Transforming Light with Metamaterials

Part 1: Electrical & Magnetic Metamaterials

Part 2: Negative-Index Metamaterials, NLO, and super/hyper-lens

Part 3: Cloaking and Transformation Optics

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<u>Part 1: Electrical & Magnetic Metamaterials</u>

Part 2: Negative-Index Metamaterials, NLO, and super/hyper-lens

Part 3: Cloaking and Transformation Optics

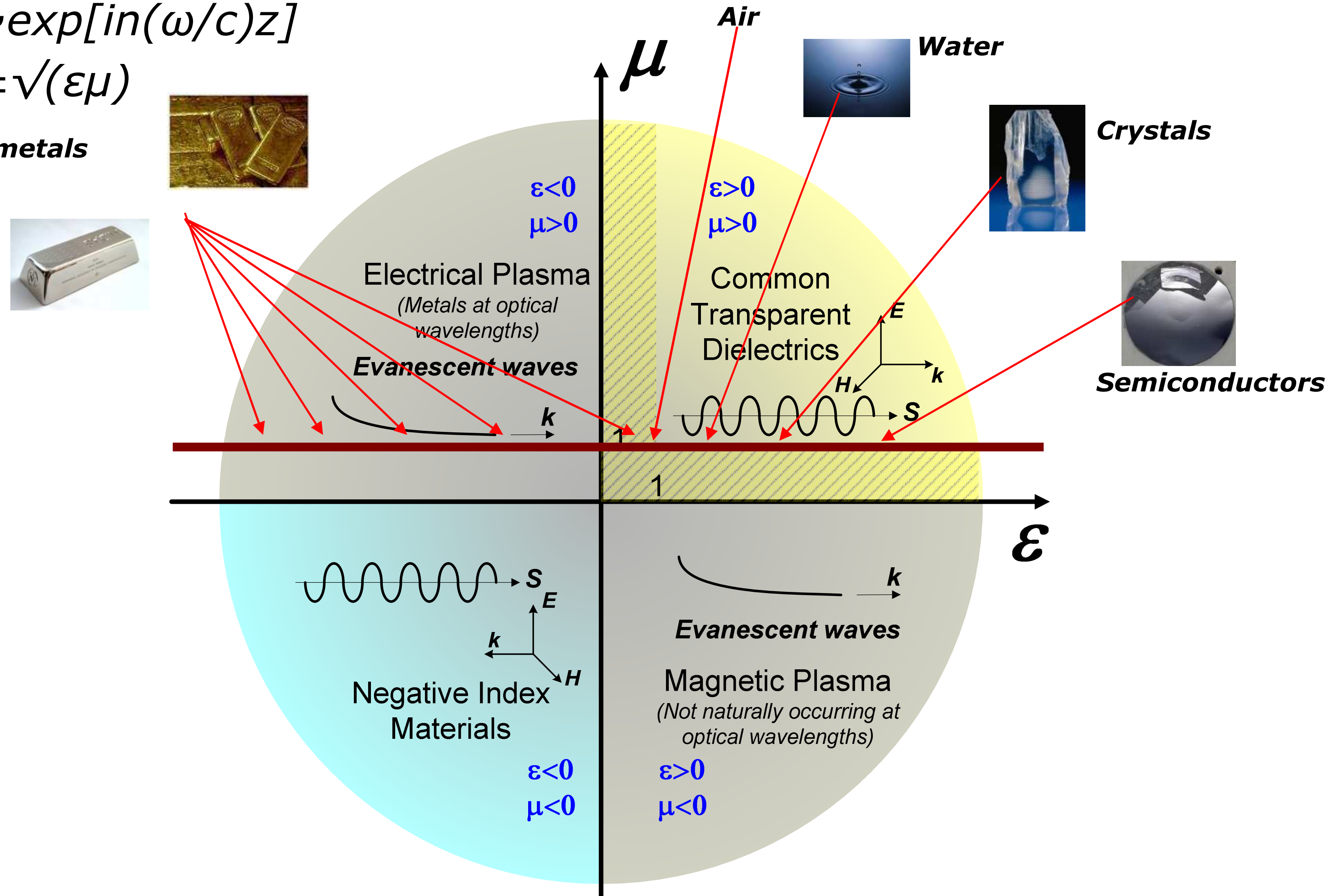
Outline

- What are metamaterials?
- Early electrical metamaterials
- Magnetic metamaterials
- Negative-index metamaterials
- Chiral metamaterials
- Nonlinear optics with metamaterials
- Super-resolution
- Optical Cloaking and Transformation Optics

Natural Optical Materials

E,H ~exp[in(ω/c)z]

$n = \pm\sqrt{(\varepsilon\mu)}$

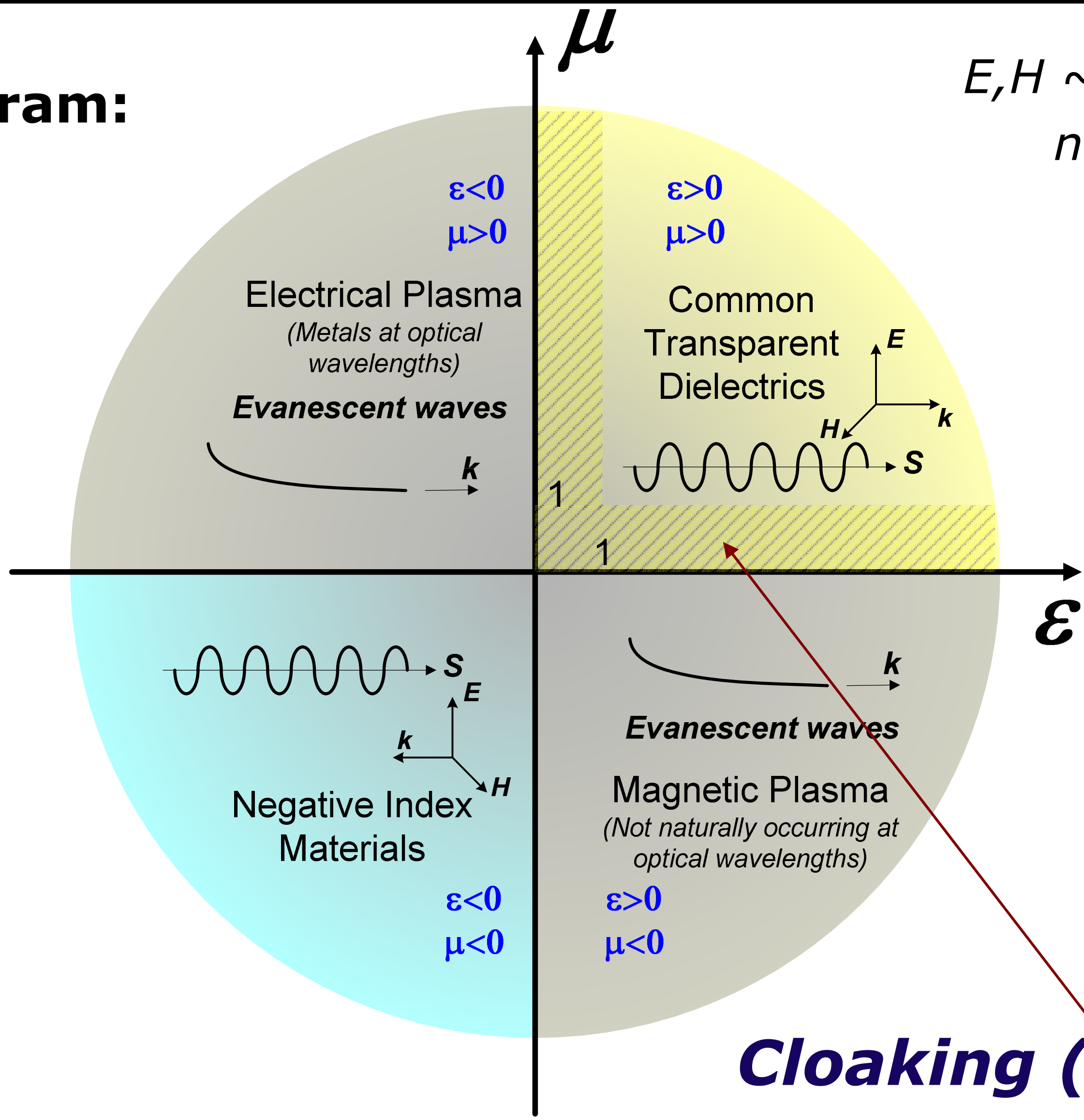


Materials & Metamaterials

ε, μ diagram:

$E,H \sim \exp[in(\omega/c)z]$

$n = \pm\sqrt{(\varepsilon\mu)}$

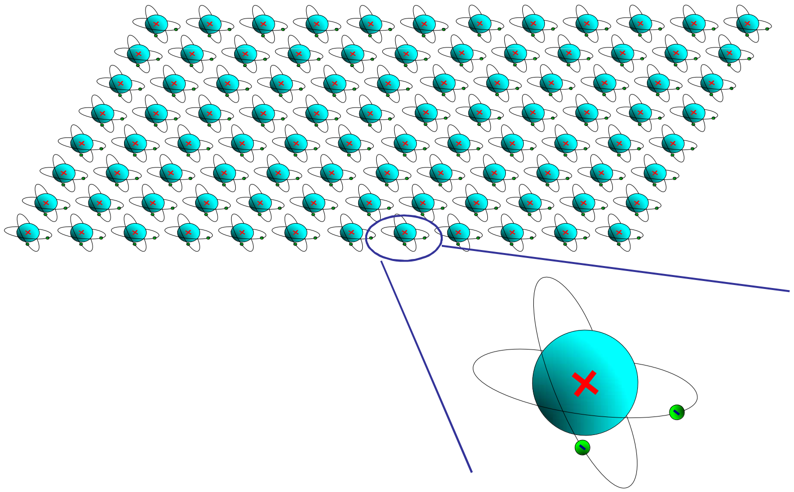


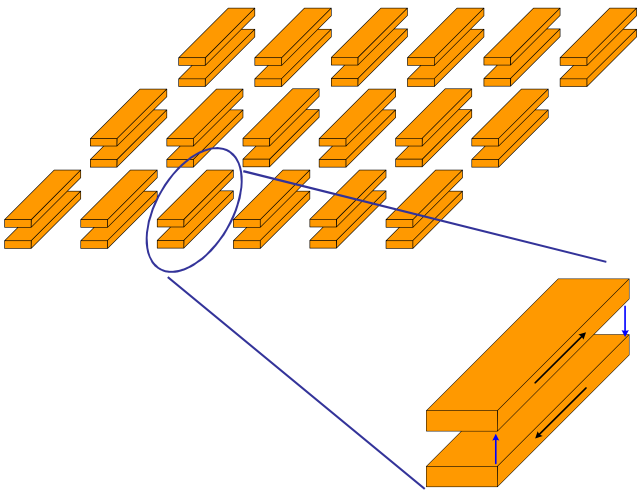
Cloaking (TO) area

What is a metamaterial?

