Lecture Set 5:
Distributed Windings and Rotating MMF

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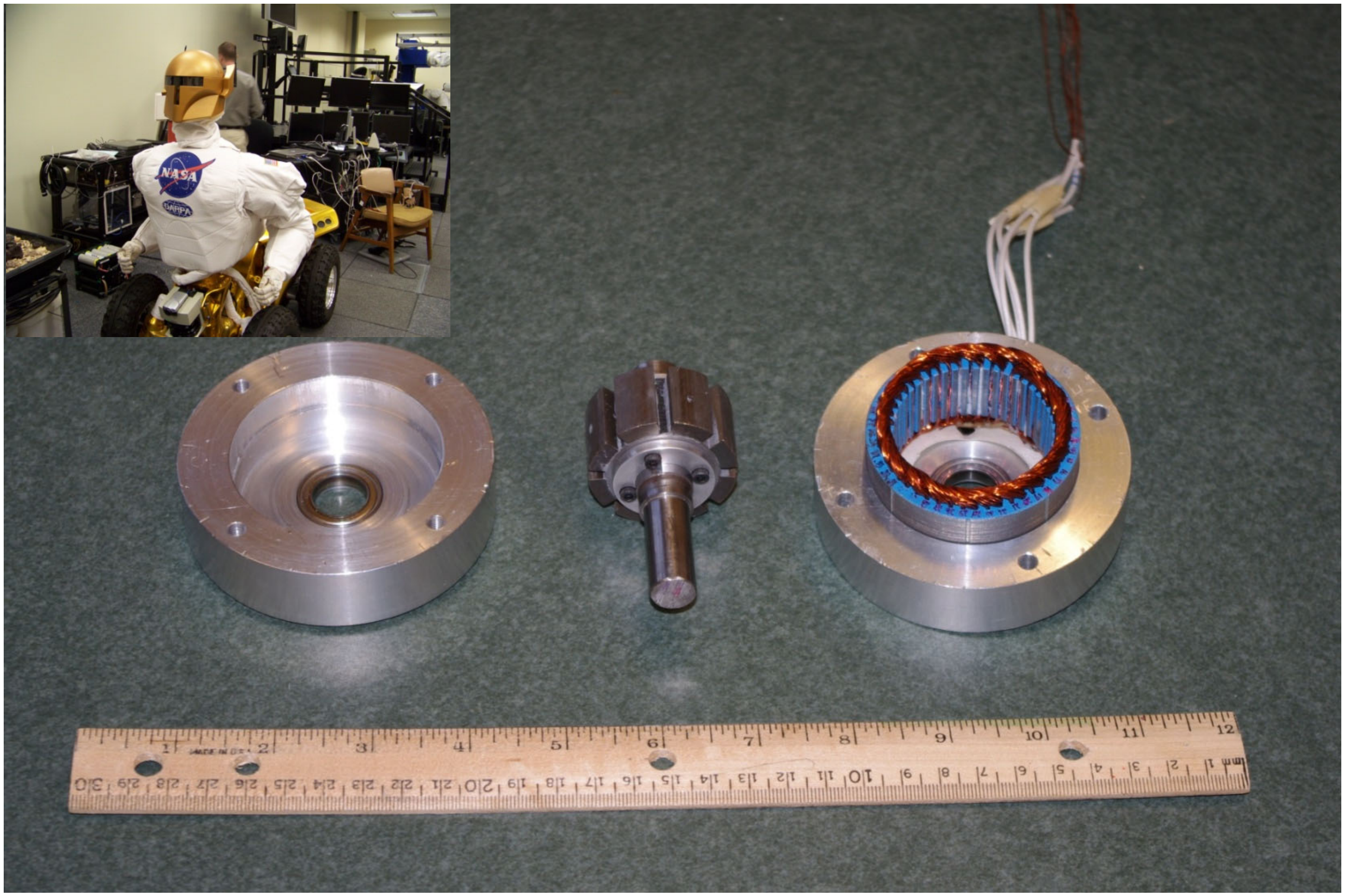
About this Lecture Set

- Reading
  - *Power Magnetic Devices: A Multi-Objective Design Approach*, 8.1-8.5, (Available through IEEE)
- Goal
  - Set the stage to study ac machinery including permanent magnet synchronous machines as well as induction machines

