Lecture Set 1:
Introduction to Magnetic Circuits

Lecture 1

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Spring 2021

Lecture Set 1 Goals

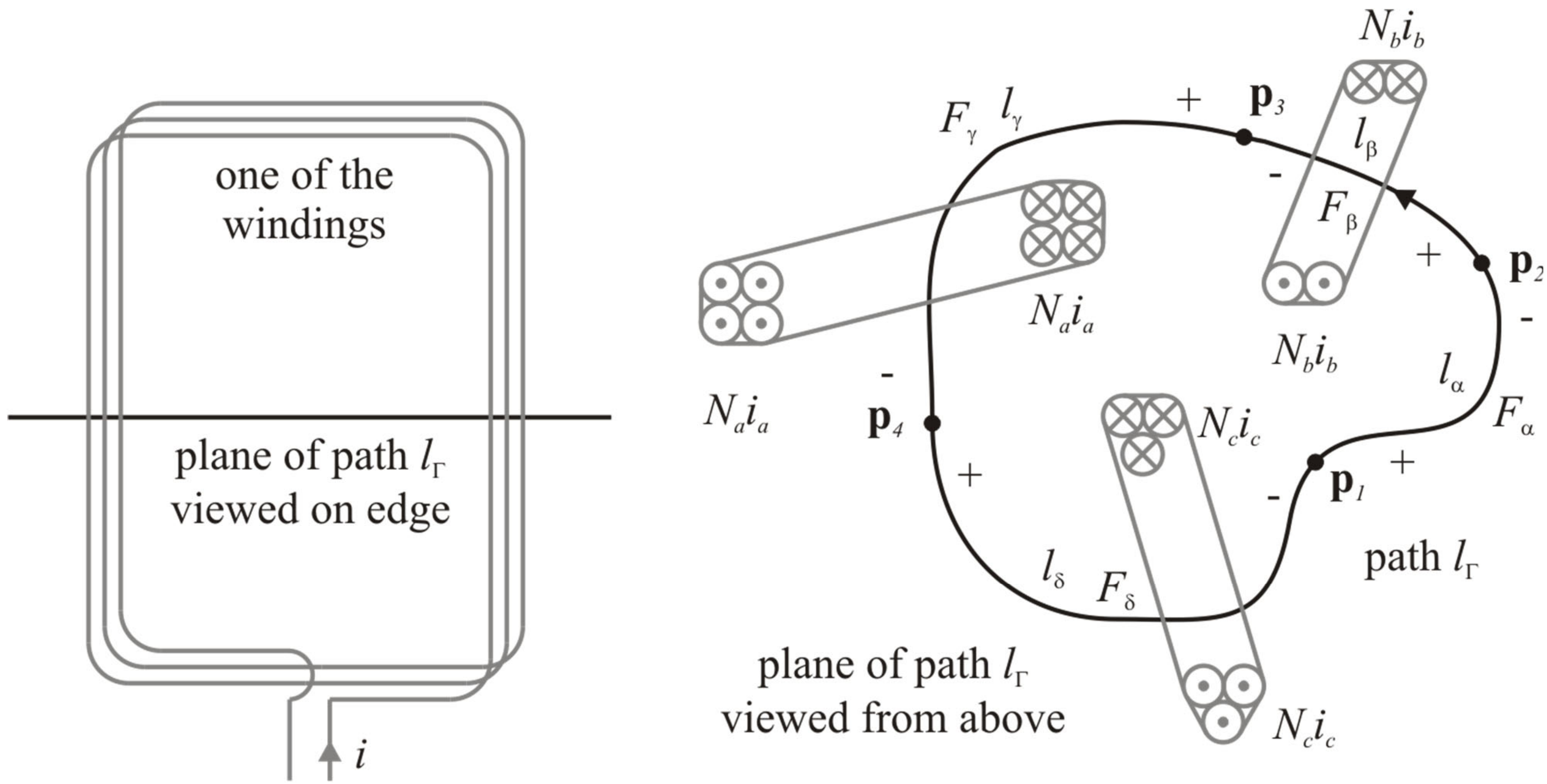
- Review physical laws pertaining to analysis of magnetic systems
- Introduce concept of magnetic circuit as means to obtain an electric circuit
- **Learn how to determine the flux-linkage versus current characteristic because it will be key to understanding electromechanical energy conversion**

Review of Symbols

- Flux Density in Tesla: B (T)
- Flux in Webers: Φ (Wb)
- Flux Linkage in Volt-seconds: λ (Vs,Wb-t)
- Current in Amperes: i (A)
- Field Intensity in Ampere/meters: H (A/m)
- Permeability in Henries/meter: μ (H/m)
- Magneto-Motive Force: F (A,A-t)
- Permeance in Henries: P (H)
- Reluctance in inverse Henries: R (1/H)
- Inductance in Henries: L (H)
- Current (fields) into page: $\otimes$
- Current (fields) out of page: $\odot$

Ampere’s Law, MMF, and Kirchoff’s MMF Law for Magnetic Circuits

Consider a Closed Path …



(a) winding configuration

(b) path of integral

Enclosed Current and MMF Sources

- Enclosed Current

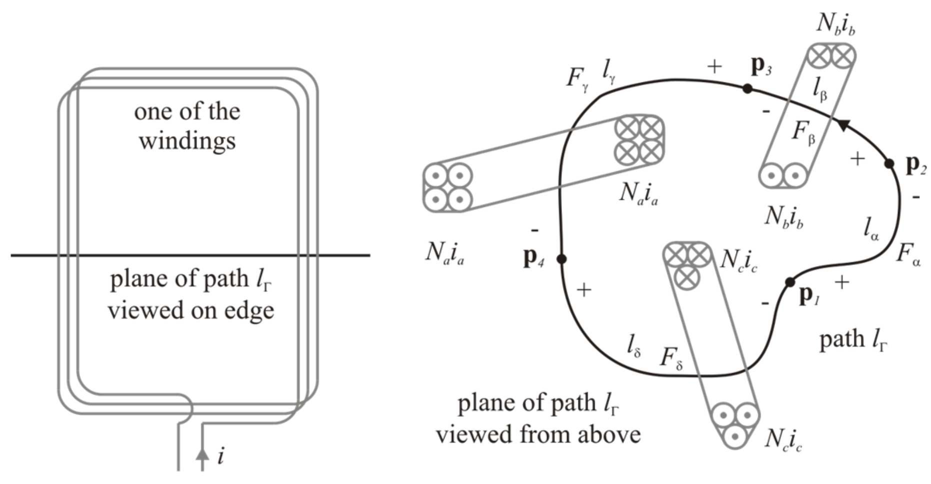
$$i_{enc,\Gamma} = -N_a i_a + N_b i_b - N_c i_c$$

- MMF Sources

$$F_a = -N_a i_a$$

$$F_b = N_b i_b$$

$$F_c = -N_c i_c$$



(a) winding configuration (b) path of integral

- Conclusion

$$i_{enc,\Gamma} = \sum_{s \in S_\Gamma} F_s \quad S_\Gamma = \{'a','b','c'\}$$

MMF Drops

- Going Around the Loop

$$\oint_{l_\Gamma} \mathbf{H} \cdot d\mathbf{l} = \int_{l_\alpha} \mathbf{H} \cdot d\mathbf{l} + \int_{l_\beta} \mathbf{H} \cdot d\mathbf{l} + \int_{l\gamma} \mathbf{H} \cdot d\mathbf{l} + \int_{l_\delta} \mathbf{H} \cdot d\mathbf{l}$$

- MMF Drop

$$F_y = \int_{l_y} \mathbf{H} \cdot d\mathbf{l}$$

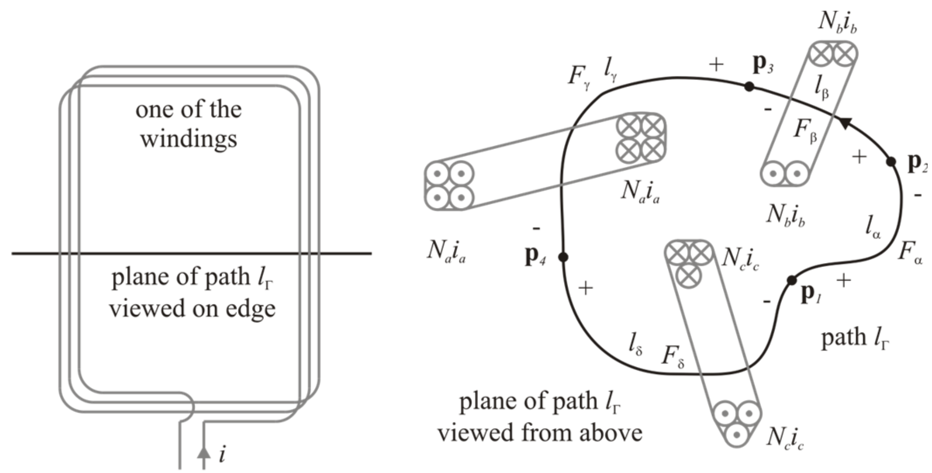
- Thus

$$\oint_{l_{\mathrm{T}}} \mathbf{H} \cdot d\mathbf{l} = F_\alpha + F_\beta + F_\gamma + F_\delta$$

- Or

$$\oint_{l_\Gamma} \mathbf{H} \cdot d\mathbf{l} = \sum_{d \in D_\Gamma} F_d$$

$$D_\Gamma = \{'\alpha','\beta','\gamma','\delta'\}$$



(a) winding configuration (b) path of integral

Ampere's Law

- Ampere's Law

$$\oint_{l_\Gamma} \mathbf{H} \cdot d\mathbf{l} = i_{enc,\Gamma}$$

- Recall

$$i_{enc,\Gamma} = \sum_{s \in S_\Gamma} F_s \qquad \oint_{l_\Gamma} \mathbf{H} \cdot d\mathbf{l} = \sum_{d \in D_\Gamma} F_d$$

- Thus

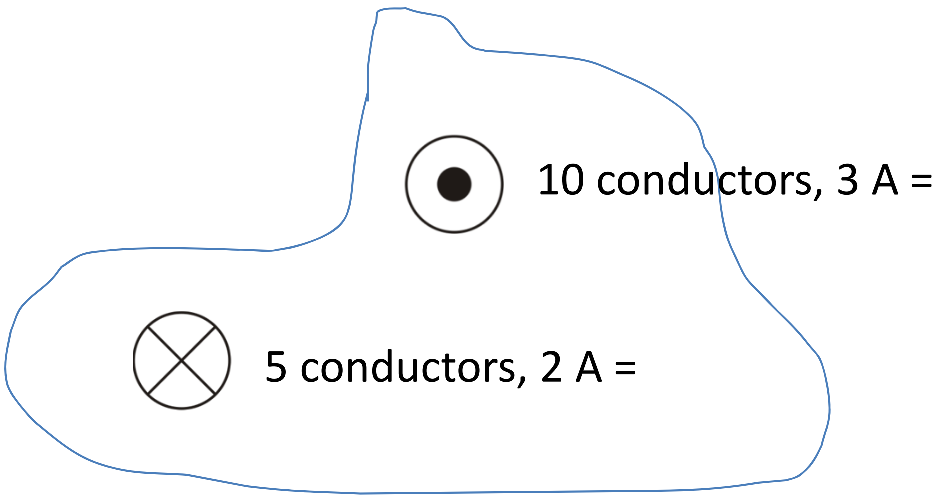
$$\sum_{s \in S_\Gamma} F_s = \sum_{d \in D_\Gamma} F_d$$

Kirchoff's MMF Law

Kirchoff's MagnetoMotive Force Law:

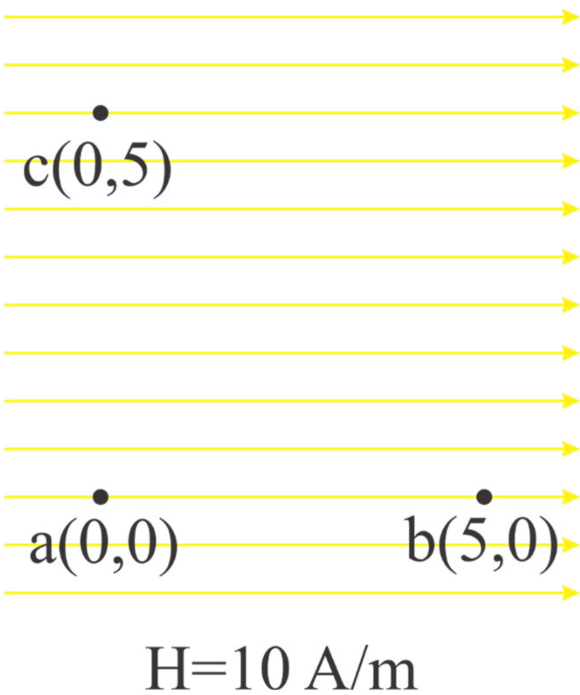
The Sum of the MMF Drops Around a Closed Loop is Equal to the Sum of the MMF Sources for That Loop

Example: MMF Sources



Example: MMF Drops

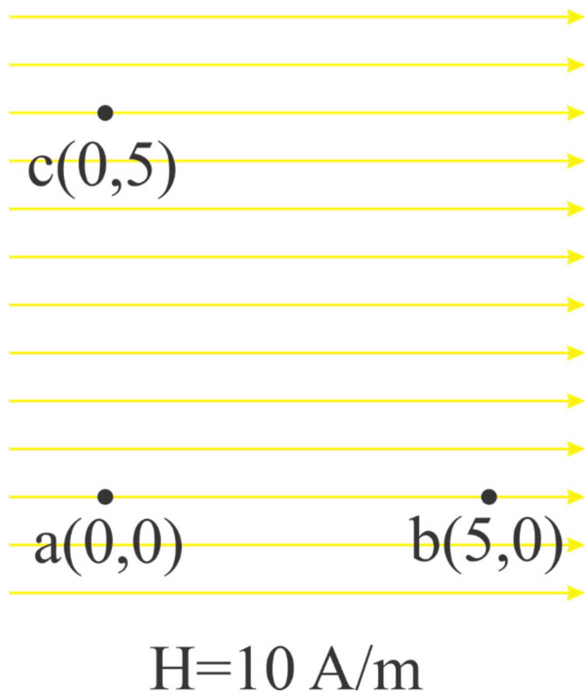
- A quick example on MMF drop:



$F_{ab}=$
$F_{ac}=$

Another Example: MMF Drops

- How about F_{cb} ?