Metamaterial is an arrangement of artificial structural elements, designed to achieve advantageous and unusual electromagnetic properties.

μετα = **meta = beyond (Greek)**





A natural material with its atoms

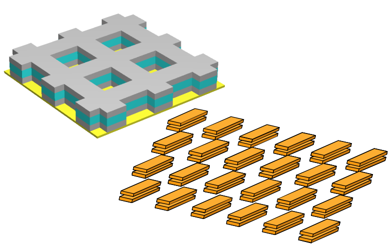
A metamaterial with artificially structured "atoms"

Photonic crystals *vs.* Optical metamaterials: connections and differences

0 — 1 — ∞ — a/λ

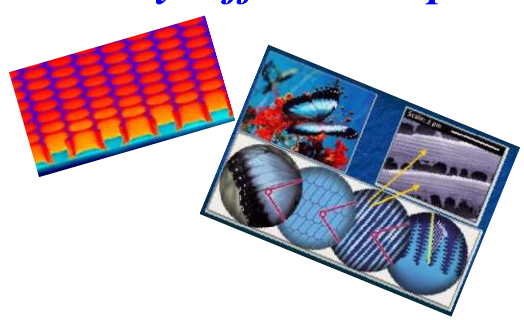
a<<λ.
Effective medium description using Maxwell equations with μ, ε, n, Z

Example:
Optical crystals
Metamaterials



a~λ.
Structure dominates. Properties determined by diffraction and interference

Example:
Photonics crystals
Phased array radar
X-ray diffraction optics



a>>λ.
Properties described using geometrical optics and ray tracing

Example:
Lens system
Shadows

Natural Crystals

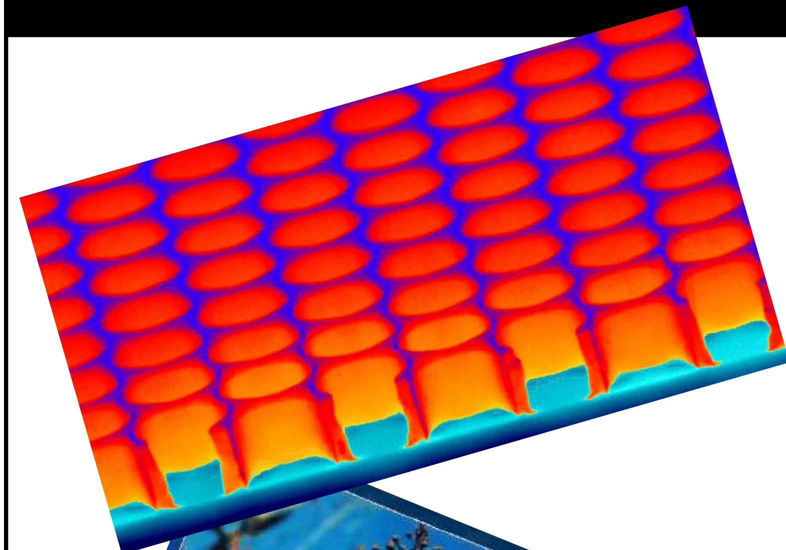
… have lattice constants much smaller than light wavelengths: $a \ll \lambda$

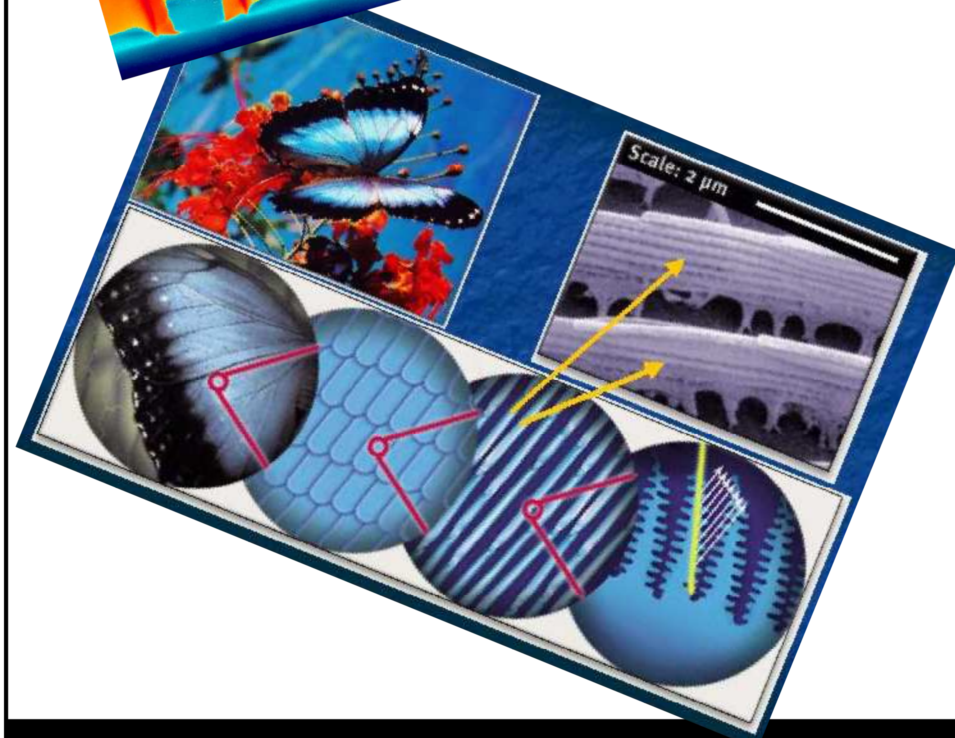
… are treated as homogeneous media with parameters ε, μ, n, Z **(tensors in anisotropic crystals)**

… have a positive refractive index: $n > 1$

… show no magnetic response at optical wavelengths: $\mu = 1$

Photonic crystals





... have lattice constants comparable to light wavelengths: $a \sim \lambda$

... can be artificial or natural

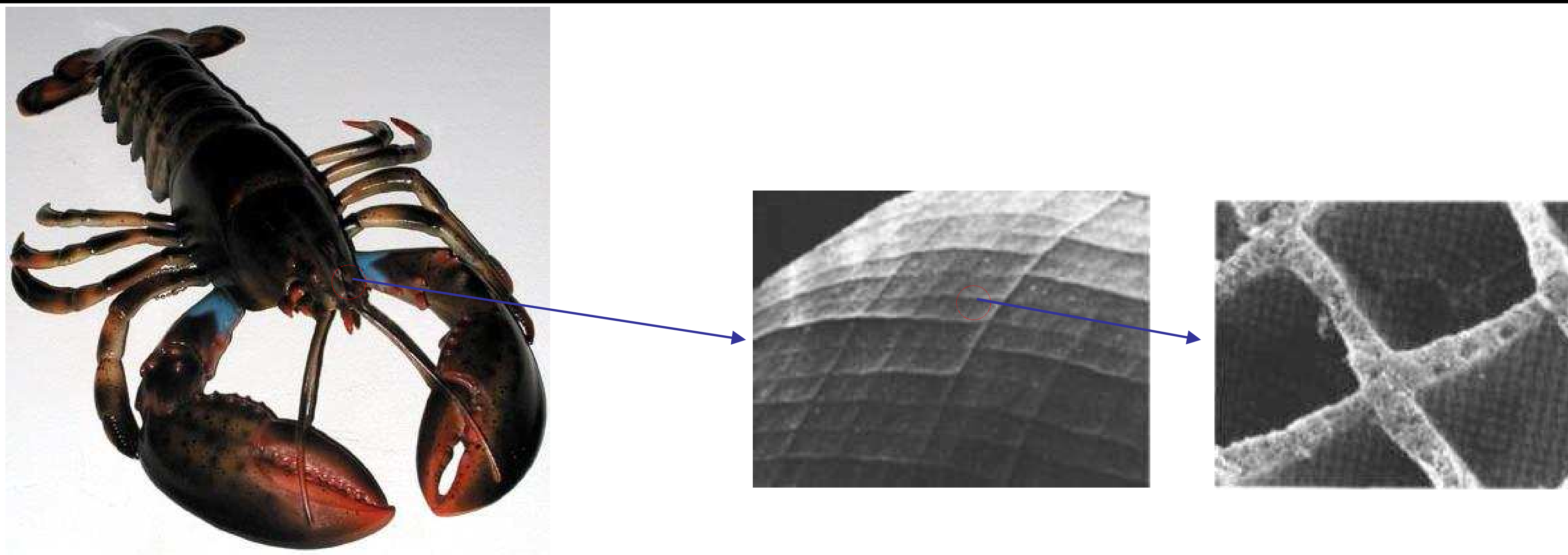
... have properties governed by the diffraction of the periodic structures

... may exhibit a bandgap for photons

... typically are *not* well described using effective parameters ε, μ, n, Z

... often behave like but they are *not* true metamaterials

Metamaterials: Properties not found in nature?