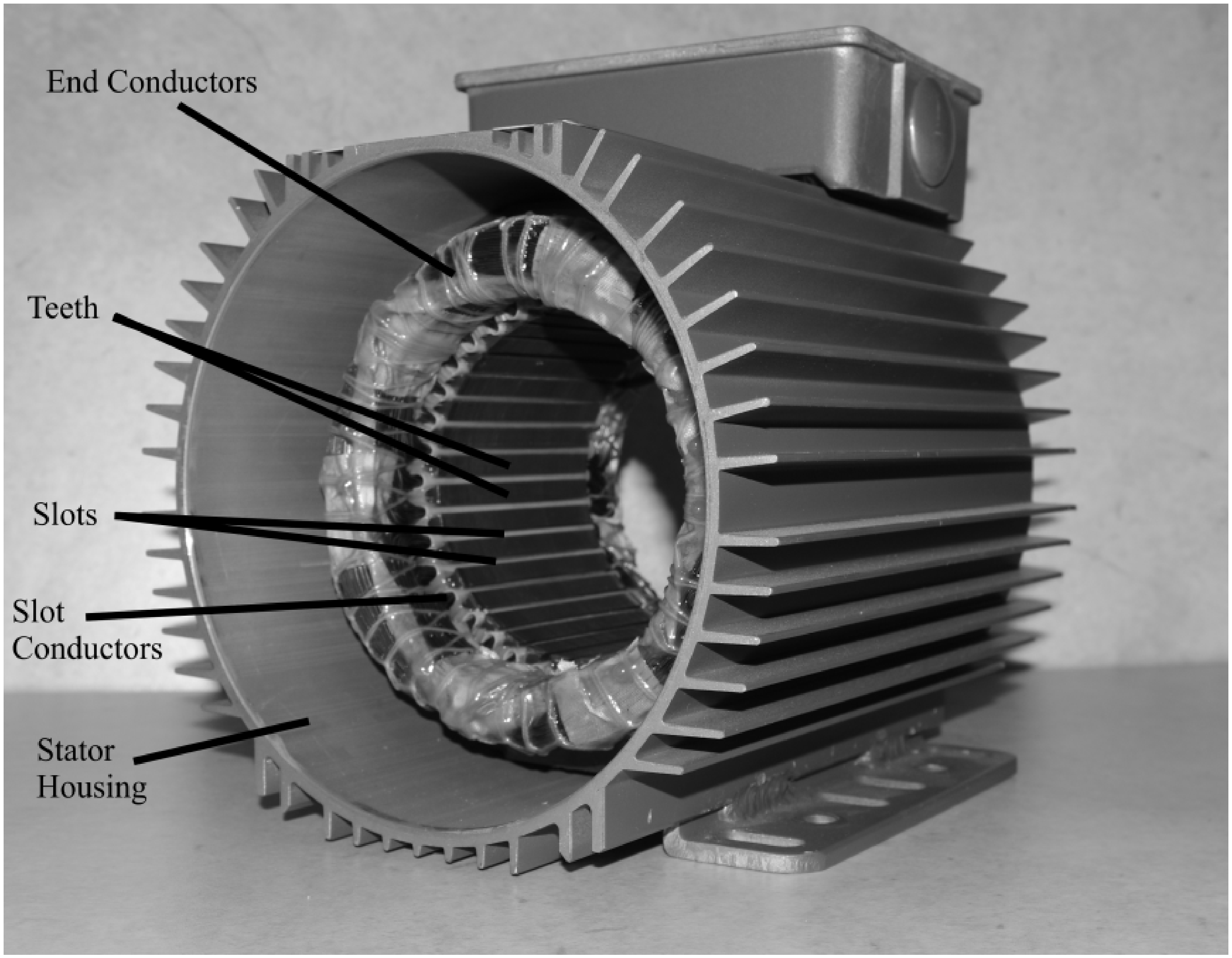
Lecture 42

Introduction to AC Machines

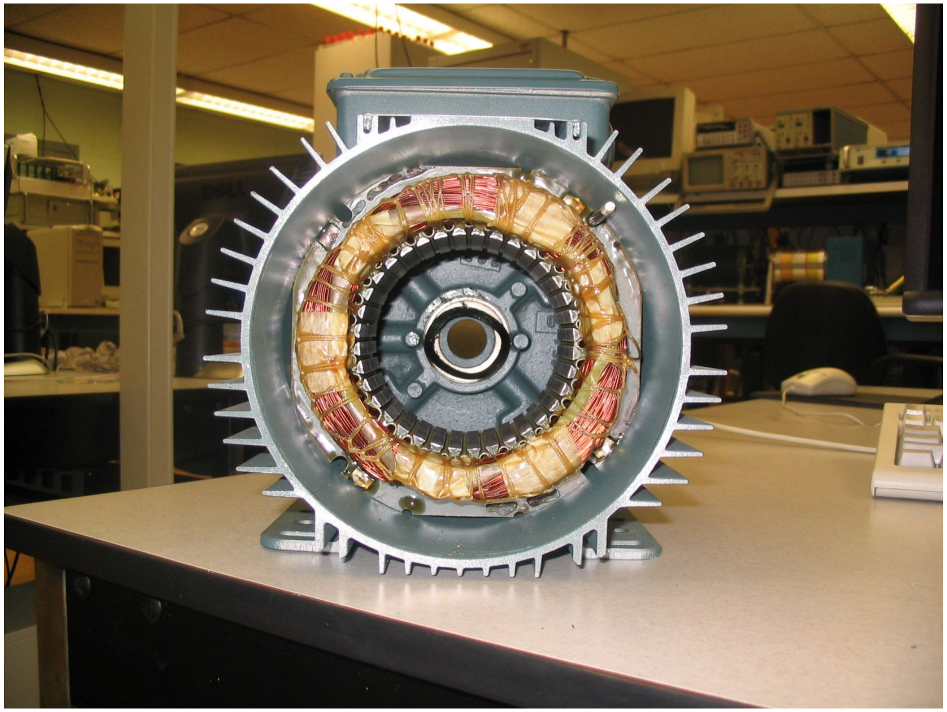
PM AC Machine

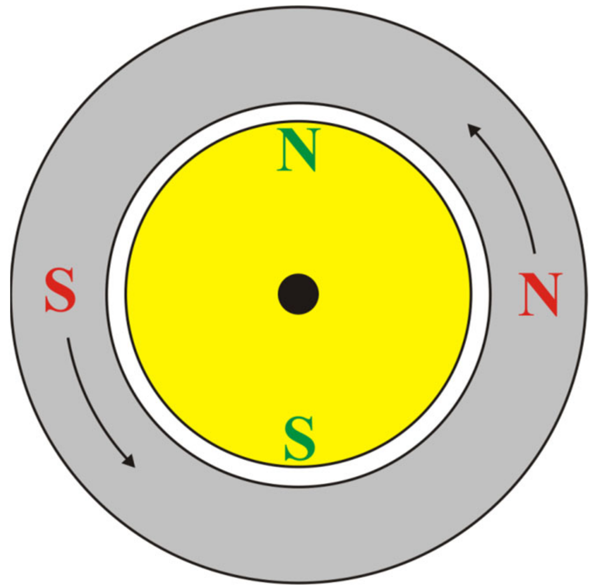


Induction Machine Stator



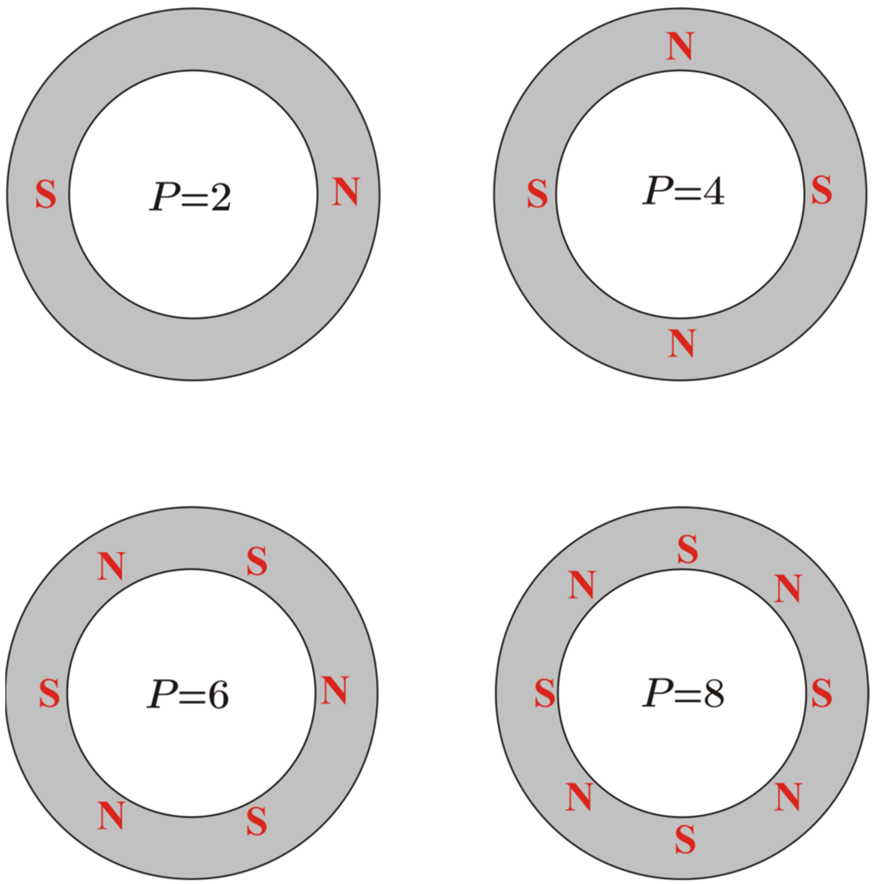
Operation of AC Machines





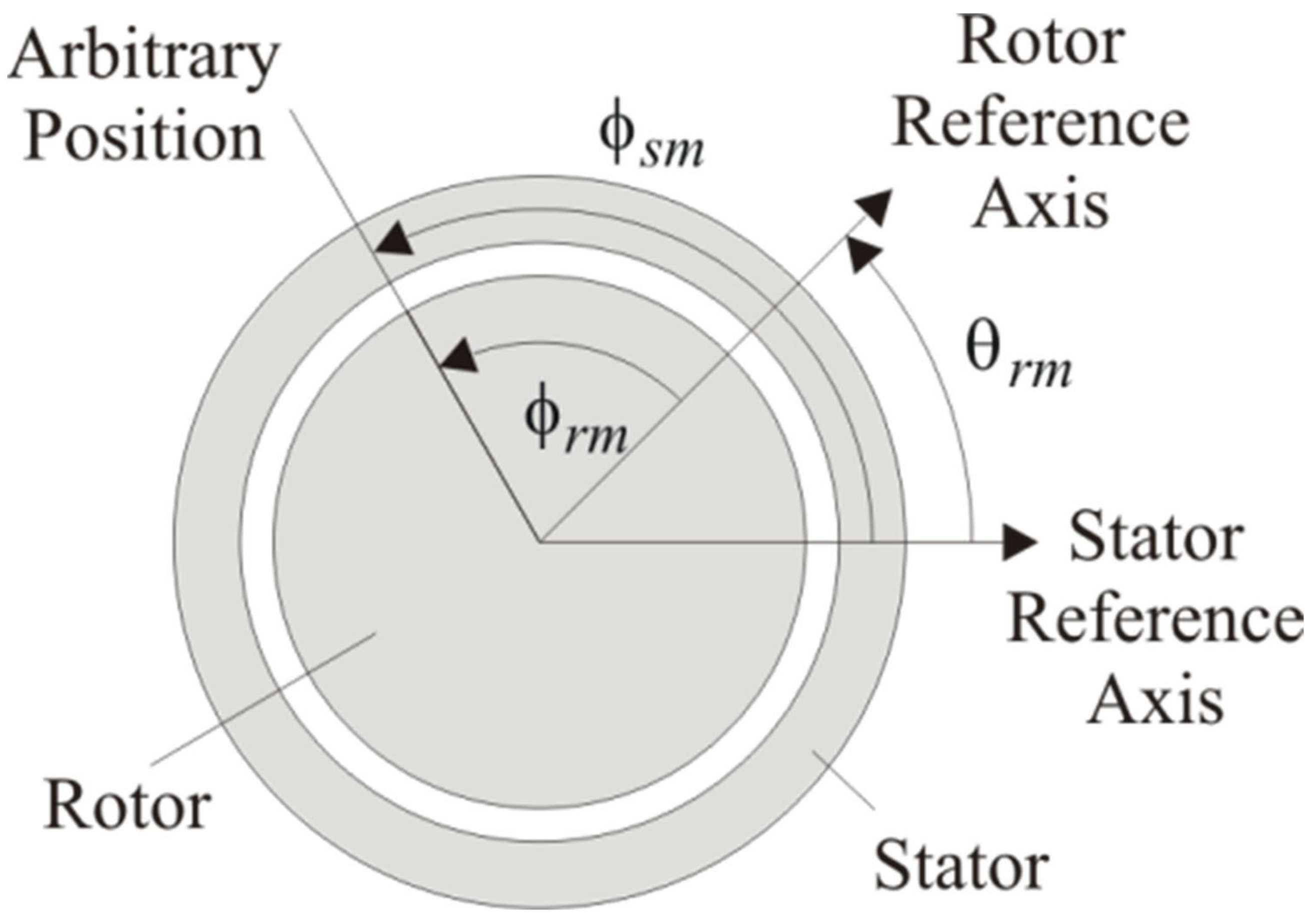
N-Pole: Flux leaves material
S-Pole: Flux enters material

