Lecture Set 2:
Energy Conversion

S.D. Sudhoff

Spring 2021

Reading

- Sections 2.1-2.6

Lecture Set 2 Goal

- We know how to
  - (1) flux linkage in terms of current and position
  - (2) current in terms of flux linkage and position

- Using either (1) or (2), we can find an expression for force or torque for an electromechanical device! No other information needed.

Lecture 14

Magnetic System Description and Energy Flow

Single Winding Magnetic System

- For single winding systems, we may describe our magnetics (ignoring hysteresis) with <u>one</u> of the following
  - Form 1:

$$i = f_\lambda(\lambda, x)$$

  - Form 2:

$$\lambda = f_i(i, x)$$

Multi-Winding Magnetic System

- For multi-winding systems, we may describe our magnetics (ignoring hysteresis) with <u>one</u> of the following
  - Form 1:

$$\mathbf{i} = \mathbf{f}_{\lambda}(\boldsymbol{\lambda}, x)$$

  - Form 2:

$$\boldsymbol{\lambda} = \mathbf{f}_{i}(\mathbf{i}, x)$$

- Form 2 is most common for both single and multi-winding systems

Energy Flow

Energy Flow

Field Energy

- Physical definition of the coupling field…

- Energy into the coupling field

$$W_f = W_e + W_m$$

Lecture 15

Calculation of Field Energy

Assumptions

- To find the field energy we will assume a lossless field
  - Implications
  - Concerns

Derivation of Field Energy (Part 1)

Derivation of Field Energy (Part 1)

- Thus

$$W_f = \sum_{j=1}^{J} \int_{\lambda_{j,0}}^{\lambda_{j,f}} i_j d\lambda_j - \int_{x_0}^{x_f} f_e dx$$

Calculating Field Energy (Part 2)

- A mathematical experiment ….

Calculation of Field Energy (Part 2)

- Positioning the mechanical system first

$$W_f = \sum_{j=1}^{J} \int_{\lambda_{j,0}}^{\lambda_{j,f}} i_j d\lambda_j$$

- Replacing the current in term of flux linkage

$$W_f = \sum_{j=1}^{J} \int_{\lambda_{j,0}}^{\lambda_{j,f}} f_{i,j}(\boldsymbol{\lambda}, x_f) d\lambda_j$$

Example 1

- Example: find the field energy for a system described by:

$$i = (5 + 2x)\lambda^2$$

Example 1

Example 2

- Example: find the field energy for the system

$$i_1 = 5x\lambda_1 + (10 + 2x)e^{2\lambda_1 + 2\lambda_2}$$

$$i_2 = 7\lambda_2 + (10 + 2x)e^{2\lambda_1 + 2\lambda_2}$$

Example 2

Example 2

Example 2

Lecture 16

Calculation of Force Using Field Energy

Derivation of Force from Field Energy

- From the field energy, we can find force

Derivation of Force from Field Energy

Derivation of Force from Field Energy

- In summary:

$$f_e = -\frac{\partial W_f(\boldsymbol{\lambda}, x)}{\partial x}$$

Example

- Example: suppose

$$i_1 = 5x\lambda_1 + (10 + 2x)e^{2\lambda_1 + 2\lambda_2}$$

$$i_2 = 7\lambda_2 + (10 + 2x)e^{2\lambda_1 + 2\lambda_2}$$

- Then

$$W_f = \frac{5}{2}x\lambda_1^2 + \frac{1}{2}(10 + 2x)\left(e^{2\lambda_1 + 2\lambda_2} - 1\right) + \frac{7}{2}\lambda_2^2$$