Journal of the European Optical Society - Rapid Publications 1, 06010 (2006)
www.jeos.org

JOURNAL OF THE EUROPEAN OPTICAL SOCIETY

Invertebrate superposition eyes-structures that behave like metamaterial with negative ~~refractive index~~

(refraction!)

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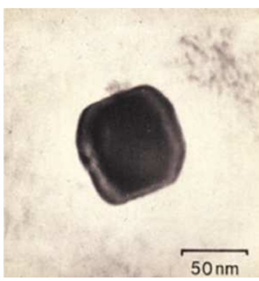
Metamaterials: Artificial periodic structures?

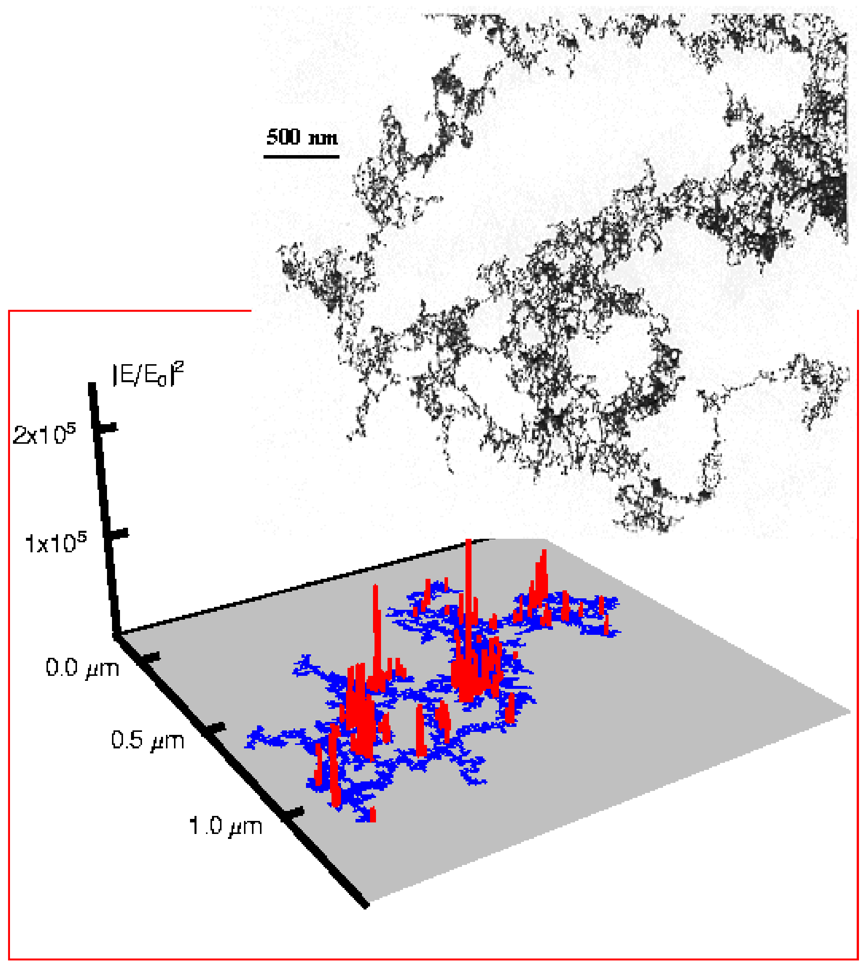
Lycurgus Cup (4th century AD)





Ancient (first?) random metamaterial (carved in Rome) with gold nano particles

"Hot-spots" in fractals



Shalaev, Nonlinear Optics of Random Media, Springer, 2000

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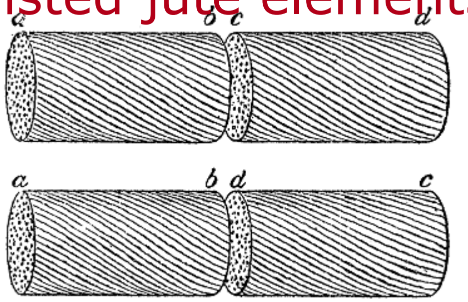
Early (first?) Example of Meta-Atoms

Twisted jute elements



Artificial chiral molecules

In order to imitate the rotation produced by liquids like sugar solutions, I made small elements or "molecules" of twisted jute, of two varieties, one kind being twisted to the right (positive) and the other twisted to the left (negative). I now interposed a number of, say, the positive variety, end to end, between the crossed polariser and analyser; this produced a restoration of the field. The same was the case with the negative variety. *I now mixed equal numbers of the two varieties, and there was now no restoration of the field, the rotation produced by one variety being counteracted by the opposite rotation produced by the other.*

Jagadis C. Bose, *Proceeding of Royal Soc. London*, 1898

"On the Rotation of Plane of Polarization of Electric Waves by a Twisted Structure"

Early Electric Metamaterial: Artificial Dielectrics

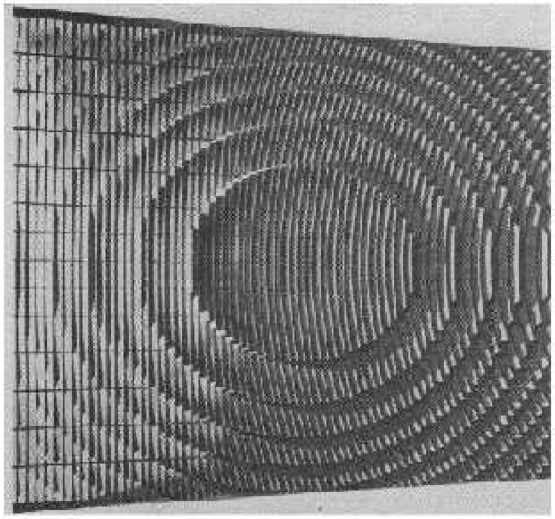
Periodic metal-dielectric plates with effective index of less than 1

828 *Proceedings of the I.R.E. and Waves and Electrons* *November*

Metal-Lens Antennas*

WINSTON E. KOCK†, SENIOR MEMBER, I.R.E.

velocity.[1] This same property is acquired by waves confined between conducting plates which are parallel to the electric vector and spaced apart a distance greater than one half wavelength. A row of such parallel plates accordingly constitutes a refractive medium with index of refraction less than unity. Such a medium, when cut to the proper profile, can be used to produce a focusing or lens effect in a manner similar to that of a dielectric lens.



W. E. Kock, *Proc. IRE*, Vol. 34, 1946

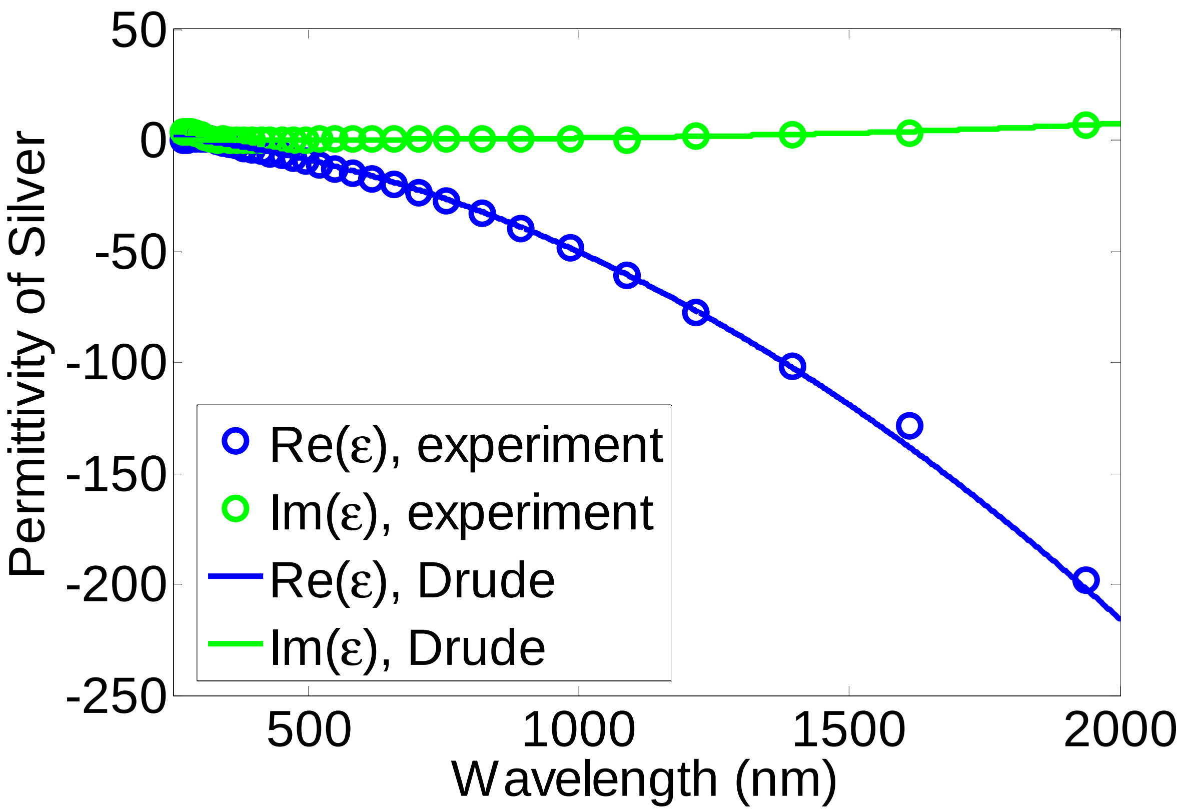
Noble metal: ε < 1 in nature

Drude model for permittivity:

$$\varepsilon(\omega) = \varepsilon_0 - \frac{\omega_p^2}{\omega(\omega + i\Gamma)}$$

