Lecture Set 3:
Variable Reluctance Stepper Motors

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About this Lecture Set

- Reading
  - *Electromechanical Motion Devices, 2nd Edition,* Sections 9.1-9.5
- Goal
  - Apply what we have learned so far to our first practical electromechanical device – the VR Stepper Motor!

Lecture 21

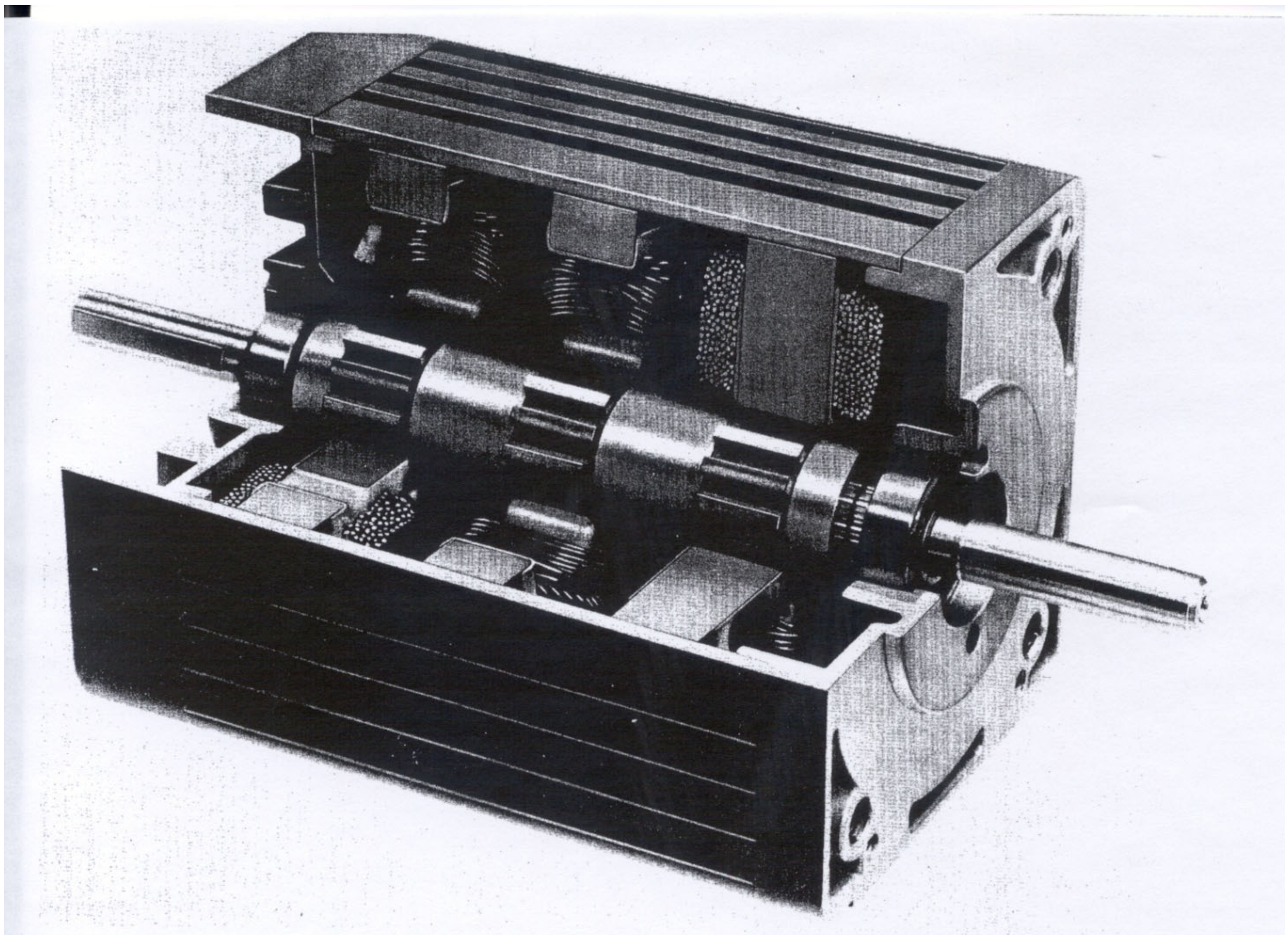
Physical Configuration of the VR Stepper Motor