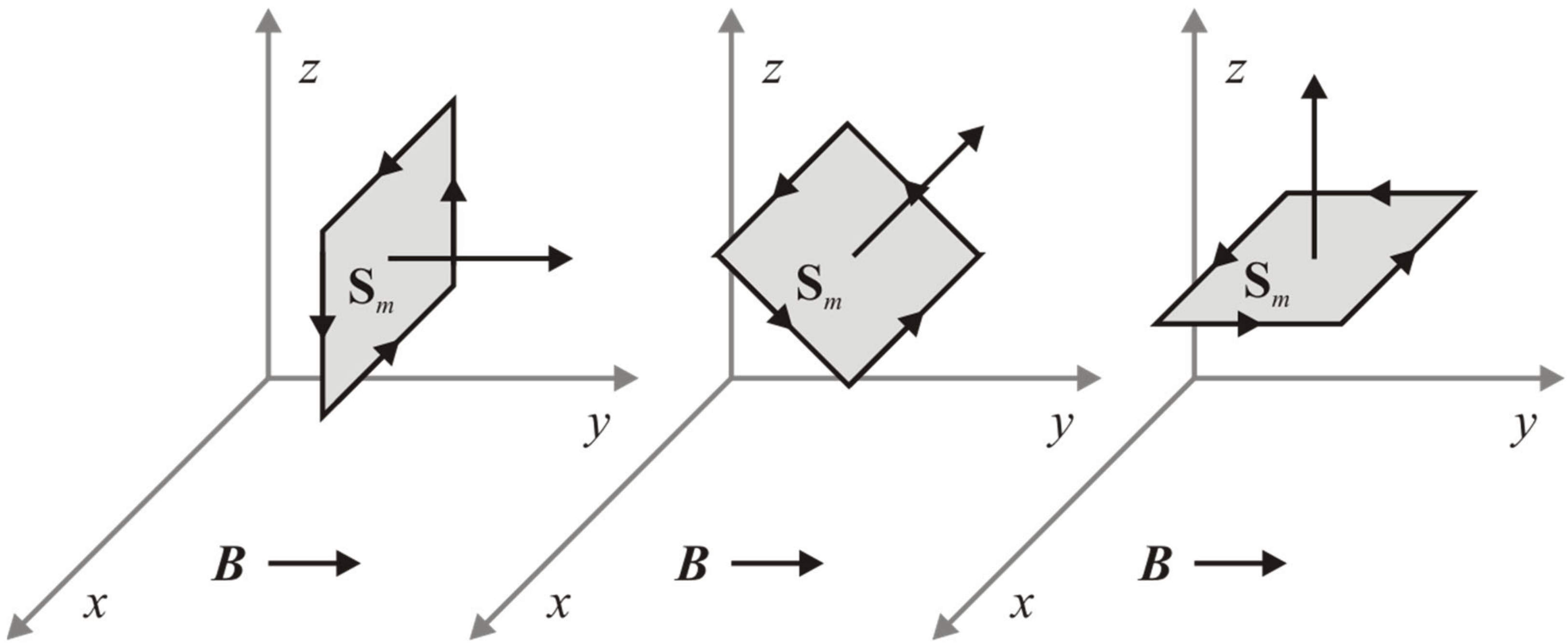


Lecture 2

Magnetic Flux, Gauss’s Law, and Kirchoff’s Flux Law for Magnetic Circuits

Magnetic Flux

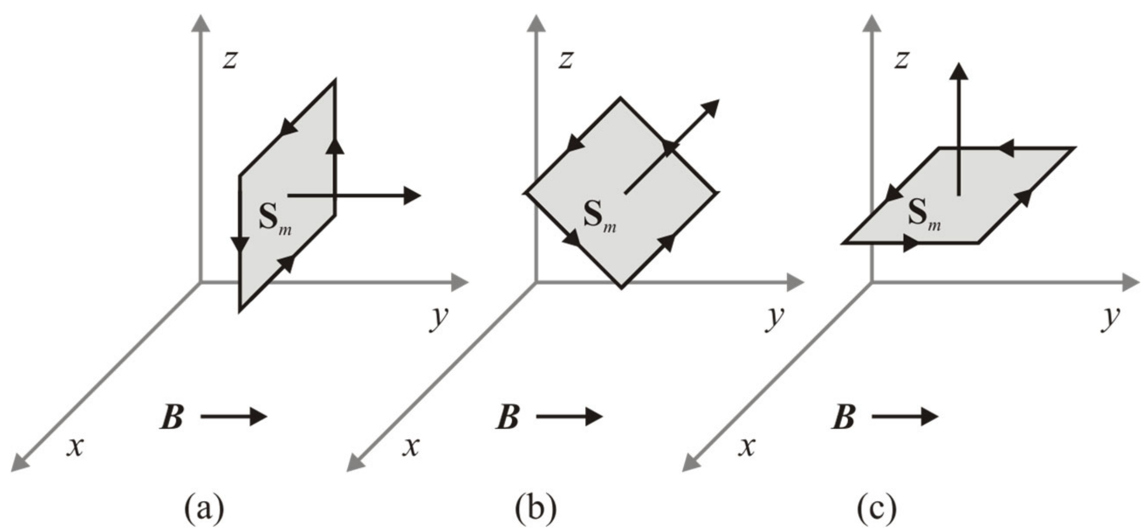
$$\Phi_m = \int_{\mathbf{S}_m} \mathbf{B} \cdot d\mathbf{S}$$



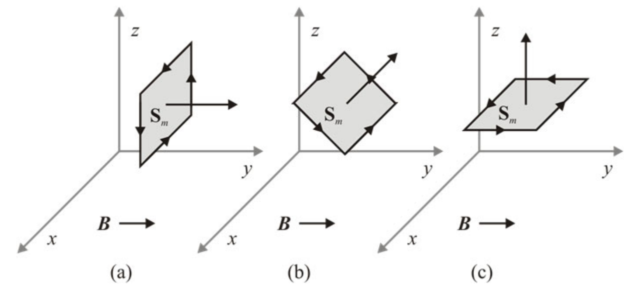
(a) (b) (c)

Example: Magnetic Flux

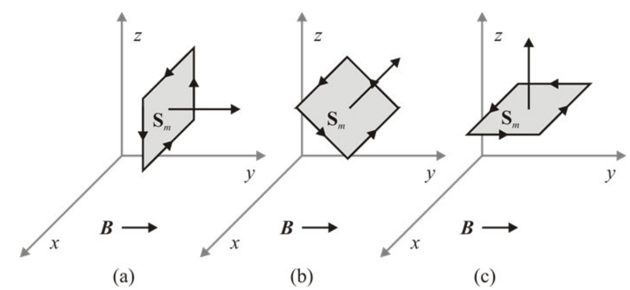
- Suppose there exist a uniform flux density field of $\mathbf{B}=1.5\mathbf{a}_\mathbf{y}$ T where $\mathbf{a}_\mathbf{y}$ is a unit vector in the direction of the *y*-axis. We wish to calculate the flux through open surface $\mathbf{S}_\mathbf{m}$ with an area of 4 m^2, whose periphery is a coil of wire wound in the indicated direction.



Example: Magnetic Flux



Example: Magnetic Flux



Kirchoff's Flux Law

- Gauss's Law

$$\oint_{\mathbf{S}_\Gamma} \mathbf{B} \cdot d\mathbf{S} = 0$$

- Thus

$$\sum_{o \in \mathbf{O}_\Gamma} \int_{\mathbf{S}_o} \mathbf{B} \cdot d\mathbf{S} \qquad \mathbf{O}_\Gamma = \{'\alpha','\beta','\gamma',\cdots\}$$

- Or

$$\sum_{o \in \mathbf{O}_\Gamma} \Phi_o = 0$$

Kirchoff’s Flux Law

- Kirchoff’s Flux Law:

The Sum of the Flux Leaving A Node of a Magnetic Circuit is Zero

Kirchoff’s Flux Law

- A quick example on Kirchoff’s Flux Law

Lecture 3

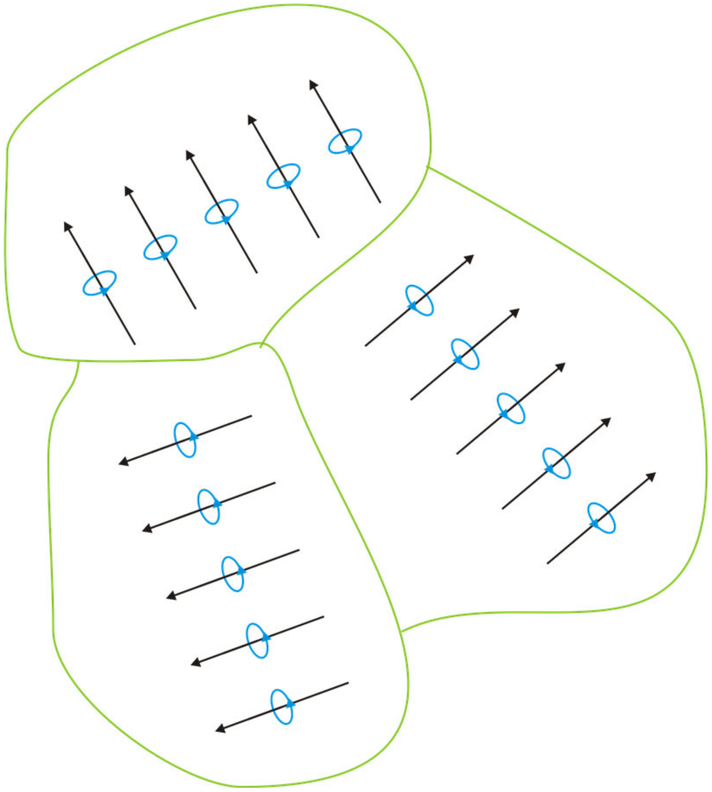
Magnetically Conductive Materials

Types of Magnetic Materials

- Interaction of Magnetic Materials with Magnetic Fields
  - Magnetic Moment of Electron Spin
  - Magnetic Moment of Electron in Shell
- These Effects Lead to Six Material Types
  - Diamagnetic
  - Paramagnetic
  - **Ferromagnetic**
  - Antiferromagnetic
  - **Ferrimagnetic**
  - Superparamagnetic

