Lecture Set 1

Introduction to Magnetic Circuits

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Goals

- Review physical laws pertaining to analysis of magnetic systems
- Introduce concept of magnetic circuit as means to obtain an electric circuit
- Learn how to determine the flux-linkage versus current characteristic because it will be key to understanding electromechanical energy conversion
Review of Symbols

• Flux Density in Tesla: \( B \) (T)
• Flux in Webers: \( \Phi \) (Wb)
• Flux Linkage in Volt-seconds: \( \lambda \) (Vs,Wb-t)
• Current in Amperes: \( i \) (A)
• Field Intensity in Ampere/meters: \( H \) (A/m)
• Permeability in Henries/meter: \( \mu \) (H/m)
• Magneto-Motive Force: \( F \) (A,A-t)
• Permeance in Henries: \( P \) (H)
• Reluctance in inverse Henries: \( R \) (1/H)
• Inductance in Henries: \( L \) (H)
• Current (fields) into page: \( \times \)
• Current (fields) out of page: \( \circ \)
Ampere’s Law, MMF, and Kirchoff’s
MMF Law for Magnetic Circuits
Consider a Closed Path …
Enclosed Current and MMF Sources

• Enclosed Current
\[ i_{enc, \Gamma} = -N_a i_a + N_b i_b - N_c i_c \]

• MMF Sources
\[ F_a = -N_a i_a \]
\[ F_b = N_b i_b \]
\[ F_c = -N_c i_c \]

• Conclusion
\[ i_{enc, \Gamma} = \sum_{s \in S_\Gamma} F_s \quad S_\Gamma = \{a', b', c'\} \]
MMF Drops

• Going Around the Loop

\[ \oint_{l_{\Gamma}} \mathbf{H} \cdot d\mathbf{l} = \int_{l_{\alpha}} \mathbf{H} \cdot d\mathbf{l} + \int_{l_{\beta}} \mathbf{H} \cdot d\mathbf{l} + \int_{l_{\gamma}} \mathbf{H} \cdot d\mathbf{l} + \int_{l_{\delta}} \mathbf{H} \cdot d\mathbf{l} \]

• MMF Drop

\[ F_y = \int_{l_y} \mathbf{H} \cdot d\mathbf{l} \]

• Thus

\[ \oint_{l_{\Gamma}} \mathbf{H} \cdot d\mathbf{l} = F_{\alpha} + F_{\beta} + F_{\gamma} + F_{\delta} \]

• Or

\[ \oint_{l_{\Gamma}} \mathbf{H} \cdot d\mathbf{l} = \sum_{d \in D_{\Gamma}} F_d \quad D_{\Gamma} = \{ \alpha', \beta', \gamma', \delta' \} \]
Ampere’s Law

• Ampere’s Law
\[ \oint_{l_{\Gamma}} \mathbf{H} \cdot d\mathbf{l} = i_{enc,\Gamma} \]

• Recall
\[ i_{enc,\Gamma} = \sum_{s \in S_{\Gamma}} F_s \quad \oint_{l_{\Gamma}} \mathbf{H} \cdot d\mathbf{l} = \sum_{d \in D_{\Gamma}} F_d \]

• Thus
\[ \sum_{s \in S_{\Gamma}} F_s = \sum_{d \in D_{\Gamma}} F_d \]
Kirchoff’s MMF Law

Kirchoff’s MagnetoMotive Force Law:

The Sum of the MMF Drops Around a Closed Loop is Equal to the Sum of the MMF Sources for That Loop
Example: MMF Sources

10 conductors, 3 A =

5 conductors, 2 A =
Example: MMF Drops

- A quick example on MMF drop:

\[
F_{ab} = \quad F_{ac} =
\]

- Vector diagram with points a(0,0), b(5,0), and c(0,5)
- Magnetic field strength \( H = 10 \text{ A/m} \)
Another Example: MMF Drops

• How about $F_{cb}$?

\[ \text{a}(0,0) \quad \text{b}(5,0) \]

\[ c(0,5) \]

$H=10 \text{ A/m}$
Magnetic Flux, Gauss’s Law, and Kirchoff’s Flux Law for Magnetic Circuits
Magnetic Flux

\[ \Phi_m = \int _{S_m} B \cdot dS \]
Example: Magnetic Flux

- Suppose there exist a uniform flux density field of $B = 1.5a_y$ T where $a_y$ is a unit vector in the direction of the $y$-axis. We wish to calculate the flux through open surface $S_m$ with an area of $4 \text{ m}^2$, whose periphery is a coil of wire wound in the indicated direction.
Example: Magnetic Flux
Example: Magnetic Flux
Kirchoff’s Flux Law

• Gauss’s Law
\[ \oint_{\Sigma} \mathbf{B} \cdot d\mathbf{S} = 0 \]

• Thus
\[ \sum_{o \in O_\Gamma} \oint_{S_o} \mathbf{B} \cdot d\mathbf{S} = 0 \]

• Or
\[ \sum_{o \in O_\Gamma} \Phi_o = 0 \]
Kirchoff’s Flux Law

• Kirchoff’s Flux Law:

The Sum of the Flux Leaving A Node of a Magnetic Circuit is Zero
Kirchoff’s Flux Law

- A quick example on Kirchoff’s Flux Law
Magnetically Conductive Materials and Ohm’s Law for Magnetic Circuits
Ohm’s Law

- On Ohm’s Law Property for Electric Circuits

- On Ohm’s Law Property for Magnetic Circuits
Types of Magnetic Materials

• Interaction of Magnetic Materials with Magnetic Fields
  ➢ Magnetic Moment of Electron Spin
  ➢ Magnetic Moment of Electron in Shell

• These Effects Lead to Six Material Types
  ➢ Diamagnetic
  ➢ Paramagnetic
  ➢ Ferromagnetic
  ➢ Antiferromagnetic
  ➢ Ferrimagnetic
  ➢ Superparamagnetic
Ferromagnetic Materials

- Iron
- Nickel
- Cobalt
Ferrimagnetic Materials

- Iron-oxide ferrite \((\text{Fe}_3\text{O}_4)\)
- Nickel-zinc ferrite \((\text{Ni}_{1/2}\text{Zn}_{12}\text{Fe}_2\text{O}_4)\)
- Nickel ferrite \((\text{NiFe}_2\text{O}_4)\)
Behavior of Ferromagnetic and Ferrimagnetic Materials

![Diagram of magnetic behavior with labels: $B_r$, $B_{sat}$, $H_c$, minor loop, anhysteretic curve.](image)
Behavior of Ferromagnetic and Ferrimagnetic Materials
Modeling Magnetic Materials

- **Free Space**
  \[ B = \mu_0 H \]

- **Magnetic Materials**
  \[ B = \mu_0 (H + M) \quad M = \chi H \]
  \[ B = \mu_0 H + M \quad M = \mu_0 \chi H \]

- **Either Way**
  \[ B = \mu_0 (1 + \chi) H \quad B = \mu_0 \mu_r H \quad \mu_r = 1 + \chi \]

- **Or**
  \[ B = \mu H \quad \mu = \mu_0 \mu_r \]
Modeling Magnetic Materials

\[ B = \mu_H(H)H \]

\[ B = \mu_B(B)H \]
Back to Ohm’s Law
Relationship Between MMF Drop and Flux

\[ F_a = R_a(\xi)\Phi_a \quad R_a(\xi) = \begin{cases} \frac{l_a}{A_a\mu_H(F_a/l_a)} \\ \frac{l_a}{A_a\mu_B(\Phi_a/A_a)} \end{cases} \]

Form 1: \( \xi = F_a \)
Form 2: \( \xi = \Phi_a \)

\[ \Phi_a = P_a(\xi)F_a \quad P_a(\xi) = \begin{cases} \frac{A_a\mu_H(F_a/l_a)}{A_a\mu_B(\Phi_a/A_a)} \\ \frac{l_a}{l_a} \end{cases} \]

Form 1: \( \xi = F_a \)
Form 2: \( \xi = \Phi_a \)
Derivation
Derivation
Construction of the Magnetic Equivalent Circuit
Consider a UI Core Inductor
Inductor Applications
UI Core Inductor MEC

(a) cross section showing nodes
(b) magnetic equivalent circuit
Reluctances
Reduced Equivalent Circuit

(a) cross section showing nodes
(b) magnetic equivalent circuit
Example

- Let us consider a UI core inductor with the following parameters: $w_i = 25.1$ mm, $w_b = w_e = 25.3$ mm, $w_s = 51.2$ mm, $d_s = 31.7$ mm, $l_c = 101.2$ mm, $g = 1.00$ mm, $w_w = 38.1$ mm, and $d_w = 31.7$ mm. The winding is composed of 35 turns. Suppose the magnetic core material has a constant relative permeability of 7700, and that a current of 25.0 A is flowing. Compute the flux through the magnetic circuit and the flux density in the I-core.
Example
Example
Solving the Nonlinear Case

\[ R_{eq}(\Phi) = R_{ic}(\Phi) + R_{buc}(\Phi) + 2R_{luc}(\Phi) + 2R_g \]

\[ \Phi = \frac{Ni}{R_{eq}(\Phi)} \]
Translation of Magnetic Circuits to Electric Circuits: Flux Linkage and Inductance
Flux Linkage

\[ \lambda_x = N_x \Phi_x \]
Inductance

\[ L_{x,abs} = \left. \frac{\lambda_x}{i_x} \right|_{i_{x1}, \lambda_{x1}} \]

\[ L_{inc} = \left. \frac{\partial \lambda_x}{\partial i_x} \right|_{i_{x1}} \]
Example

• Suppose

\[ \lambda = 0.05(1 - e^{-0.2i}) + 0.03i \]

• Find the absolute and incremental inductance at 5 A
Computing Inductance

\[ N_x i_x \pm \Phi_x \trianglelefteq R_x \]
Nonlinear Flux Linkage Vs. Current
Nonlinear Flux Linkage Vs. Current