VR Stepper Motor

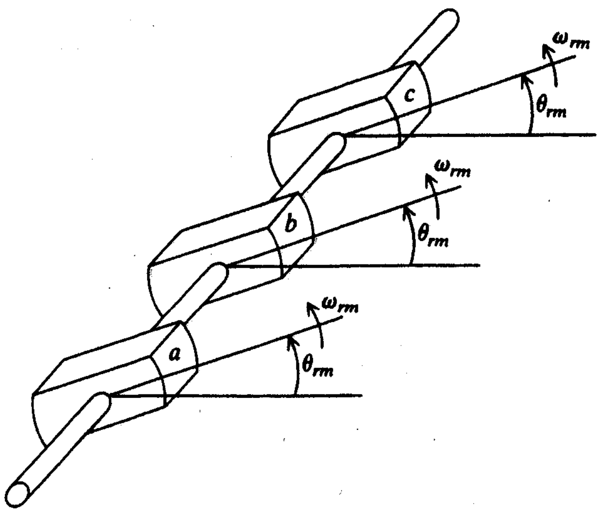
- Characteristics

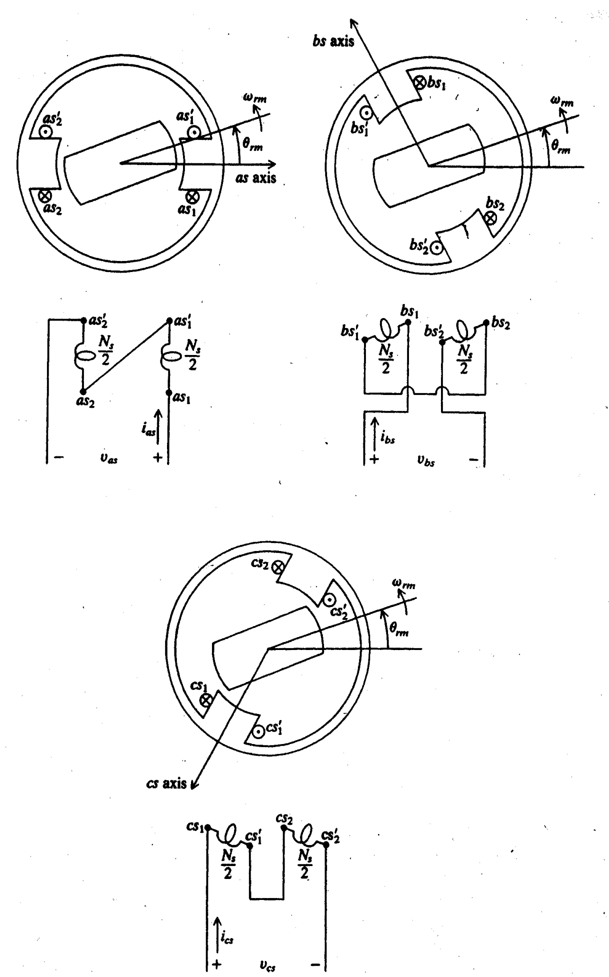
- Uses

Construction



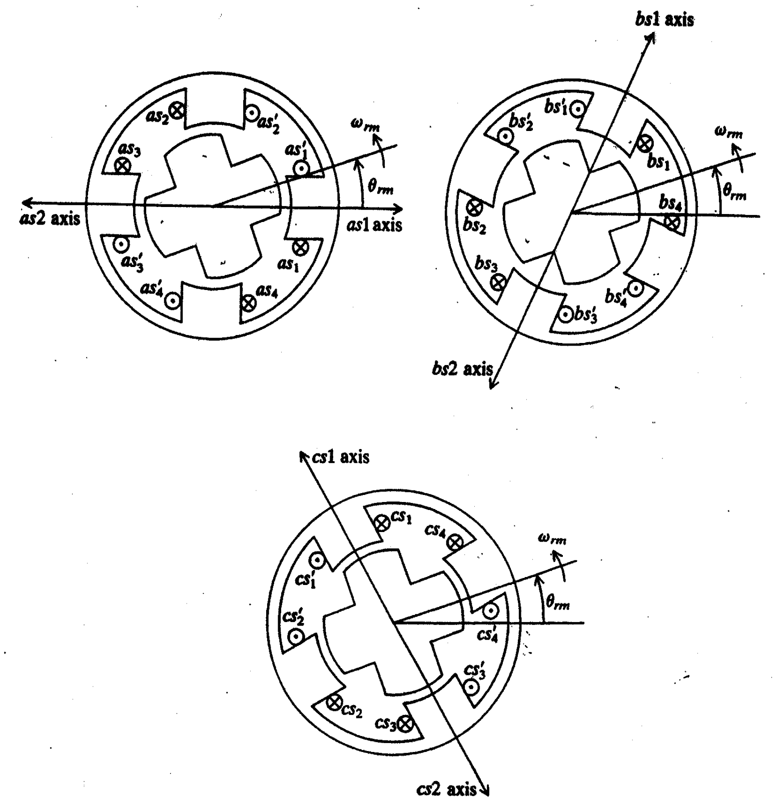
Physical Configuration





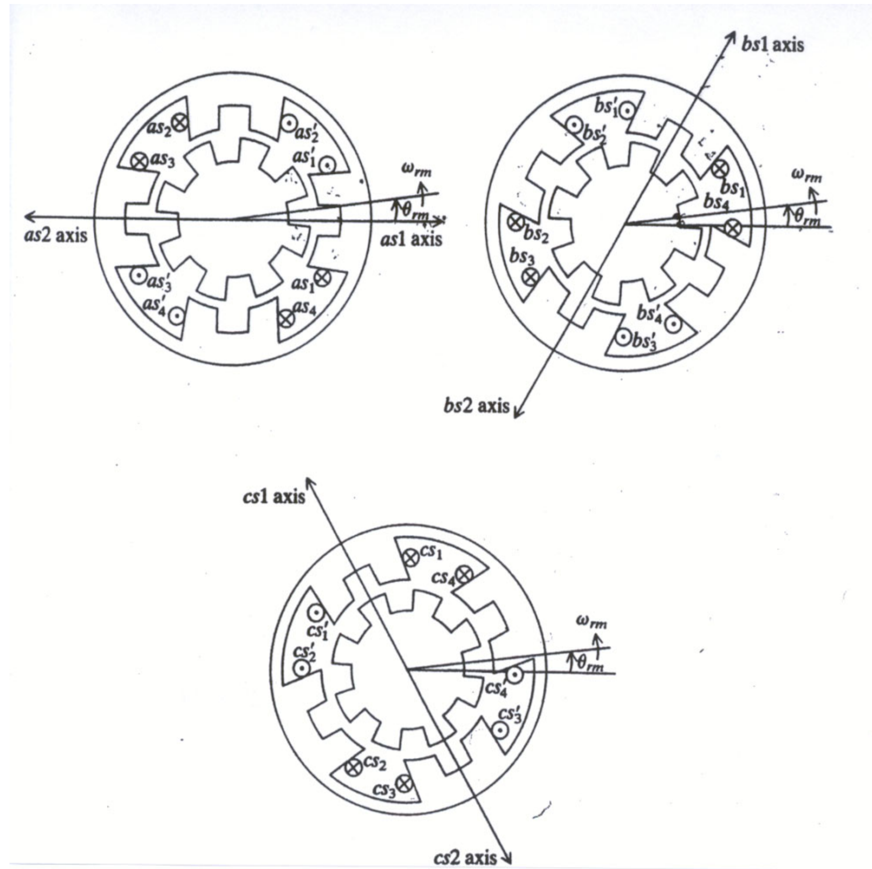
Excitation

Some Definitions



Rotor Teeth (RT):
Tooth Pitch (TP):
Number of Phases (N):
Step Length (SL):

Example



Rotor Teeth (RT):
Tooth Pitch (TP):
Number of Phases (N):
Step Length (SL):

Lecture 22

Modeling of the VR Stepper Motor Part 1:
Setup to the A-Phase Flux Linkage Equation

Analysis of VR Stepper

- Voltage Equations
- Flux Linkage Equations
- Torque Equation
- Mechanical Dynamics

VR Stepper Voltage Equations

- Voltage Equations

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

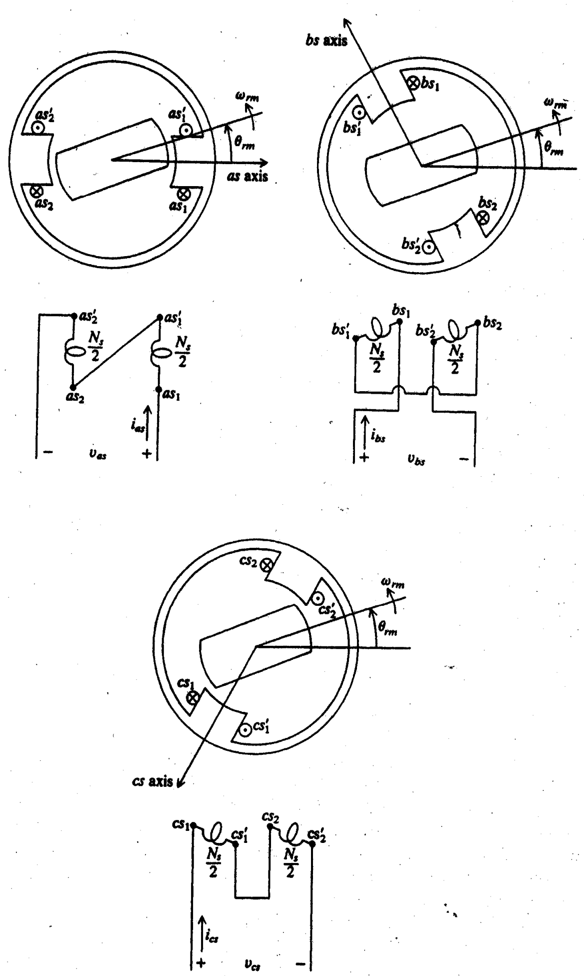
$$v_{cs} = r_s i_{cs} + \frac{d\lambda_{cs}}{dt}$$