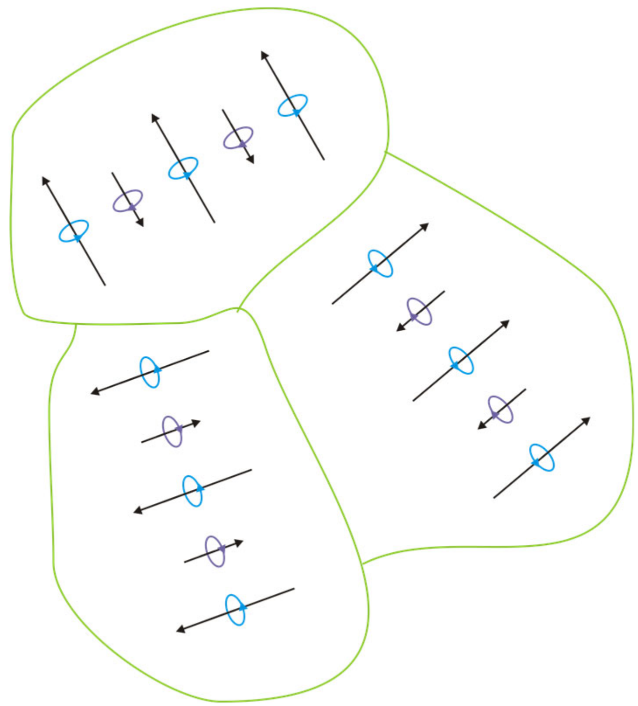
Ferromagnetic Materials

- Iron
- Nickel
- Cobalt

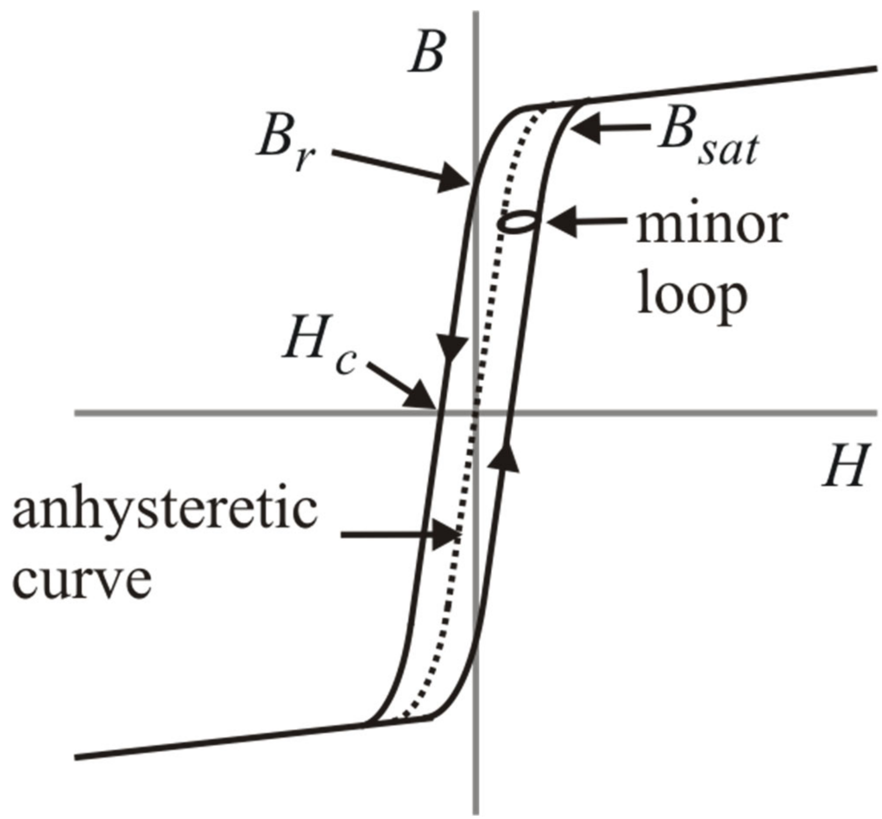


Ferrimagnetic Materials

- Iron-oxide ferrite (Fe_3O_4)
- Nickel-zinc ferrite ($Ni_{1/2}Zn_{12}Fe_2O_4$)
- Nickel ferrite ($NiFe_2O_4$)