Silver parameters:

$$\varepsilon_0 = 5.0$$
$$\omega_p = 9.216 \quad eV$$
$$\Gamma = 0.0212 \quad eV$$

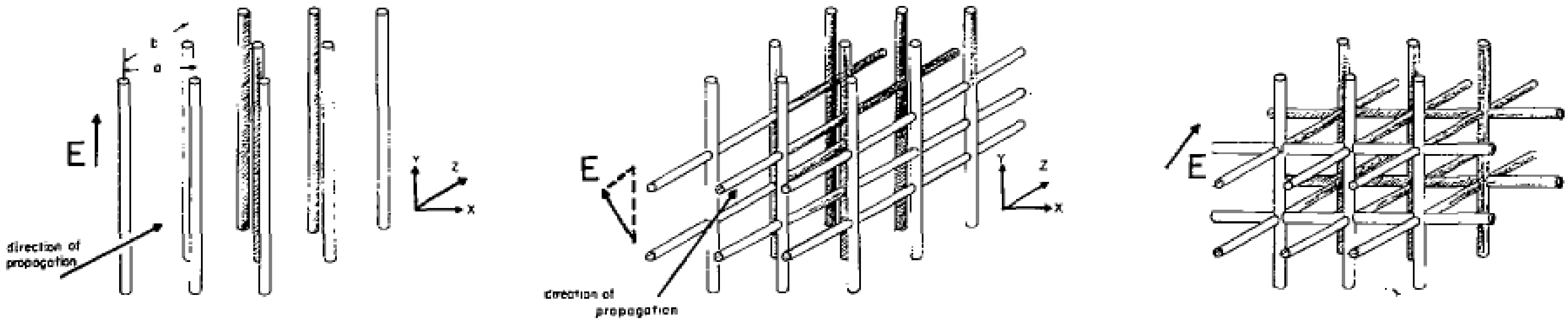


Experimental data from Johnson & Christy, PRB, 1972

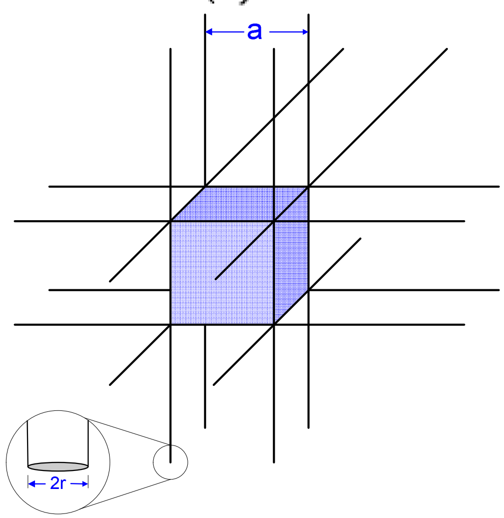
Array of Thin Wires and Tunable Plasma Frequency

1962 IRE TRANSACTIONS ON ANTENNAS AND PROPAGATION 83

Plasma Simulation by Artificial Dielectrics and Parallel-Plate Media*

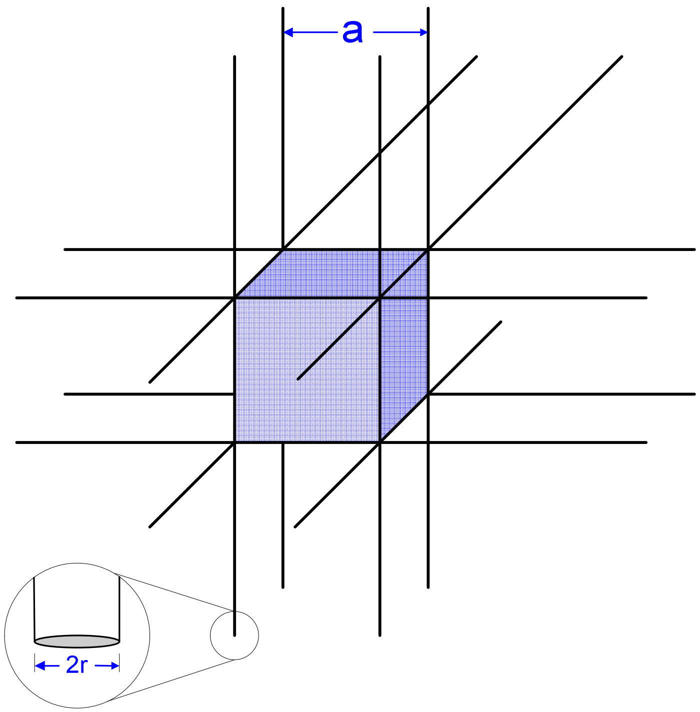


(a) (b) (c)



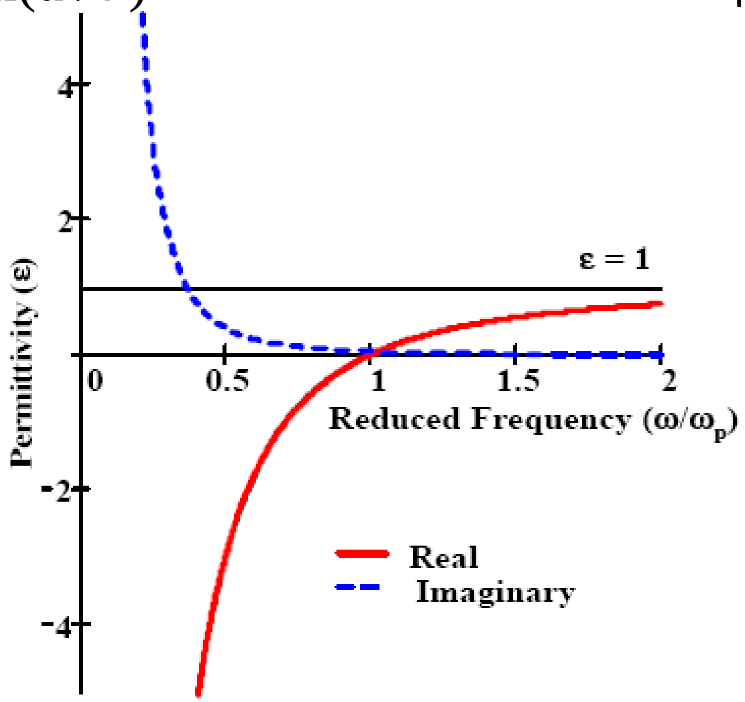
J. Brown, Proc. IEE 100 (1953)
W. Rotman, Trans. IRE AP 10 (1962)
J.B. Pendry, et al., Phys. Rev. Lett. (1996)

Electrical metamaterials:
metal wires arrays with tunable plasma frequency



$$\varepsilon = \varepsilon' + i\varepsilon'' = 1 - \frac{\omega_p^2}{\omega(\omega + i\varepsilon_0 a^2 \omega_p^2 / \pi r^2 \sigma)}$$

$$\omega_p^2 = \frac{2\pi c^2}{a^2 \ln(a/r)}$$



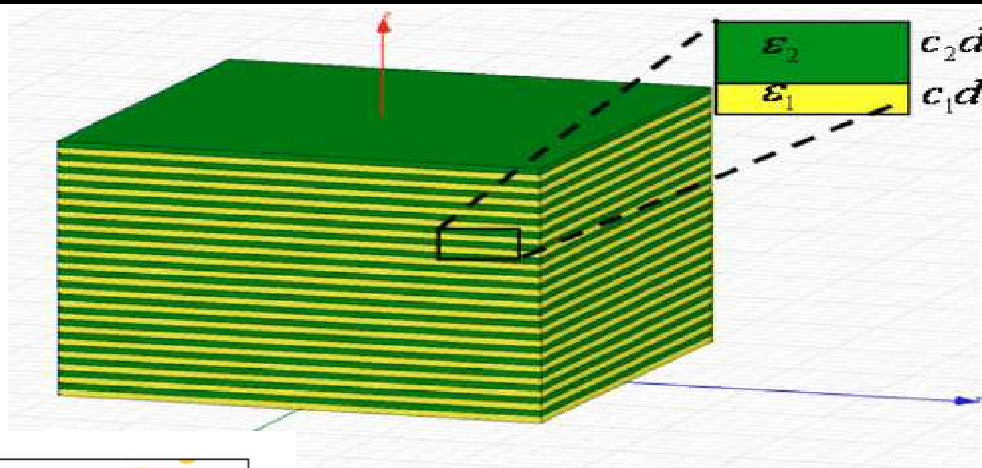
A periodic array of thin metal wires with r<<a<<λ acts as a low frequency plasma

The effective ε is described with modified ω_p

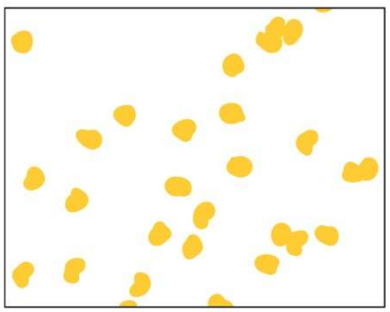
Plasma frequency depends on geometry rather than on material properties

Pendry, PRL (1996)