- In Matrix Form

$$\mathbf{v}_{abcs} = r_s \mathbf{i}_{abcs} + p\boldsymbol{\lambda}_{abcs}$$

$$(\mathbf{f}_{abcs})^T = [f_{as} \quad f_{bs} \quad f_{cs}]$$

Flux Linkage Equations



Flux Linkage Equations

Flux Linkage Equations

Flux Linkage Equations

Flux Linkage Equations

- For any multi-stack VR stepper

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} L_{asas} & 0 & 0 \\ 0 & L_{bsbs} & 0 \\ 0 & 0 & L_{cscs} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

- For this machine

$$L_{asas} = L_{ls} + L_A + L_B \cos 2\theta_{rm}$$

$$L_{bsbs} = L_{ls} + L_A + L_B \cos 2(\theta_{rm} - \frac{2}{3}\pi)$$

$$L_{cscs} = L_{ls} + L_A + L_B \cos 2(\theta_{rm} - \frac{4}{3}\pi)$$

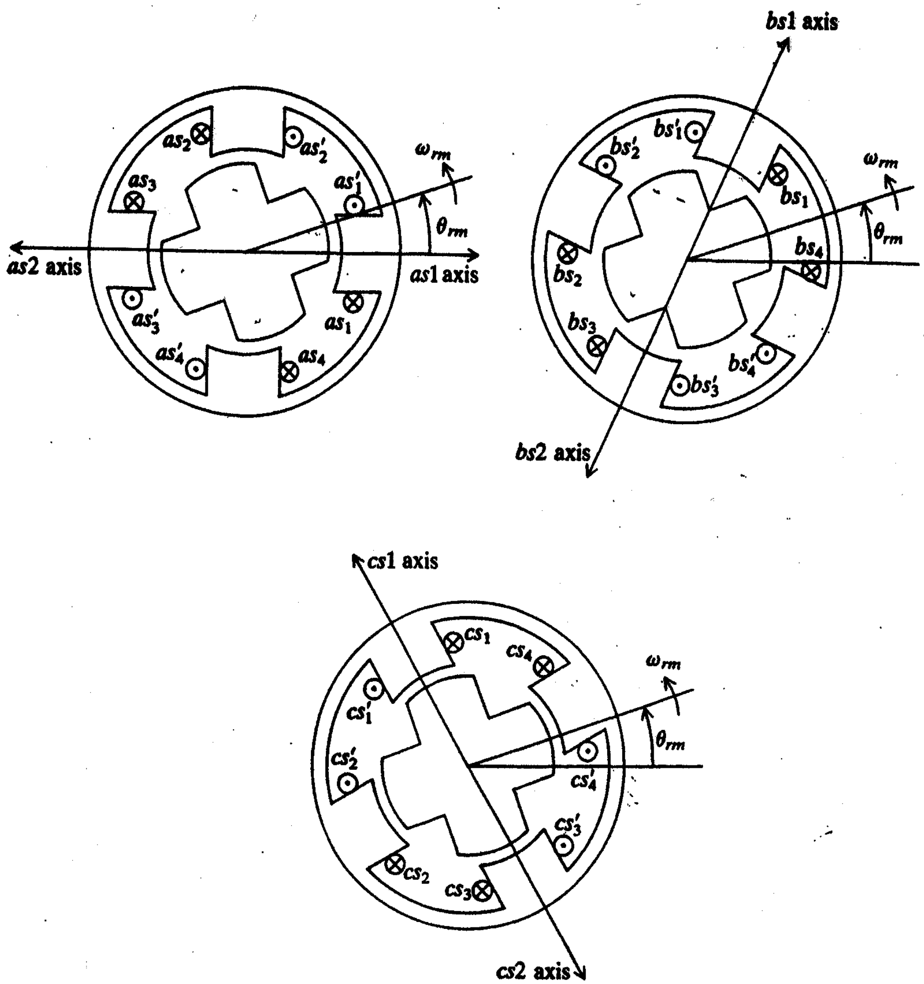
Lecture 23

Modeling of the VR Stepper Motor Part 2: Remaining Flux Linkage Equations and Torque

Flux Linkage Equations

Flux Linkage Equations

Flux Linkage Equations (Figure 8.2-3)



Flux Linkage Equations (Figure 8.2-3)

- We have…

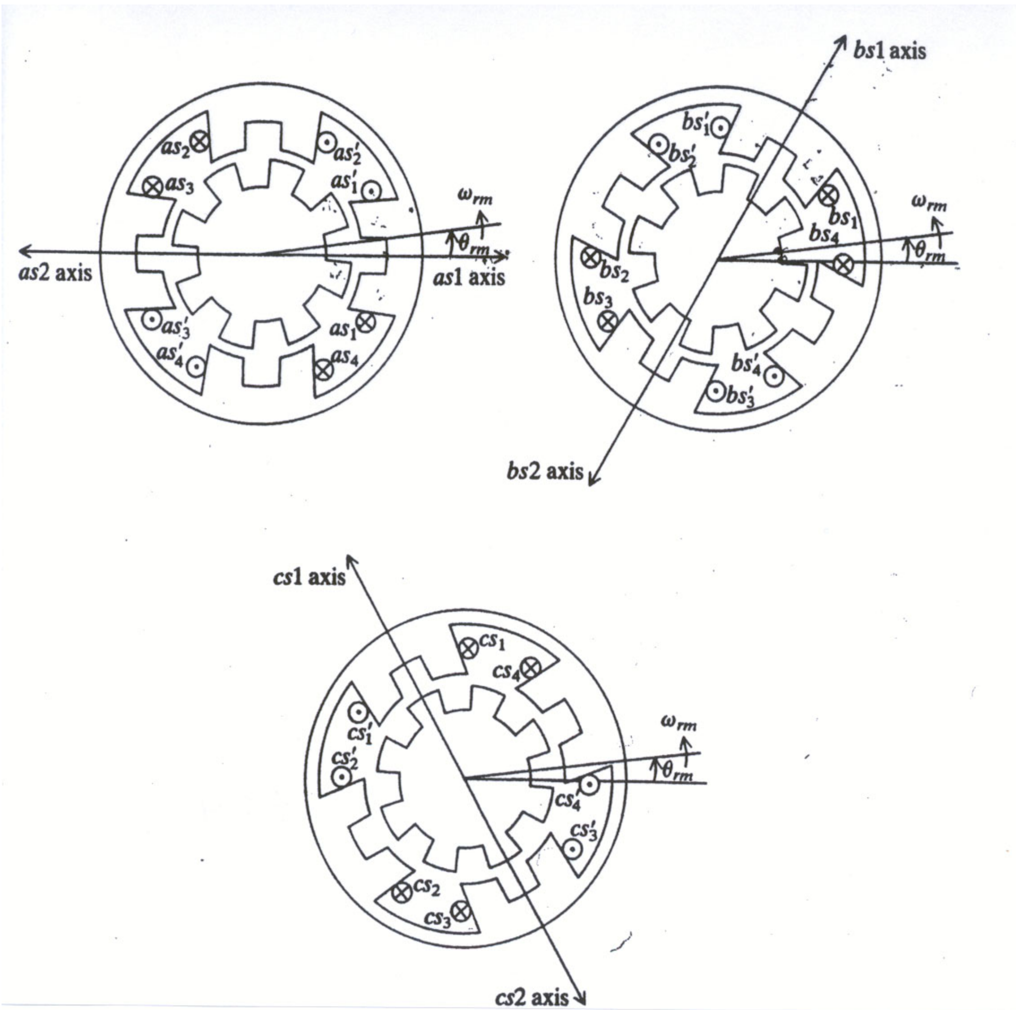
$$L_{asas} = L_{ls} + L_A + L_B \cos 4\theta_{rm}$$