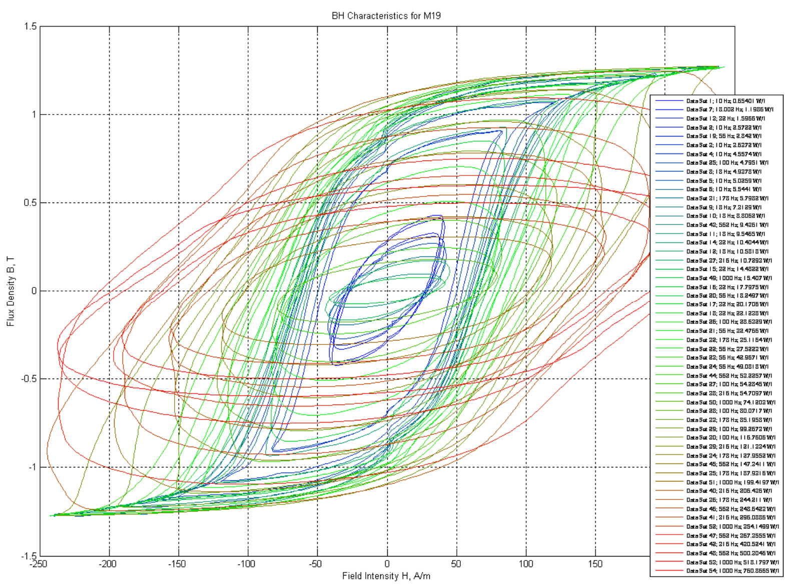


Behavior of Ferro/Ferrimagnetic Materials

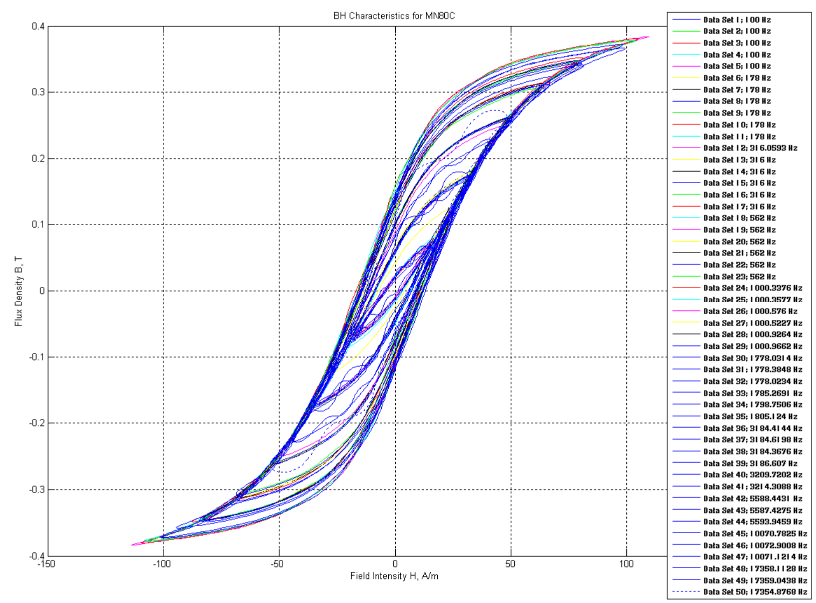


Behavior of Ferro/Ferrimagnetic Materials

M19



MN80C



Modeling Magnetic Materials

- Free Space

$$B = \mu_0 H$$

- Magnetic Materials

$$B = \mu_0 (H + M) \qquad M = \chi H$$

$$B = \mu_0 H + M \qquad M = \mu_0 \chi H$$

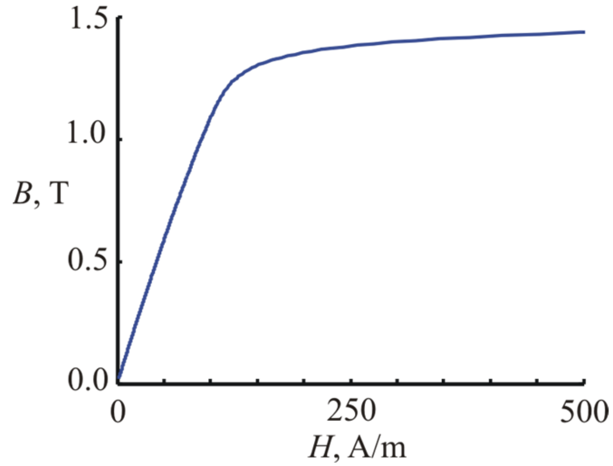
- Either Way

$$B = \mu_0 (1 + \chi) H \qquad B = \mu_0 \mu_r H \qquad \mu_r = 1 + \chi$$

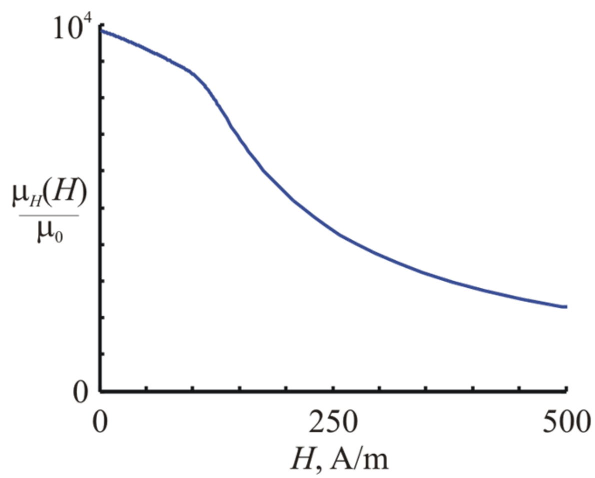
- Or

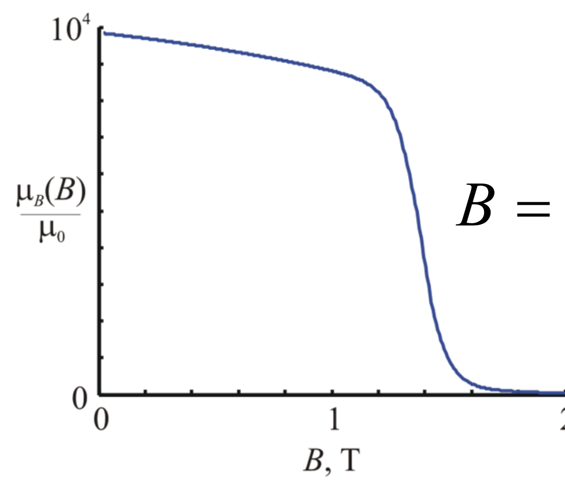
$$B = \mu H \qquad \mu = \mu_0 \mu_r$$

Modeling Magnetic Materials



$$B = \mu_H(H)H$$





$$B = \mu_B(B)H$$

Lecture 4

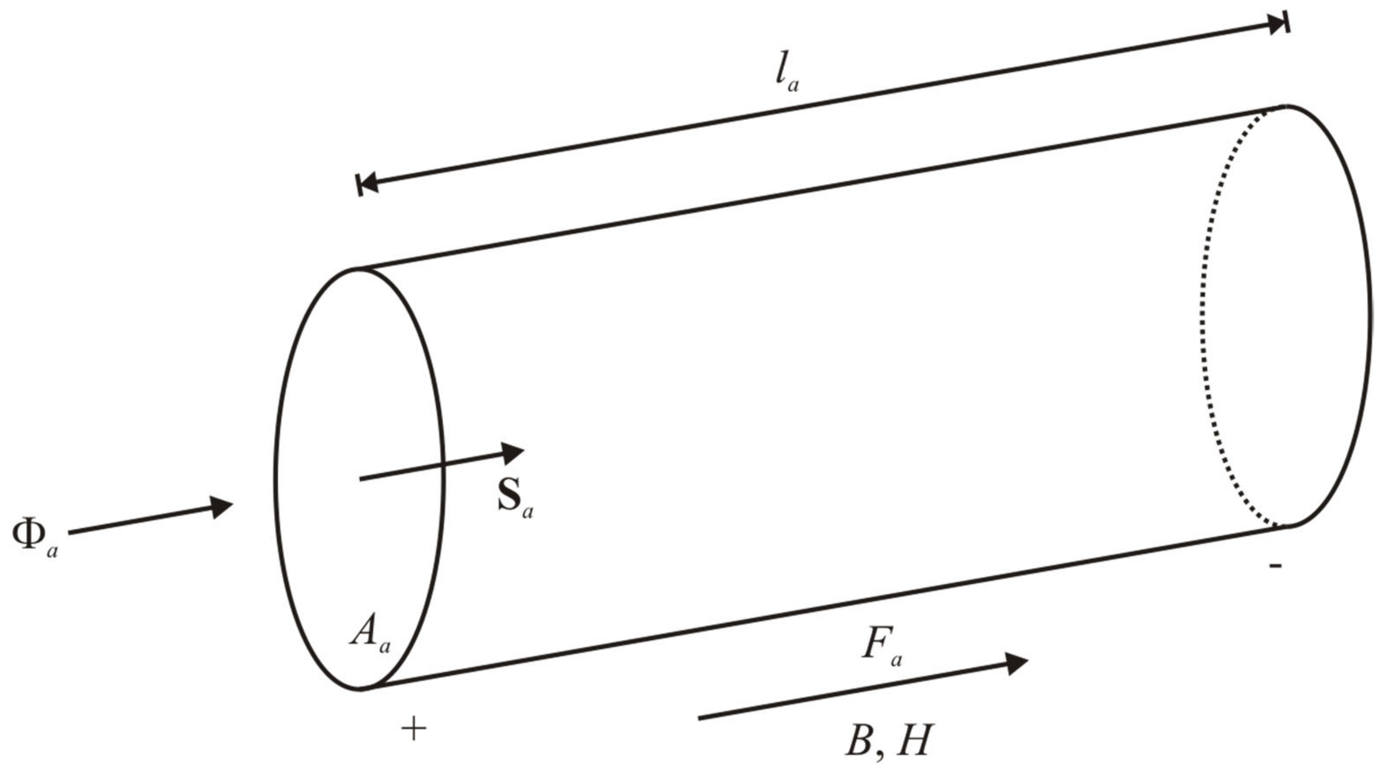
Ohm’s Law for Magnetic Circuits

Ohm's Law

- On Ohm's ~~Law~~ Property for Electric Circuits

- On Ohm's ~~Law~~ Property for Magnetic Circuits

Back to Ohm's Law

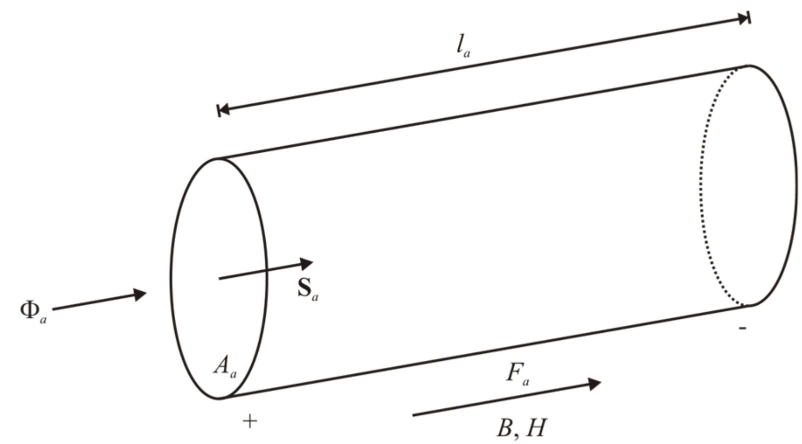


Relationship Between MMF Drop and Flux

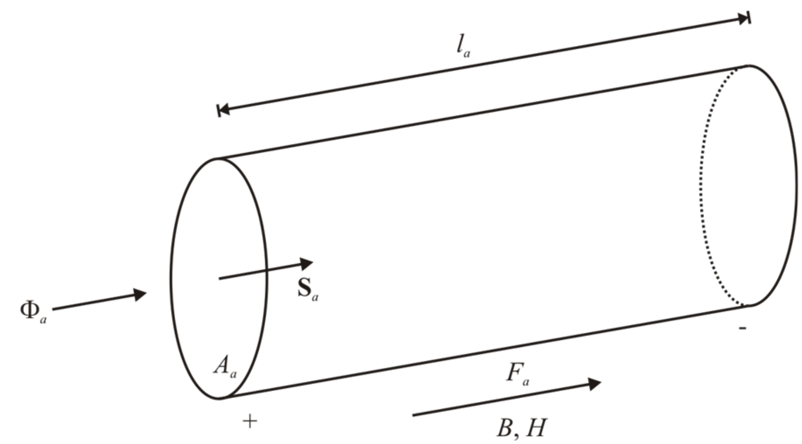
$$F_a = R_a(\xi)\Phi_a \qquad R_a(\xi) = \begin{cases} \dfrac{l_a}{A_a \mu_H (F_a / l_a)} & \text{Form 1}: \xi = F_a \\ \dfrac{l_a}{A_a \mu_B (\Phi_a / A_a)} & \text{Form 2}: \xi = \Phi_a \end{cases}$$

$$\Phi_a = P_a(\xi)F_a \qquad P_a(\xi) = \begin{cases} \dfrac{A_a \mu_H (F_a / l_a)}{l_a} & \text{Form 1}: \xi = F_a \\ \dfrac{A_a \mu_B (\Phi_a / A_a)}{l_a} & \text{Form 2}: \xi = \Phi_a \end{cases}$$

Derivation



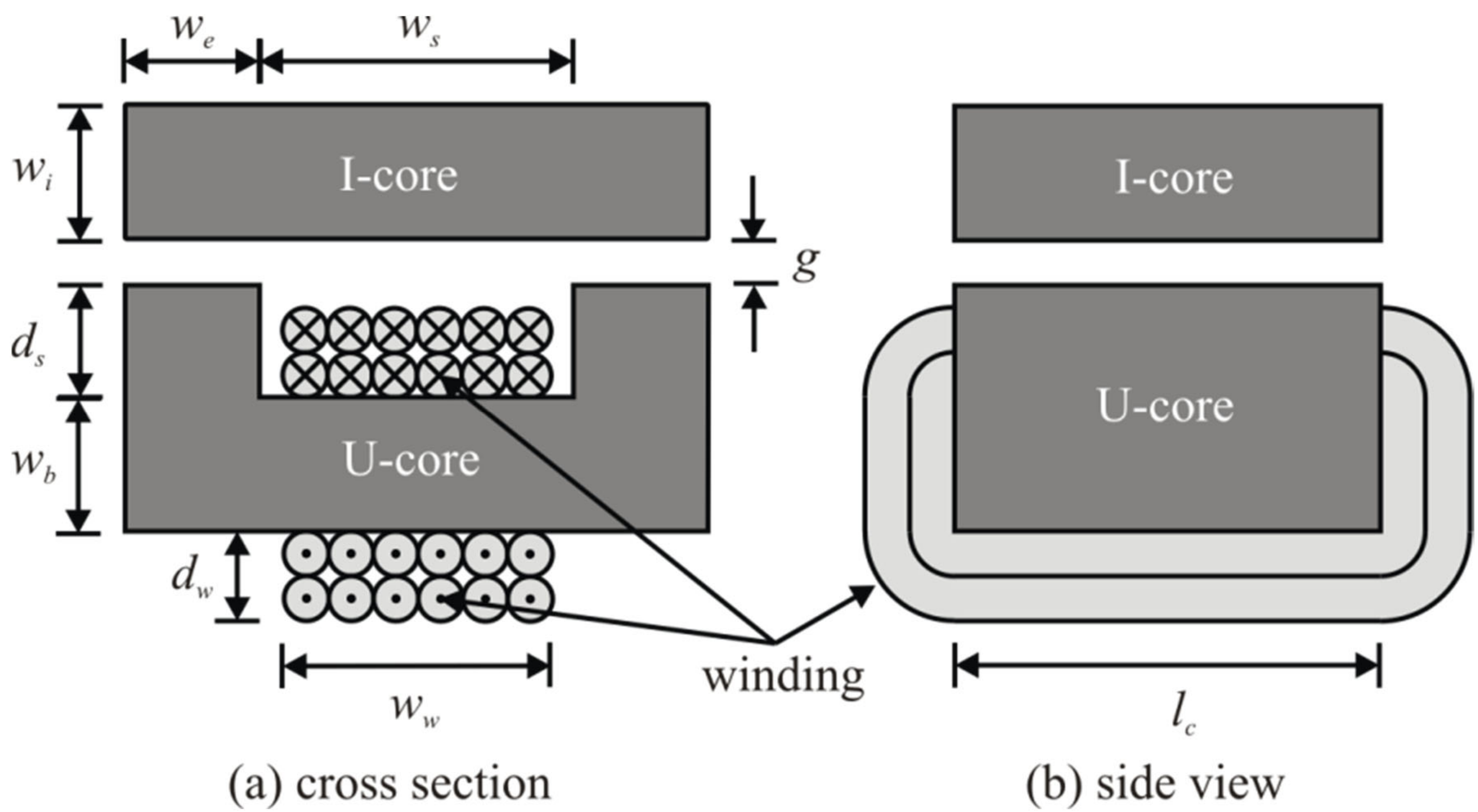
Derivation



Lecture 5

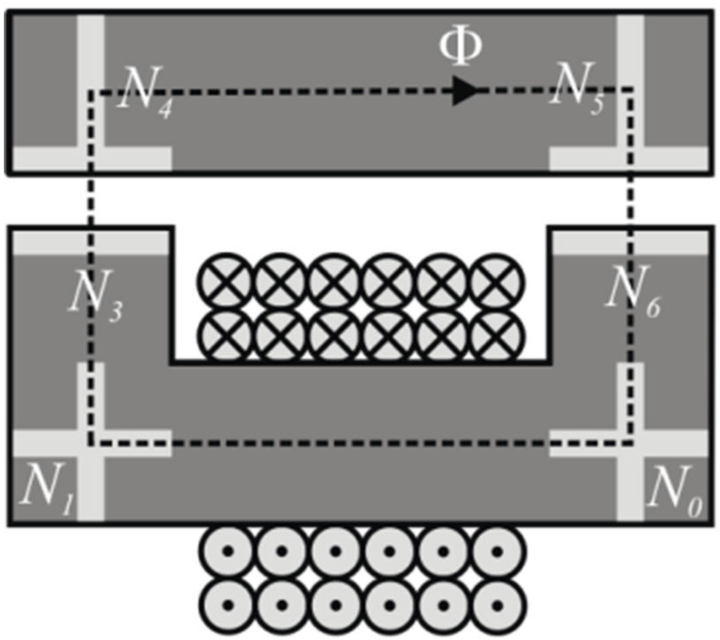
Construction of the Magnetic Equivalent Circuit

