Transportation Decision-making – Principles of Project Evaluation and Programming

Estimating Transportation Demand, Part 2
Contents of Today’s Lecture

- Example of Demand Estimation based on Attributes of Entire Network
- Transportation Supply
- Equilibration of Transportation Demand and Supply
- Elasticity of Transportation Demand
- Consumer Surplus and Latent Demand
Example of Demand Estimation based on Attributes of Entire Network

(The Transportation Planning Model)
The TPM - Steps

1. Trip Generation:
   What generates the trips? – Trip productions.

2. Trip Distribution:
   For the trips generated, how are they distributed (shared) among the various destination points?

3. Traffic Assignment
   Which routes are taken by the travelers from any origin to any destination?

4. Mode Choice or Mode Split
   For a given set of travelers on each chosen route, what fraction takes which mode (auto, bus, walk, rail, air, etc.)
1. Trip Generation

What generates (produces) the trips?
How many trip are generated (produced)?

Zone 1

Example: 1000
Trips Produced
2. Trip Distribution - For the trips generated, how are they distributed (shared) among the various destination points?

Zone 1 → Zone 2
Zone 1 → Zone 3
Zone 1 → Zone 4
Zone 1 → Zone 5

1000 Trips Produced
2. Trip Distribution (continued)

Many ways exist to determine the shares of trips produced among the trip destination zones

Most popular way is analogous to Newton’s Law of Gravitation:

“The force of attraction between any 2 bodies is directly proportional to their masses and inversely proportional to the distance between them.”

\[ F = G \frac{m_1 m_2}{d^2} \]

\( F = \) gravitational force between \( m_1 \) and \( m_2 \),
\( G \) is the gravitational constant
\( m_1 \) is the mass of the first body,
\( m_2 \) is the mass of the second body,
\( d \) is the distance between the two bodies.
Newton’s Law | Travel Attraction between 2 Zones
---|---
\( m_1, m_2 \) | Physical bodies with mass \( m_1 \) and \( m_2 \) | Zones with a set of socio-economic activities \( m_1 \) and \( m_2 \) (residences, schools, work places, etc.)
\( d \) | Distance, \( d \), between the 2 bodies | Some characteristic of the link that discourages people from traveling from Point 1 to Point 2 (distance, travel time, inconvenience, lack of safety, etc.)

\[
F_{12} = A \frac{m_1 m_2}{f(d_{12})}
\]
Travel Attraction between 2 Zones

For Only 2 bodies:

\[ F_{12} = A \frac{m_1 m_2}{f(d_{12})} \]

For more than 2 bodies, it can be shown that:

\[ T_{12} = P_1 \frac{A_2 F_{12}}{A_2 F_{12} + A_3 F_{13} + \ldots + A_5 F_{15}} \]

Generally, …

\[ T_{ij} = P_j \frac{A_j F_{ij}}{\sum_{j=1}^{J} (A_j F_{ij})} \]
2. Steps in Trip Distribution (continued)

\[ T_{ij} = P_i \times \frac{A_j F_{ij}}{\sum_j A_j F_j} \]

- Step 1: Observe \( T_{ij} \) for the Base Year
- Step 2: Determine \( P_{ij} \) and \( A_{ij} \) for each zone using Productions and Attractions Models
- Step 3: Trip Balancing
- Step 4: Calibrate Gravity model by estimating the function for Friction Factor
2. Trip Distribution (continued)

\( F \) is the “friction factor” or a factor that describes the travel impedance between any two points.

\[ F_{ij} = \frac{1}{t_{ij}^\alpha} \quad \text{(in general, there are other variations of this model as well)} \]

\[ t_{ij} = \text{travel\_time\_between\_i\_and\_j}, \quad \alpha = \text{a constant} = 2.00 \text{ generally} \]

Determining the value of alpha means “calibrating the gravity model”

We first calibrate the Gravity Model using known values of the variables (Trip Productions, Trip Attractions, Travel Times and Zone-to-Zone Trips).

Then we use the calibrated model to determine the future year trip distributions.
# Example 1

**Table 1: Base Year (2000) Zone-to-zone Person Trips (100’s), Auto Travel Time and Friction Factors**

<table>
<thead>
<tr>
<th>To From</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Total Trip Productions</th>
</tr>
</thead>
</table>
| 1       | $TT = 1$
         | $NT = 40$
         | $FF = 0.753$ | $TT = 9$
         | $NT = 110$
         | $FF = 1.597$ | $TT = 4$
         | $NT = 150$
         | $FF = 0.753$ | 300 |
| 2       | $TT = 11$
         | $NT = 50$
         | $FF = 0.987$ | $TT = 2$
         | $NT = 20$
         | $FF = 0.753$ | $TT = 17$
         | $NT = 30$
         | $FF = 0.765$ | 100 |
| 3       | $TT = 6$
         | $NT = 110$
         | $FF = 1.597$ | $TT = 12$
         | $NT = 30$
         | $FF = 0.765$ | $TT = 3$
         | $NT = 10$
         | $FF = 0.753$ | 150 |

**Total Trip Attractions**

200 160 190 550

$TT$ – Travel Time in minutes, $NT$ – Number of Trips, $FF$ – Friction Factor ($\alpha = 2$).
Table 2 below shows the calculated trip interchanges between the various zones after row and column factoring.