Metal-Dielectric Composites and Mixing Rules



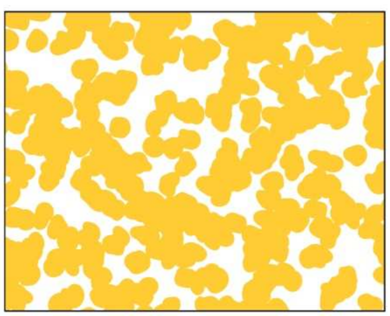
$$\begin{cases} \varepsilon_{\parallel} = c_1\varepsilon_1 + c_2\varepsilon_2 \\ \varepsilon_{\perp} = \varepsilon_1\varepsilon_2 / \left(c_1\varepsilon_2 + c_2\varepsilon_1\right) \end{cases}$$



Maxwell-Garnett (MG) theory:

$$\frac{\varepsilon_{MG}(\omega) - \varepsilon_h(\omega)}{\varepsilon_{MG}(\omega) + 2\varepsilon_h(\omega)} = f\,\frac{\varepsilon_i(\omega) - \varepsilon_h(\omega)}{\varepsilon_i(\omega) + 2\varepsilon_h(\omega)}$$

f « 1



Effective-Medium Theory (EMT):

$$f\,\frac{\varepsilon_m - \varepsilon_{eff}}{\varepsilon_m + (d-1)\varepsilon_{eff}} + (1-f)\frac{\varepsilon_d - \varepsilon_{eff}}{\varepsilon_d + (d-1)\varepsilon_{eff}} = 0$$

Composites with “elongated” inclusions

Depolarization factor:

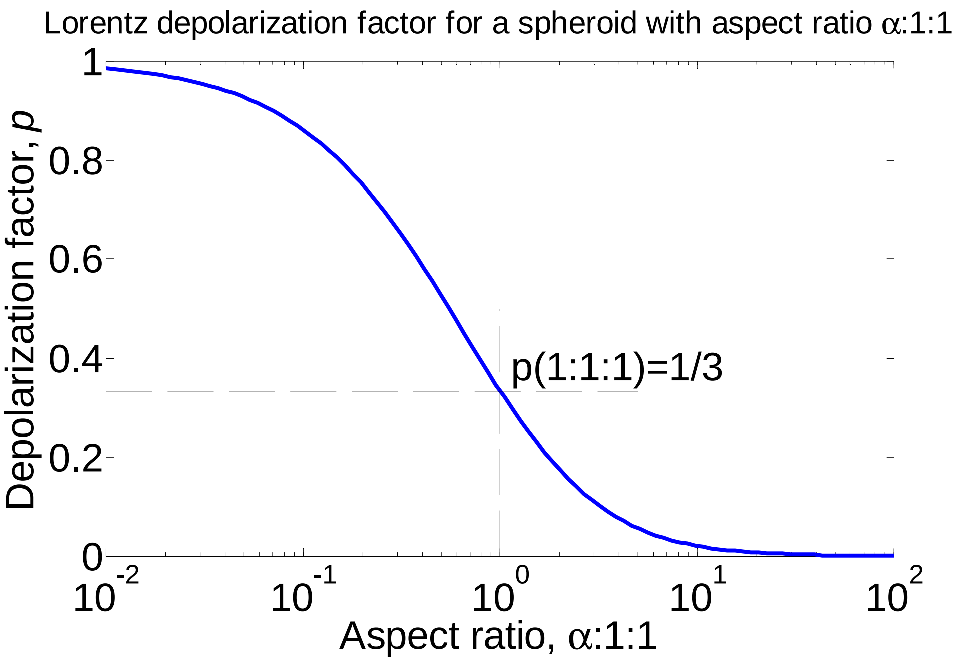
$$q_i = \int_0^\infty \frac{a_i a_j a_k ds}{2(s+a_i^2)^{3/2}(s+a_j^2)^{1/2}(s+a_k^2)^{1/2}}$$

Screening factor:

$$\kappa = (1-q)/q$$

Clausius-Mossotti yields shape-dependent EMT:

$$f\frac{\varepsilon_m - \varepsilon_{eff}}{\varepsilon_m + \kappa\varepsilon_{eff}} + (1-f)\frac{\varepsilon_d - \varepsilon_{eff}}{\varepsilon_d + \kappa\varepsilon_{eff}} = 0$$



$$\varepsilon_{eff} = \frac{1}{2\kappa}\left\{\bar{\varepsilon} \pm \sqrt{\bar{\varepsilon}^2 + 4\kappa\varepsilon_m\varepsilon_d}\right\}$$

$$\bar{\varepsilon} = [(\kappa+1)f - 1]\varepsilon_m + [\kappa - (\kappa+1)f]\varepsilon_d$$

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Absence (or very weak: μ≈1) Optical Magnetism in Nature

Magnetic coupling to an atom: ~ $\mu_B = e\hbar / 2m_e c = \alpha e a_0$ ***(Bohr magneton)***

Electric coupling to an atom: ~ $e a_0$

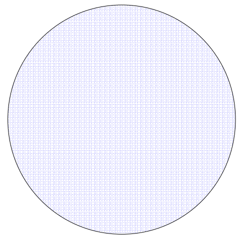
Magnetic effect / electric effect $\approx \alpha^2 \approx (1/137)^2 < 10^{-4}$

"… the magnetic permeability μ(ω) ceases to have any physical meaning at relatively low frequencies…there is certainly no meaning in using the magnetic susceptibility from optical frequencies onwards, and in discussion of such phenomena we must put μ=1."

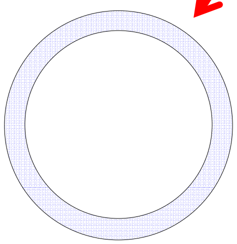
Landau and Lifshitz, ECM, Chapter 79.

SRRs: first magnetic metamaterials

Make it a ring: loop current induced by H

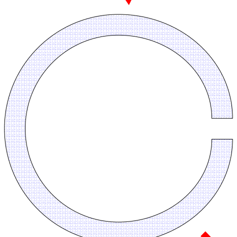


A bulk metal has no magnetism in optics



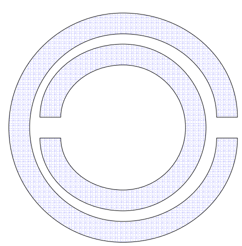
A metal ring: weak magnetic response

Cut the ring to introduce resonance



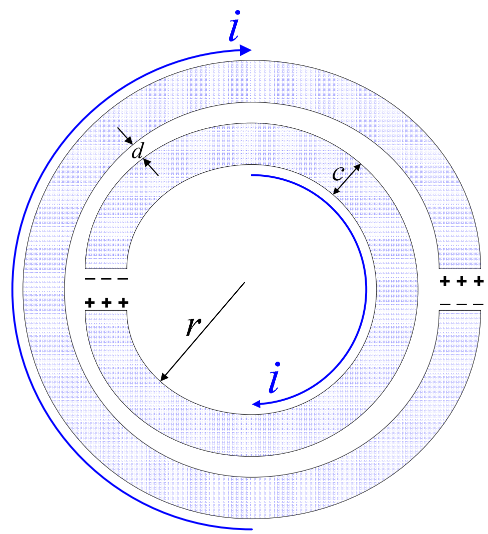
A split ring: magnetic resonance

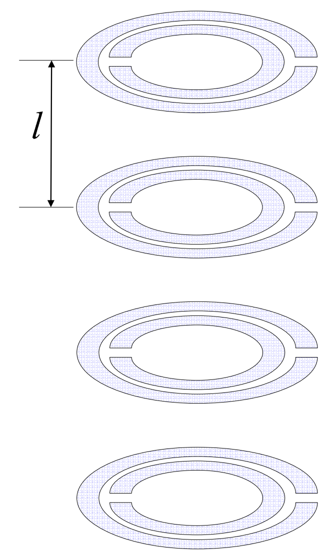
Double the ring to strengthen the resonance



Double SRR: enhanced magnetic resonance

Split-ring resonator (SRR)





Theory: Pendry et al., 1999.
Experiment: Smith et al., 2000.

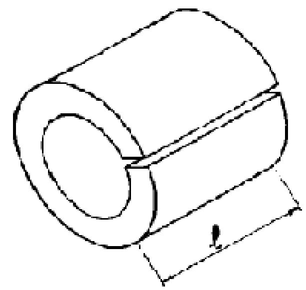
Artificial magnetic resonators: Earlier form and Today's design

SRR for GHz magnetic resonance (Hardy et al., 1981):

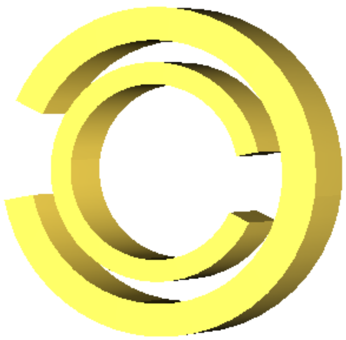
213 Rev. Sci. Instrum. 52(2), Feb. 1981 0034-6748/81/020213-04$00.60 © 1981 American Institute of Physics 213

Split-ring resonator for use in magnetic resonance from 200–2000 MHz

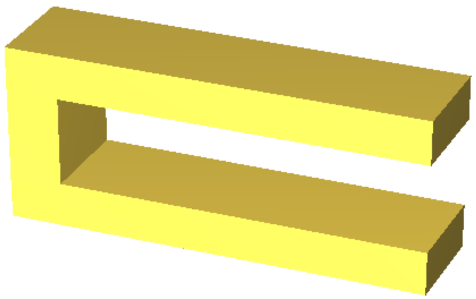
W. N. Hardy and L. A. Whitehead



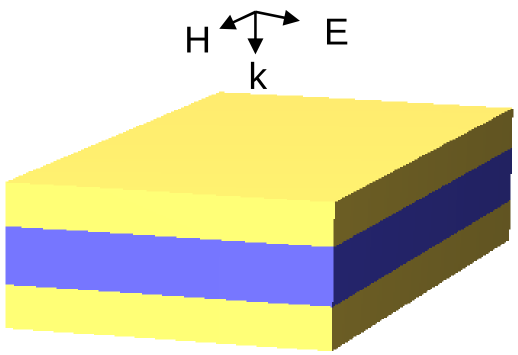
Modern magnetic units for optical metamagnetism:



SRR



C-shaped Rod



Nanostrip (or nanorod) Pair

Limits of size scaling in SRRs

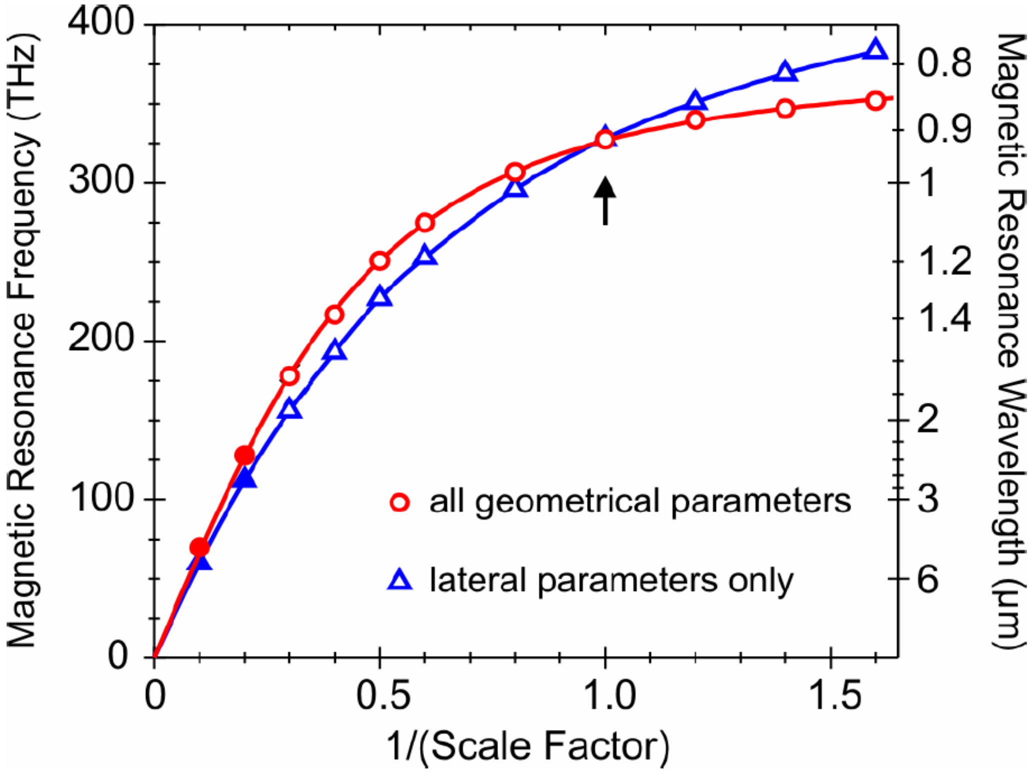
Direct scaling-down the SRR dimensions doesn't help much…

Loss in metal gives kinetic inductance

$$L_{coil} \propto size \qquad L_{kinetic} \propto \frac{1}{size}$$

$$L_{total} = L_{coil} + L_{kinetic}$$

$$C_{total} \propto size$$



$$\omega_{res} \propto \frac{1}{\sqrt{L_{total} \times C_{total}}} = \frac{1}{\sqrt{(A \cdot size + B / size) \cdot (C \cdot size)}} \propto \frac{1}{\sqrt{size^2 + const.}}$$

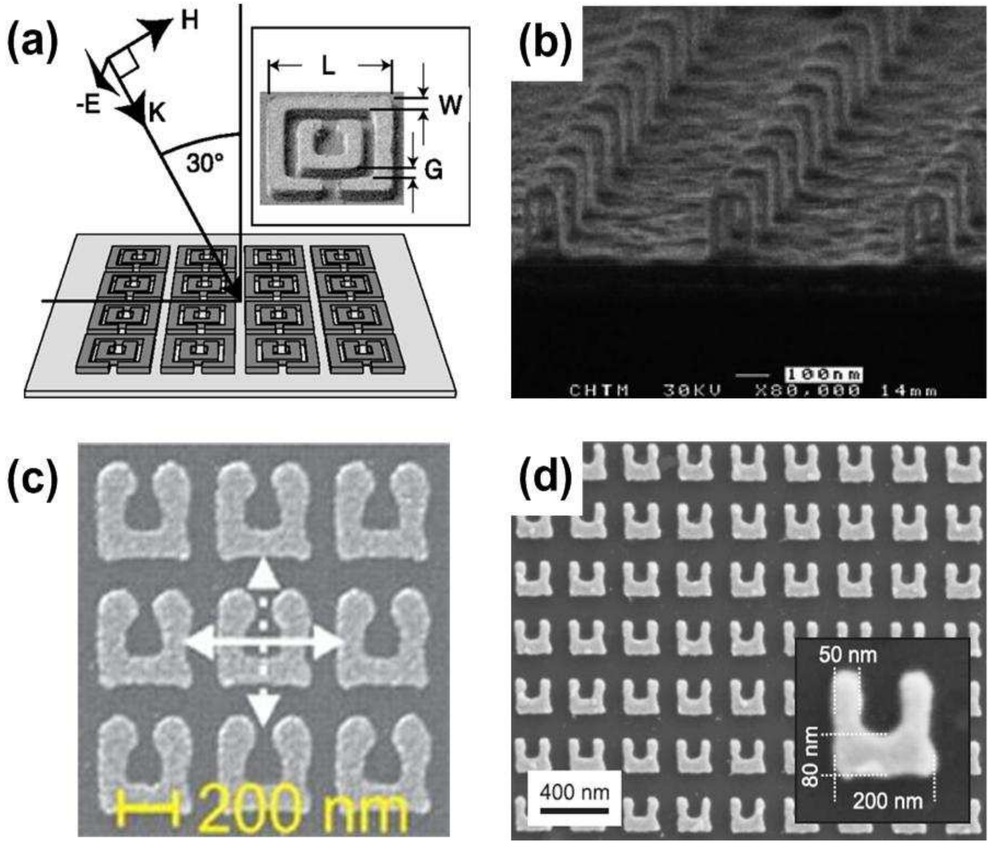
Saturation

Zhou et al, PRL (2005); Klein, et al., OL (2006)

Progress in Optical Magnetism Metamaterials

Terahertz magnetism

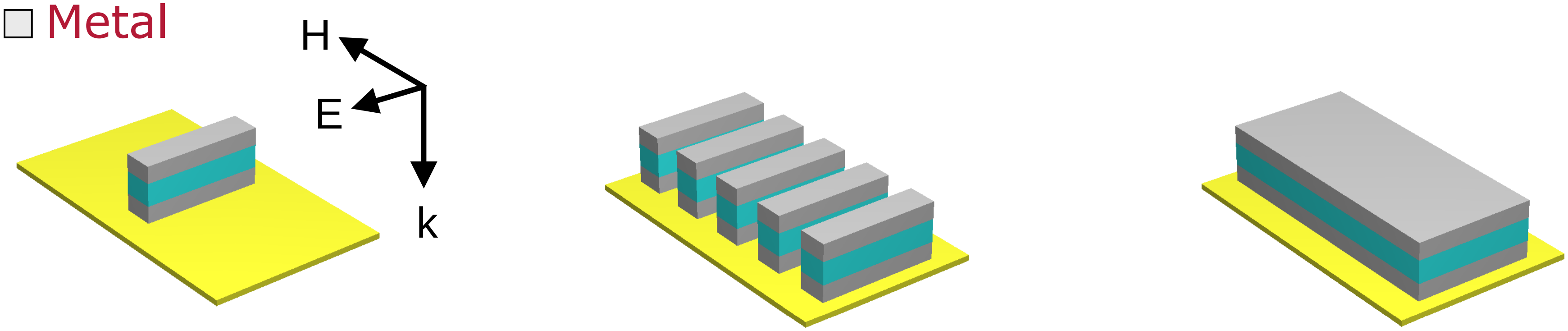
a) Yen, et al. ~ 1THz (2-SRR) – 2004
Katsarakis, et al (SRR – 5 layers) - 2005
b) Zhang et al ~50THz (SRR+mirror) - 2005
c) Linden, et al. 100THz (1-SRR) -2004
d) Enkrich, et al. 200THz (u-shaped)-2005