Consider a UI Core Inductor

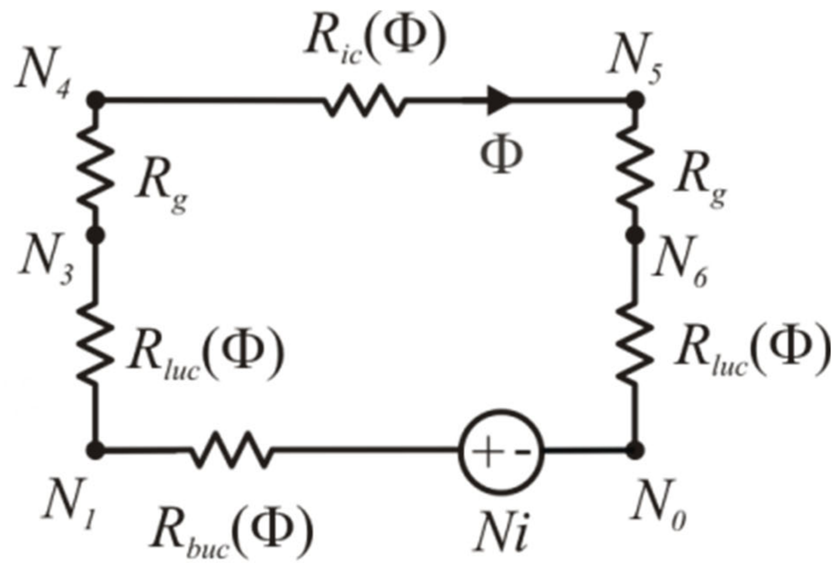


Inductor Applications

UI Core Inductor MEC

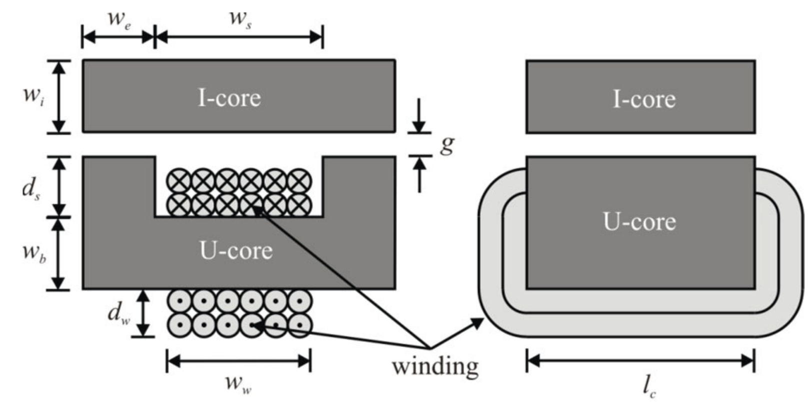


(a) cross section showing nodes

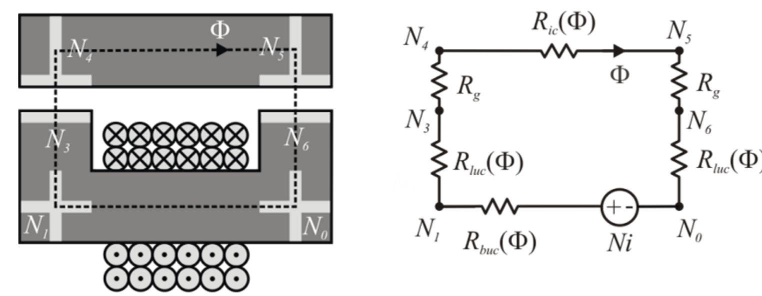


(b) magnetic equivalent circuit

Reluctances

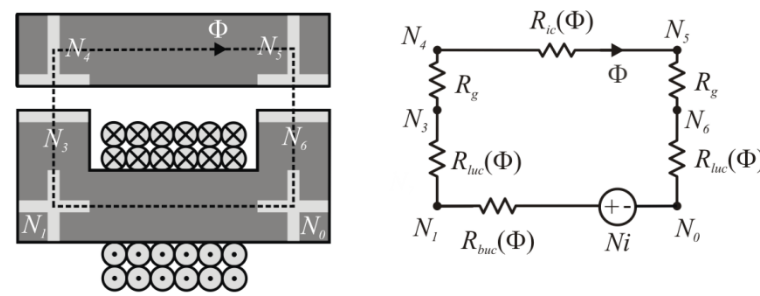


(a) cross section (b) side view

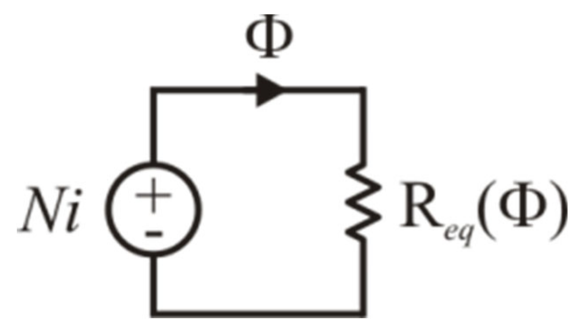


(a) cross section showing nodes (b) magnetic equivalent circuit

Reduced Equivalent Circuit



(a) cross section showing nodes (b) magnetic equivalent circuit



Example

- Let us consider a UI core inductor with the following parameters: w_i = 25.1 mm, $w_b = w_e$ = 25.3 mm, w_s = 51.2 mm, d_s = 31.7 mm, l_c = 101.2 mm, g =1.00 mm, w_w = 38.1 mm, and d_w = 31.7 mm. The winding is composed of 35 turns. Suppose the magnetic core material has a constant relative permeability of 7700, and that a current of 25.0 A is flowing. Compute the flux through the magnetic circuit and the flux density in the I-core.

Example

Example

Solving the Nonlinear Case

$$R_{eq}(\Phi) = R_{ic}(\Phi) + R_{buc}(\Phi) + 2R_{luc}(\Phi) + 2R_g$$

$$\Phi = \frac{Ni}{R_{eq}(\Phi)}$$

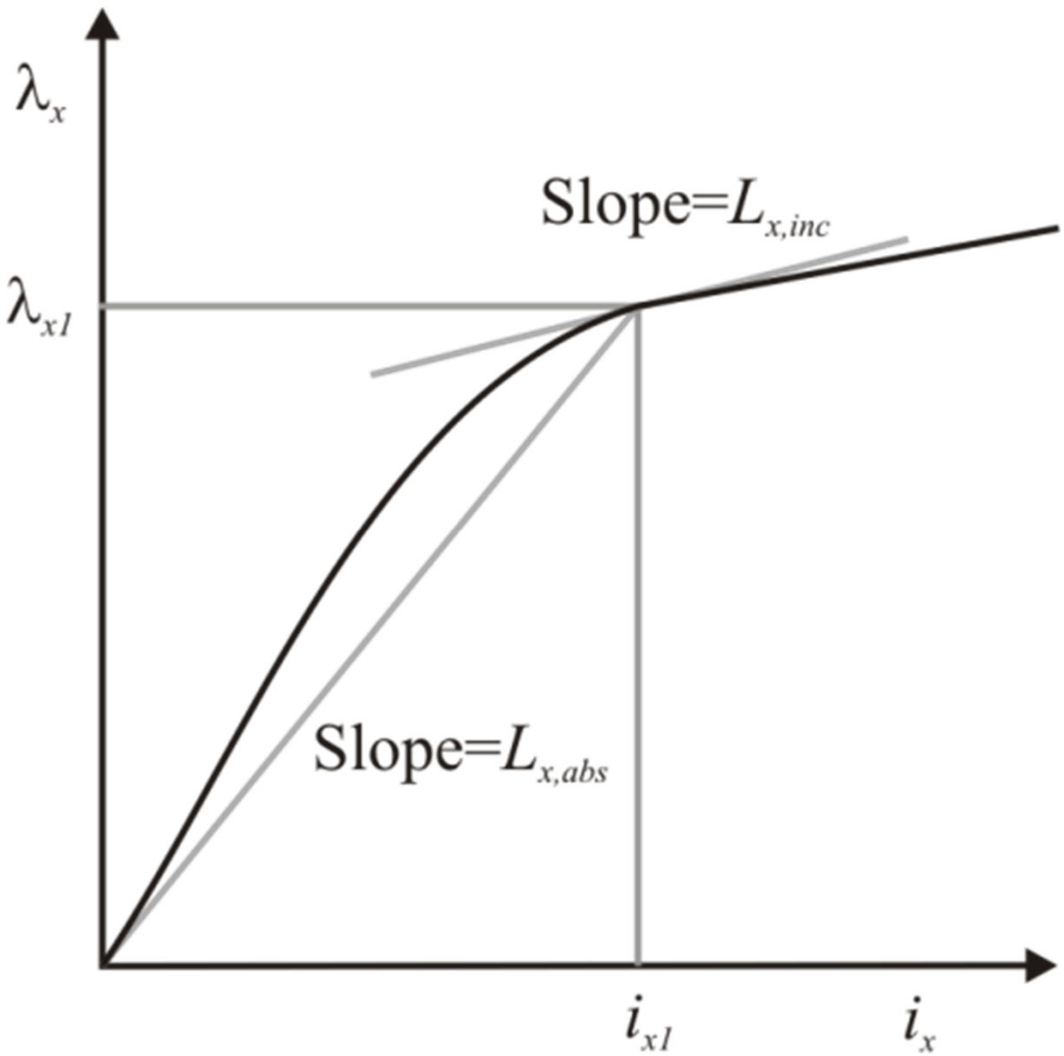
Lecture 6

Flux Linkage and Inductance

Flux Linkage

$$\lambda_x = N_x \Phi_x$$

Inductance



$$L_{x,abs} = \left.\frac{\lambda_x}{i_x}\right|_{i_{x1},\lambda_{x1}}$$

$$L_{inc} = \left.\frac{\partial \lambda_x}{\partial i_x}\right|_{i_{x1}}$$

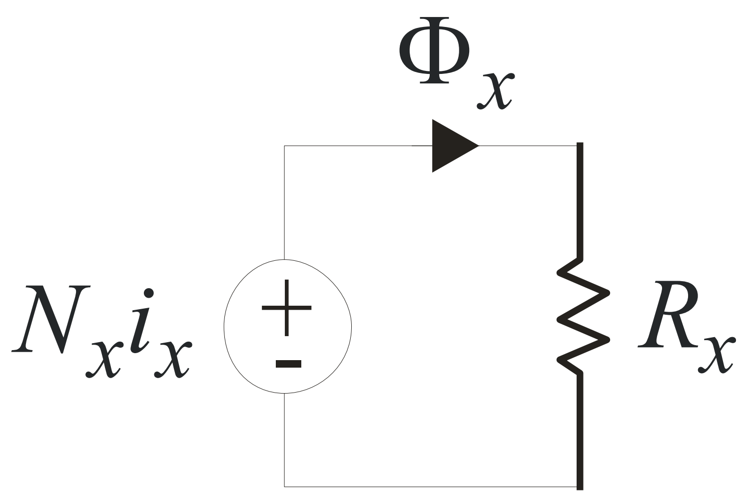
Example

- Suppose

$$\lambda = 0.05(1 - e^{-0.2i}) + 0.03i$$

- Find the absolute and incremental inductance at 5 A

Computing Inductance



Mutual Inductance

Lecture 7

Nonlinear Flux-Linkage Versus Current

Approach

Matlab Code

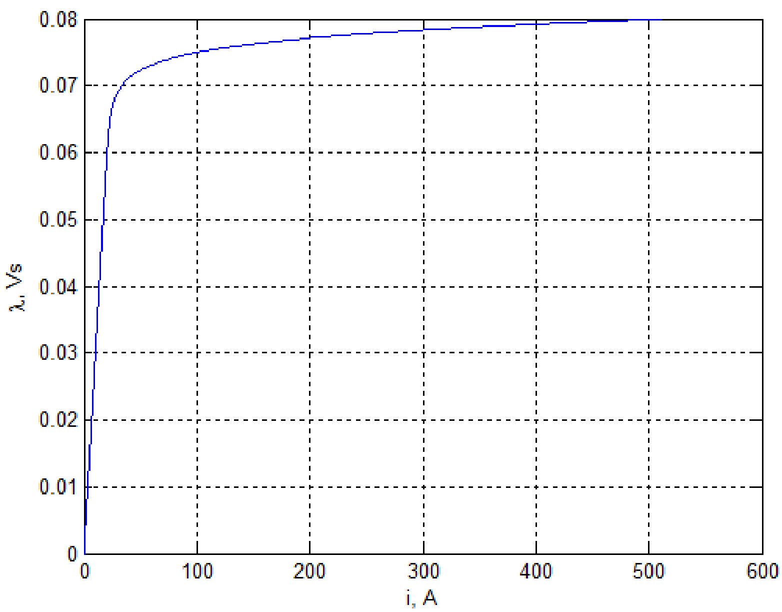
```
% Computation of lambda-i curve
% for UI core inductor
% using a simple but nonlinear MEC
%
% S.D. Sudhoff for ECE321
% January 13, 2012

% clear work space
clear all
close all

% assign parameters
cm = 1e-2;              % a centimeter
mm = 1e-3;              % a millimiter
w=1*cm;                 % core width (m)
ws=5*cm;                % slot width (m)
ds=2*cm;                % slot depth (m)
d=5*cm;                 % depth (m)
g=1*mm;                 % air gap (m)
N=100;                  % number of turns
mu0=4e-7*pi;            % perm of free space (H/m)
Bsat=1.3;               % sat flux density (T)
mur=1500;               % init rel permeability
Am=w*d;                 % mag cross section (m^2)

% start with flux linkage
lambda=linspace(0,0.08,1000);

% find flux and B and mu
phi=lambda/N;
B=phi/(w*d);
mu=mu0*(mur-1)./(exp(10*(B-Bsat)).^2+1)+mu0;
```

Matlab Code

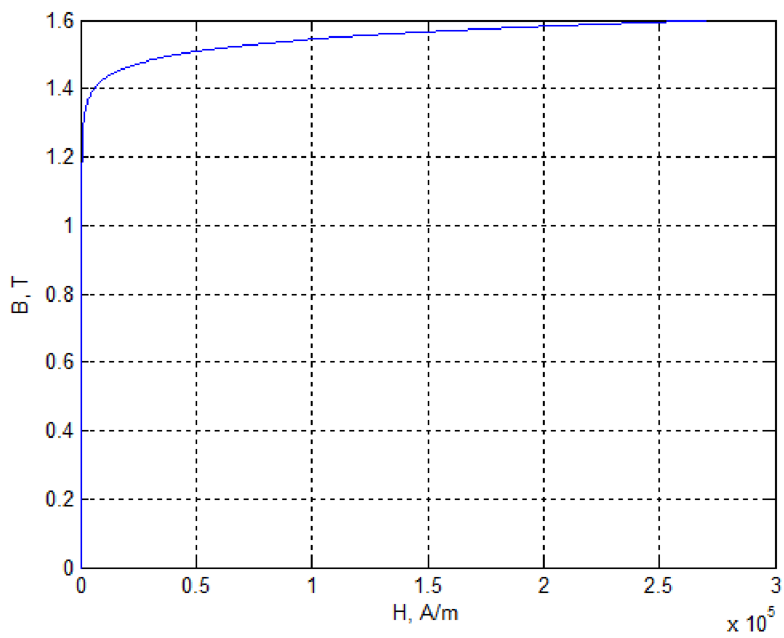
```
% Assign reluctances;
Ric=(ws+w)./(Am*mu);
Rbuc=(ws+w)./(Am*mu);
Rg=g/(Am*mu0);
Rluc=(ds+w/2)./(Am*mu);

% compute effective reluctances
Reff=Ric+Rbuc+2*Rluc+2*Rg;

% compute MMF and current
F=Reff.*phi;
i=F/N;
```

```
% find lambda-i characteristic
figure(1)
plot(i,lambda);
xlabel('i, A');
ylabel('\lambda, Vs');
grid on;

% show B-H characteristic
H=B./mu;
figure(2)
plot(H,B)
xlabel('H, A/m');
ylabel('B, T');
grid on;
```

Results



λ, Vs
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
100
200
300
400
500
600
i, A