Table 2: Calculated Trip Table (2000) Using Gravity Model*

<table>
<thead>
<tr>
<th>ZONE</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>( P_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>85</td>
<td>111</td>
<td>104</td>
<td>300</td>
</tr>
<tr>
<td>2</td>
<td>39</td>
<td>19</td>
<td>42</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
<td>31</td>
<td>44</td>
<td>150</td>
</tr>
<tr>
<td>( A_j )</td>
<td>199</td>
<td>161</td>
<td>190</td>
<td>550</td>
</tr>
</tbody>
</table>

* shown in equation 3-1
Zone-to-Zone adjustment factors are calculated for taking into account the effect on travel patterns of social and economic linkages not otherwise incorporated in the gravity model.

- The adjustment factors $K_{ij}$ are calculated as follows:

\[ K_{ij} = \frac{T_{ij}(\text{observed})}{T_{ij}(\text{calculated})} \]

- $T_{ij}(\text{observed})$ and $T_{ij}(\text{calculated})$ are determined from Table 3.

<table>
<thead>
<tr>
<th>ZONE</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.47</td>
<td>0.99</td>
<td>1.45</td>
</tr>
<tr>
<td>2</td>
<td>1.27</td>
<td>1.06</td>
<td>0.72</td>
</tr>
<tr>
<td>3</td>
<td>1.47</td>
<td>0.98</td>
<td>0.23</td>
</tr>
</tbody>
</table>
Trip Generation in the Horizon Year

2. Horizon Year 2020: Provision of transit service
   Trip Generation Models (from trip generation phase)

   Productions: \( P_i = -10 + 2.0X_1 + 1.0X_2 \)
   (where \( X_1 \) = number of cars, \( X_2 \) = number of households)

   Attractions: \( A_j = -30 + 1.4X_3 + 0.04X_4 \)
   (where \( X_3 \) = employment, \( X_4 \) = commercial area in ha.)

   Next table shows the socio-economic characteristics of each zone in terms of the number of cars, number of households, employment, and area of commercial activity.
Table 4: Zonal Socio-economic Characteristics (Horizon Year 2020)

<table>
<thead>
<tr>
<th>Zone</th>
<th>Cars ($X_1$)</th>
<th>Households ($X_2$)</th>
<th>Employment ($X_3$)</th>
<th>Commercial Area ($X_4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>280</td>
<td>200</td>
<td>420</td>
<td>4100</td>
</tr>
<tr>
<td>2</td>
<td>220</td>
<td>150</td>
<td>560</td>
<td>800</td>
</tr>
<tr>
<td>3</td>
<td>190</td>
<td>110</td>
<td>220</td>
<td>600</td>
</tr>
</tbody>
</table>

The travel time and friction factors between zone centroids for the year 2020 are shown in next table.
Trip Generation in the Horizon Year

Step 1 (*Trip Generation*)

- The projected trip productions $P_i$ and attractions $A_j$ for each zone for the year 2020 are shown in Table below.
- Total number of trips produced = Total number of trips attracted = 1810 (Trip Balancing)

**Table 5: Trip Productions and Attractions for the 3-Zone Study Area in the Horizon Year (2020)**

<table>
<thead>
<tr>
<th>ZONE</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trip Productions ($P_i$)</td>
<td>750</td>
<td>580</td>
<td>480</td>
</tr>
<tr>
<td>Trip Attractions ($A_j$)</td>
<td>722</td>
<td>786</td>
<td>302</td>
</tr>
</tbody>
</table>
Trip Distribution for the Horizon Year

Step 2: Trip Distribution

- Calculate the zone-to-zone trips for the horizon year using the calibrated Gravity Model

\[ T_{ij} = P_i \times \frac{A_j F_{ij} K_{ij}}{\sum_j A_j F_j K_{ij}} \]

Table 6: Horizon Year Zone-to-zone Auto Travel Time and Friction Factors

<table>
<thead>
<tr>
<th>To From</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( TT = 2 ) ( FF = 0.753 )</td>
<td>( TT = 12 ) ( FF = 0.987 )</td>
<td>( TT = 7 ) ( FF = 1.597 )</td>
</tr>
<tr>
<td>2</td>
<td>( TT = 13 ) ( FF = 0.987 )</td>
<td>( TT = 3 ) ( FF = 0.753 )</td>
<td>( TT = 19 ) ( FF = 0.765 )</td>
</tr>
<tr>
<td>3</td>
<td>( TT = 9 ) ( FF = 1.597 )</td>
<td>( TT = 16 ) ( FF = 0.765 )</td>
<td>( TT = 4 ) ( FF = 0.753 )</td>
</tr>
</tbody>
</table>
Example 1 solution

- Apply the gravity model (Equation 1) to estimate zone to zone trips for the horizon year 2020. Friction factors are obtained from Table 6. The $K_{ij}$ values are used from Table 3. The final trip interchange matrix for the horizon year is shown in Table 7.

