Lecture Set 6:
Permanent Magnet AC Machines

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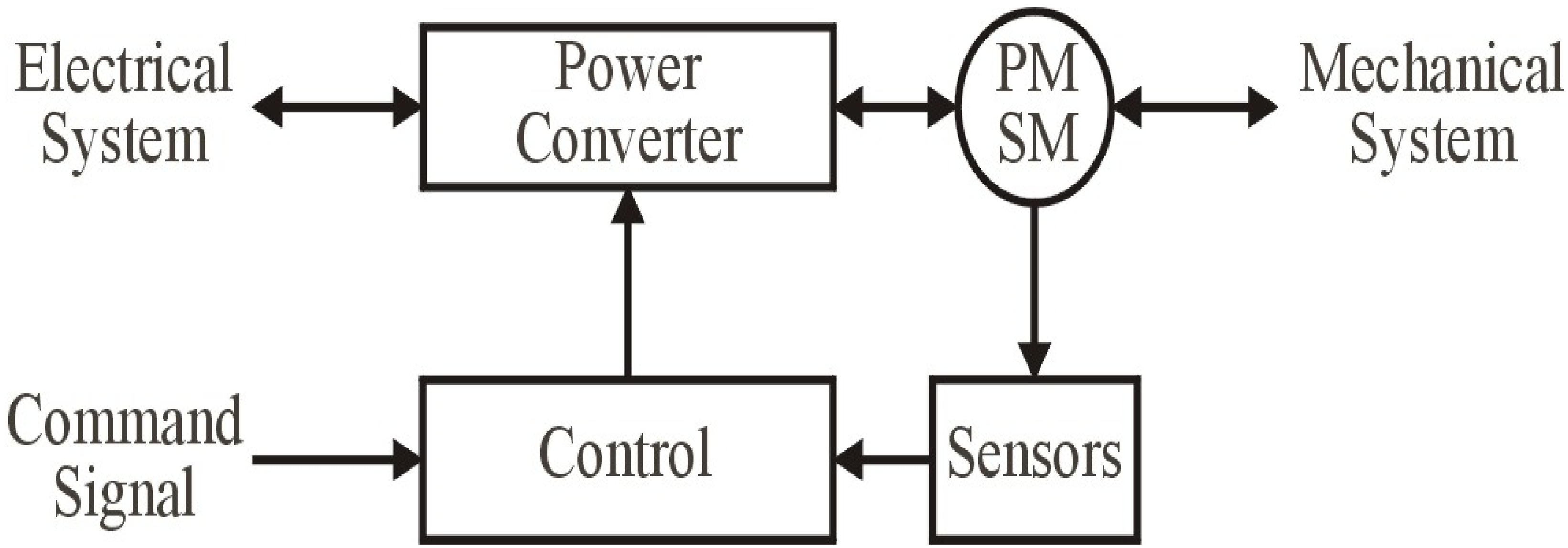
About this Lecture Set

- Reading
  - *Electromechanical Motion Devices, 2nd Edition,* Chapter 8
- Goal
  - Understand the theory and operation of the permanent magnet ac machine

Lecture 55

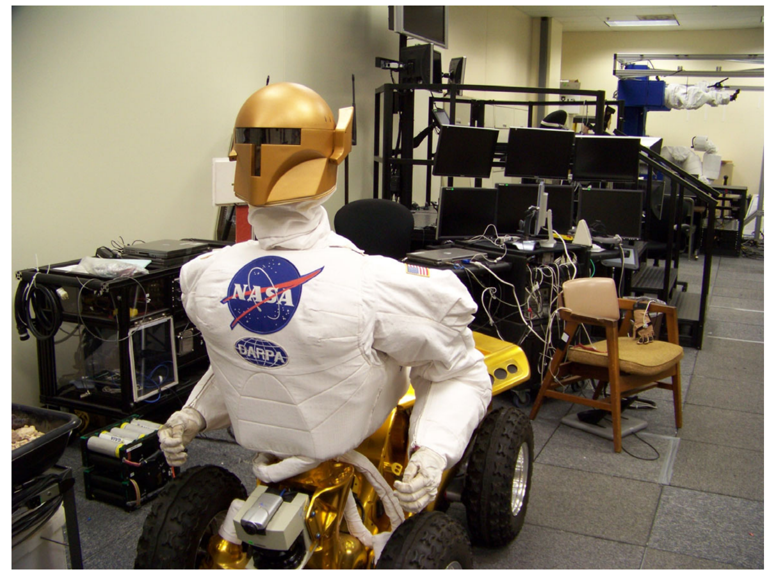
Topology

A Brushless DC Machine

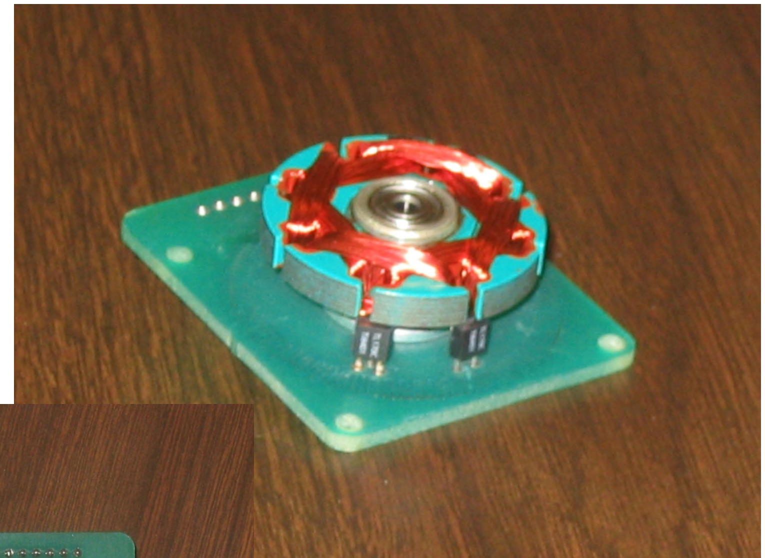


Sample Applications

- Low Power:
  - Disk drive motors
- Medium Power:
  - Robot manipulators
  - Servo systems
  - Hybrid/electric vehicles
- High Power:
  - Ship and submarine propulsion
  - Wind turbines