$$L_{bsbs} = L_{ls} + L_A + L_B \cos 4(\theta_{rm} - \frac{1}{3}\pi)$$

$$L_{cscs} = L_{ls} + L_A + L_B \cos 4(\theta_{rm} - \frac{2}{3}\pi)$$

- Or …

Flux Linkage Equations (Figure 8.2-4)



Flux Linkage Equations (Figure 8.2-4)

- We have…

$$L_{asas} = L_{ls} + L_A + L_B \cos 8\theta_{rm}$$

$$L_{bsbs} = L_{ls} + L_A + L_B \cos 8(\theta_{rm} - \frac{1}{3}\pi)$$

$$L_{\mathrm{cscs}} = L_{ls} + L_A + L_B \cos 8(\theta_{rm} - \frac{2}{3}\pi)$$

- Or …

Generalized Flux Linkage Eqns

- Thus in general, we have

$$L_{asas} = L_{ls} + L_A + L_B \cos(RT\,\theta_{rm})$$

$$L_{bsbs} = L_{ls} + L_A + L_B \cos[RT(\theta_{rm} + SL)]$$

$$L_{cscs} = L_{ls} + L_A + L_B \cos[RT(\theta_{rm} - SL)]$$

- Or

$$L_{asas} = L_{ls} + L_A + L_B \cos(RT\,\theta_{rm})$$

$$L_{bsbs} = L_{ls} + L_A + L_B \cos[RT(\theta_{m} - SL)]$$

$$L_{cscs} = L_{ls} + L_A + L_B \cos[RT(\theta_{rm} + SL)]$$

Derivation of Torque

Derivation of Torque

Electromagnetic Torque

- Thus

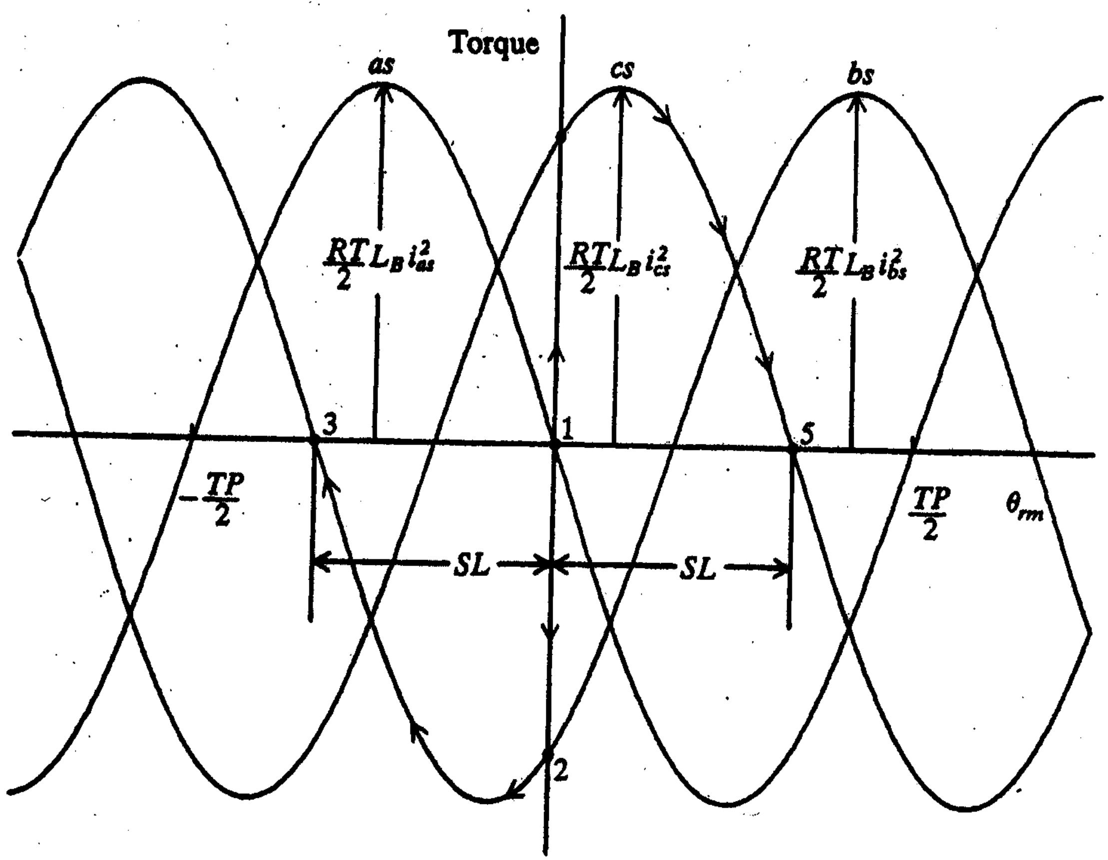
$$T_e = -\frac{RT}{2} L_B \left\{ i_{as}^2 \sin(RT\ \ \theta_{rm}) + i_{bs}^2 \sin[RT(\theta_{rm} \pm SL)] \right.$$
$$\left. + \mathrm{i}_{\mathrm{cs}}^2 \sin[RT(\theta_{rm}\ \mathrm{b}\ SL)] \right\}$$

$$T_e = -\frac{RT}{2} L_B \left\{ i_{as}^2 \sin\left(\frac{2\pi}{TP}\theta_{rm}\right) + i_{bs}^2 \sin\left[\frac{2\pi}{TP}\left(\theta_{rm} \pm \frac{TP}{3}\right)\right] \right.$$
$$\left. + \mathrm{i}_{\mathrm{cs}}^2 \sin\left[\frac{2\pi}{TP}\left(\theta_{rm}\ \mathrm{b}\ \frac{TP}{3}\right)\right] \right\}$$

