**FIRST\(_k\) set construction**

**input:** context-free grammar \(G = \{N, T, P, S\}\)

**private:** \(F',\) the FIRST set from the previous iteration

**private:** \(F,\) the current FIRST set

1. for each \(\alpha \in T\) do \(F'(\alpha) := \{\alpha\}\)

2. for each \(A \in N\) do

   if \(A \rightarrow \lambda\) is a rule in \(P\) then
   \(F(A) := \{\lambda\}\)

   if \(F(A) := \emptyset\)

3. repeat
   3.1 for each \(A \in N,\) \(F'(A) := F(A)\)
   3.2: for each \(A \rightarrow u_1u_2\ldots u_n, n > 0 \in P\) do
   \(F(A) := F(A) \cup \text{trunc\(_k\)}(F'(u_1)F'(u_2)\ldots F'(u_n))\)
   until \(F(A) = F'(A), \forall A \in N\)

4. \(\text{FIRST}\_k(A) := F(A)\)

**return** the FIRST sets

**What does trunc\(_1\) mean?**

\[
\{t_{1,1}, t_{1,2}, \ldots, t_{1,m_1}\} = \text{trunc\(_1\)}(\{t_{1,1}, t_{1,2}, \ldots, t_{1,m_1}\}\{t_{2,1}, t_{2,2}, \ldots, t_{2,m_2}\} \ldots \{t_{n,1}, t_{n,2}, \ldots, t_{n,m_2}\})
\]

\[
\{\{t_{1,1}, t_{1,2}, \ldots, t_{1,m_1}, t_{2,1}, t_{2,2}, \ldots, t_{2,m_2}\} = \text{trunc\(_1\)}(\{t_{1,1}, t_{1,2}, \lambda, \ldots, t_{1,m_1}\}\{t_{2,1}, t_{2,2}, \ldots, t_{2,m_2}\} \ldots \{t_{n,1}, t_{n,2}, \ldots, t_{n,m_2}\})
\]

\[
\{\lambda\} \in \text{trunc\(_1\)} \text{when} \text{trunc\(_1\)}(\lambda \in \{t_{1,1}, t_{1,2}, \ldots, t_{1,m_1}\}\lambda \in \{t_{2,1}, t_{2,2}, \ldots, t_{2,m_2}\} \ldots \lambda \in \{t_{n,1}, t_{n,2}, \ldots, t_{n,m_2}\})
\]

Note that \(\text{trunc\(_k\)},\) for arbitrary \(k\) is defined similarly, except that the strings in \(F_k\) and \(F'_k\) have length \(k\).
Example

Let $G = \{N, T, P, S\}, N = \{S, A, B, X\}, T = \{1, 2, 3, 4, 5, \$\}$

$P = \{$

\[
\begin{align*}
S & \rightarrow AB \$ \\
A & \rightarrow B 1 \\
A & \rightarrow X 2 \\
A & \rightarrow \lambda \\
B & \rightarrow A 3 \\
B & \rightarrow 4 \\
X & \rightarrow 5 \\
\end{align*}
\]

$\}$

1. $F'_1(1) = \{1\}, F'_1(2) = \{2\}, \ldots F'_1(5) = \{5\}, F'_1(\$) = \{\$\}$

2. $F_1(S) = \emptyset, F_1(A) = \{\lambda\}, F_1(B) = \emptyset, F_1(X) = \emptyset$