Disk Drive Motor



4 Hp BDC Machine

Characteristics

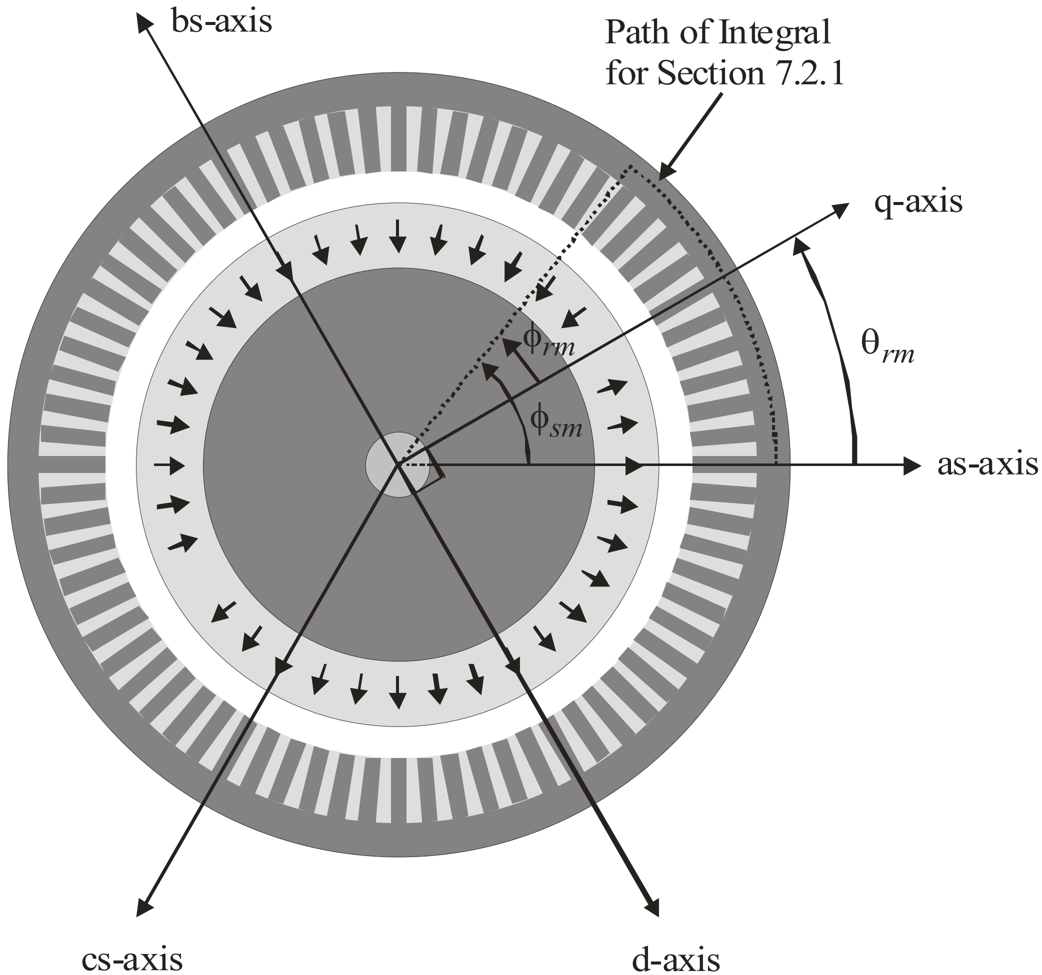
- The Good

- The Bad

Permanent Magnet Synchronous Machines

- **Radial** Versus Axial
- **Surface Mounted** Versus Buried Magnet
- **Sinusoidal** Versus Non-Sinusoidal