Lecture 24

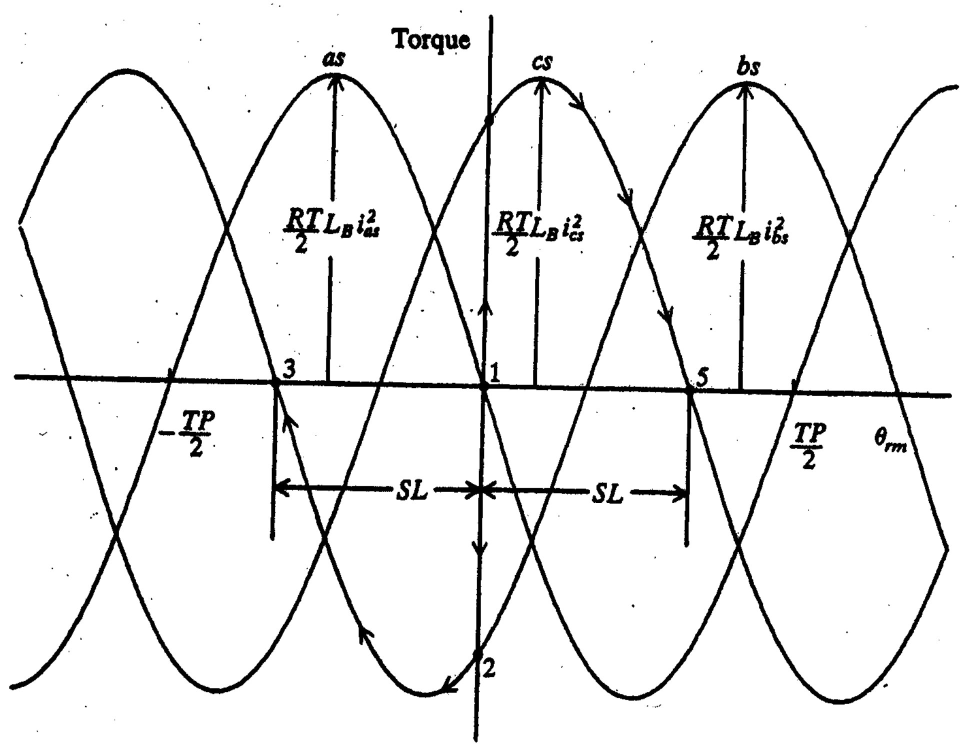
Torque vs Position Characteristics

Torque Position Characteristics



Mechanical Dynamics

Stable and Unstable Equilibrium



Example

- Consider a VR stepper with the following parameters
  - R_s: 2 Ohms
  - L_A=12 mH
  - L_B=10 mH
  - RT=8
  - N=3

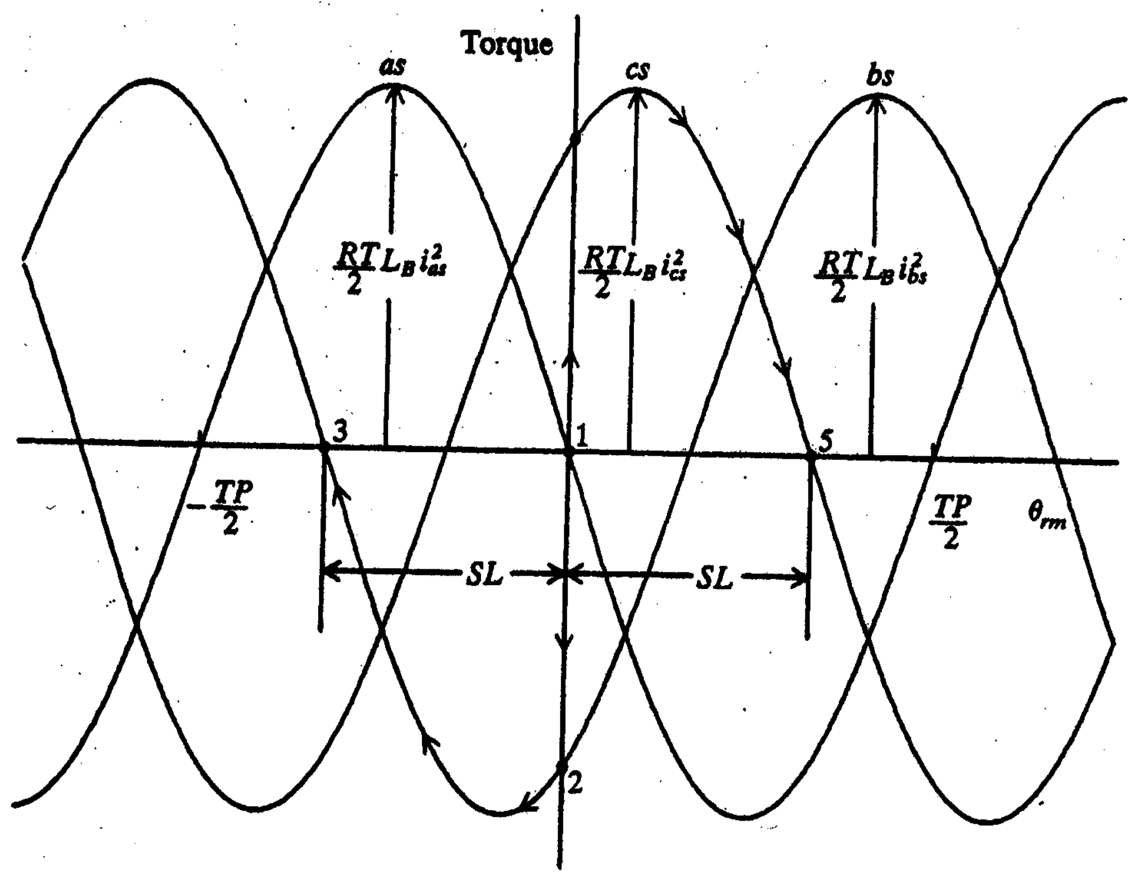
- Suppose 6 V applied to a-phase with load torque of 0.25 Nm. Compute position.