<table>
<thead>
<tr>
<th>ZONE</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>$P_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>105</td>
<td>396</td>
<td>249</td>
<td>750</td>
</tr>
<tr>
<td>2</td>
<td>288</td>
<td>247</td>
<td>45</td>
<td>580</td>
</tr>
<tr>
<td>3</td>
<td>329</td>
<td>143</td>
<td>9</td>
<td>480</td>
</tr>
<tr>
<td>$A_j$</td>
<td>722</td>
<td>786</td>
<td>303</td>
<td>1810</td>
</tr>
</tbody>
</table>

Table 7: Trip Table For Horizon Year (2020)
Mode Split

The zone-to-zone travel times and costs for auto and transit are given in Table 8.
The utility functions for auto and transit, which are used in the mode choice models, are as follows:

Auto: \[ U_{\text{AUTO}} = 2.50 - 0.5CT_A - 0.010TT_A \]
Transit: \[ U_{\text{TRANSIT}} = -0.4CT_T - 0.012TT_T \]

where \( CT_A, TT_A \) are the cost and travel time for auto travel, respectively, and
\( CT_T, TT_T \) are the cost and travel time for transit travel, respectively.
Where \( TC = \) travel costs in dollars; \( TT = \) travel time in minutes.
## Mode Split

Table 8: Zone-to-zone Travel Time and Cost for Auto and Transit

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Auto</td>
<td>Transit</td>
<td>Auto</td>
</tr>
<tr>
<td>From</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Auto</td>
<td>TT = 2</td>
<td>TT = 5</td>
<td>TT = 12</td>
</tr>
<tr>
<td></td>
<td>CT = $0.5</td>
<td>CT = $1.0</td>
<td>CT = $1.0</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Auto</td>
<td>TT = 13</td>
<td>TT = 15</td>
<td>TT = 3</td>
</tr>
<tr>
<td></td>
<td>CT = $1.2</td>
<td>CT = $1.8</td>
<td>CT = $0.8</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Auto</td>
<td>TT = 9</td>
<td>TT = 20</td>
<td>TT = 16</td>
</tr>
<tr>
<td></td>
<td>CT = $1.7</td>
<td>CT = $2.0</td>
<td>CT = $1.5</td>
</tr>
</tbody>
</table>

*TT – Travel Time in minutes, CT – Travel Cost ($)
Example 1 solution (continued)

Table 8: Zone-to-zone Utilities for Auto and Transit

<table>
<thead>
<tr>
<th>To From</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( U_{\text{AUTO}} = 2.23 ) &lt;br&gt;( U_{\text{TRANSIT}} = -0.46 )</td>
<td>( U_{\text{AUTO}} = 1.88 ) &lt;br&gt;( U_{\text{TRANSIT}} = -0.78 )</td>
<td>( U_{\text{AUTO}} = 1.73 ) &lt;br&gt;( U_{\text{TRANSIT}} = -0.94 )</td>
</tr>
<tr>
<td>2</td>
<td>( U_{\text{AUTO}} = 1.77 ) &lt;br&gt;( U_{\text{TRANSIT}} = -0.90 )</td>
<td>( U_{\text{AUTO}} = 2.07 ) &lt;br&gt;( U_{\text{TRANSIT}} = -0.55 )</td>
<td>( U_{\text{AUTO}} = 1.71 ) &lt;br&gt;( U_{\text{TRANSIT}} = -1.07 )</td>
</tr>
<tr>
<td>3</td>
<td>( U_{\text{AUTO}} = 1.56 ) &lt;br&gt;( U_{\text{TRANSIT}} = -1.04 )</td>
<td>( U_{\text{AUTO}} = 1.59 ) &lt;br&gt;( U_{\text{TRANSIT}} = -1.05 )</td>
<td>( U_{\text{AUTO}} = 2.11 ) &lt;br&gt;( U_{\text{TRANSIT}} = -0.54 )</td>
</tr>
</tbody>
</table>

\( U_{\text{AUTO}} \) – Auto Utility, \( U_{\text{TRANSIT}} \) – Transit Utility
Example 1 solution (continued)

• **Step 3: Mode Choice**

• Use the utility functions to estimate the utilities for auto and transit.

• For example, the utility of auto, \( P(Auto) \), is given by equation below:

\[
P(Auto) = \frac{e^{U_{AUTO}}}{e^{U_{AUTO}} + e^{U_{TRANSIT}}}
\]
Example 1 solution (continued)

Use the Logit model to determine the percent of zone to zone trips by auto and transit, as follows:

Table 9: Zone-to-zone mode split (% trips) at horizon year

<table>
<thead>
<tr>
<th>To</th>
<th>From</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>P(Auto) = 0.94, P(Transit) = 0.06</td>
<td>P(Auto) = 0.93, P(Transit) = 0.07</td>
<td>P(Auto) = 0.94, P(Transit) = 0.06</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>P(Auto) = 0.94, P(Transit) = 0.06</td>
<td>P(Auto) = 0.93, P(Transit) = 0.07</td>
<td>P(Auto) = 0.94, P(Transit) = 0.06</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>P(Auto) = 0.93, P(Transit) = 0.07</td>
<td>P(Auto) = 0.93, P(Transit) = 0.07</td>
<td>P(Auto) = 0.93, P(Transit) = 0.07</td>
</tr>
</tbody>
</table>
Example 1 solution (continued)

The trip interchange matrix obtained from trip distribution in the previous step and the utility functions yield:

Table 9: Zone-to-zone mode split (nr. of trips) at horizon year

<table>
<thead>
<tr>
<th>To</th>
<th>From</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>Auto Trips = 98 Transit Trips = 7</td>
<td>Auto Trips = 370 Transit Trips = 26</td>
<td>Auto Trips = 233 Transit Trips = 16</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>Auto Trips = 269 Transit Trips = 19</td>
<td>Auto Trips = 230 Transit Trips = 17</td>
<td>Auto Trips = 42 Transit Trips = 3</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>Auto Trips = 306 Transit Trips = 23</td>
<td>Auto Trips = 133 Transit Trips = 10</td>
<td>Auto Trips = 8 Transit Trips = 1</td>
</tr>
</tbody>
</table>
Example 1 solution (continued)