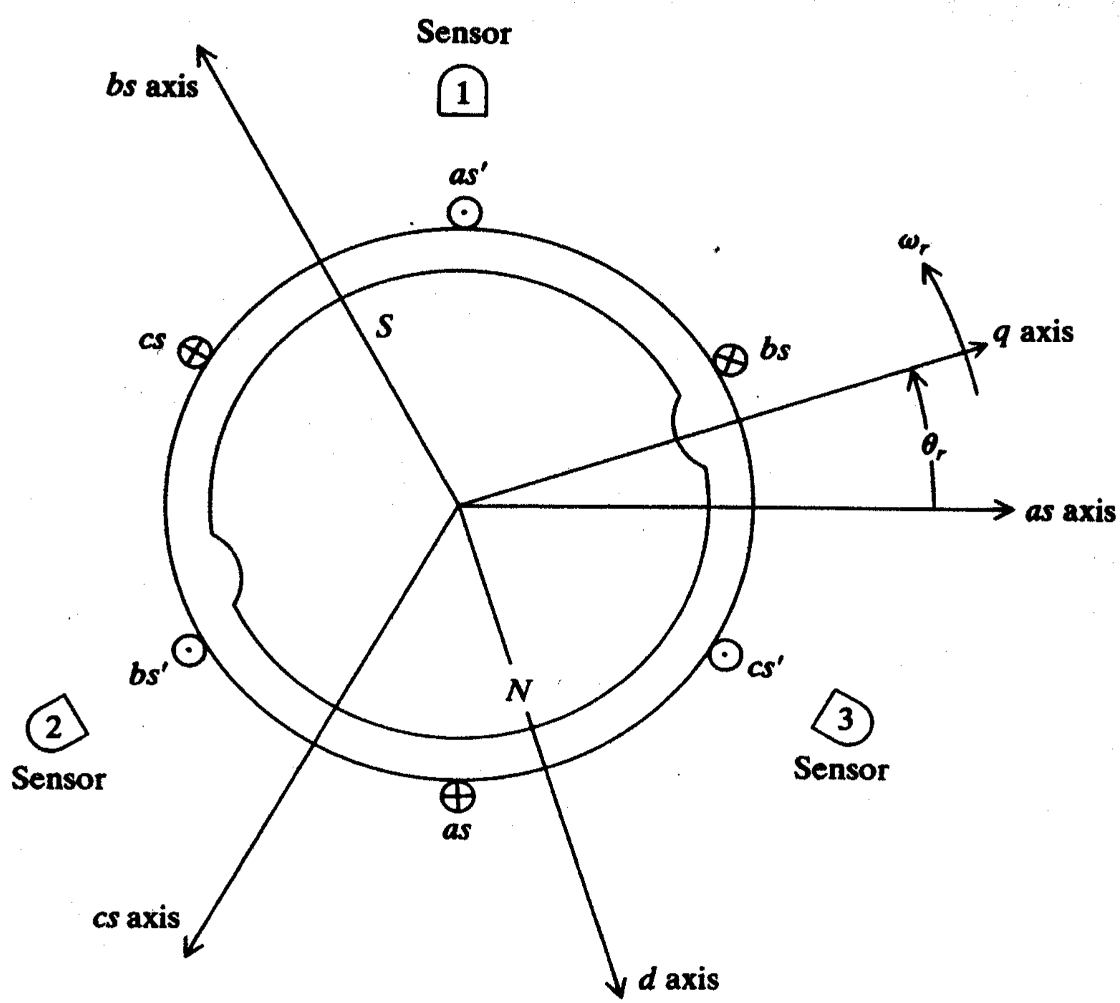
Radial Surface Mounted PMSM



Lecture 56

Machine Variable Model Voltage and Flux Linkages

3-Phase PMSM



Sensor
1
bs axis
as'
ωr
q axis
cs
S
bs
θr
as axis
bs'
N
cs'
2
3
Sensor
Sensor
as
cs axis
d axis

3-Phase PMSM

- Notation

$$(\mathbf{f}_{abcs})^T = [f_{as} \quad f_{bs} \quad f_{cs}]$$

- Voltage equations

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

$$v_{cs} = r_s i_{cs} + \frac{d\lambda_{cs}}{dt}$$

$$\mathbf{v}_{abcs} = r_s \mathbf{i}_{abcs} + p\boldsymbol{\lambda}_{abcs}$$

3-Phase PMSM

- Flux Linkage Equations

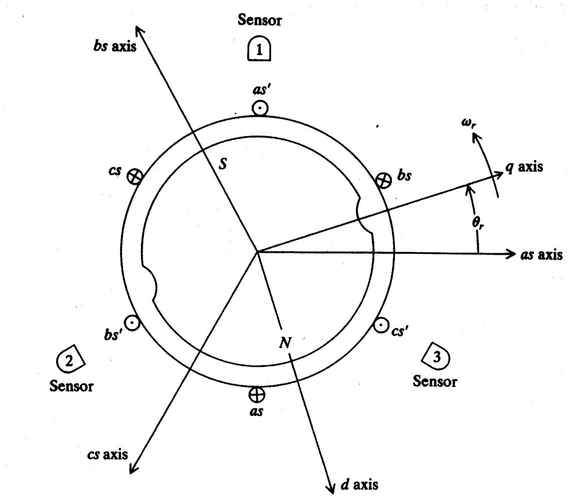
$$\boldsymbol{\lambda}_{\text{abcs}} = \mathbf{L}_s \mathbf{i}_{abcs} + \boldsymbol{\lambda}'_m$$

- where

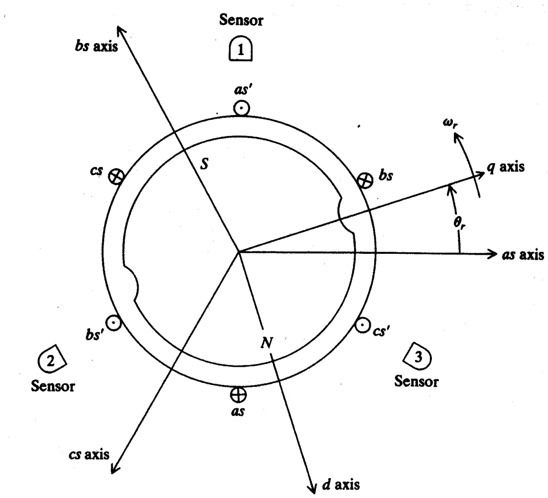
$$\boldsymbol{\lambda}'_m = \begin{bmatrix} \lambda_{asm} \\ \lambda_{bsm} \\ \lambda_{csm} \end{bmatrix} = \lambda'_m \begin{bmatrix} \sin\theta_r \\ \sin(\theta_r - \frac{2}{3}\pi) \\ \sin(\theta_r + \frac{2}{3}\pi \end{bmatrix}$$

$$\mathbf{L}_s = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} \end{bmatrix}$$

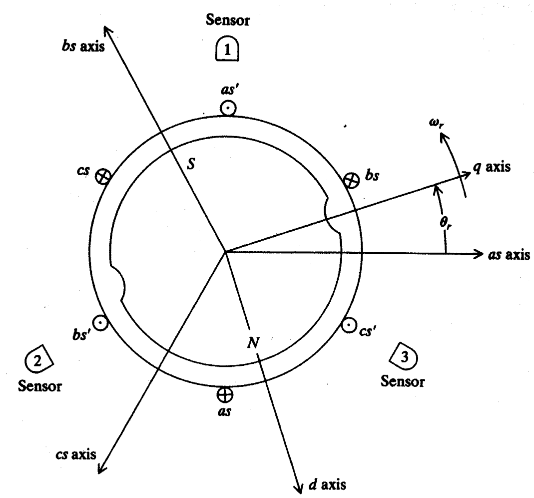
PM Terms – Intuitive Approach



PM Terms – Intuitive Approach



PM Terms – Intuitive Approach



Inductances

- We will assume the following

$$n_{as} = N_s \sin(P\phi_{sm} / 2)$$

$$n_{bs} = N_s \sin(P\phi_{sm} / 2 - 2\pi/3)$$

$$n_{cs} = N_s \sin(P\phi_{sm} / 2 + 2\pi/3)$$

- It follows that

$$w_{as} = \frac{2N_s}{P} \cos(P\phi_{sm} / 2)$$

$$w_{bs} = \frac{2N_s}{P} \cos(P\phi_{sm} / 2 - 2\pi / 3)$$

$$w_{cs} = \frac{2N_s}{P} \cos(P\phi_{sm} / 2 + 2\pi / 3)$$

Inductances

- Recall

$$\frac{\lambda_{\alpha,m}}{i_\beta} = L_{m,\alpha\beta} = \mu_0 r L \int_0^{2\pi} \frac{w_\alpha(\phi) w_\beta(\phi)}{g(\phi)} d\phi$$