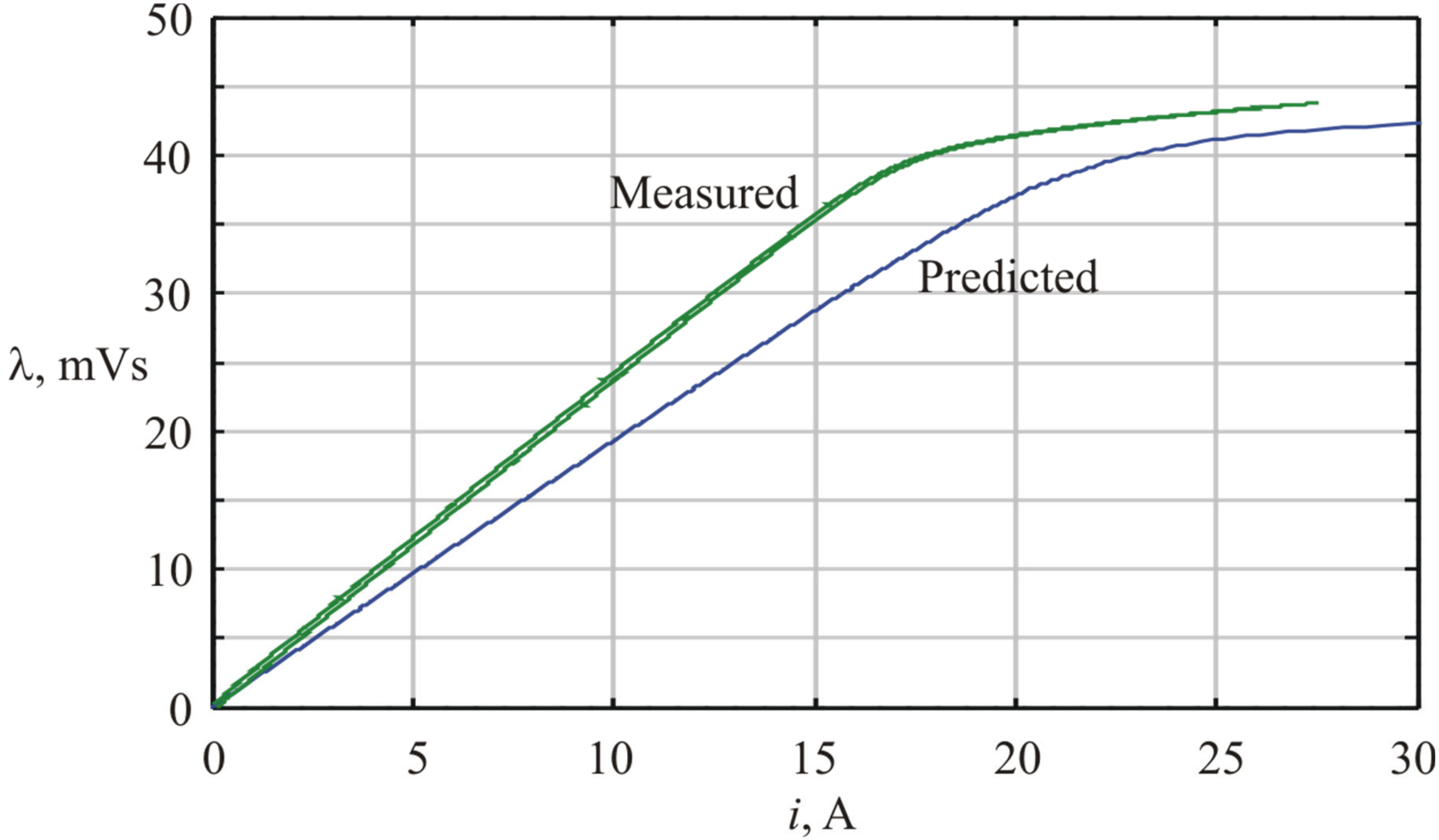


B, T
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
H, A/m
x 10^5

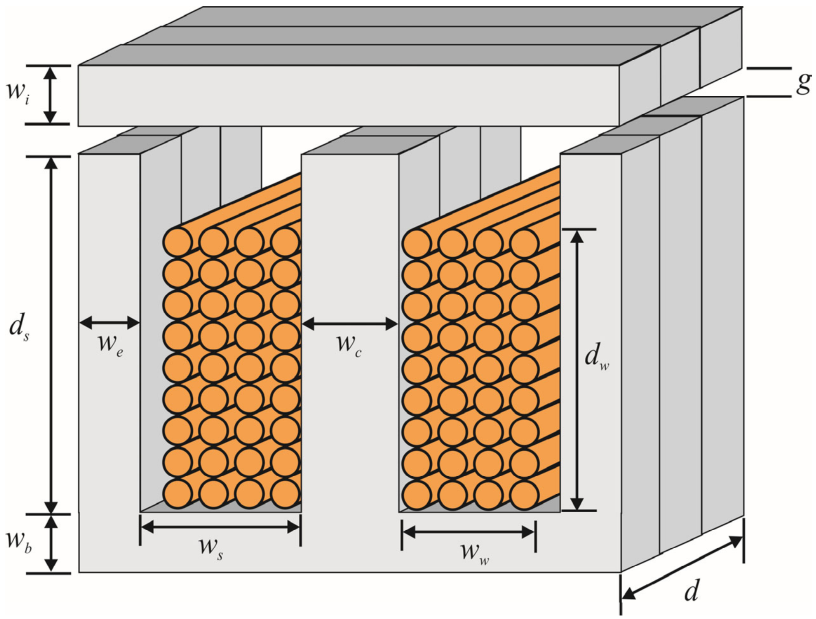
Lecture 8

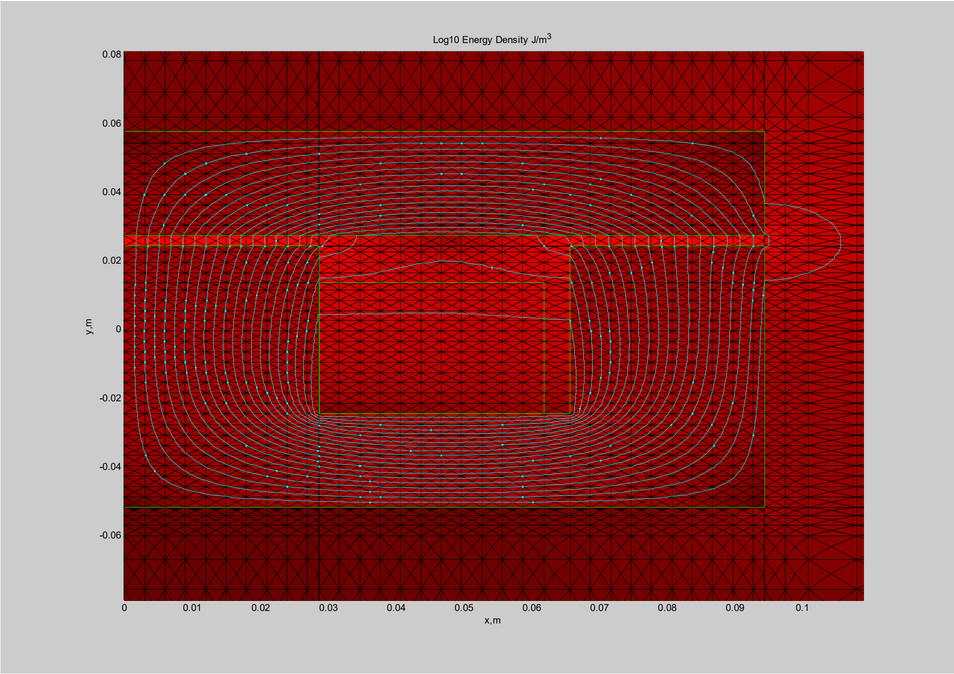
Accuracy of the MEC

Hardware Example – Our MEC

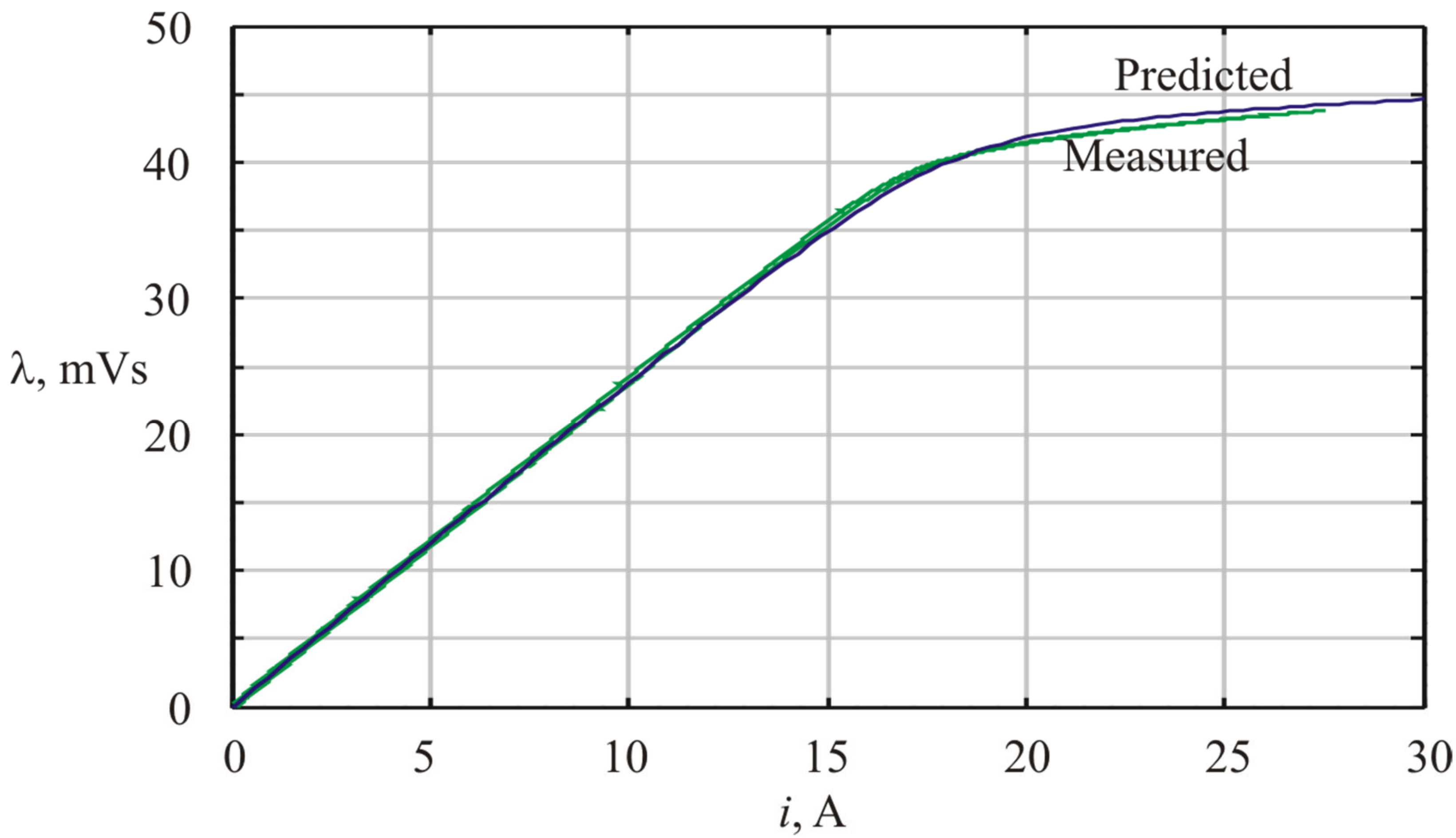


Non-Idealities: Fringing and Leakage Flux





Our MEC – With Leakage and Fringing



Lecture 9

Mapping Magnetic Circuits to Electrical Circuits

Faraday's Law

- Faraday's Law, voltage equation for winding

$$v_x = r_x i_x + \frac{d\lambda_x}{dt}$$

- If inductance is constant

$$\lambda_x = L_x i_x$$

- Combining above yields

$$v_x = r_x i_x + L_x \frac{di_x}{dt}$$

Energy in a Magnetically Linear Inductor

- We can show

$$E_x = \frac{1}{2} L_x i_x^2$$

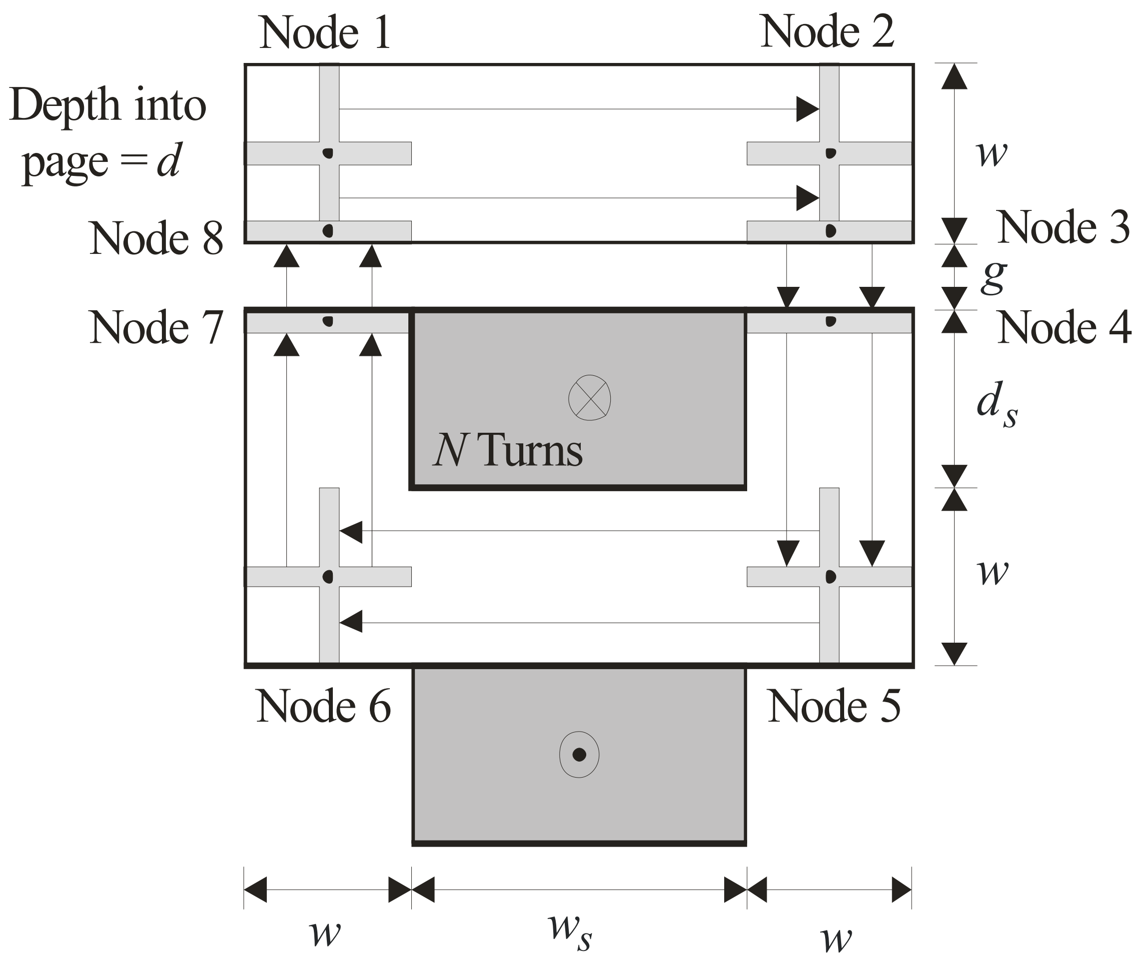
- Proof

Faraday’s Law with Varying Inductance

Lecture 10

Case Study: UI Core Inductor – Variables and Constraints

Architecture



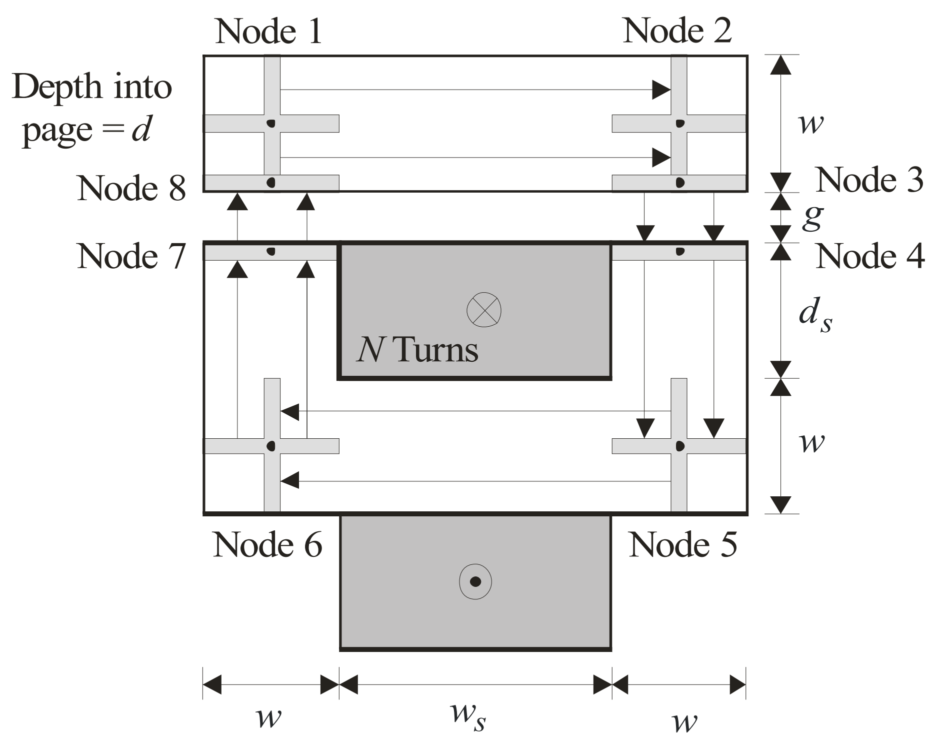
Design Requirements

- Current Level: 40 A
- Inductance: 5 mH
- Packing Factor: 0.7
- Slot Shape η_{dw}: 1