- **Step 4: Traffic Assignment**
- The minimum path (all-or-nothing) method is used for loading the trips on the highway and transit networks to yield:

<table>
<thead>
<tr>
<th>Route</th>
<th>Auto and Transit Travel Time</th>
<th>Auto and Transit Trips</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>12* (15*)</td>
<td>370 (26)</td>
</tr>
<tr>
<td>1-3</td>
<td>7* (12*)</td>
<td>233 (16)</td>
</tr>
<tr>
<td>1-2-3</td>
<td>31 (41)</td>
<td></td>
</tr>
<tr>
<td>1-3-2</td>
<td>23 (33)</td>
<td></td>
</tr>
<tr>
<td>2-1</td>
<td>13* (15*)</td>
<td>269 (19)</td>
</tr>
<tr>
<td>2-3</td>
<td>19* (26*)</td>
<td>42 (3)</td>
</tr>
<tr>
<td>2-3-1</td>
<td>28 (46)</td>
<td></td>
</tr>
<tr>
<td>2-1-3</td>
<td>20* (27*)</td>
<td></td>
</tr>
<tr>
<td>3-1</td>
<td>9* (20*)</td>
<td>306 (23)</td>
</tr>
<tr>
<td>3-2</td>
<td>16* (21*)</td>
<td>133 (10)</td>
</tr>
<tr>
<td>3-2-1</td>
<td>29 (36)</td>
<td></td>
</tr>
<tr>
<td>3-1-2</td>
<td>21 (35)</td>
<td></td>
</tr>
</tbody>
</table>
Transportation Supply
Elements of Transportation Supply

**Quantity**
- Nr. of highway lanes, rail tracks
- Size of runway area, harbor area, etc.
- Capacity of transportation facilities

**Quality**
- Traveling comfort, safety, convenience, etc.
- Non-physical systems and operational features that increase facility capacity (ITS initiatives, etc.)
- Can help increase the flow of traffic even when the physical capacity is constant
How to Estimate Transportation Supply

- **Transportation Supply**
  The quantity (or quality) of transportation facilities that facility producers are willing to provide under a given set of conditions.

- **Supply functions or supply models**
  Mathematical expressions that describe transportation supply.

\[ S = f(X_1, X_2, \ldots X_n) \]
• When trip price increases, the supply of transportation services increases
• When trip price decreases, the supply of transportation service decreases
• In classical economics, this is known as *The Law of Supply*
Shifts in the Supply Curve

- A change in supply can occur even when the price is constant

 Causes of shifts in supply curve

- Number of competing transportation modes that are available
- Changes in prices of using any alternative transportation modes
- Changes in technology
Shifts in the Supply Curve

• A change in supply can occur even when the price is constant

Causes of shifts in supply curve

Number of competing transportation modes that are available
Changes in prices of using any alternative transportation modes
Changes in technology
Equilibration of Transportation Demand and Supply
Demand-Supply Equilibration

Socio-economic Activities
- Going to/from work
- Going to/from school
- Leisure/Entertainment
- Visiting restaurants
- Shopping
- Meetings
- Etc.

Flow of Traffic

Facility Supply

Demand and Supply Equilibration

Transportation Demand
An Instance of Demand-Supply Equilibration – How it happens

Demand function: \( f_D(p, V) \)

Supply Function: \( f_S(p, V) \)

At equilibrium, Demand = Supply

\[ f_D(p, V) = f_S(p, V) \]

**Solving simultaneously**, we get:

\( p = p^* \) (equilibrium trip price) and \( V = V^* \) (equilibrium demand)
Demand-Supply Equilibration – Example 1 (Rail transit)

- **Demand function**
  \[ V = 5500 - 22p \]

- **Supply function**
  \[ p = 1.50 + 0.003V \]

- **Solving the above 2 equations simultaneously yields: the demand/supply equilibrium conditions**

  - \( V = 5,431 \) passengers daily
  - \( P = $3.13 \) fare (trip price) per passenger
Demand-Supply Equilibration – Example 2 (Air Travel)

Supply Function

\[
\text{price per seat} = 200 + 0.02 \times (\text{nr. of airline seats sold per day})
\]

\[p = 200 + 0.02q\]

Demand Function

\[
\text{Nr. of seats demanded per day} = 5000 - 20 \times (\text{price per seat})
\]

\[q = 200 + 0.02p\]

Solving simultaneously ....

Equilibrium price, \( p^* = 214.28 \)

Nr. of seats demanded and sold at equilibrium, \( q^* = 714 \)
Demand-Supply Equilibration – Example 3 (Freeway travel)

**Supply Function**

Travel Time = 15 + 0.02*traffic volume

\( t = 15 + 0.02*q \)

**Demand Function**

Traffic volume = 4,000 – 120* Travel time

\( q = 4000 - 120*t \)

**Equilibrium conditions:**

Travel time = 27.94 mins.
Traffic Volume = 647 vehs/hr
Is there always only one instance of equilibration?
Recall Demand-Supply Equilibration

**Socio-economic Activities**
- Going to/from work
- Going to/from school
- Leisure/Entertainment
- Visiting restaurants
- Shopping
- Meetings
- Etc.