Solution

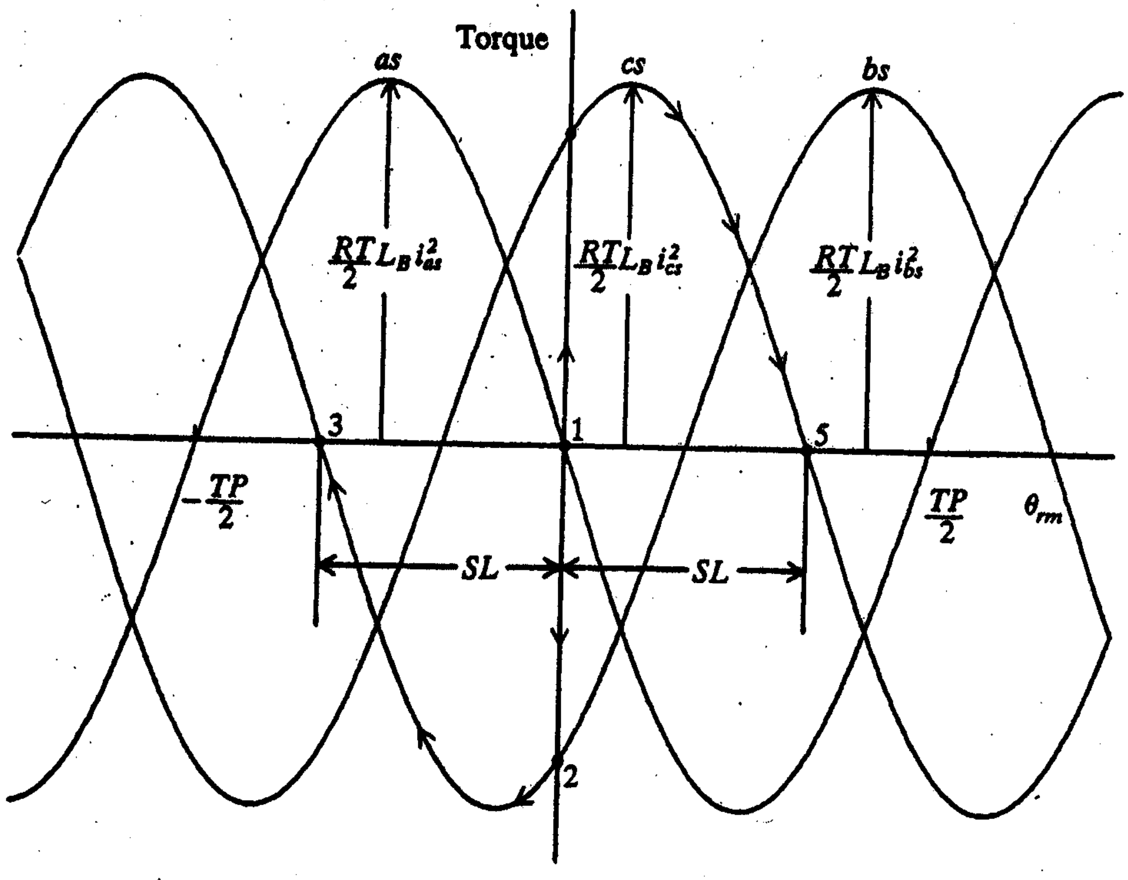
Lecture 25

Stepping

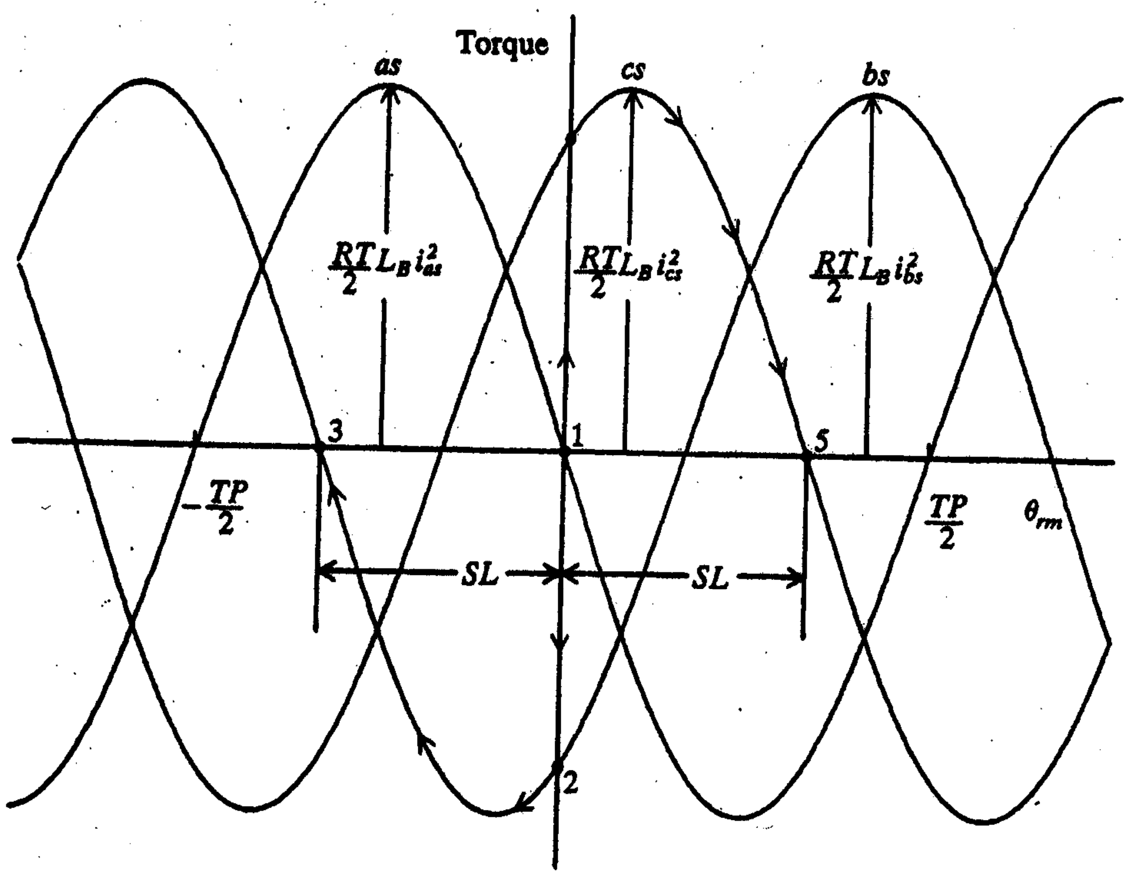
Basic Stepping Operation: Forward



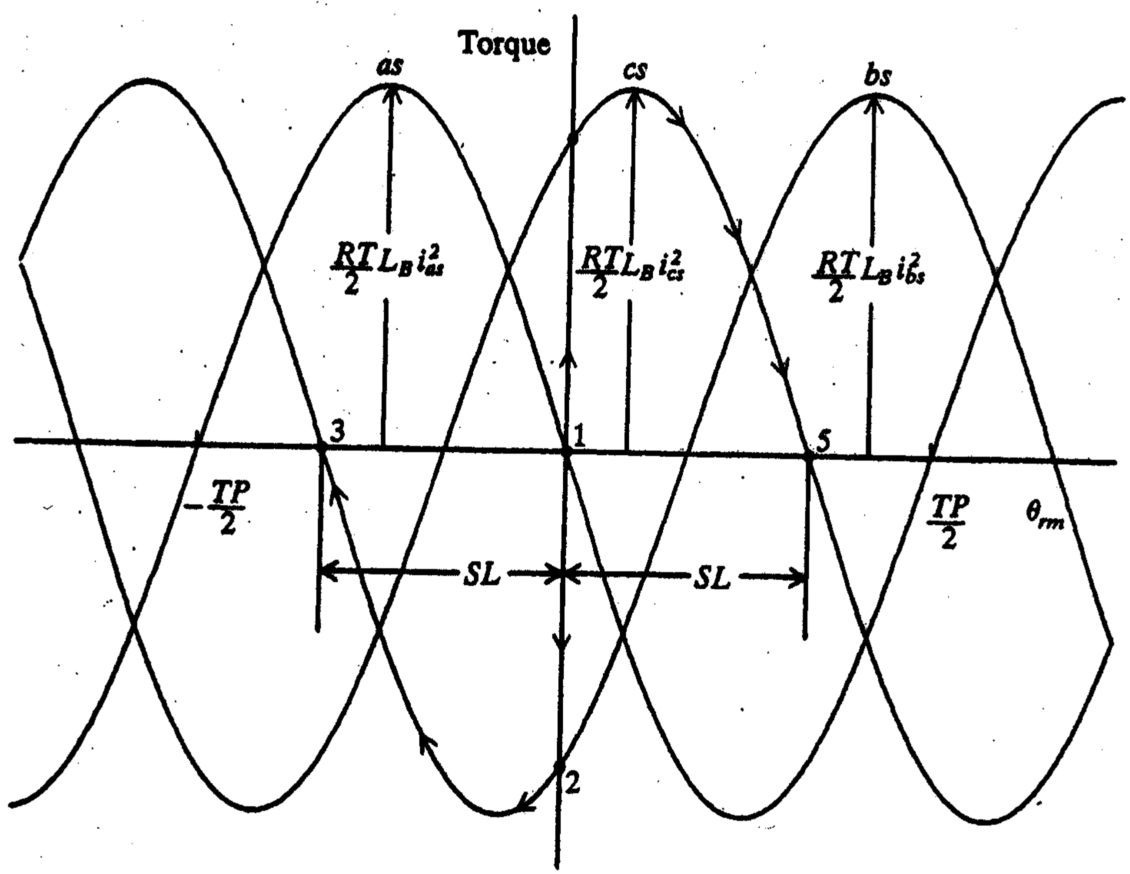
Basic Stepping Operation: Backward



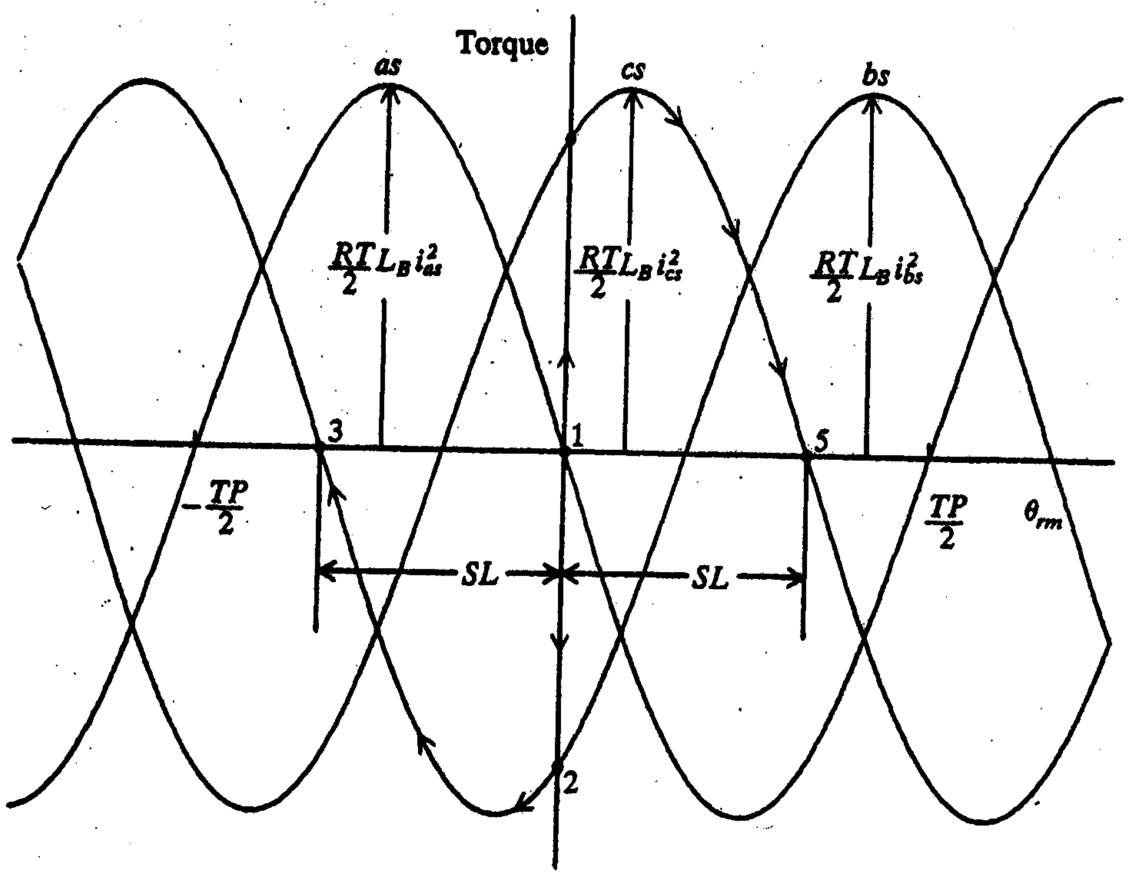
Basic Stepping: Irregular



Stepping Operation: Missing a Step



Stepping: Forward Load Torque Range



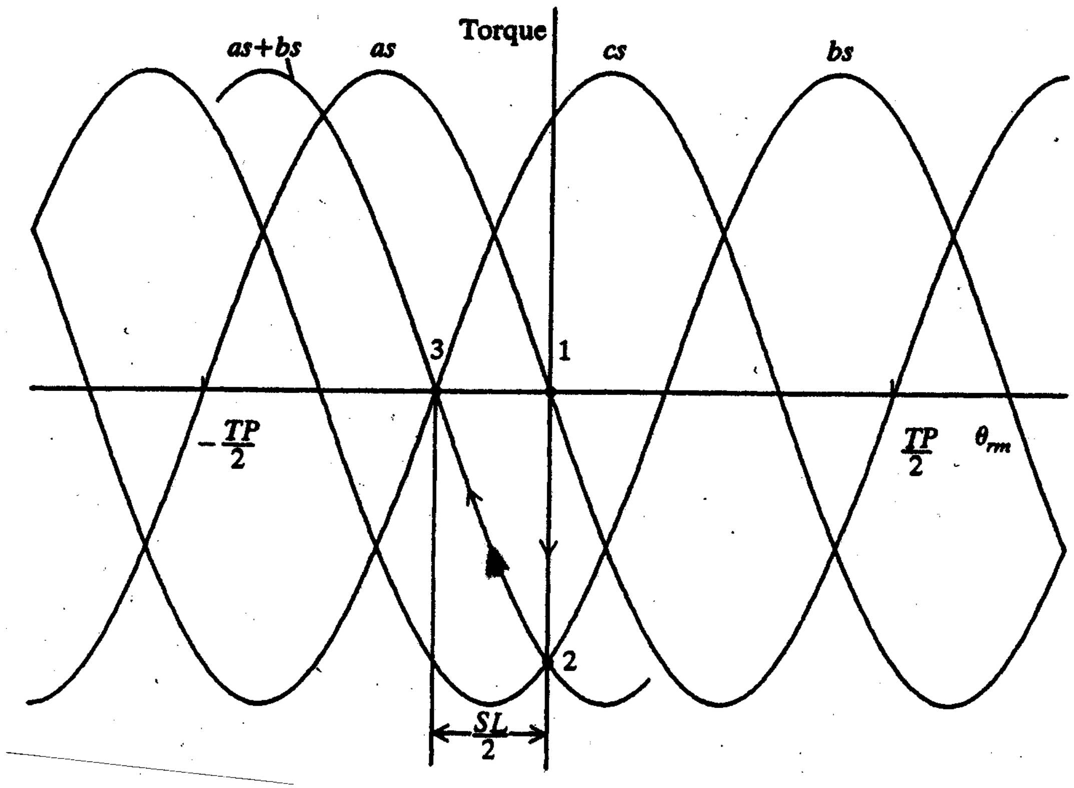
Advance Torque

- Start with