- Maximum Resistance: 0.1Ω

Information

- Cost of Ferrite: 230 k$/$m^3$
- Density of Ferrite: 4740 Kg/m^3
- Saturation Flux Density: 0.5 T
- Cost of Copper: 556 $/$m^3$
- Density of Copper: 8960 Kg/m^3
- Resistivity of Copper: 1.7e-8 Ωm

Step 1: Design Variables



Step 2: Design Constraints

- Constraint 1: Flux Density

Step 2: Design Constraints

- Constraint 2: Inductance

Step 2: Design Constraints

- Constraint 3: Slot Shape

Step 2: Design Constraints

- Constraint 4: Packing Factor

Step 2: Design Constraints

- Constraint 5: Resistance

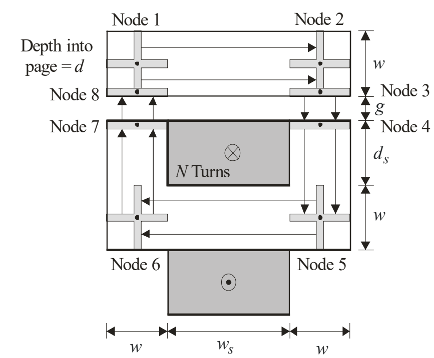
Step 2: Design Constraints

Lecture 11

Case Study: UI Core Inductor – Metrics and Results

Step 3: Design Metrics

- Magnetic Material Volume

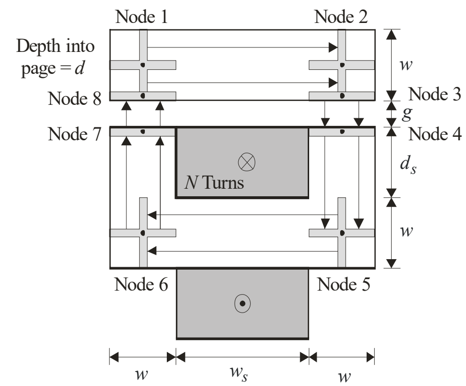


Step 3: Design Metrics

- Wire Volume

Step 3: Design Metrics

- Circumscribing Box



Step 3: Design Metrics

- Material Cost

- Mass

Lecture 12

Case Study: UI Core Inductor – Approach

Approach (1/6)

Approach (2/6)

Approach (3/6)

Approach (4/6)

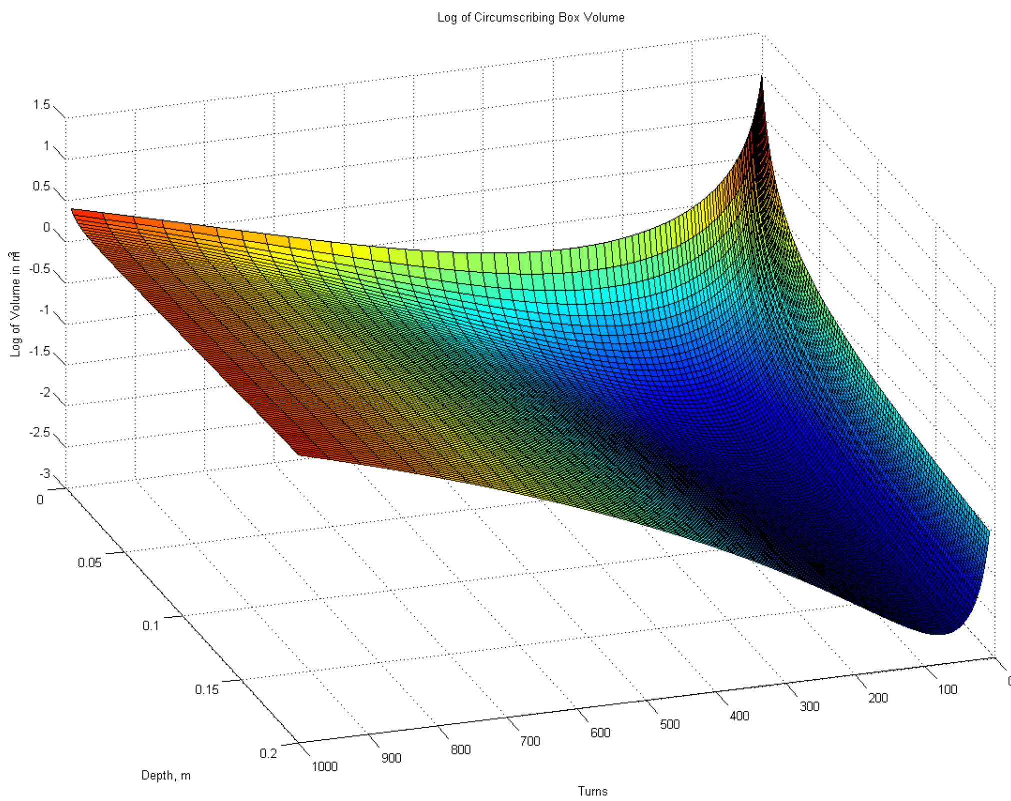
Approach (5/6)

Approach (6/6)