- From which we obtain

$$L_{asas} = \frac{4\pi\mu_0 r L N_s^2}{P^2 g} = L_{ms}$$

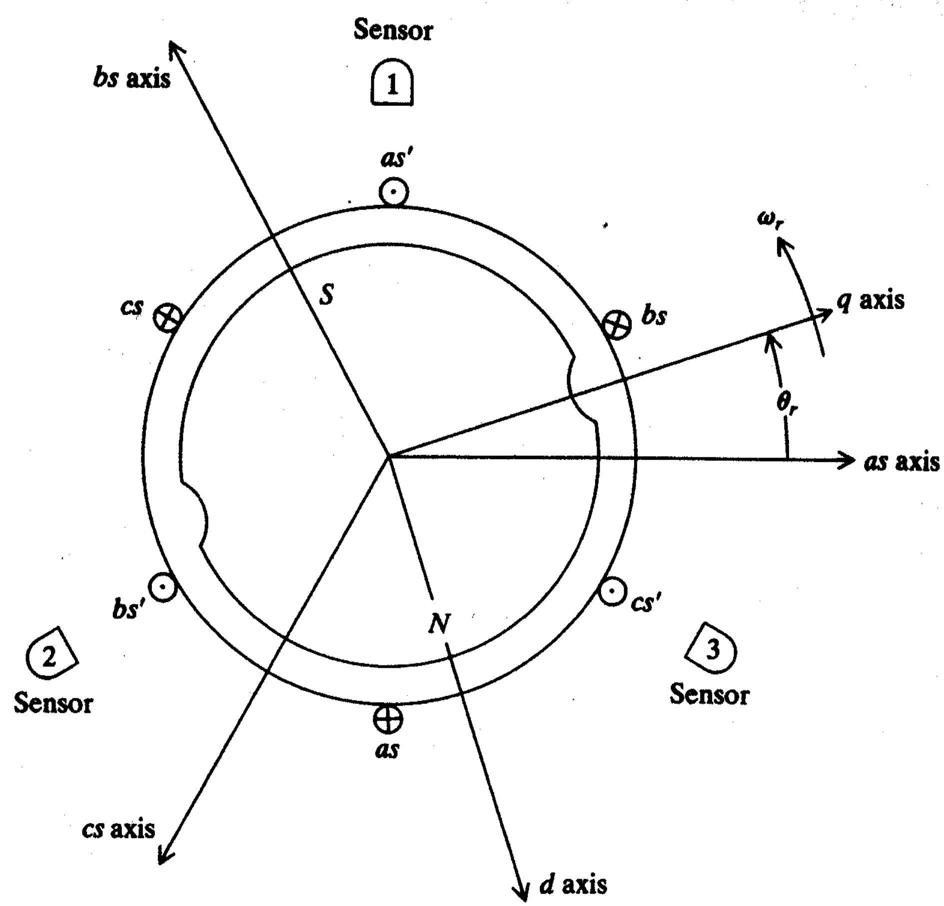
$$L_{asbs} = -\frac{2\pi\mu_0 r L N_s^2}{P^2 g} = -\frac{1}{2} L_{ms}$$

Doesn't include leakage

Lecture 57

Detailed Treatment of PMs

PM Terms – Analytical Approach



PM Terms – Def. of Elec. Quantities

$$\theta_r = P\theta_{rm} / 2$$

$$\omega_r = P\omega_{rm} / 2$$

$$\phi_s = P\phi_{sm} / 2$$

$$\phi_r = P\phi_{rm} / 2$$

PM Terms

- Suppose the B field due to the PM may be expressed

$$B\big|_{due\ to\ PM} = \begin{cases} -B_{pm} & 0 \le \phi_r \le \pi \\ B_{pm} & \pi \le \phi_r \le 2\pi \end{cases}$$

and suppose

$$w_{as} = \frac{2N_s}{P}\cos(P\phi_{sm}/2) = \frac{2N_s}{P}\cos(\phi_s)$$

PM Terms

- It can be shown that

$$\lambda_{as}\Big|_{due\ to\ PM} = \lambda_m \sin\theta_r$$

where

$$\lambda_m = \frac{8rLB_{pm}N_s}{P}$$

PM Terms

PM Terms

PM Terms

PM Terms

PM Terms

- Comment: The sinusoidal turns distribution gives rise to a sinusoidal flux linkage versus electrical rotor position characteristic

Lecture 57

Torque

Expression for Torque

- It can be shown that

$$T_e = \frac{P}{2}\lambda_m\left[i_{as}\cos(\theta_r) + i_{bs}\cos(\theta_r - 2\pi/3) + i_{cs}\cos(\theta_r + 2\pi/3)\right]$$

Expression for Torque

Expression for Torque

Expression for Torque

Expression for Torque

Expression for Torque

Lecture 58

Reference Frame Analysis

Machine Equations in Rotor Ref. Frame

- Consider the transformation

$$\mathbf{f}_{qd0s}^{r} = \mathbf{K}_s^r \mathbf{f}_{abcs}$$

- Where

$$(\mathbf{f}_{qd0s}^{r})^T = [f_{qs}^r \quad f_{ds}^r \quad f_{0s}]$$

$$\mathbf{K}_s^r = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos(\theta_r - \frac{2}{3}\pi) & \cos(\theta_r + \frac{2}{3}\pi) \\ \sin\theta_r & \sin(\theta_r - \frac{2}{3}\pi) & \sin(\theta_r + \frac{2}{3}\pi) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Machine Equations in Rotor Ref. Frame