**Transportation Demand**

**Facility Supply**

**Demand and Supply Equilibration**

**Estimating Transportation Demand**
Demand-Supply Equilibration – Series of Instances

Socio-economic Activities
- Going to/from work
- Going to/from school
- Leisure/Entertainment
- Visiting restaurants
- Shopping
- Meetings
- Etc.

Transportation Demand

Demand And Supply Equilibration

Facility Supply

Demand And Supply Equilibration

Change in Facility Supply

Demand And Supply Equilibration

Change in Transportation Demand

Change in Transportation Demand
Demand-Supply Equilibration – A Series of 3 Instances (Graphical Illustration)

Hence:

Equilibration occurs continually, because of the dynamic nature of socio-economic activity and transportation decisions.
Elasticity of Transportation Demand
Elasticity of Demand

• Definition:
  Change in demand in response to a unit change in a demand factor or service (supply) attribute, $X$

• Demand factors and service attributes include:
  – Price
    Total trip price
    Parking price
    Fuel price
    Congestion price (price for entering the CBD)
    Transit fare, etc.
  – Travel Time
  – Trip comfort, safety, convenience, etc.

• Mathematical formula:

$$e_{v,x} = \frac{\text{Change in demand} / \text{Original demand}}{\text{Change in x} / \text{Original value of x}} = \frac{\partial V / V}{\partial X / x} = \left( \frac{X}{V} \right) \left( \frac{\partial V}{\partial x} \right)$$
Elasticity of Demand

\[ e_{V,x} = \left( \frac{X}{V} \right) \left( \frac{\partial V}{\partial x} \right) \]

Obviously, the actual expression for \( e_{V,x} \) will depend on the functional form of the demand model \( V \) as a function of \( x \), as seen in table below.

<table>
<thead>
<tr>
<th>Shape of Demand Function</th>
<th>Elasticity Function ((X/V) \times (\partial V/\partial X))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear ( V = \alpha + \beta X )</td>
<td>( e = \frac{\beta X}{V} = \frac{1}{1 + \left( \frac{\alpha}{\beta X} \right)} )</td>
</tr>
<tr>
<td>Product ( V = \alpha X^\beta )</td>
<td>( e = \beta )</td>
</tr>
<tr>
<td>Exponential ( V = \alpha e^{\beta X} )</td>
<td>( e = \beta X )</td>
</tr>
<tr>
<td>Logistic ( V = \frac{\alpha}{1 + e^{\beta X}} )</td>
<td>( e = -\beta X \left( 1 - \frac{V}{\alpha} \right) = -\frac{\beta y X e^{\beta X}}{1 + \gamma e^{\beta X}} )</td>
</tr>
<tr>
<td>Logistic-Product ( V = \frac{\alpha}{1 + \gamma X^\beta} )</td>
<td>( e = -\beta \left( 1 - \frac{V}{\alpha} \right) = -\frac{\beta y X^\beta}{1 + \gamma X^\beta} )</td>
</tr>
</tbody>
</table>

Source: Manheim, 1979
Point Elasticity vs. Arc Elasticity

Point Elasticity

\[
\frac{dV}{dx} \div x = \frac{dV}{x} \div V = \frac{dV}{dx} \cdot \frac{x}{V} = f'(x^*) \times \frac{x^*}{f(x^*)}
\]

Arc Elasticity
Point Elasticity vs. Arc Elasticity

**Point Elasticity**

Factor or Attribute, $x$

$V = f(x)$

$V^* = f(x^*)$

Quantity of trips demanded, $V$

**Arc Elasticity**

Factor or Attribute, $x$

$V = f(x)$

$x_0$

$x_1$

$V_0$

$V_1$

Quantity of trips demanded, $V$

$$
\text{Arc Elasticity} = \frac{(V_1 - V_o)(x_1 + x_o)}{(x_1 - x_o)(V_1 + V_0)}/2
$$

$$
= \frac{\left(f(x_1) - f(x_o)\right)(x_1 + x_o)}{(x_1 - x_o)\left(f(x_1) + f(x_o)\right)}/2
$$
**Point Elasticity vs. Arc Elasticity**

**Point Elasticity**

\[
\frac{dV}{V} = \frac{dx}{x} \times \frac{x}{V} = f'(x^*) \times \frac{x^*}{f(x^*)}
\]

**Arc Elasticity**

\[
= \frac{(V_1 - V_o)(x_1 + x_0)}{(x_1 - x_o)(V_1 + V_0)} / 2
\]

\[
= \frac{(f(x_1) - f(x_o))(x_1 + x_0)}{(x_1 - x_o)(f(x_1) + f(x_0))} / 2
\]
Example of Point Elasticity Calculation

An aggregate demand function for a rail transit service from a suburb to downtown is represented by the equation \( V = 500 - 20p^2 \), where \( V \) is the number of trips made per hour and \( p \) is the trip fare.

At a certain time when the price was $1.50, 2,000 trips were made. What is the elasticity of rail transit demand with respect to price?

• Solution:

Point price elasticity = \( e_p(V) = \frac{\partial V}{\partial p / p} = \frac{\partial V}{\partial p} \times \frac{p}{V} \)

\[ = (-20)(2)(1.50)(1.50/2000) = -0.045 \]
Example of Arc Elasticity Calculation

Two years ago, the average air fare between two cities was $1,000 per trip, 45,000 people made the trip per year. Last year, the average fare was $1,200 and 40,000 people made the trip.

Assuming no change in other factors affecting trip-making (such as security, economy, etc.), what is the elasticity of demand with respect to price of travel?