$$T_e = -\frac{RT}{2} L_B \left\{ i_{as}^2 \sin\left(\frac{2\pi}{TP}\theta_{rm}\right) + i_{bs}^2 \sin\left[\frac{2\pi}{TP}\left(\theta_{rm} + \frac{TP}{3}\right)\right] + i_{cs}^2 \sin\left[\frac{2\pi}{TP}\left(\theta_{rm} - \frac{TP}{3}\right)\right]\right\}$$

Advance Torque

Half-Step Operation



Lecture 26

Drive Circuitry

Driving VR Stepper Motor

- Common solution

Driving VR Stepper

- Typical waveforms

Driving VR Stepper

- Better (but more expensive) solution

Driving VR Stepper

- Typical waveforms

Example

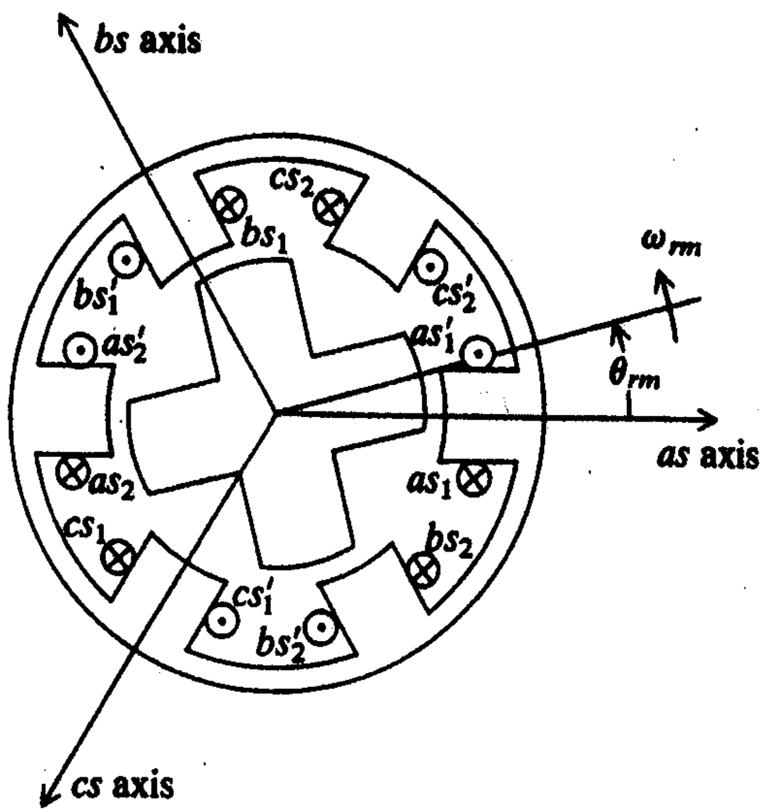