Machine Equations in Rotor Ref. Frame

- Voltage

$$v_{qs}^r = r_s i_{qs}^r + \omega_r \lambda_{ds}^r + p\lambda_{qs}^r$$

$$v_{ds}^r = r_s i_{ds}^r - \omega_r \lambda_{qs}^r + p\lambda_{ds}^r$$

- Flux Linkage

$$\lambda_{qs}^r = L_{ss} i_{qs}^r$$

$$\lambda_{ds}^r = L_{ss} i_{ds}^r + \lambda_m'$$

- Torque

$$T_e = \frac{3}{2}\frac{P}{2}\lambda_m' i_{qs}^r$$

Aside: Some Shorthand

$$c = \cos(\theta_r)$$

$$c^- = \cos(\theta_r - 2\pi/3)$$

$$c^+ = \cos(\theta_r + 2\pi/3)$$

$$s = \sin(\theta_r)$$

$$s^- = \sin(\theta_r - 2\pi/3)$$

$$s^+ = \sin(\theta_r + 2\pi/3)$$

Aside: Some Trig IDs

$$\cos(x)+\cos(x-2\pi/3)+\cos(x+2\pi/3)=0$$

$$\sin(x)+\sin(x-2\pi/3)+\sin(x+2\pi/3)=0$$

$$\cos(x)\cos(y)+\cos(x-2\pi/3)\cos(y-2\pi/3)+\cos(x+2\pi/3)\cos(y+2\pi/3)=\frac{3}{2}\cos(x-y)$$

$$\sin(x)\sin(y)+\sin(x-2\pi/3)\sin(y-2\pi/3)+\sin(x+2\pi/3)\sin(y+2\pi/3)=\frac{3}{2}\cos(x-y)$$

$$\sin(x)\cos(y)+\sin(x-2\pi/3)\cos(y-2\pi/3)+\sin(x+2\pi/3)\cos(y+2\pi/3)=\frac{3}{2}\sin(x-y)$$

Lecture 59

Transformation of Voltage Equations

Transformation of Voltage Equations

Transformation of Voltage Equations

Transformation of Voltage Equations

Transformation of Voltage Equations

Transformation of Voltage Equations

Transformation of Voltage Equations

- This yields

$$\mathbf{v}_{qd0s}^{r} = \mathbf{r}_s \mathbf{i}_{qd0s}^{r} + \omega_r \boldsymbol{\lambda}_{dqs}^{r} + p\boldsymbol{\lambda}_{qd0s}^{r}$$

- Where

$$(\boldsymbol{\lambda}_{dqs}^{r})^T = [\lambda_{ds}^{r} \quad -\lambda_{qs}^{r} \quad 0]$$

- In expanded form

$$v_{qs}^{r} = r_s i_{qs}^{r} + \omega_r \lambda_{ds}^{r} + p\lambda_{qs}^{r}$$

$$v_{ds}^{r} = r_s i_{ds}^{r} - \omega_r \lambda_{qs}^{r} + p\lambda_{ds}^{r}$$

$$v_{0s} = r_s i_{0s} + p\lambda_{0s}$$

Lecture 60

Transformation of Flux Linkage Equations

Transformation of Flux-Linkage Equations

Transformation of Flux-Linkage Equations

Transformation of Flux-Linkage Equations

Transformation of Flux-Linkage Equations

Transformation of Flux-Linkage Equations

Transformation of Flux-Linkage Equations

Transformation of Flux-Linkage Equations

- This yields

$$\boldsymbol{\lambda}_{qd0s}^{r} = \begin{bmatrix} L_{ls} + \frac{3}{2}L_{ms} & 0 & 0 \\ 0 & L_{ls} + \frac{3}{2}L_{ms} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} i_{qs}^{r} \\ i_{ds}^{r} \\ i_{0s} \end{bmatrix} + \lambda'_{m} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

- Or in expanded form

$$\lambda_{qs}^{r} = L_{ss} i_{qs}^{r} \qquad \lambda_{ds}^{r} = L_{ss} i_{ds}^{r} + \lambda'_{m} \qquad \lambda_{0s} = L_{ls} i_{0s}$$

- Where

$$L_{ss} = L_{ls} + \frac{3}{2} L_{ms}$$

Lecture 61

Transformation of Torque Equation

Transformation of Torque Equation

- Start with

$$T_e = \frac{P}{2}\lambda_m\left[i_{as}\cos(\theta_r) + i_{bs}\cos(\theta_r - 2\pi/3) + i_{cs}\cos(\theta_r + 2\pi/3)\right]$$