P-Pole Machines



For given ac frequency having more than 2 poles reduces machine speed by P/2

Position Measurement



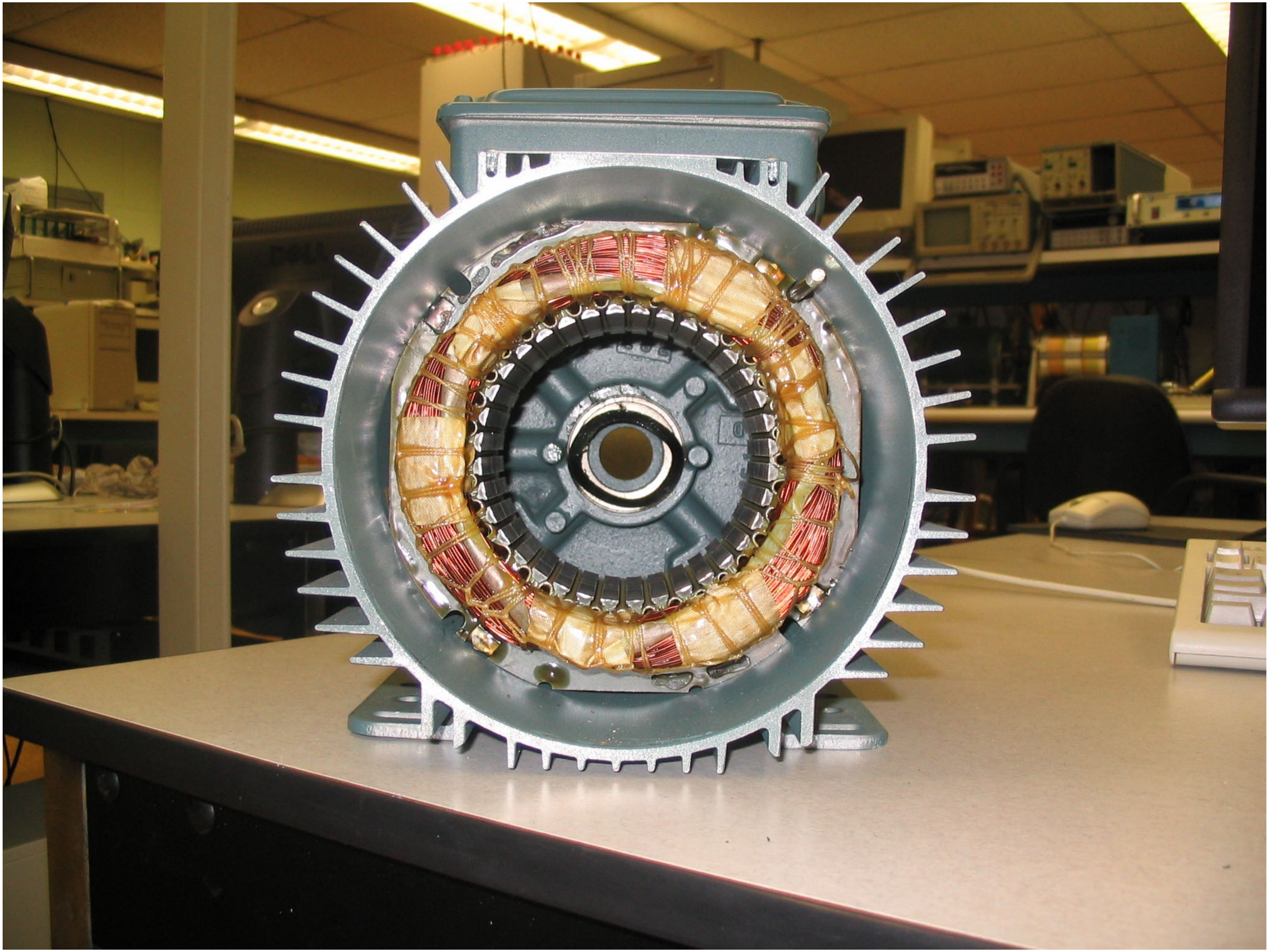
Electrical Angles

Lecture 43

Distributed Windings

Distributed Windings

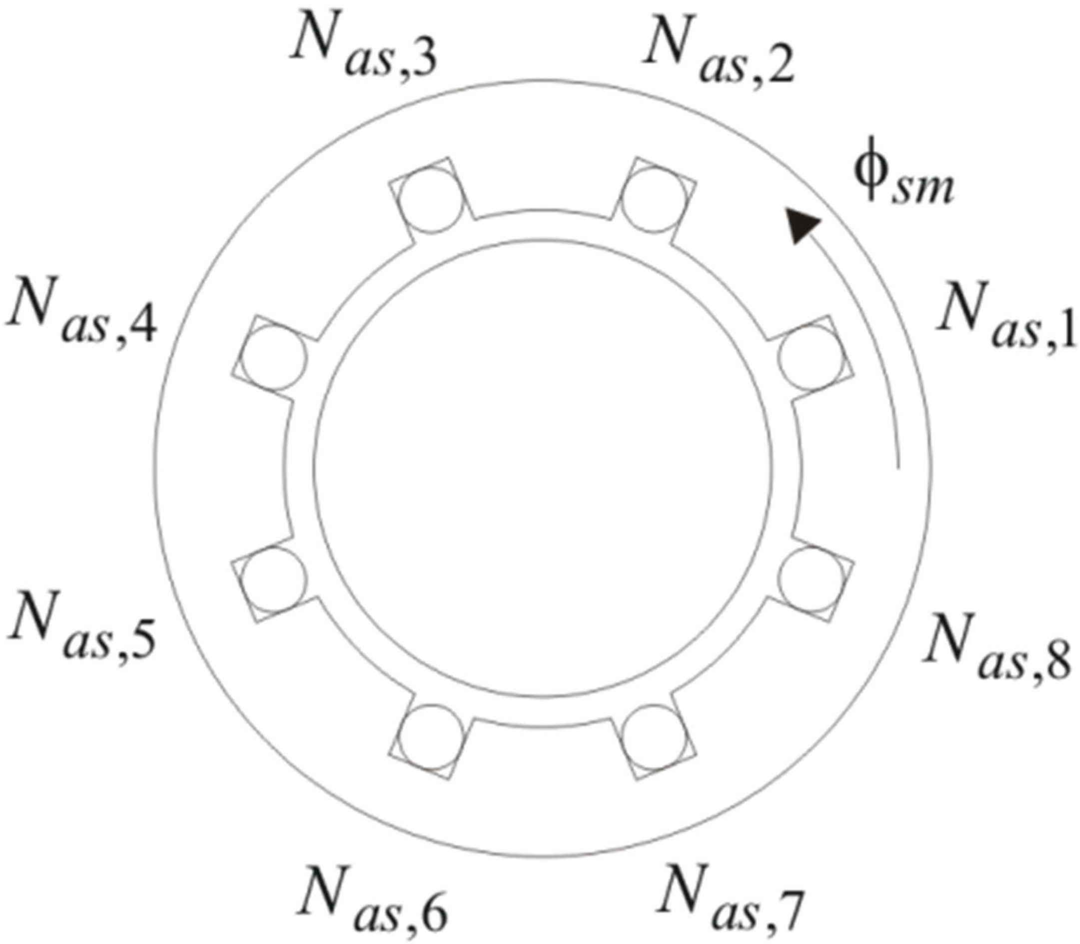
- Thus far, we have mostly considered lumped windings. We must now consider distributed winding. This is a more difficult concept.
- Two Representations
  - Discrete Winding Representation
  - Continuous Winding Representation