Solution:
Arc price elasticity, $e_p =$

$$\frac{\Delta V \times (p_1 + p_2) / 2}{\Delta p \times (V_1 + V_2) / 2} = \frac{(45,000 - 40,000) \times (1,000 + 1,200) / 2}{(1,000 - 1,200) \times (45,000 + 40,000) / 2} = -0.647$$
Interpretation of Elasticity Values

- A unit increase in $x$ results in a very large decrease in demand, $V$
- A unit increase in $x$ results in a very small decrease in demand, $V$
- A unit increase in $x$ results in no change in decrease in demand, $V$
- A unit increase in $x$ results in a very small increase in demand, $V$
- A unit increase in $x$ results in a very large increase in demand, $V$
Interpretation of Demand Elasticity Values - Example:

When the fare, \( p \), on a bus route was $1, the daily ridership, \( q \), was 2000. By reducing the fare to $0.50, the ridership increased by 600. What is the elasticity of demand with respect to trip fare?

**Solution:**

\[
e = \frac{(q_1 - q_0)(p_1 + p_0)}{(p_1 - p_0)(q_1 + q_0)}/2
\]

\[
e = \frac{600}{0.5} \times \frac{(1+.5)/2}{(2000 + 2600)/2} = 0.39
\]
Interpretation of Demand Elasticity Values - Example:

When the fare, \( p \), on a bus route was $1, the daily ridership, \( q \), was 2000. By reducing the fare to $0.50, the ridership increased by 600. What is the elasticity of demand with respect to trip fare?

**Solution:**

\[
e = \frac{(q_1 - q_o)(p_1 + p_o)/2}{(p_1 - p_o)(q_1 + q_o)/2}
\]

\[
e = \frac{600}{0.5} \times \frac{(1 + .5)/2}{(2000 + 2600)/2} = 0.39
\]

Because the resulting elasticity is less than 1.00, the demand is considered **inelastic**.
Applications of the Concept of Elasticity

- **Prediction of expected demand** in response to a change in trip prices, out-of-pocket costs, fuel costs, etc. Thus helps in evaluating policy decisions.

- For transit agencies, elasticities help predict the expected change in demand (and therefore, *predict expected change in revenue*) in response to changes in transit service attributes (trip time, safety, comfort, security, etc). Thus helps agencies examine the potential impact of their transit investment (enhancements) or increases or decreases in transit fare.

- Elasticities therefore are generally useful for evaluating the impact of changes in transportation systems on travel demand.
Applications of the Concept of Elasticity (cont’d)

Prediction of revenue changes

*How does demand elasticity affect revenues from fare-related transportation systems?*

Elasticity of transit demand with respect to price is given by:

$$ e = \frac{\% \text{ change in ridership}}{\% \text{ change in price}} $$

If $e > 1$, demand is elastic
- increase in price will reduce revenue
- decrease in price will increase revenue

If $e < 1$, opposite effect on revenue

$e = 1$, revenue will remain the same irrespective of the change in price
Example 1:
The demand for a certain transit system is governed by the power functional form \( q = \alpha (p)\beta \)
The agency is considering increasing the transit fare to 70 cents. If \( e_p = -2.75 \), \( q_0 = 12,500/\text{day} \), \( p_0 = $0.50 \), \( p_1 = $0.70 \)
What policy should be adopted (Should \( p \) stay as it is or should be increased to 70 cents)?

Solution
\[
q = \alpha (p)^\beta
\]

\( \alpha \) \& \( \beta \) are model parameters (i.e., constants)

\( \beta \) is the elasticity of demand (\( q \)) with respect to an attribute (\( p \))

\[
\frac{dq}{dp} = \alpha \beta p^{\beta-1}
\]

\[
e_p = \frac{dq}{dp} \cdot \frac{p}{q} = \alpha \beta p^{\beta-1} \cdot \frac{p}{q} = \alpha \beta p^{\beta-1} \cdot p \cdot q^{-1} = \beta
\]

Thus, it can be seen that for the power form: the exponent of the price variable is the price elasticity.
Solution (continued)

\[ q = \alpha p^\beta \]

\[ 12,500 = \alpha (50)^{-2.75} \]

\[ \alpha = 5.876 \times 10^8 \]

For \( p = 70 \) cents, \( q = 4,955 \)

Loss of ridership = 12,500 – 4,955 = 7,545

Loss of revenue = 12,500 x 0.5 -4,955 x 0.7 = $3,406 daily.

Clearly, the loss in transit demand due to the price increase will lead to a very large reduction in revenue that will not be offset by the increase in transit fare.

So the agency should not increase the transit fare.
Direct and Cross Elasticities

• **Direct Elasticity** – the effect of change in the price of a good on the demand for the same good.

• **Cross Elasticity** – the effect on the demand for a good due to a change in the price of another good.
1. Direct and Cross Elasticities

- **Direct Elasticity** – the effect of change in the price of a good on the demand for the same good.

- **Cross Elasticity** – the effect on the demand for a good due to a change in the price of another good.

<table>
<thead>
<tr>
<th>Demand for <strong>Auto</strong> Travel</th>
<th>Demand for <strong>Bus Transit</strong> Travel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parking Price</td>
<td>Transit Fare</td>
</tr>
<tr>
<td>Travel Time</td>
<td>Travel Time</td>
</tr>
<tr>
<td>Fuel</td>
<td>Safety</td>
</tr>
</tbody>
</table>
Examples of Direct Elasticities

• **Direct Elasticity** – the effect of change in the price of a good on the demand for the same good.
Examples of Cross Elasticities

- **Cross Elasticity** – the effect on the demand for a good due to a change in the price of another good.