2004-2007 years:
from 10 GHz to 500 THZ

Magnetic Metamaterial: Nanorod to Nanostrip

- Dielectric
- Metal



Nanorod pair

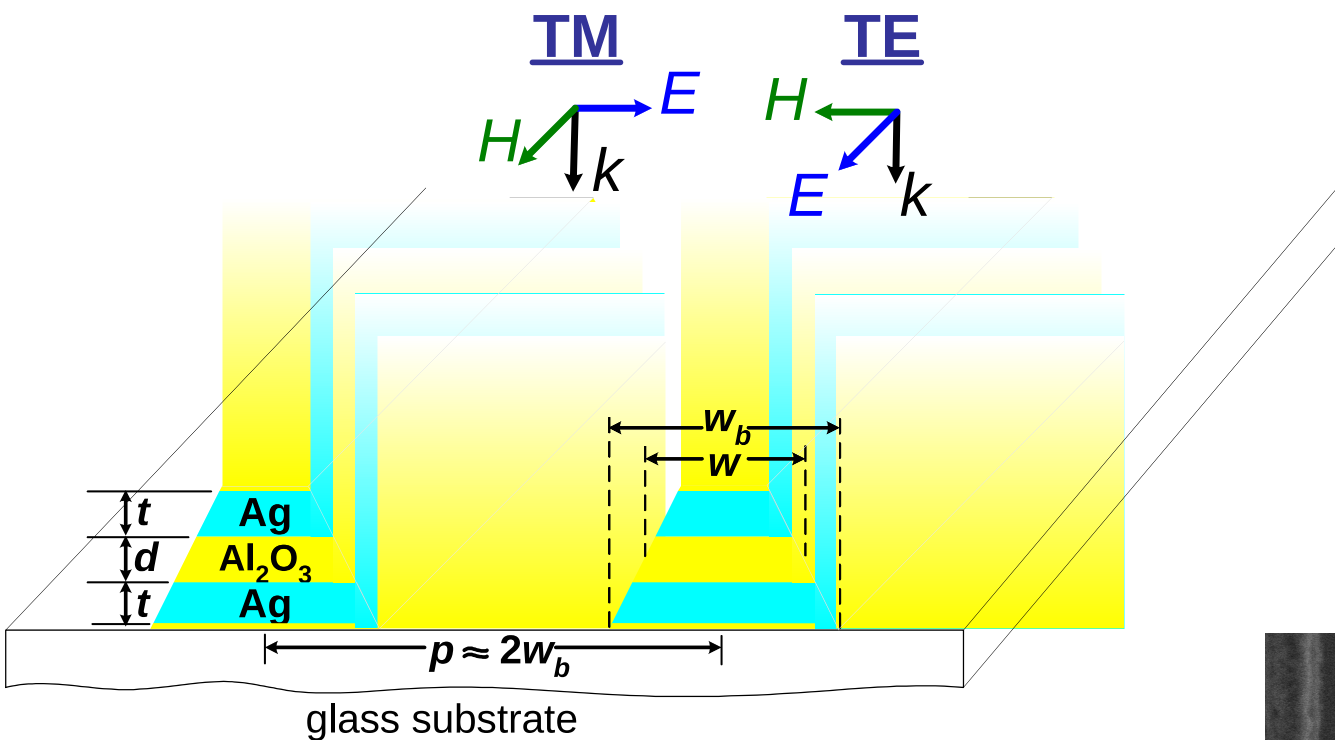
Nanorod pair array

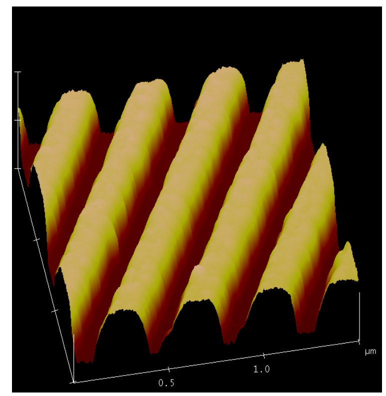
Nanostrip pair

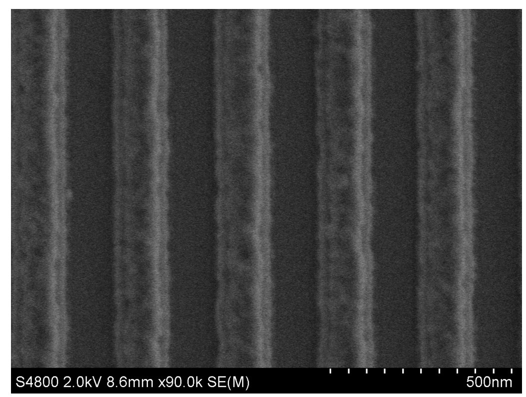
Nanostrip pair has a much stronger magnetic response

Lagar'kov, Sarychev PRB (1996) - μ > 0
Podolskiy, Sarychev & Shalaev, *JNOPM* (2002) - μ < 0 & n < 0
Kildishev et al, *JOSA B* (2006); Shvets et al JOSA (2006) – strip pairs
(Svirko, et al, APL (2001) - "crossed" rods for chirality)

Visible magnetism: structure and geometries





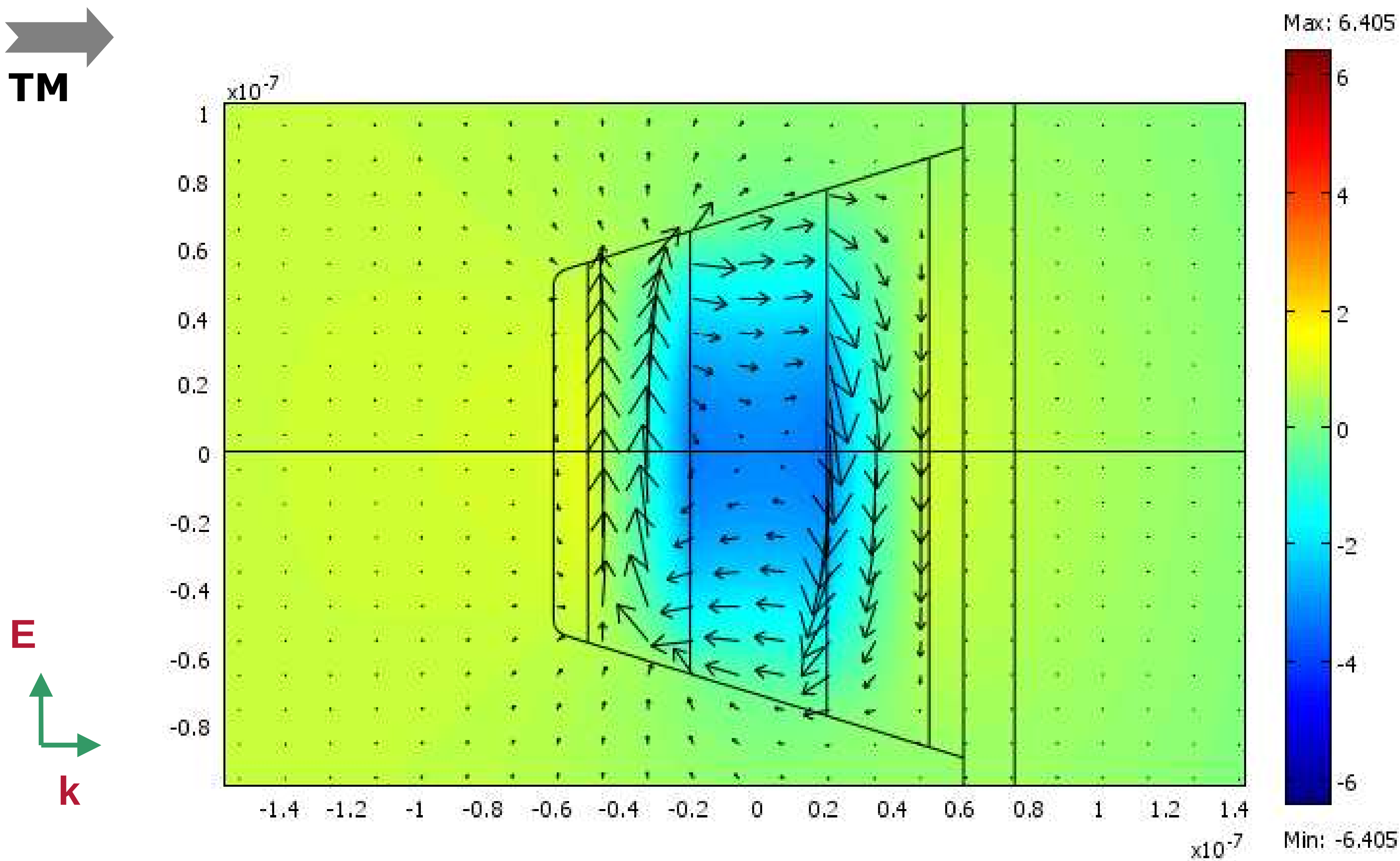


$t = 35\ nm \quad d = 40\ nm \quad p \approx 2w_b$

Width varies from 50 nm to 127 nm

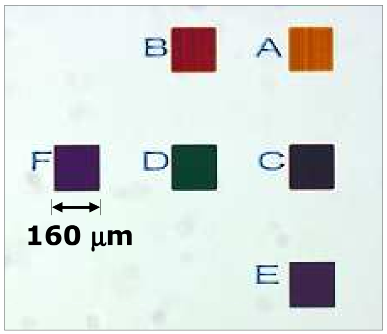
Purdue group
Yuan, et al., Opt. Expr., 2007 – red light
Cai, et al., Opt. Expr., 2007 – all the visible

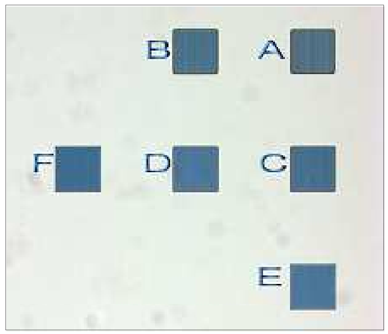
Negative Magnetic Response

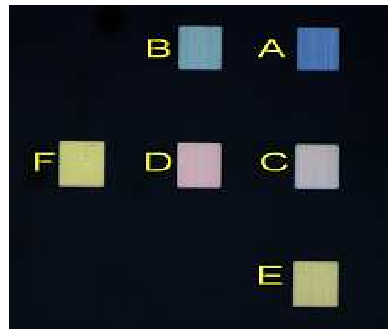


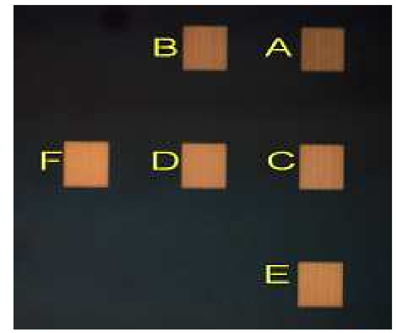
Magnetic Colors: visualizing magnetism

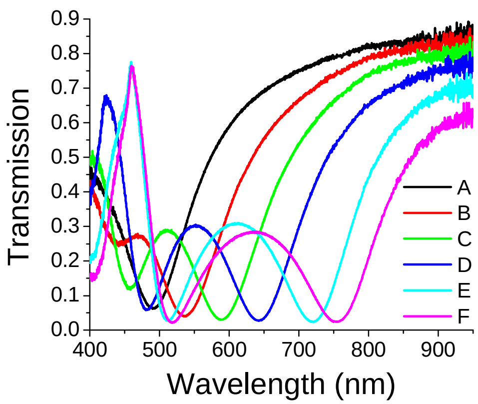
Resonant TM Transmission	***Non-resonant TE Transmission***	***Resonant TM Reflection***	***Non-resonant TE Reflection***