Discrete Winding Representation

- Let $N_{x,i}$ denote number of conductors of a x-phase winding in the i'th slot
  - 'i' designates the slot number
  - 'x' denotes winding or phase and is typically 'as', 'bs', or 'cs'
  - 'y' is 's' for stator or 'r' for rotor
  - A positive value indicates the conductor directed out of the page

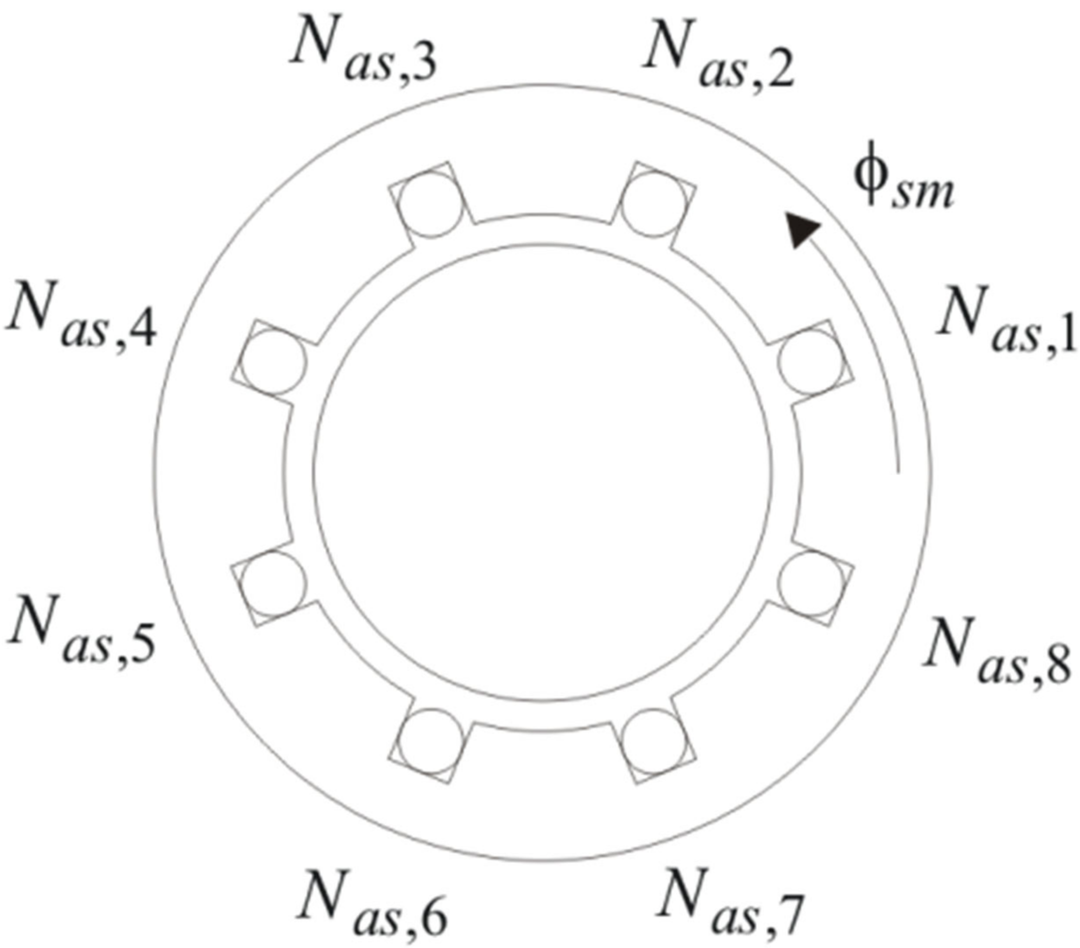
Slot and Tooth Locations



$$\phi_{ys,i} = \pi(2i-2)/S_y + \phi_{ys,1}$$

$$\phi_{yt,i} = \pi(2i-3)/S_y + \phi_{ys,1}$$

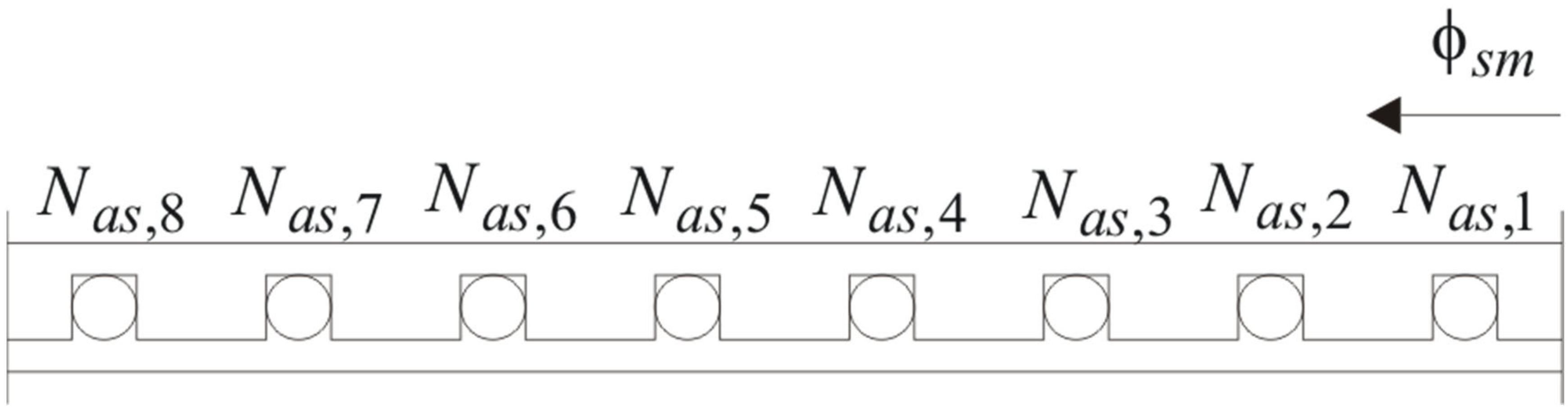
Discrete Winding Representation



$$\sum_{i=1}^{S_y} N_{x,i} = 0$$

$$N_x = \sum_{i=1}^{S_y} N_{x,i} \; \mathrm{u}(N_{x,i})$$

Developed Diagram



Continuous Winding Description

- Conductor density (conductors/rad): $n_x(\phi_{ym})$
  - ‘x’ denotes winding/phase ‘as’, ‘bs’, or ‘cs’
  - ‘y’ is location (‘s’ for stator, ‘r’ for rotor)
  - positive value indicates conductors out of the page
- For example, the a-phase conductor density may be given by