<table>
<thead>
<tr>
<th>Demand for <strong>Auto</strong> Travel</th>
<th>Demand for <strong>Bus Transit</strong> Travel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parking Price</td>
<td>Transit Fare</td>
</tr>
<tr>
<td>Travel Time</td>
<td>Travel Time</td>
</tr>
<tr>
<td>Fuel</td>
<td>Safety</td>
</tr>
</tbody>
</table>
Examples of Cross Elasticities

- **Cross Elasticity** – the effect on the demand for a good due to a change in the price of another good.

<table>
<thead>
<tr>
<th>Demand for <strong>Auto</strong> Travel</th>
<th>Demand for <strong>Bus Transit</strong> Travel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parking Price</td>
<td>Transit Fare</td>
</tr>
<tr>
<td>Travel Time</td>
<td>Travel Time</td>
</tr>
<tr>
<td>Fuel</td>
<td>Safety</td>
</tr>
</tbody>
</table>
Example 1:

A 15% increase in gasoline price has resulted in a 7% increase in bus ridership and a 9% decrease in gasoline consumption.

\[ p_0 = \text{gasoline price before} \]
\[ p_1 = \text{gasoline price after} \]
\[ B_0 = \text{Bus ridership before} \]
\[ B_1 = \text{Bus ridership after} \]
Cross Elasticity

\[ p_1 = p_0 \times 1.15 \]

\[
e = \frac{dB}{B} \frac{dp}{p} = \left[ \frac{0.07}{(1+1.07)} \right] \left[ \frac{0.15}{(1+1.15)} \right] = 0.48
\]
Example 2:

The demand for a certain bus transit system is given by:

$$q = T^{-0.3} p^{-0.2} A^{0.1} l^{-0.25}$$

Where:

- $q$ = transit ridership/hr
- $T$ = transit travel time (hrs)
- $p$ = transit fare ($)
- $A$ = cost of auto trip ($)
- $l$ = average income ($)
Question (a):
When \( q_0 = 10,000/\text{hr} \), \( p = \$1 \). \( e_p = -0.2 \). What is \( q_1 \) when \( p = \$0.9 \)?
Also, what is the gain in revenue?

Solution
\( e_p = -0.2 \quad 1\% \) reduction in fare \( \rightarrow \) \( 0.2\% \) increase in ridership
Fare reduction = \( (100-90)/100 = 10\% \)
Corresponding increase in ridership = \( 2\% \)
New demand = \( q_1 = 200 + 10,000 = 10,200/\text{hr} \)
Revenue Gain = \( 10,200 \times 0.9 - 10,000 \times 1 = -\$820/\text{hr} \)
Question (b). Current auto trip cost $3 (including parking). The cross elasticity of transit demand with respect to auto cost is 0.1. If the parking charge were raised by 30 cents, the impact on transit ridership?

Solution:

Auto cost cross elasticity $\approx 0.1$

30 cents/$3 \approx 10\%$ increase in auto cost

Ridership increase $1\% \rightarrow$ addl. 100 riders / hr
**Question (c).** Average income of travelers $15,000/yr. Income elasticity is -0.25. What additional income is necessary to offset the auto cost increase?

**Solution:**

1% increase in income \( \cong 0.25\% \) decrease in ridership

\[
e_{i} = 0.25 = \frac{dq}{q} / \frac{dl}{l} = 0.01 / \frac{dl}{l} \quad \frac{dq}{q} = 1\%
\]

\[
\frac{dl}{l} = \frac{.01}{0.25} = 0.04 \cong 4\%
\]

An increase in income of 4\% \((15,000 \times .04 = $600)\) would cover a 30 cent increase (10\% increase) in auto cost. That means a $600 income increase would prevent 100 riders from switching to transit.
Recall Factors that Affect Demand

Demand factors and service attributes include:

- **Price**
  - Total trip price
  - Parking price
  - Fuel price
  - Congestion price (price for entering the CBD)
  - Transit fare, etc.

- **Travel time**

- **Trip comfort, safety, convenience, etc.**

Research has established some values of elasticity
### 2. Some Elasticity Values

Demand Elasticities with respect to Parking Price, by Mode

<table>
<thead>
<tr>
<th>Trip Purpose</th>
<th>Car Driver</th>
<th>Car Passenger</th>
<th>Public Transportation</th>
<th>Walking and Cycling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commuting</td>
<td>-0.08</td>
<td>+0.02</td>
<td>+0.02</td>
<td>+0.02</td>
</tr>
<tr>
<td>Business</td>
<td>-0.02</td>
<td>+0.01</td>
<td>+0.01</td>
<td>+0.01</td>
</tr>
<tr>
<td>Education</td>
<td>-0.10</td>
<td>+0.00</td>
<td>+0.00</td>
<td>+0.00</td>
</tr>
<tr>
<td>Other</td>
<td>-0.30</td>
<td>+0.04</td>
<td>+0.04</td>
<td>+0.05</td>
</tr>
</tbody>
</table>