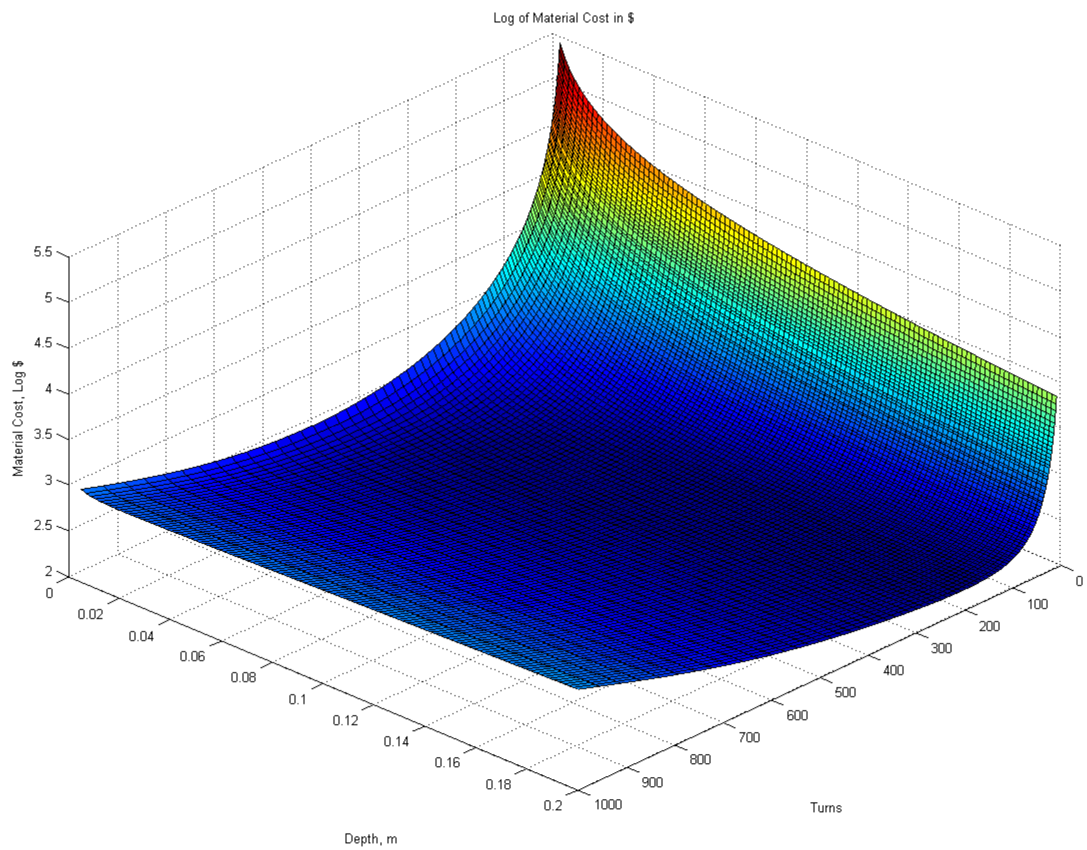
Lecture 13

Case Study: UI Core Inductor – Results and Reflections

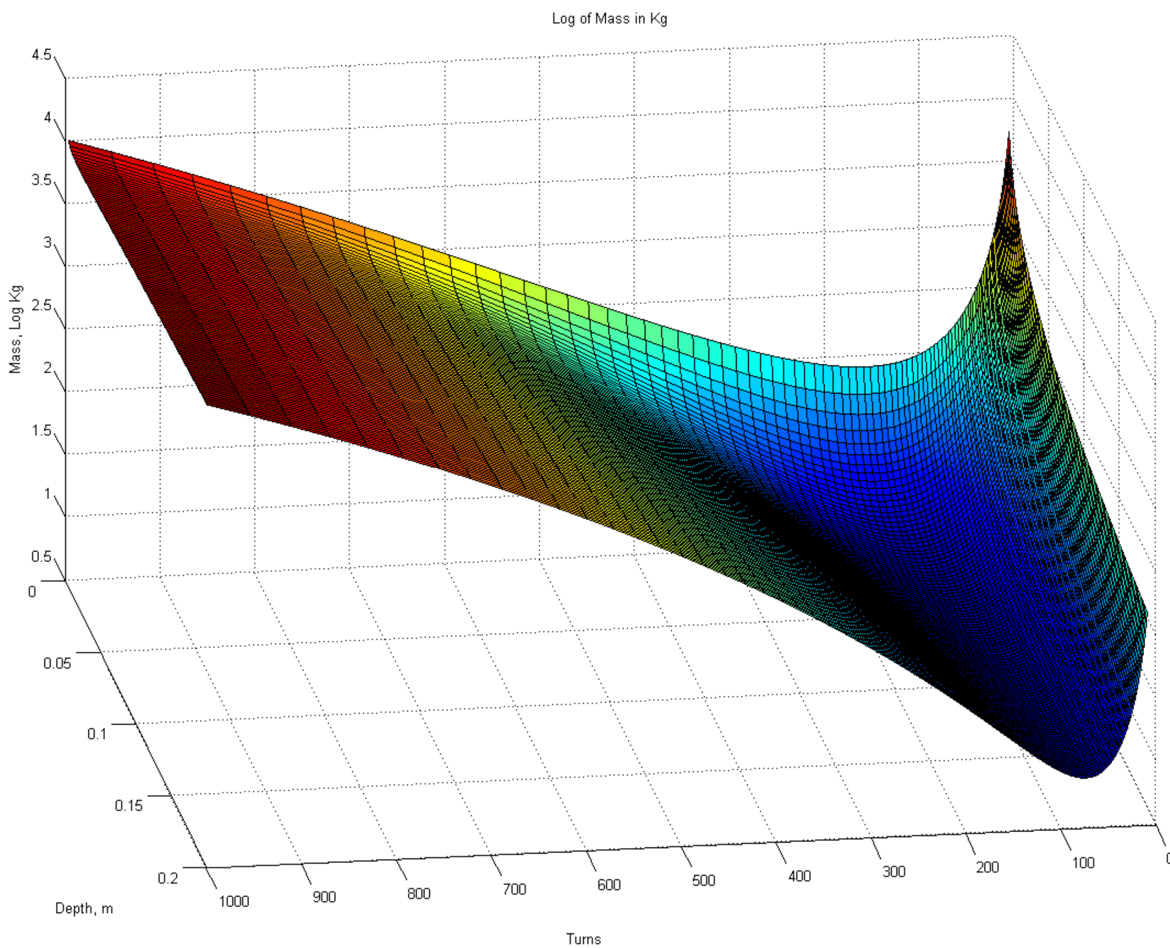
Circumscribing Box Volume



Material Cost



Mass


Log of Mass in Kg
Mass, Log Kg
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0.05
0.1
0.15
0.2
Depth, m
1000
900
800
700
600
500
400
300
200
100
0
Turns

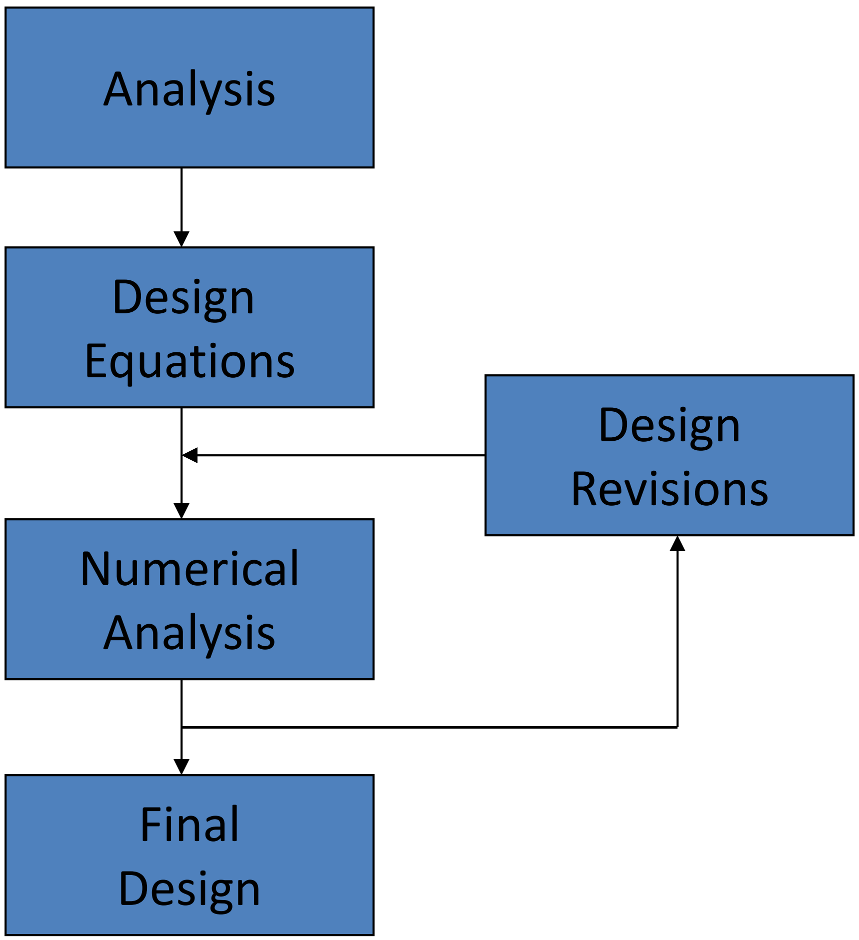
Least Cost Design

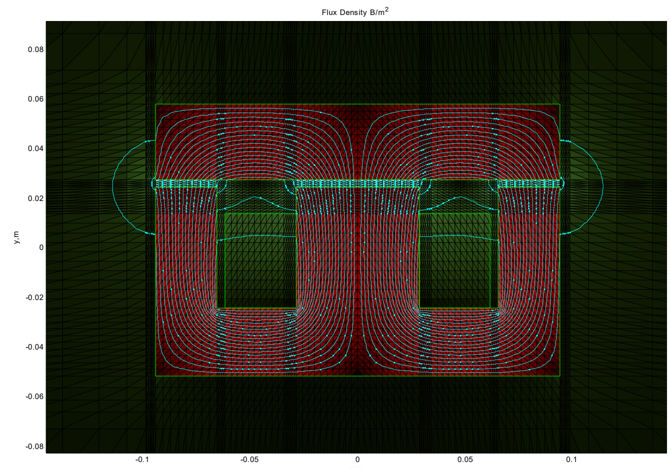
- N = 260 Turns
- d = 8.4857 cm
- g = 13.069 mm
- w = 1.813 cm
- a_w = 21.5181 (mm)^2
- d_s = 8.94 cm
- w_s = 8.94 cm
- V_{mm} = 0.00066173 m^3
- V_{cu} = 0.0027237 m^3
- V_{bx} = 0.0075582 m^3
- Cost = 153.7112 $
- Mass = 27.5408 Kg

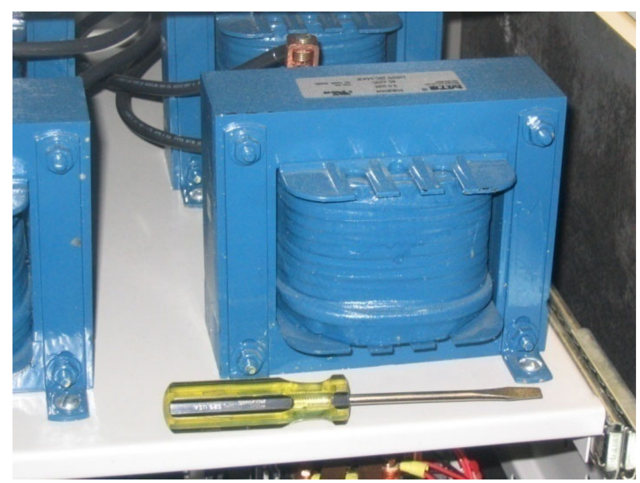
Loss and Mass

Design Criticisms

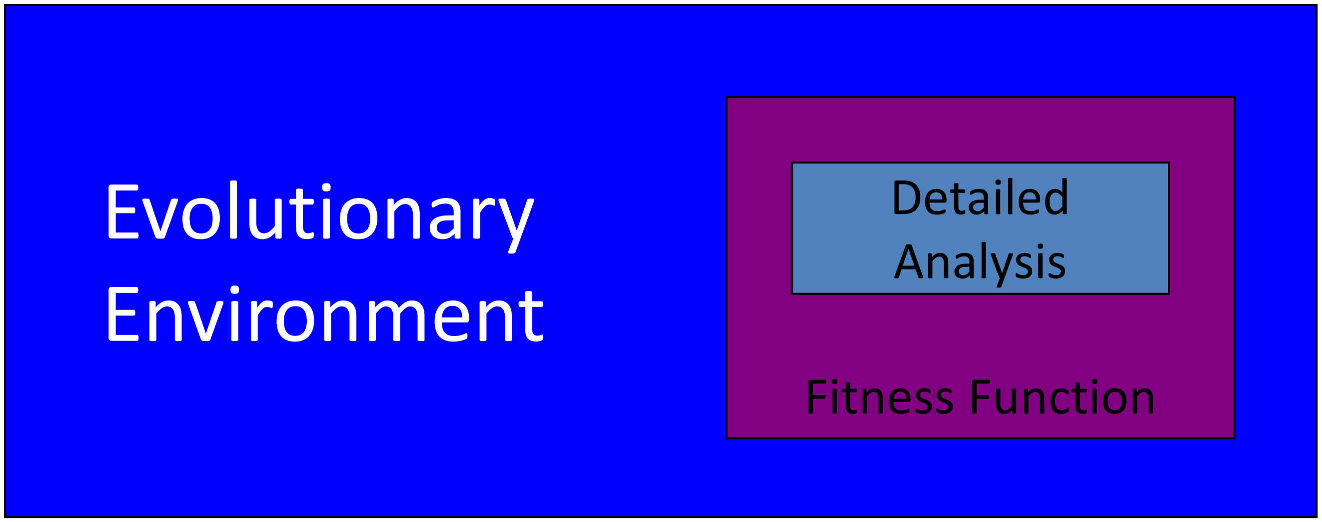
Manual Design Approach

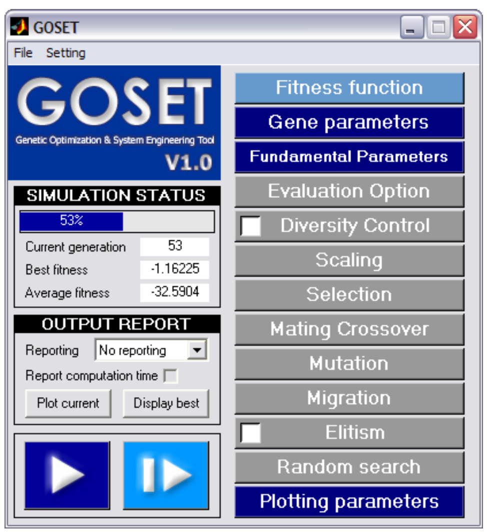






Formal Design Optimization





Pareto-Optimal Front