$$n_{as}(\phi_{sm}) = N_{s1}\sin(P\phi_{sm}/2) - N_{s3}\sin(3P\phi_{sm}/2)$$

- Total conductors per phase

$$N_{xy} = \int_0^{2\pi} n_{xy}(\phi)\,u(n_{xy}(\phi))\,d\phi$$

Lecture 44

Properties of Distributed Windings

Symmetry Conditions

$$N_{x,i+2S_y/P} = N_{x,i}$$

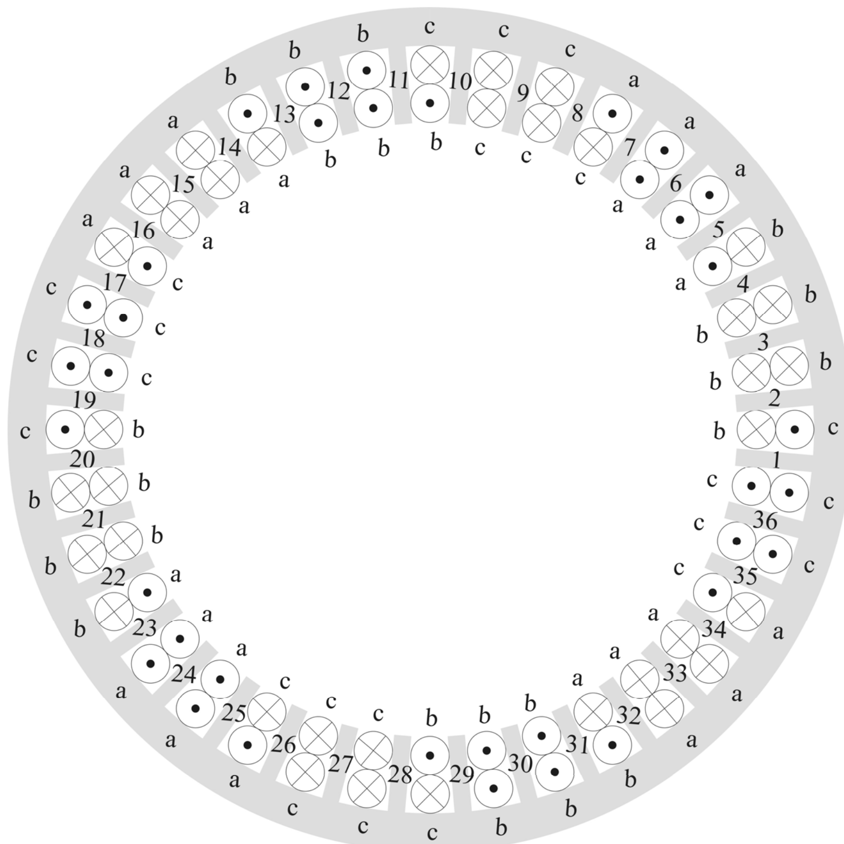
$$N_{x,i+S_y/P} = -N_{x,i}$$

$$\mathrm{n}_x(\phi_m + 4\pi / P) = \mathrm{n}_x(\phi_m)$$

$$\mathrm{n}_x(\phi_m + 2\pi / P) = -\mathrm{n}_x(\phi_m)$$

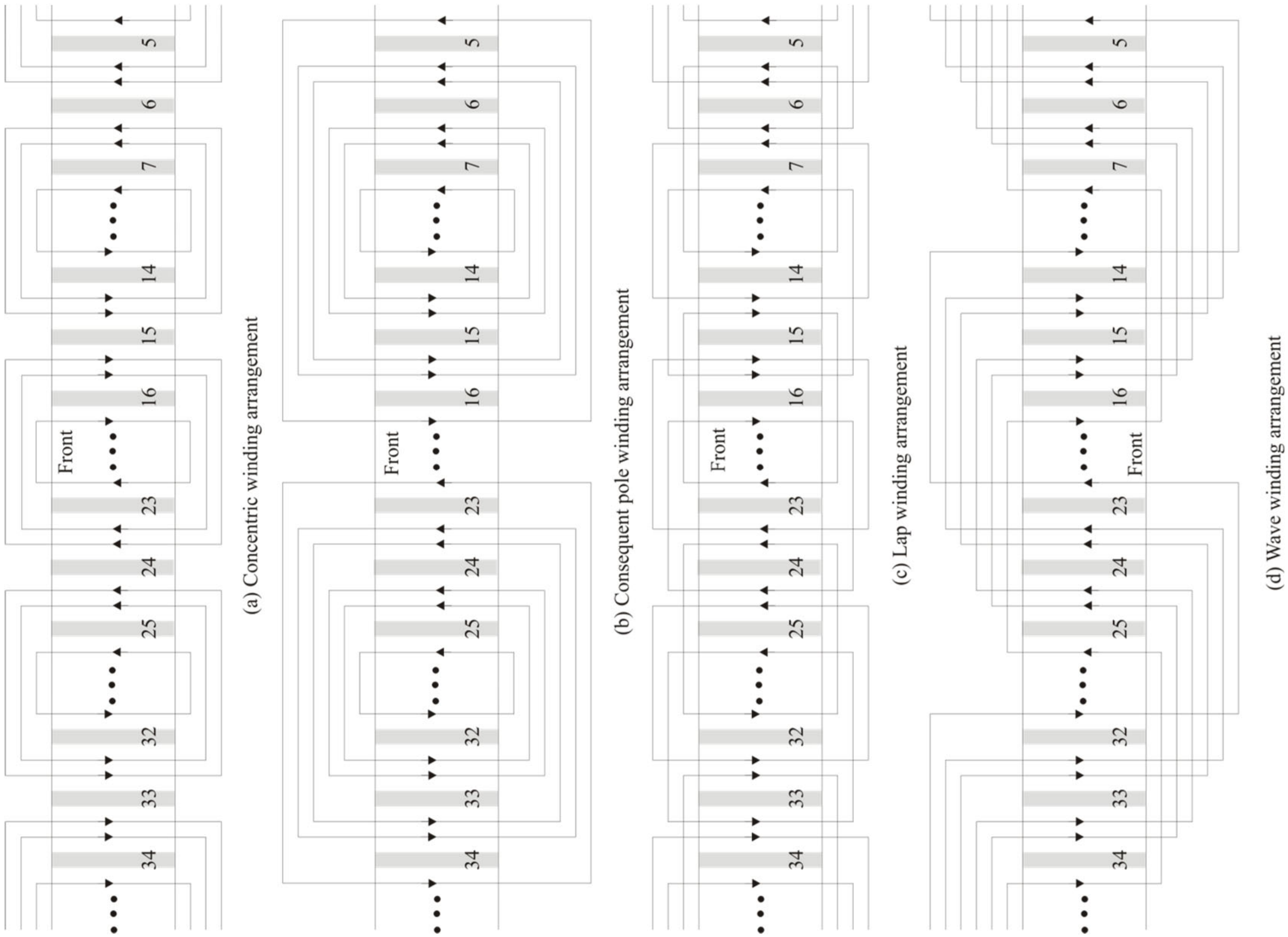
Symmetry Conditions

Winding Arrangements



$$\mathbf{N}_{as}\Big|_{1-18} = N\left[0\ 0\ 0\ 1\ 2\ 2\ 1\ 0\ 0\ 0\ 0\ 0\ -1\ -2\ -2\ -1\ 0\ 0\right]$$

Winding Arrangements



(a) Concentric winding arrangement

(b) Consequent pole winding arrangement

(c) Lap winding arrangement

(d) Wave winding arrangement

Lecture 45

The Discrete Winding Function

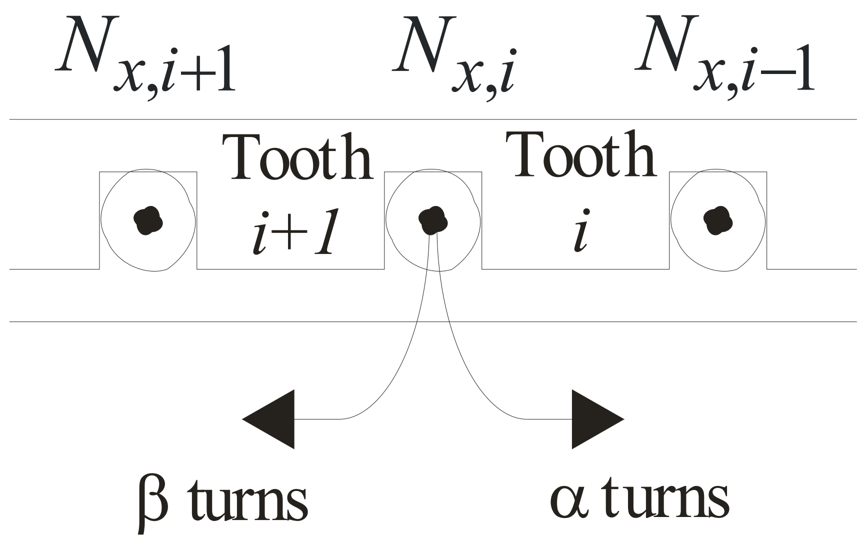
Introduction to Winding Functions

- The winding function is a measure of how many times a winding links flux at a given place
- The winding function will be used to
  - Find self-inductance of a distributed winding
  - Find mutual-inductance between distributed windings
  - Find the mmf associated with a distributed winding
  - It can be expressed in terms of both discrete and continuous winding descriptions

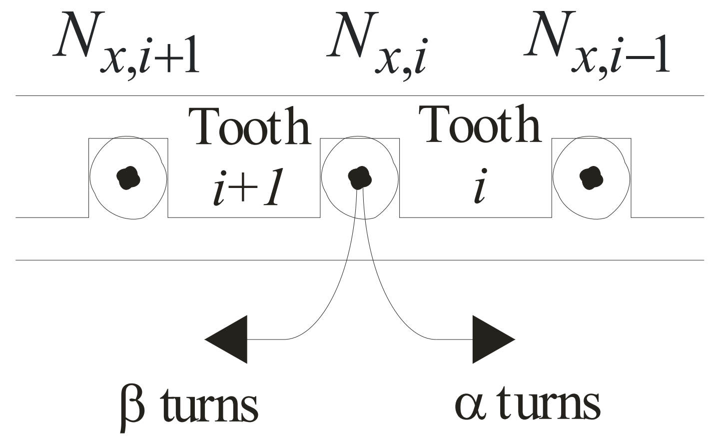
The Discrete Winding Function

- $W_{x,i}$ is the discrete winding function of phase ‘x’ of the stator or rotor
- It is the number of times the winding links flux through the *i*’th tooth
- It is the number of turns around tooth *i*