Transformation of Torque Equation

- Finally, we arrive at

$$T_e = \frac{3}{2}\frac{P}{2}\lambda'_m i_{qs}^r$$

Lecture 62

Handy Stuff

Zero Sequence

Zero Sequence

RMS and QD Components

RMS and QD Components

RMS and QD Components

RMS and QD Components

Lecture 62

Voltage Source Operation

Voltage Source Operation

- In this mode, idealized voltage applied is

$$v_{as} = \sqrt{2} v_s \cos\theta_{esv}$$

$$v_{bs} = \sqrt{2} v_s \cos(\theta_{esv} - \frac{2}{3}\pi)$$

$$v_{cs} = \sqrt{2} v_s \cos(\theta_{esv} + \frac{2}{3}\pi)$$

- Where

$$\theta_{esv} = \theta_r + \phi_v$$

Applied Voltage in QD Variables

- We can show that

$$v_{qs}^{r} = \sqrt{2} v_{s} \cos \phi_{v}$$

$$v_{ds}^{r} = -\sqrt{2} v_{s} \sin \phi_{v}$$

Applied Voltage in ABC Variables

Applied Voltage in QD Variables

Analysis of Steady State Operation

- Prediction of Q- and D-Axis Currents

Analysis of Steady State Operation

Analysis of Steady State Operation

Analysis of Steady State Operation

Example 1

- Consider a machine with the following parameters

  $r_s = 3.1\ \Omega$

  $P = 4$

  $L_{ss} = 12.1$ mH

  $\lambda_m = 0.156$ Vs

  N =3

- Further suppose

  $V_s = 100$