Source: TRACE (1999)
## Some Elasticity Values

### Estimated Fuel Price Elasticities by Mode and Fuel Type

<table>
<thead>
<tr>
<th></th>
<th>Short Run Elasticity</th>
<th>Long Run Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>Base</td>
</tr>
<tr>
<td>Road Gasoline</td>
<td>-0.10</td>
<td>-0.15</td>
</tr>
<tr>
<td>Road Diesel - Truck</td>
<td>-0.05</td>
<td>-0.10</td>
</tr>
<tr>
<td>Road Diesel - Bus</td>
<td>-0.05</td>
<td>-0.10</td>
</tr>
<tr>
<td>Road Propane</td>
<td>-0.10</td>
<td>-0.15</td>
</tr>
<tr>
<td>Road CNG</td>
<td>-0.10</td>
<td>-0.15</td>
</tr>
<tr>
<td>Rail Diesel</td>
<td>-0.05</td>
<td>-0.10</td>
</tr>
<tr>
<td>Aviation Turbo</td>
<td>-0.05</td>
<td>-0.10</td>
</tr>
<tr>
<td>Aviation Gasoline</td>
<td>-0.10</td>
<td>-0.15</td>
</tr>
<tr>
<td>Marine Diesel</td>
<td>-0.02</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

Source: Hagler Bailly (1999)
Some Elasticity Values

Transit Demand Elasticities (wrt to Fare) by Time-of-Day and City Size

<table>
<thead>
<tr>
<th></th>
<th>Large Cities (More than One Million Population)</th>
<th>Smaller Cities (Less than One Million Population)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average for All Hours</td>
<td>-0.36</td>
<td>-0.43</td>
</tr>
<tr>
<td>Peak Hour</td>
<td>-0.18</td>
<td>-0.27</td>
</tr>
<tr>
<td>Off-Peak</td>
<td>-0.39</td>
<td>-0.46</td>
</tr>
<tr>
<td>Off-peak Average</td>
<td></td>
<td>-0.42</td>
</tr>
<tr>
<td>Peak Hour Average</td>
<td></td>
<td>-0.23</td>
</tr>
</tbody>
</table>

Kain and Liu (1999)
3. Consumer Surplus and Latent Demand
Consumer Surplus - Conceptual Illustration

• Consider you go to the mall and you see a flashy new i-pod
• Display price is $75.00
• But you like it (or need it) so much that you are even prepared to pay $200 for it
• Your individual consumer surplus = $125 (= 200-75)
• Others may have a CS that is less or more than yours.
• Let’s say the average consumer surplus of potential buyers is $100.
• What does this tell the producer?
Consumer Surplus – In the Context of Transportation Demand

• Existing transit fare is $p$ dollars.

• Some people are prepared to pay up to $q$ dollars for the trip, where $q > p$

• Then, maximum consumer surplus = $q - p$

  minimum consumer surplus = 0

  the average consumer surplus = $((q - p) + 0) / 2$

  $= 0.5(q - p)$

• For all the travelers that demand that trip at equilibrium conditions, $V_{p*}$, the total consumer surplus is

  $= V_{p*} \times 0.5(q - p) = 0.5 V_{p*} (q - p)$

• If consumer surplus is large, what does that tell the transit service provider?
### Consumer Surplus

The graph illustrates the concept of consumer surplus, which is the area between the demand curve and the price line, up to the quantity demanded. The formula for consumer surplus is:

\[
\text{Consumer surplus} = 0.5V_p^*(q - p)
\]
Change in Consumer Surplus

A change in transportation supply (e.g., increased quantity, increased capacity, increased quality of service (comfort, safety, convenience, etc.) can lead to a change in the consumer surplus.

The change in consumer surplus, which is a measure of the beneficial impact of the improvement, is given by:

\[
= (p_1 - p_2)V_1 + \frac{1}{2}(p_1 - p_2)(V_2 - V_1)
= (p_1 - p_2)(V_1 + V_2)/2
\]
Consumer Surplus - Example:

Urban bus service
Current: 100 @ 40 – capacity buses, 90% load factor
  fare $1
Proposed: 20% increase in fleet size
  95% load factor
  fare $0.9
Calculate the change in consumer surplus.
Determine if there is a revenue gain.
Solution

Existing situation

q₁ = 100 buses x 40 seats x 0.9 (load factor)
= 3600 persons/hr
Rev = 3600 x $1
= $3600/hr
Proposed Situation

\[ q_2 = 120 \text{ buses} \times 40 \text{ seats} \times 0.95 \text{ (load factor)} \]

\[ = 4560 \text{ persons/hr} \]

Rev = 4560 x 0.9

\[ = \$4140/\text{hr} \]

Change in Consumer Surplus

\[ = (1.0 - 0.9)(3600 + 4560)/2 \]

\[ = \$408/\text{hr} \]

Rev Gain = (4140 - 3600) = \$504/\text{hr}
Latent Demand - Conceptual Illustration

• Again, consider you go to the mall and you see that flashy new i-pod

• Display price is $75.00

• Assume the producer is willing to give it out free to anyone who is interested.

• How many people would demand it?

• This is the latent demand.

• What does this tell the producer? Gives indication of demand under pro-bono conditions.
  
  – Useful for sales promotions of the good (example, free good for limited period to help advertise)

  – Useful for sales promotion of complementary goods (e.g., free phone but you pay for the service.)
Latent Demand – In the Context of Transportation Demand

• Existing transit fare is $p$ dollars.

• On the average, $V_L$ people are prepared to use the transit service if it were free-of-charge.

• Then the latent demand is $V_L - V_{p^*}$

where $V_{p^*}$ is the demand at equilibrium conditions,
Latent demand = $V_L - V_{P^*}$