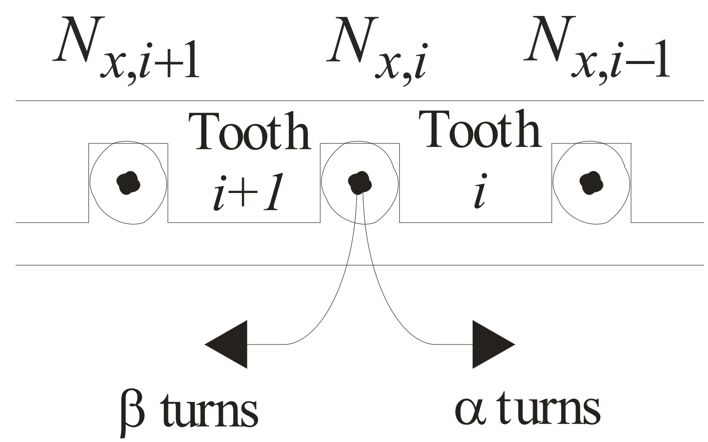
Derivation of Discrete Winding Function



Derivation of Discrete Winding Function



Derivation of Discrete Winding Function



Derivation of Discrete Winding Function

Derivation of Discrete Winding Function

- Summarizing:

$$W_{x,1} = \frac{1}{2} \sum_{i=1}^{S_y/P} N_{x,i}$$

$$W_{x,i+1} = W_{x,i} - N_{x,i}$$

Example

- Suppose we have

$$\mathbf{N}_x = \begin{bmatrix} 10 & 20 & 10 & -10 & -20 & -10 & 10 & 20 & 10 & -10 & -20 & -10 \end{bmatrix}^T$$

Example

- Finally we obtain …

$$\mathbf{w}_x = \begin{bmatrix} 20 & 10 & -10 & -20 & -10 & 10 & 20 & 10 & -10 & -20 & -10 & 10 \end{bmatrix}^T$$

Lecture 46

The Continuous Winding Function

Notation

- $\phi = \phi_{rm}$ for rotor windings
- $\phi = \phi_{sm}$ for stator windings
- $w_{\mathrm{x}}(\phi)$ = winding function
  - Number of times a winding links flux at position ϕ

Derivation of The Cont. Winding Function

Derivation of The Cont. Winding Function

Derivation of The Cont. Winding Function

Derivation of The Cont. Winding Function

- Thus we have

$$\mathrm{w}_x(\phi_m) = \frac{1}{2}\int_0^{2\pi/P} \mathrm{n}_x(\phi_m)d\phi_m - \int_0^{\phi_m} \mathrm{n}_x(\phi_m)d\phi_m$$

Example

- Consider the following geometry

- We might have

$$n_{as} = 100\sin(\phi_{sm})$$

- Find the winding function

Example

Example

Lecture 47

Comparison of Winding Functions

Example

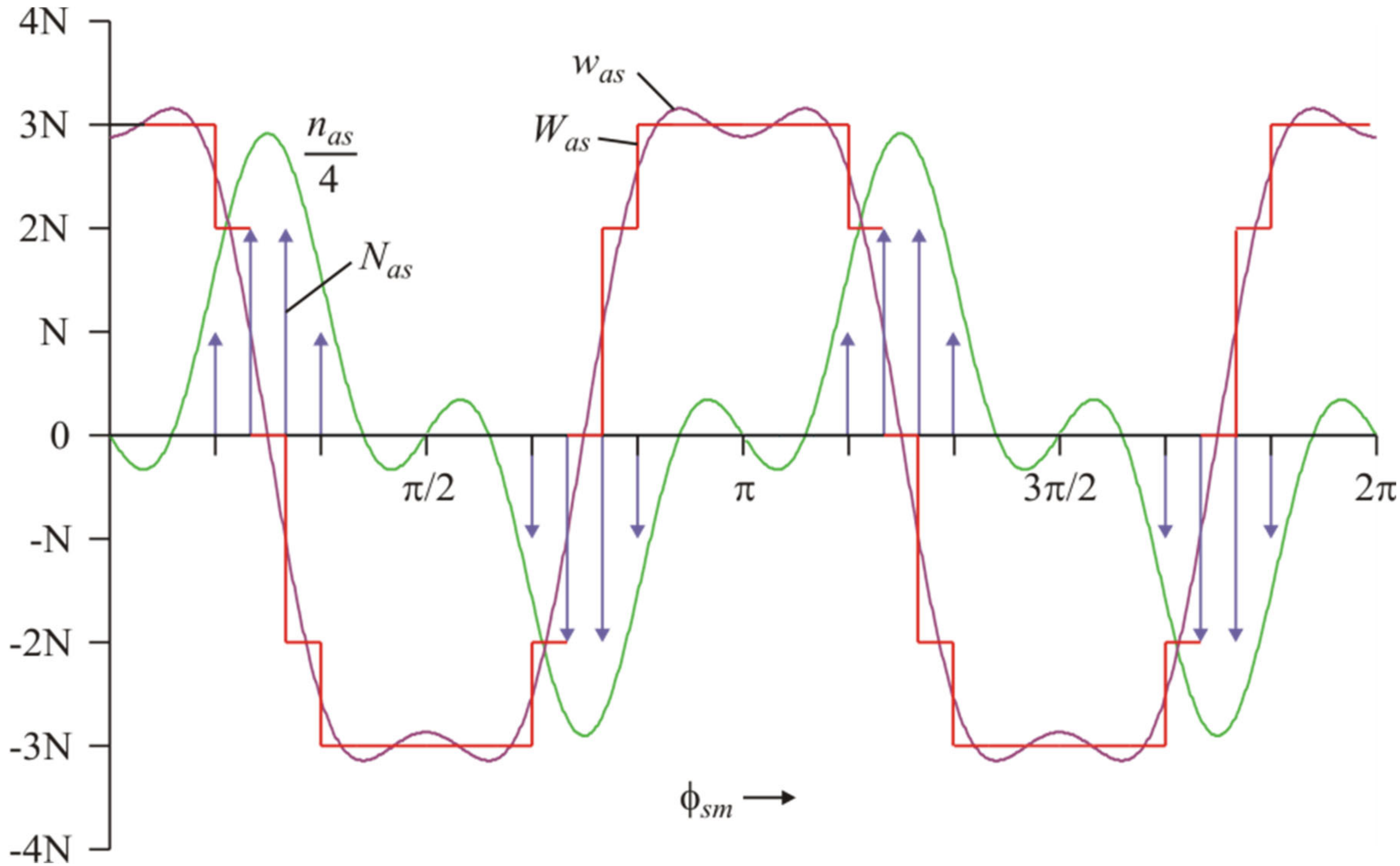
$$\mathbf{N}_{as}\Big|_{1-18} = N\left[0\;0\;0\;1\;2\;2\;1\;0\;0\;0\;0\;0\;-1\;-2\;-2\;-1\;0\;0\right]$$

$$\mathbf{W}_{as}\Big|_{1-18} = N\left[3\;3\;3\;3\;2\;0\;-2\;-3\;-3\;-3\;-3\;-3\;-3\;-2\;0\;2\;3\;3\right]$$

$$n_{as} = N\left(7.221\sin(2\phi_{sm}) - 4.4106\sin(6\phi_{sm})\right)$$

$$w_{as} = N\left(\frac{7.221}{2}\cos(2\phi_{sm}) - \frac{4.4106}{6}\cos(6\phi_{sm})\right)$$

Comparison of Winding Functions



Lecture 48

Distributed MMF

The Idea

- In a distributed machine, the MMF source is associated with a winding is a function of position as well as the current
- We will focus on the continuous winding distribution
- We can show

$$F_S(\phi_m) = \sum_{x \in X_S} w_x(\phi_m) i_x \qquad F_R(\phi_m) = \sum_{x \in X_R} w_x(\phi_m) i_x$$

$$F_g(\phi_m) = F_S(\phi_m) + F_R(\phi_m)$$

Lumped vs Distributed MMF

Airgap MMF

- Before we start, define airgap MMF in a rotating machine:

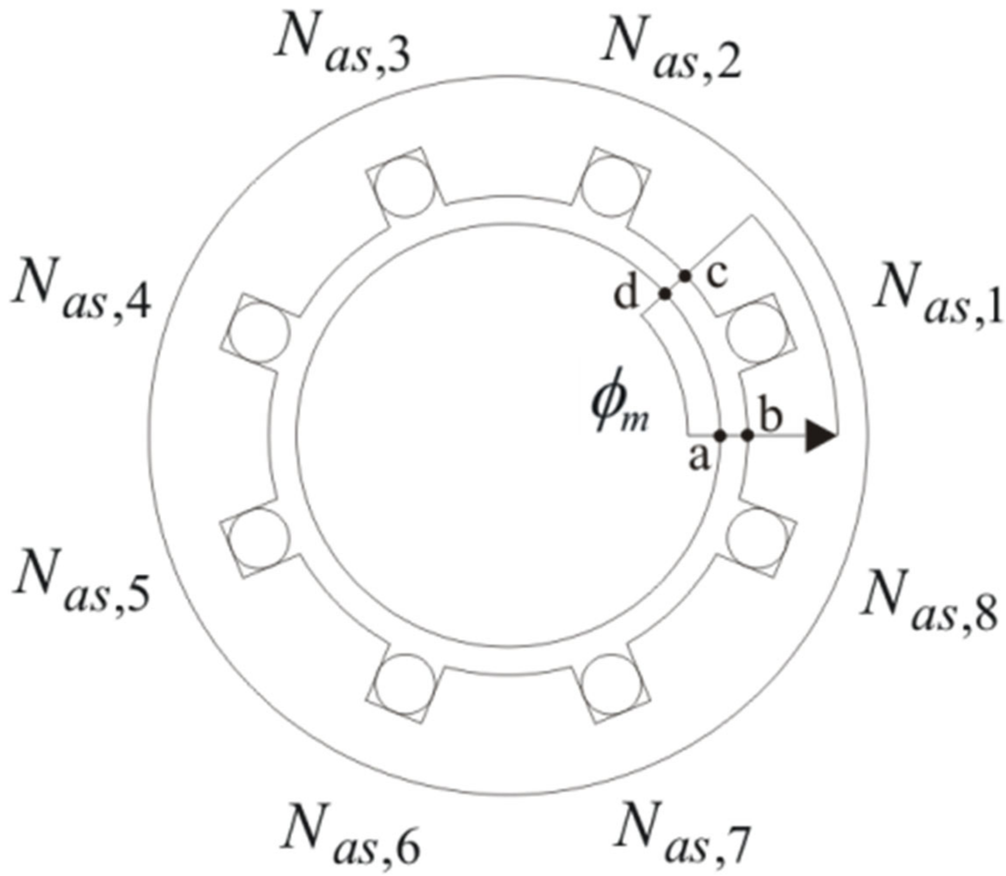
$$\mathrm{F}_g(\phi_m) = \int_{rotor}^{stator} \mathbf{H}(\phi_m) \cdot d\mathbf{l}$$

- Since the integration is in radial direction

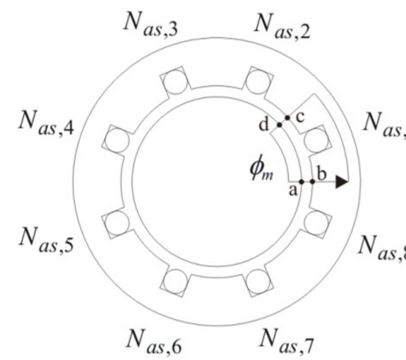
$$\mathrm{F}_g(\phi_m) = \int_{rotor}^{stator} \mathrm{H}_r(\phi_m) dl$$

Air Gap MMF

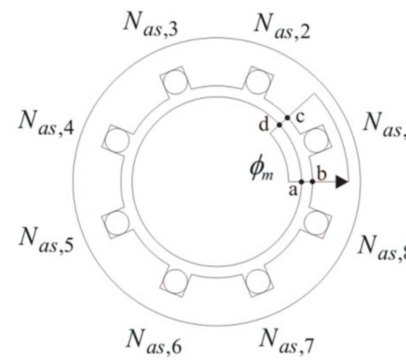
- Now consider the following



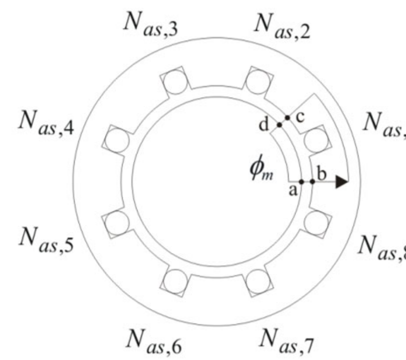
Air Gap MMF



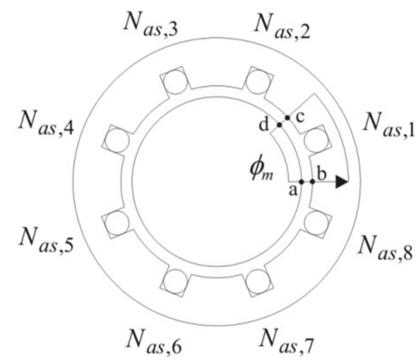
Air Gap MMF



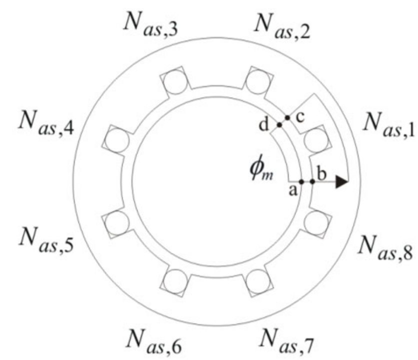
Air Gap MMF



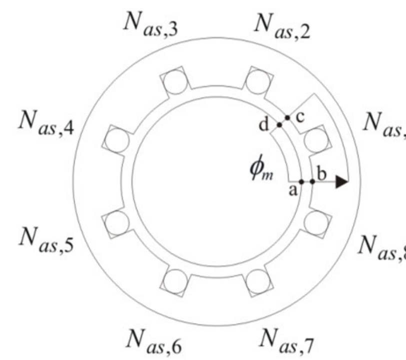
Air Gap MMF



Air Gap MMF



Air Gap MMF



Lecture 49

Reasons to Care About Airgap MMF

Air Gap Flux Density

- Thus we have

$$B_g(\phi_m) = \frac{\mu_0 \mathrm{F}_g(\phi_m)}{\mathrm{g}(\phi_m)}$$

Total Field

$$F(\phi) = \sum_{x \in X} F_x(\phi)$$

$$B = \frac{\mu_0 F(\phi)}{g(\phi)}$$

Field Associate With a Winding

$$B_x = \frac{\mu_0 F_x(\phi)}{g(\phi)}$$

Example

- Suppose
  - w_{as}=100cos($4\phi_{sm}$); i_{as}=5 A
  - w_{bs}=50sin($4\phi_{sm}$); i_{bs}= 10 A
  - g = 1 mm
- What is the peak B-field ?
- At what position is the peak B-field ?

Example

Example

Lecture 50

Rotating MMF

Rotating MMF

- Another concept from the MMF is that the fields rotate

Rotating MMF in Two-Phase Systems

- Suppose

$$n_{as} = N_s \sin\left(P\phi_{sm}/2\right) \qquad w_{as} = \frac{2N_s}{P}\cos\left(P\phi_{sm}/2\right)$$

$$n_{bs} = -N_s \cos\left(P\phi_{sm}/2\right) \qquad w_{bs} = \frac{2N_s}{P}\sin\left(P\phi_{sm}/2\right)$$

- And that

$$i_{as} = \sqrt{2}I_s \cos(\omega_e t + \phi_i)$$

$$i_{bs} = \sqrt{2}I_s \sin(\omega_e t + \phi_i)$$

- What is the MMF ?