  $\phi_v = 0$

  $\omega_{rm} = 1800$ RPM

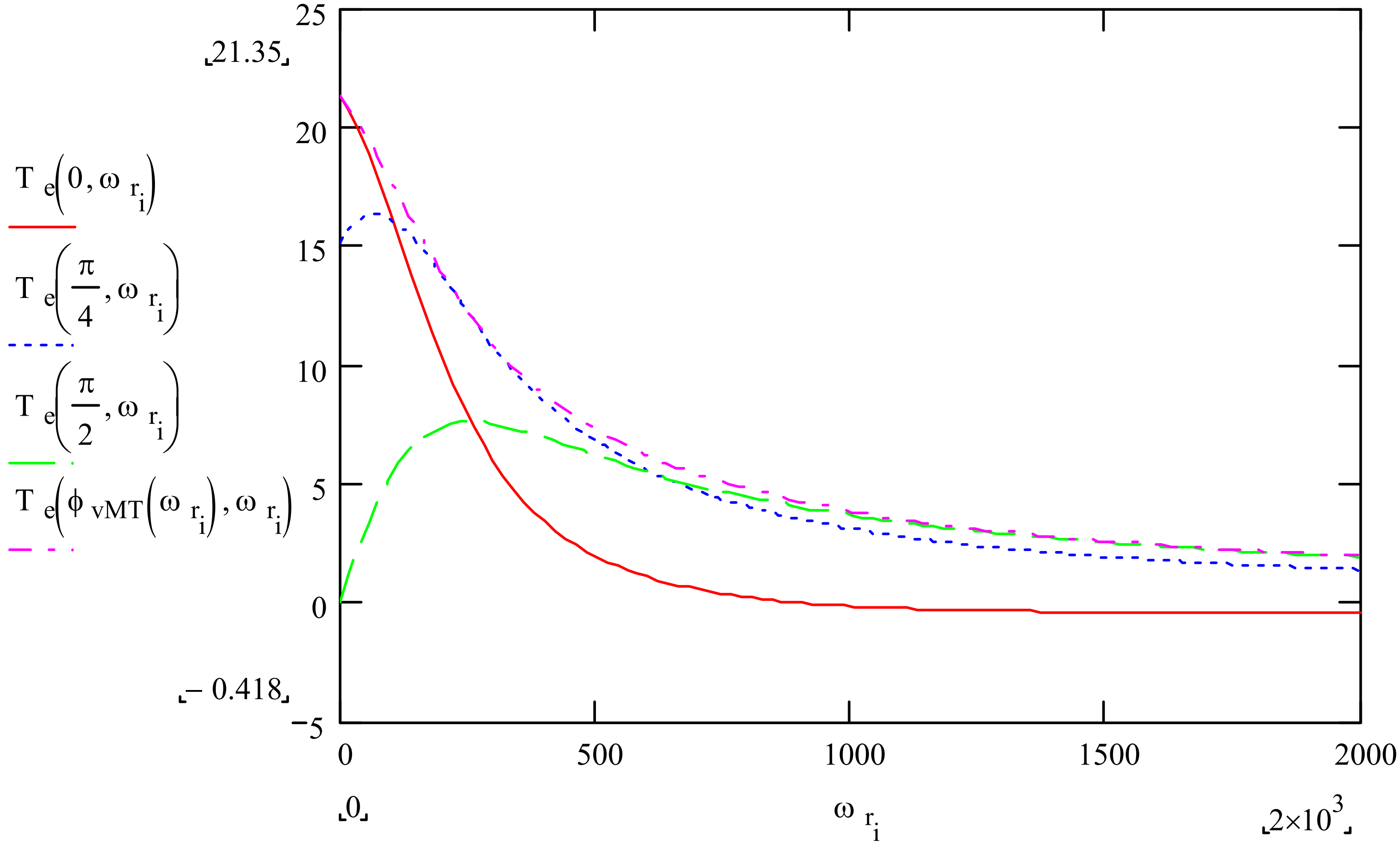
- Find the torque and efficiency

Example 1

Example 2

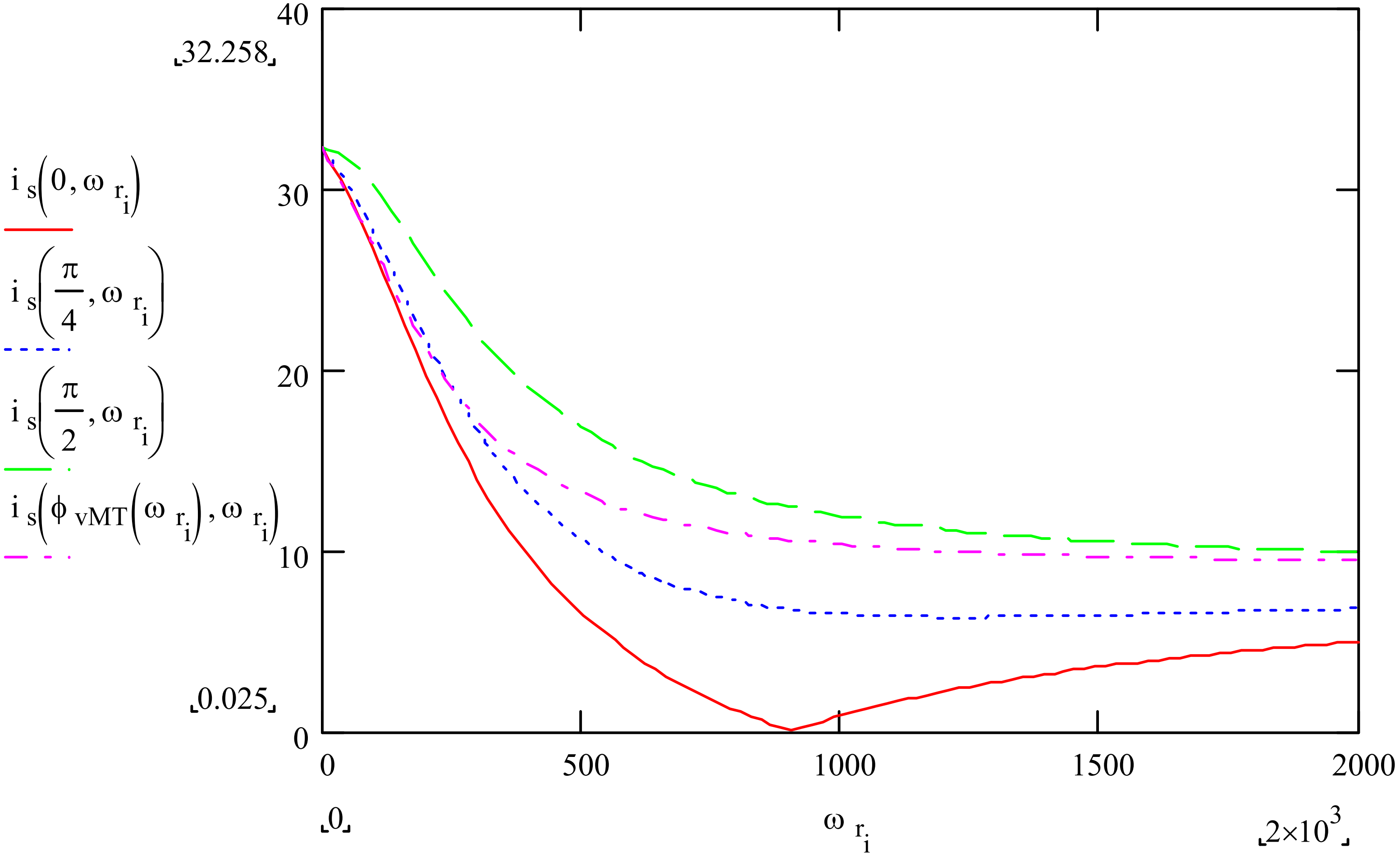
- Consider the machine with parameters of example 1.
- Plot the torque speed and rms current speed curves

Example 2



25
21.35
20
15
10
5
0
– 0.418
−5
0
500
1000
1500
2000
0
ω r i
2×10^3
T e(0, ω r i)
T e(π/4, ω r i)
T e(π/2, ω r i)
T e(ϕ vMT(ω r i), ω r i)

Example 2



40
⌞32.258⌟
30
20
10
⌞0.025⌟
0
0
500
1000
1500
2000
⌞0⌟
ω r_i
⌞2×10^3⌟
i_s(0, ω r_i)
i_s(π/4, ω r_i)
i_s(π/2, ω r_i)
i_s(φ vMT(ω r_i), ω r_i)

Optimization of Phase Advance

Optimization of Phase Advance

Lecture 63

Current Source Operation

Current Source Operation

- Interpretation 1 (ABC Variable)

Current Source Operation

- Interpretation 2 (Torque Transducer)

Desired D-Axis Current

Example 3

- Consider a machine with the following parameters

  $r_s = 3.1\ \Omega$

  $P = 4$

  $L_{ss} = 12.1$ mH

  $\lambda_m = 0.156$ Vs

  N =3

- Plot the voltage required and efficiency for the following conditions

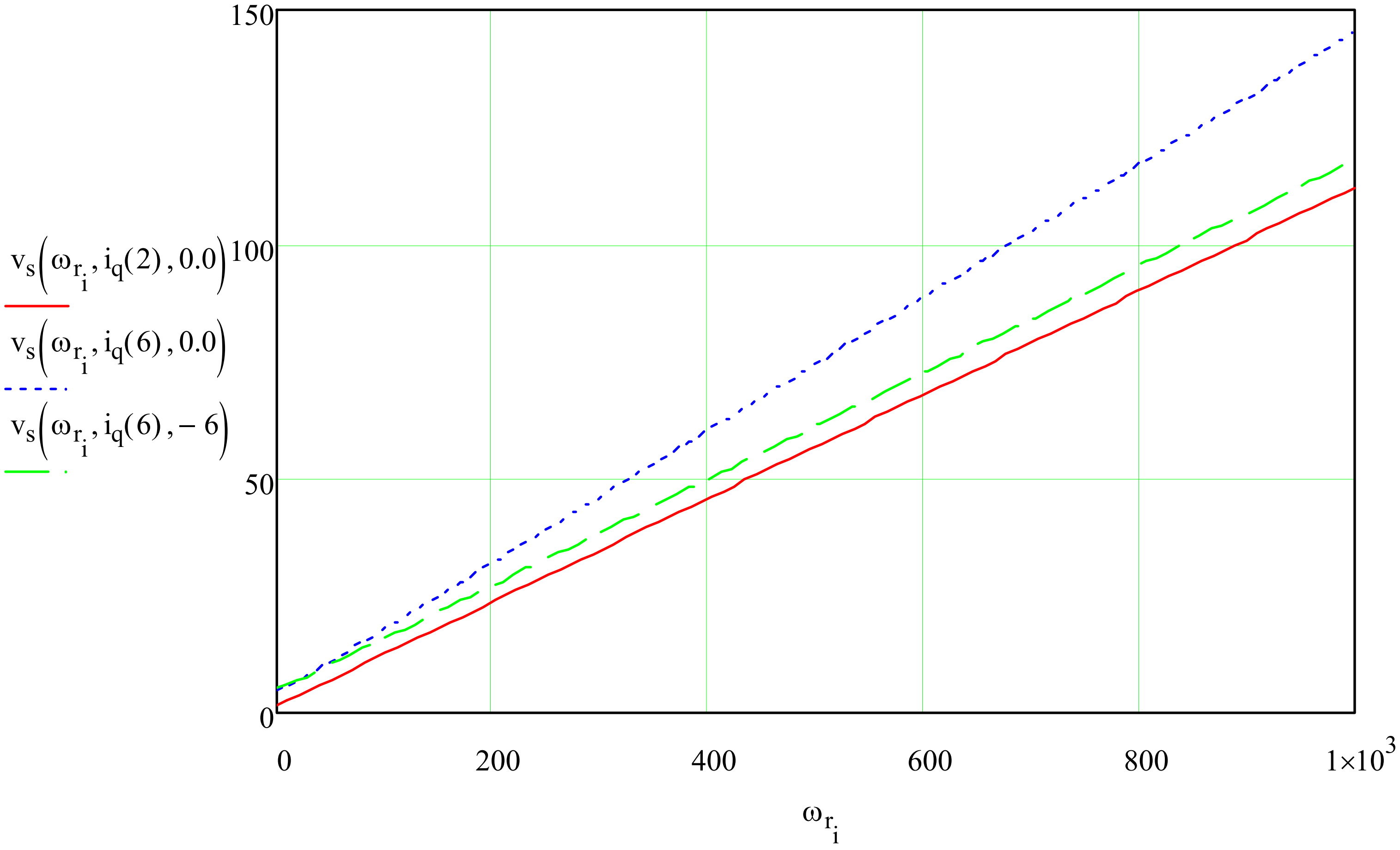
  Torque command: 2 Nm, d-axis current 0 A