- Consider a multistack VR stepper motor with N=3 and 8 rotor teeth being fed from a single transistor per stack circuit. If the dc voltage is 12 V, the transistor drop is 1 V, and the diode drop is 2 V, sketch the a-phase voltage waveform if the machine is traveling at 20 rad/s (average speed).

Solution

Lecture 27

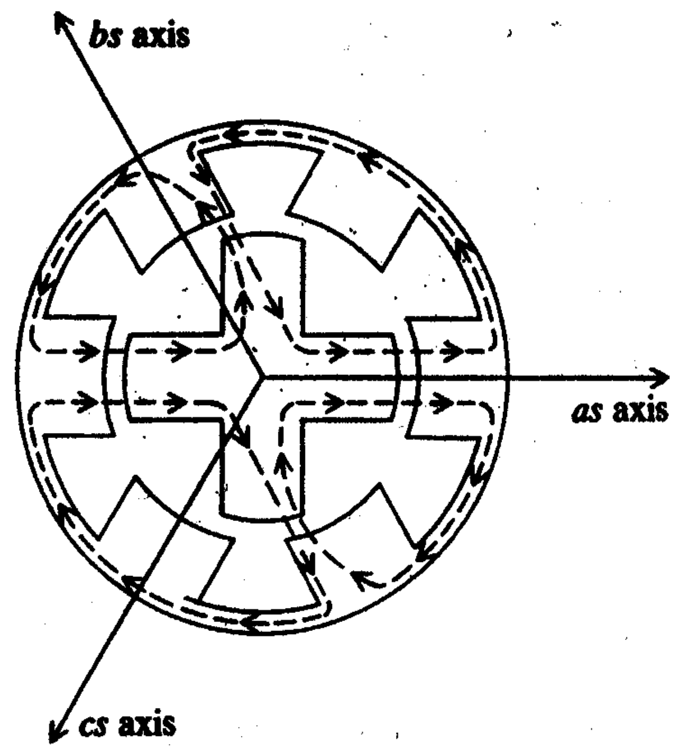
Single-Stack Machines

Single Stack VR Stepper



Modeling the VR Stepper

- Primary issue: mutual inductance



Single Stack VR Stepper