Rotating MMF in Two-Phase Systems

Rotating MMF in Two-Phase Systems

Rotating MMF in Two-Phase Systems

- Result

$$\mathrm{F}_s = \frac{2\sqrt{2}N_s I_s}{P}\cos\left(P\phi_{sm}/2 - \omega_e t - \varphi_i\right)$$

Affect of Number of Poles

$$F_s = \frac{2\sqrt{2}N_s I_s}{P}\cos\left(P\phi_{sm}/2 - \omega_e t - \varphi_i\right)$$

Lecture 51

Unbalance

Unbalanced Two-Phase Systems

- Let's consider another case. Again starting with

$$F_s = \frac{2N_s}{P}\left(\cos\left(P\phi_{sm}/2\right)i_{as} + \sin\left(P\phi_{sm}/2\right)i_{bs}\right)$$

- Suppose the b-phase current is zero; the a-phase current given by

$$i_{as} = \sqrt{2}I_s \cos(\omega_e t + \phi_i)$$

Unbalanced Two-Phase Systems

Unbalanced Two-Phase Systems

Unbalanced Two-Phase Systems

Lecture 52

Three Phase Systems

Rotating MMF in Three-Phase Systems

- In this case suppose

$$\mathrm{n}_{as}(\phi_{sm}) = N_{s1}\sin(P\phi_{sm}/2) - N_{s3}\sin(3P\phi_{sm}/2)$$

$$\mathrm{n}_{bs}(\phi_{sm}) = N_{s1}\sin(P\phi_{sm}/2 - 2\pi/3) - N_{s3}\sin(3P\phi_{sm}/2)$$

$$\mathrm{n}_{cs}(\phi_{sm}) = N_{s1}\sin(P\phi_{sm}/2 + 2\pi/3) - N_{s3}\sin(3P\phi_{sm}/2)$$

- Question: why would I do this ?

Rotating MMF in Three-Phase Systems

- Proceeding, we have

$$\mathrm{w}_{as}(\phi_{sm}) = \frac{2N_{s1}}{P}\cos(P\phi_{sm}/2) - \frac{2N_{s3}}{3P}\cos(3P\phi_{sm}/2)$$

$$\mathrm{w}_{bs}(\phi_{sm}) = \frac{2N_{s1}}{P}\cos(P\phi_{sm}/2 - 2\pi/3) - \frac{2N_{s3}}{3P}\cos(3P\phi_{sm}/2)$$

$$\mathrm{w}_{cs}(\phi_{sm}) = \frac{2N_{s1}}{P}\cos(P\phi_{sm}/2 + 2\pi/3) - \frac{2N_{s3}}{3P}\cos(3P\phi_{sm}/2)$$

- Now

$$\mathrm{F}_S(\phi_{sm}) = \mathrm{w}_{as}(\phi_{sm})i_{as} + \mathrm{w}_{bs}(\phi_{sm})i_{bs} + \mathrm{w}_{cs}(\phi_{sm})i_{cs}$$

Rotating MMF in Three-Phase Systems

- Now let us assume a three-phase balanced set of currents

$$i_{as} = \sqrt{2} I_s \cos(\omega_e t + \phi_i)$$

$$i_{bs} = \sqrt{2} I_s \cos(\omega_e t + \phi_i - 2\pi / 3)$$

$$i_{cs} = \sqrt{2} I_s \cos(\omega_e t + \phi_i + 2\pi / 3)$$

Rotating MMF in Three-Phase Systems

- Combining yields

$$\mathrm{F}_S = \frac{3\sqrt{2}N_{s1}I_s}{P}\cos\left(P\phi_{sm}/2 - \omega_e t - \phi_i\right)$$

Lecture 53

Flux Linkage and Inductance

Flux Linkage and Magnetizing Inductance

- Flux linkage:

$$\lambda_x = \lambda_{xl} + \lambda_{xm}$$

- Inductance

$$L_{xy} = \frac{\lambda_x \big|_{due\ to\ i_y}}{i_y}$$

Flux Linkage and Magnetizing Inductance

- Partitioning Inductance

$$L_{xy} = L_{xyl} + L_{xym}$$

- Inductance Terms

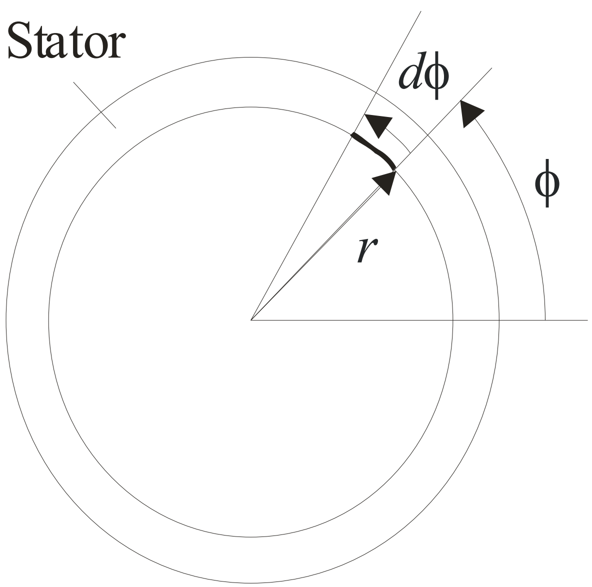
$$L_{xyl} = \frac{\lambda_{xl}\big|_{due\ to\ i_y}}{i_y} \qquad L_{xym} = \frac{\lambda_{xm}\big|_{due\ to\ i_y}}{i_y}$$

Flux Linkage and Magnetizing Inductance

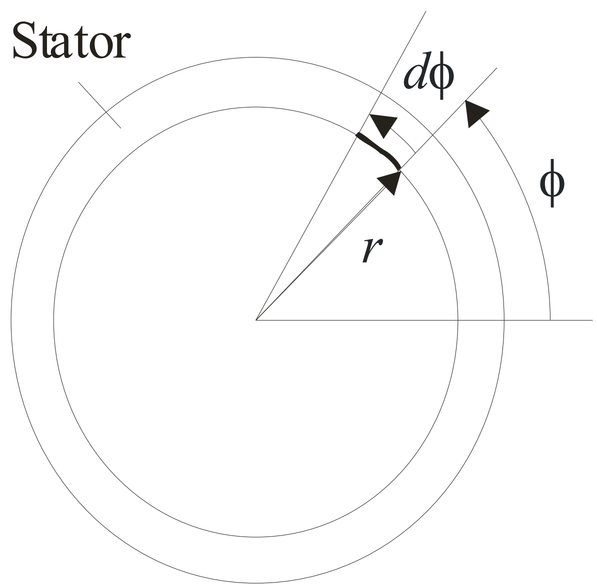
- We can show

$$\frac{\lambda_{xym}}{i_y} = L_{xym} = \mu_0 r l \int_0^{2\pi} \frac{\mathrm{w}_x(\phi_m)\,\mathrm{w}_y(\phi_m)}{\mathrm{g}(\phi_m)} d\phi_m$$

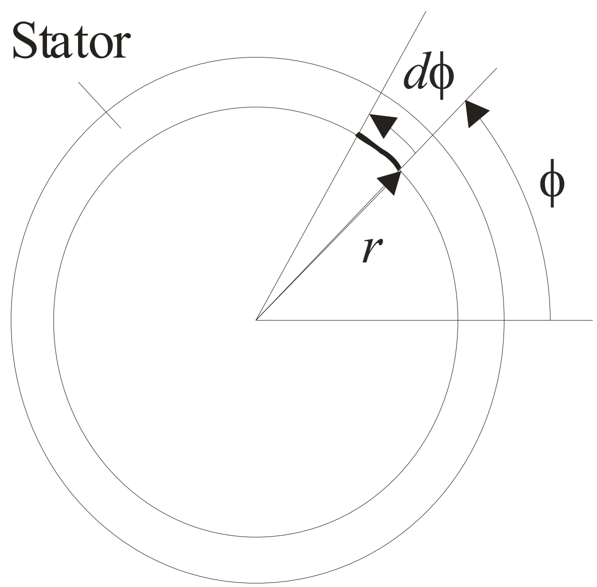
Flux Linkage and Magnetizing Inductance



Flux Linkage and Magnetizing Inductance



Flux Linkage and Magnetizing Inductance



Lecture 54

Example Inductance Calculations

Example 1

- Suppose

$$n_{as} = N_s \sin(P\phi_{sm}/2) \qquad w_{as} = \frac{2N_s}{P}\cos(P\phi_{sm}/2)$$

$$n_{bs} = N_s \sin(P\phi_{sm}/2 - 2\pi/3) \qquad w_{bs} = \frac{2N_s}{P}\cos(P\phi_{sm}/2 - 2\pi/3)$$

and that the air gap is uniform. Find the magnetizing inductance between the two phases

Example 1

- We obtain

$$L_{asbs} = -\frac{2\pi\mu_0 r L N_s^2}{P^2 g}$$

Example 2 (IM)

$$n_{as}(\phi_{sm}) = N_{s1}\sin(P\phi_{sm}/2)$$

$$n_{ar}(\phi_{rm}) = N_{r1}\sin(P\phi_{rm}/2)$$

Example 2 (Cont.)

Example 2 (Cont.)

Example 2 (Cont.)