Omega$

- Turns Ratio Test
  - $|V_1| = 110$ V
  - $|V_2| = 10.7$ V

Parameter ID Example

Parameter ID Example