% Computation of lambda-i curve
% for UI core inductor
% using a simple but nonlinear MEC
% S.D. Sudhoff for ECE321
% January 13, 2012

% clear work space
clear all
close all

% assign parameters
cm = 1e-2;                % a centimeter
mm = 1e-3;                % a millimeter
w=1*cm;                   % core width (m)
ws=5*cm;                  % slot width (m)
ds=2*cm;                  % slot depth (m)
d=5*cm;                   % depth (m)
g=1*mm;                   % air gap (m)
N=100;                    % number of turns
mu0=4e-7*pi;              % perm of free space (H/m)
Bsat=1.3;                 % sat flux density (T)
mur=1500;                 % init rel permeability
Am=w*d;                   % mag cross section (m^2)

% start with flux linkage
lambda=linspace(0,0.08,1000);

% find flux and B and mu
phi=lambda/N;
B=phi/(w*d);
mu=mu0*(mur-1)./(exp(10*(B-Bsat)).^2+1)+mu0;
Nonlinear Flux Linkage Vs. Current

% Assign reluctances;
Ric=(ws+w)./(Am*mu);
Rbuc=(ws+w)./(Am*mu);
Rg=g/(Am*mu0);
Rluc=(ds+w/2)./(Am*mu);

% compute effective reluctances
Reff=Ric+Rbuc+2*Rluc+2*Rg;

% compute MMF and current
F=Reff.*phi;
i=F/N;

% find lambda-i characteristic
figure(1)
plot(i,lambda);
xlabel('i, A');
ylabel('\lambda, Vs');
grid on;

% show B-H characteristic
H=B./mu;
figure(2)
plot(H,B)
xlabel('H, A/m');
ylabel('B, T');
grid on;
Nonlinear Flux Linkage Vs. Current
Hardware Example – Our MEC
Non-Idealities: Fringing and Leakage Flux
Our MEC – With Leakage and Fringing
Faraday’s Law

• Faraday’s Law, voltage equation for winding

\[ v_x = r_x i_x + \frac{d\lambda_x}{dt} \]

• If inductance is constant

\[ \lambda_x = L_x i_x \]

• Combining above yields

\[ v_x = r_x i_x + L_x \frac{d i_x}{dt} \]
Energy Stored in a Magnetically Linear Inductor

• We can show

\[ E_x = \frac{1}{2} L_x i_x^2 \]

• Proof
Mutual Inductance
Case Study: Let's Design a UI Core Inductor
Architecture

Diagram showing architecture with nodes and depth into page labeled as $d$. Nodes 1 and 2 are connected, as are nodes 3, 4, 5, 6, 7, and 8. Depth into page is indicated as $d$, and $w$, $g$, and $d_s$ are measured distances. The diagram includes labels for $N$ Turns.
Design Requirements

• Current Level: 40 A
• Inductance: 5 mH
• Packing Factor: 0.7
• Slot Shape $\eta_{dw}$: 1
• Maximum Resistance: 0.1$\Omega$
Information

- Cost of Ferrite: 230 k$/m$^3$
- Density of Ferrite: 4740 Kg/m$^3$
- Saturation Flux Density: 0.5 T
- Cost of Copper: 556 $/m^3$
- Density of Copper: 8960 Kg/m$^3$
- Resistivity of Copper: 1.7e-8 Ωm
Step 1: Design Variables

Depth into page = $d$

$N$ Turns
Step 2: Design Constraints

• Constraint 1: Flux Density
Step 2: Design Constraints

- Constraint 2: Inductance
Step 2: Design Constraints

- Constraint 3: Slot Shape
Step 2: Design Constraints

• Constraint 4: Packing Factor
Step 2: Design Constraints

• Constraint 5: Resistance
Step 2: Design Constraints
Approach (1/6)
Approach (2/6)
Approach (3/6)
Approach (4/6)
Approach (5/6)
Approach (6/6)
Design Metrics

- Magnetic Material Volume
Design Metrics

- Wire Volume
Design Metrics

- Circumscribing Box
Design Metrics

- Material Cost
- Mass
Circumscribing Box Volume
Material Cost
Mass
Least Cost Design

- N = 260 Turns
- d = 8.4857 cm
- g = 13.069 mm
- w = 1.813 cm
- \(a_w = 21.5181 \text{ (mm)}^2\)
- \(d_s = 8.94 \text{ cm}\)
- \(w_s = 8.94 \text{ cm}\)
- \(V_{mm} = 0.00066173 \text{ m}^3\)
- \(V_{cu} = 0.0027237 \text{ m}^3\)
- \(V_{bx} = 0.0075582 \text{ m}^3\)
- Cost = 153.7112 
- Mass = 27.5408 Kg
Loss and Mass
Design Criticisms
Manual Design Approach

Analysis

Design Equations

Numerical Analysis

Design Revisions

Final Design
Formal Design Optimization

Evolutionary Environment

Detailed Analysis

Fitness Function
Another Design Trade-Off
Lunar Rover Propulsion Drive
Pareto-Optimal Front