3. (a) $F'_1(S) = \emptyset, F'_1(A) = \{\lambda\}, F'_1(B) = \emptyset, F'_1(X) = \emptyset$
   $F_1(S) = \emptyset, F_1(A) = \{\lambda\}, F_1(B) = \{3, 4\}, F_1(X) = \{5\}$
   (b) $F'_1(S) = \emptyset, F'_1(A) = \{\lambda\}, F'_1(B) = \{3, 4\}, F'_1(X) = \{5\}$
   $F_1(S) = \{3, 4\}, F_1(A) = \{3, 4, 5, \lambda\}, F_1(B) = \{3, 4\}, F_1(X) = \{5\}$
   (c) $F'_1(S) = \{3, 4\}, F'_1(A) = \{3, 4, 5, \lambda\}, F'_1(B) = \{3, 4\}, F'_1(X) = \{5\}$
   $F_1(S) = \{3, 4, 5\}, F_1(A) = \{3, 4, 5, \lambda\}, F_1(B) = \{3, 4, 5\}, F_1(X) = \{5\}$
   (d) $F'_1(S) = \{3, 4, 5\}, F'_1(A) = \{3, 4, 5, \lambda\}, F'_1(B) = \{3, 4, 5\}, F'_1(X) = \{5\}$
   $F_1(S) = \{3, 4, 5\}, F_1(A) = \{3, 4, 5, \lambda\}, F_1(B) = \{3, 4, 5\}, F_1(X) = \{5\}$

return all computed FIRST sets.
Finding the $\text{FIRST}_k$ of a string of terminals and non-terminals

We often want to do this because we want to find the $\text{FIRST}_k$ set for the right-hand side of a production so that we know if we want to apply the production.

Consider the Production $P \rightarrow X_1 X_2 \ldots X_n$. Then

$$\text{FIRST}_k(X_1 X_2 \ldots X_n) = \text{FIRST}_k(X_1) \bigcup_{i=1}^{n} \text{FIRST}_k(X_i, \lambda \in \text{FIRST}_k(X_j), j < i) - \{\lambda\}$$

$$\text{FIRST}_k(X_1 X_2 \ldots X_n) = \text{FIRST}_k(X_1 X_2 \ldots X_n) \cup \{\lambda\} \text{iff } \lambda \in \text{FIRST}_1(X_i), 1 \leq i \leq n$$

Example

Let $G = \{N, T, P, S\}, N = \{S, A, B, X\}, T = \{1, 2, 3, 4, 5, \$\}$

$P = \{$

$$S \rightarrow A B \$$

$$A \rightarrow B 1$$

$$A \rightarrow X 2$$

$$A \rightarrow \lambda$$

$$A \rightarrow X 2$$

$$B \rightarrow A 3$$

$$B \rightarrow 4$$

$$X \rightarrow 5$$

$\}$

Applying the definition above,

$$\text{FIRST}_1(A B \$) = \text{FIRST}_1(A) \cup \text{FIRST}_1(B) - \{\lambda\}$$

If there we add a production $B \rightarrow X A 4$, to the grammar $G$, then

$$\text{FIRST}_1(X A 4) = \text{FIRST}_1(X)$$
Follow\textsubscript{k} set construction

**input:** context-free grammar \( G = \{N, T, P, S\} \),
\( FIRST_k(A) \) for every \( A \in N \)

**private:** \( FL' \), the FOLLOW sets from the previous iteration

**private:** \( FL \), the current FOLLOW set

1. \( FL(S) = \{\lambda\} \)
2. for each \( A \in N - \{S\} \), \( FL(A) = \emptyset \)
3. repeat
   (a) foreach \( A \in N \), \( FL'(A) = FL(A) \)
   (b) for each rule \( A \rightarrow \beta, \beta = u_1u_2 \ldots u_n, \beta \notin T^* \)
      i. \( L = FL'(A) \)
      ii. if \( u_n \in N \) then \( FL(u_n) = FL(u_n) \cup L \)
      iii. for \( i = n - 1 \) to 1
         A. \( L = \text{trunc}_k(FIRST_k(u_{i+1} \ldots u_n)L) \)
         B. if \( u_1 \in N \) then \( FL(u_1) = FL(u_1) \cup L \)
   until \( FL(A) = FL'(A) \) for all \( A \in V \)
4. \( FOLLOW_k(A) = FL(A) \)

return the FOLLOW sets.