- Thus

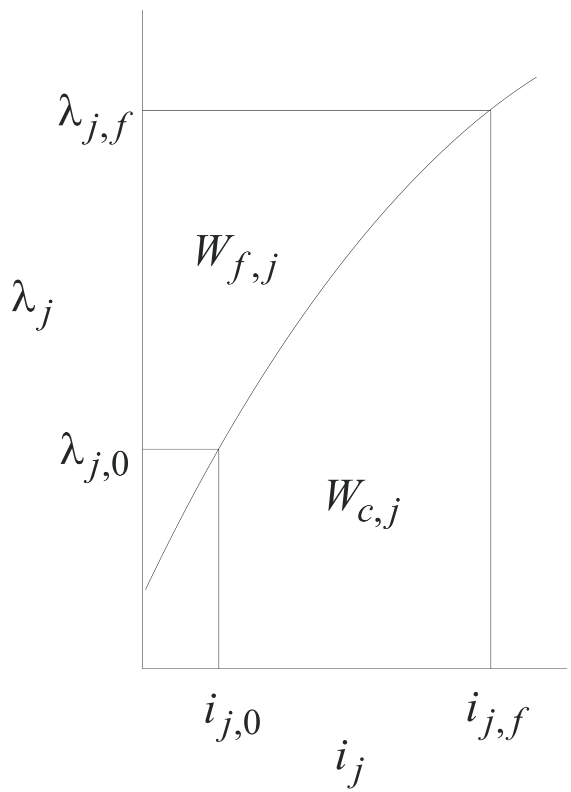
Lecture 17

Co-Energy

Why?

- Question: What’s wrong with field energy ?

Graphical View of Co-Energy



$$W_{f,j} = \int_{\lambda_{j,0}}^{\lambda_{j,f}} i_j d\lambda_j$$

$$W_{c,j} = \int_{i_{j,0}}^{i_{j,f}} \lambda_j \, di_j$$

$$W_{c,j} + W_{f,j} = \lambda_{f,j} i_{f,j} - \lambda_{0,j} i_{0,j}$$

Relationship of Field and Co-Energy

- Summing over all inputs

$$W_c + W_f = \sum_{j=1}^{J} \left(\lambda_j i_j - \lambda_{j,0} i_{j,0} \right)$$

- Observation on nature of co-energy

Example 1

- Suppose

$$\lambda_1 = 2i_1 + \frac{1}{2+x}(i_1 + i_2)^{0.5}$$

$$\lambda_2 = 5i_2^{0.4} + \frac{1}{2+x}(i_1 + i_2)^{0.5}$$

Example 1

Example 1

Example 1

Example 2

- Suppose

$$\lambda_1 = 2i_1 + \frac{1}{2+x}(i_1 + i_2)^{0.5}$$

$$\lambda_2 = 5i_2^{0.4} + \frac{1}{2+x}(i_1 + i_2)^{0.5}$$

- This time, lets try a different path

Example 2

Example 2

Example 2

Lecture 18

Calculation of Force from Co-Energy

Derivation

- Force from co-energy

Derivation

Derivation

Force from Co-Energy

- In summary

$$f_e = \frac{\partial W_c(\mathbf{i}, x)}{\partial x}$$

- This is most important and frequently used result in this chapter. Memorize it.
- Note the arguments must be as indicated.

Example 1

- Find force if

$$\lambda_1 = 2i_1 + \frac{1}{2+x}(i_1 + i_2)^{0.5}$$

$$\lambda_2 = 5i_2^{0.4} + \frac{1}{2+x}(i_1 + i_2)^{0.5}$$

- From our earlier work

$$W_c = i_1^2 + \frac{2}{3}\frac{1}{2+x}(i_1 + i_2)^{1.5} + \frac{5}{1.4}i_2^{1.4}$$

- Thus

Example 2

Example 2

Example 2

Example 2

Lecture 19

System Classifications

Conditions for Conservative Fields

- If describing the system in terms of flux linkage

$$\frac{\partial i_j}{\partial \lambda_k} = \frac{\partial i_k}{\partial \lambda_j}$$

- If describing the system in terms of current

$$\frac{\partial \lambda_j}{\partial i_k} = \frac{\partial \lambda_k}{\partial i_j}$$

- Implication of this

Example

Magnetically Linear Systems

- Definition

$$\boldsymbol{\lambda} = \mathbf{L}\mathbf{i}$$

- Field and Co-Energy

$$W_c = \frac{1}{2}\mathbf{i}^T \mathbf{L}\mathbf{i}$$

$$W_f = \frac{1}{2}\boldsymbol{\lambda}^T \mathbf{L}^{-1}\boldsymbol{\lambda}$$

Derivation

Derivation

Derivation

Derivation

Relationship of Field and Co-Energy for MLS

- Another observation for magnetically linear systems

- Thus

$$W_c = W_f$$

Lecture 20

Mechanics

Review of Translational Dynamics

Definition of Torque

Review of Rotational Dynamics

Review of Rotational Dynamics

Calculating Torque from Field and Co-Energy

- For a translational system

$$W_m = -\int_{x_0}^{x_f} f_e \, dx$$

- For a rotational system

$$W_m = -\int_{\theta_{rm,0}}^{\theta_{rm,f}} T_e \, d\theta_{rm}$$

- Thus to treat to rotational systems, we only need to …

Calculating Torque from Field and Co-Energy