  Torque command: 6 Nm, d-axis current 0 A

  Torque command: 6 Nm, d-axis current -6 A

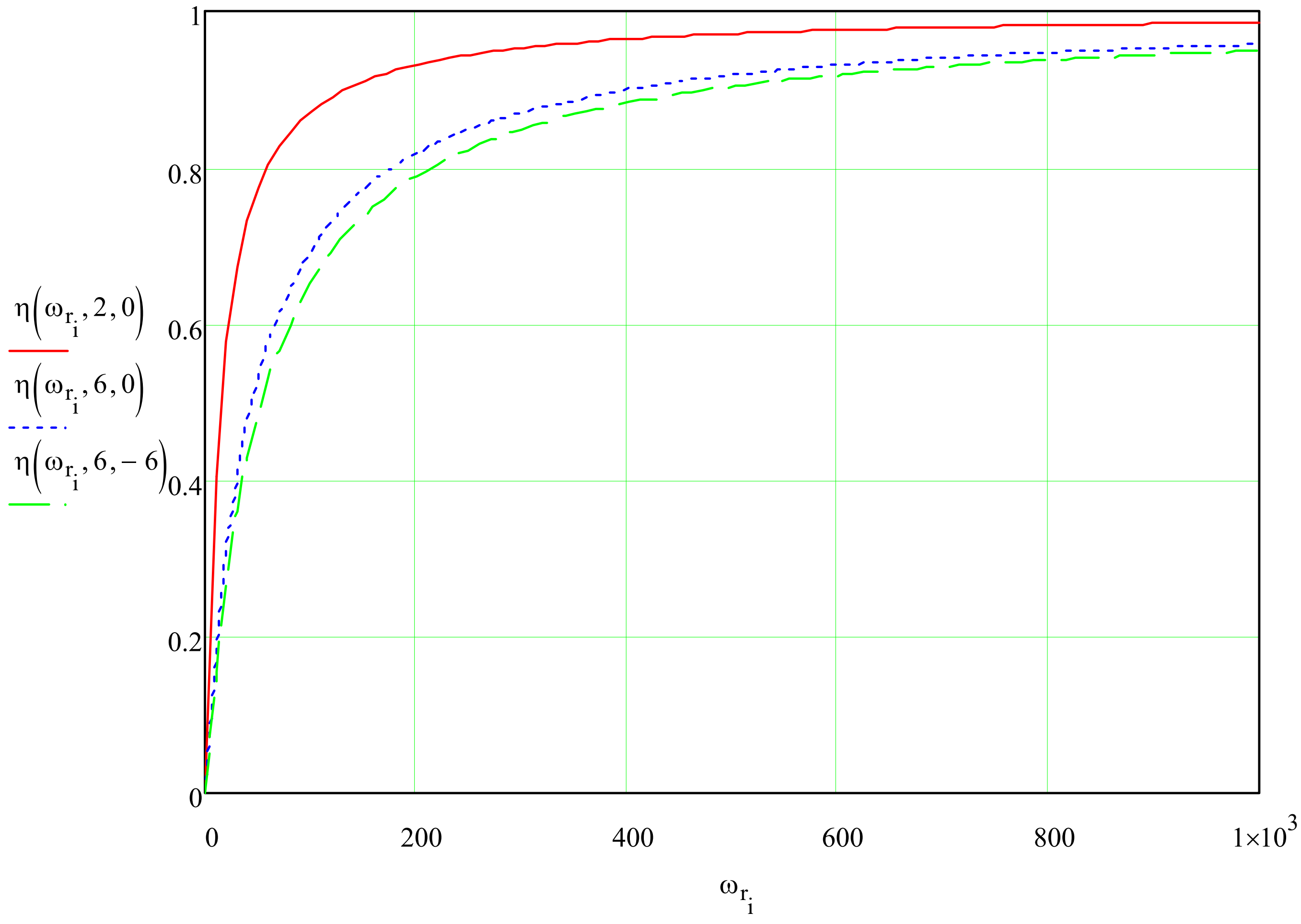
Example 3

Example 3



150
100
50
0
0
200
400
600
800
1×10^3
ω_{r_i}
$v_s\left(\omega_{r_i}, i_q(2), 0.0\right)$
$v_s\left(\omega_{r_i}, i_q(6), 0.0\right)$
$v_s\left(\omega_{r_i}, i_q(6), -6\right)$

Example 3



Lecture 63

Flux Weakening

Effect of D-Axis Current on Voltage

D-Axis Injection

D-Axis Injection

D-Axis Injection

D-Axis Injection

D-Axis Injection

Example 4

- At 2000 rpm, the zero-to-peak line-to-line voltage has a 100 V amplitude and a frequency of 100 Hz. Compute λ_m and P.

- At standstill and at 60 Hz, the impedance looking into the a- to b-phase is 0.2+2j. Find r_s and L_{ss}.

Example 4 – Part 1

Example 4 – Part 1

Example 4 – Part 1

Example 4 – Part 1

Example 4 – Part 2

Example 4 – Part 2

Example 4 – Part 2