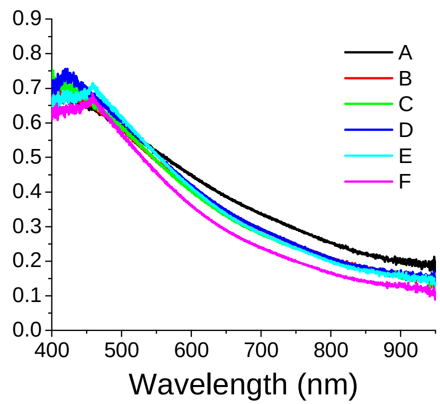


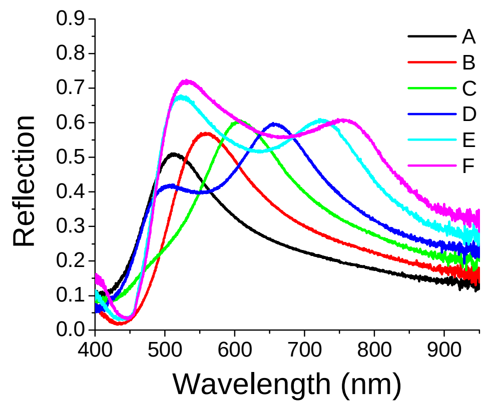


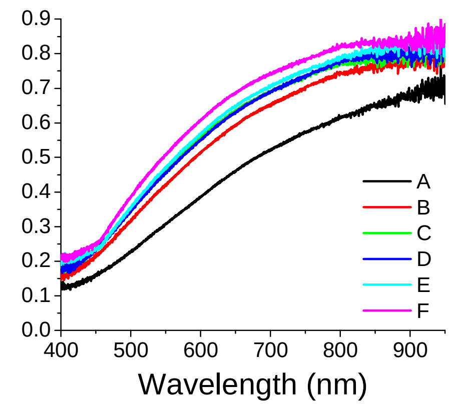








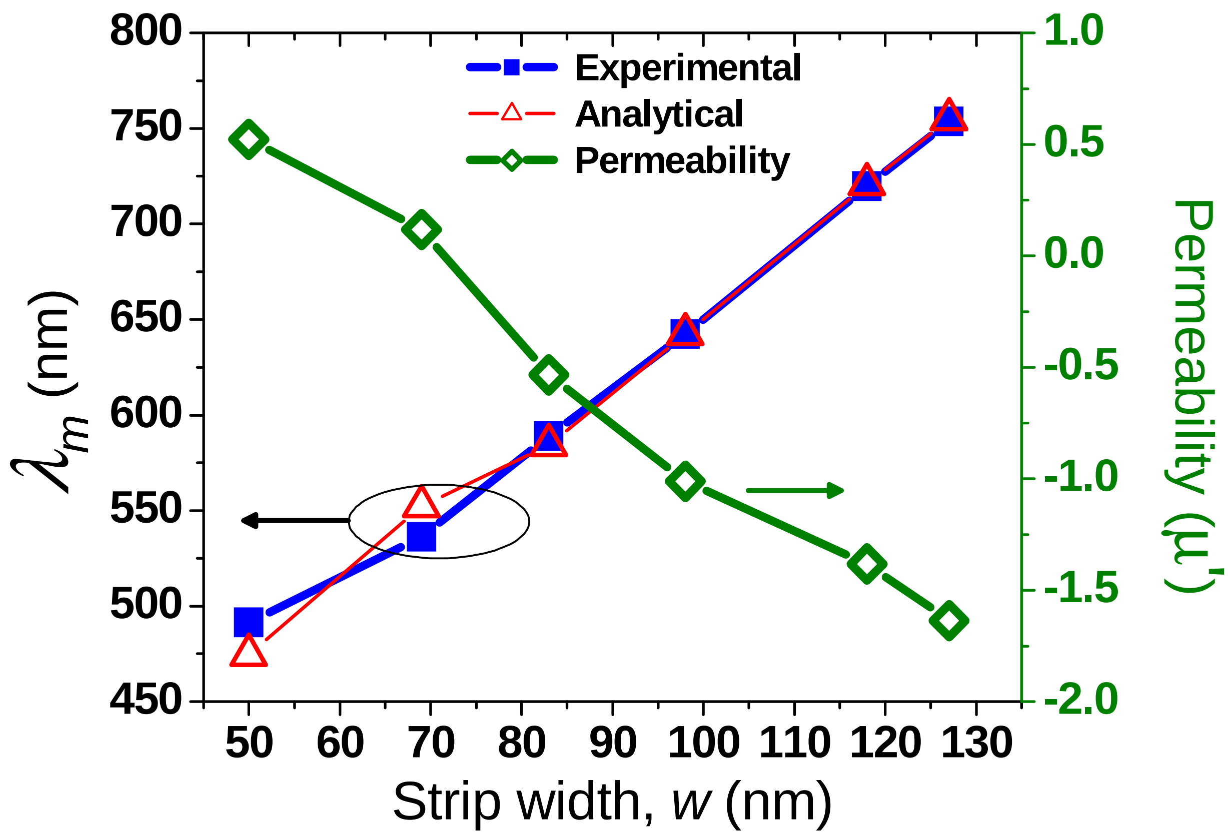




Sample #	*A*	*B*	*C*	*D*	*E*	*F*
Width *w (nm)*	**50**	**69**	**83**	**98**	**118**	**127**

Cai, et al., Opt. Expr., 15, 3333 (2007)

Meta-magnetism across the visible



λ_m as a function of strip width "w": experiment vs. theory

<u>Negligible saturation</u> effect on size-scaling (as opposed to SRRs)

Transforming Light with Metamaterials

Part 1: Electrical and Magnetic Metamaterials

<u>Part 2: Negative-Index Metamaterials, NLO, and super/hyper-lens</u>

Part 3: Cloaking and Transformation Optics

Outline

- What are metamaterials?
- Early electrical metamaterials
- Magnetic metamaterials
- Negative-index metamaterials
- Chiral metamaterials
- Nonlinear optics with metamaterials
- Super-resolution
- Optical cloaking

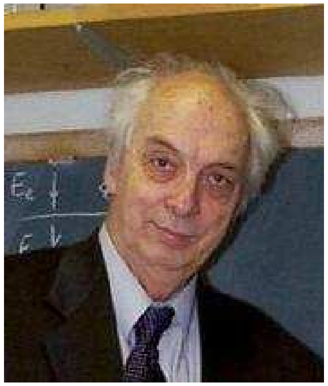
Negative refractive index: A historical review

Sir Arthur Schuster   Sir Horace Lamb

… energy can be carried forward at the group velocity but in a direction that is anti-parallel to the phase velocity…
Schuster, 1904

Negative refraction and backward propagation of waves
Mandel'stam, 1945

L. I. Mandel'stam



V. G. Veselago

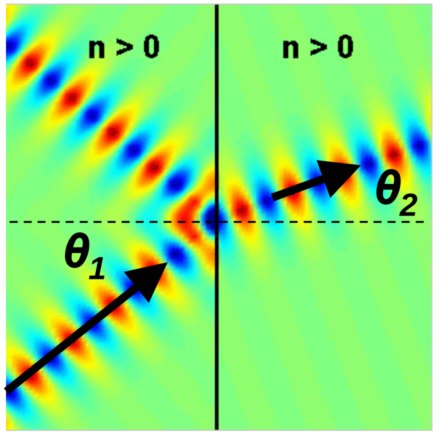
Left-handed materials: the electrodynamics of substances with simultaneously negative values of ε and μ
Veselago, 1968

Sir John Pendry

Pendry, the one who whipped up the recent boom of NIM researches

Perfect lens (2000)
EM cloaking (2006)

Metamaterials with Negative Refraction



Refraction:

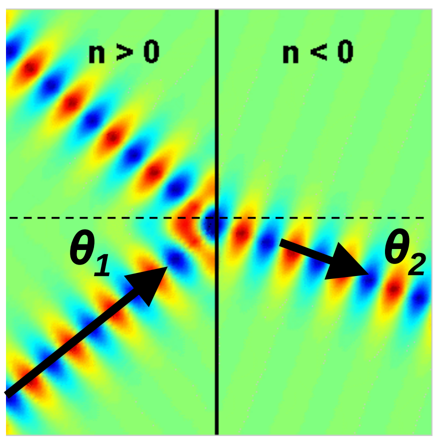
$$n^2 = \varepsilon\mu$$

$$n = \pm\sqrt{\varepsilon\mu}$$

Figure of merit

$F = |n'|/n''$

$n < 0, \quad \text{if } \varepsilon'|\mu| + \mu'|\varepsilon| < 0$



<u>Single-negative:</u>

n<0 ***when ε′ < 0 whereas μ′ > 0 (F is low)***

<u>Double-negative:</u>

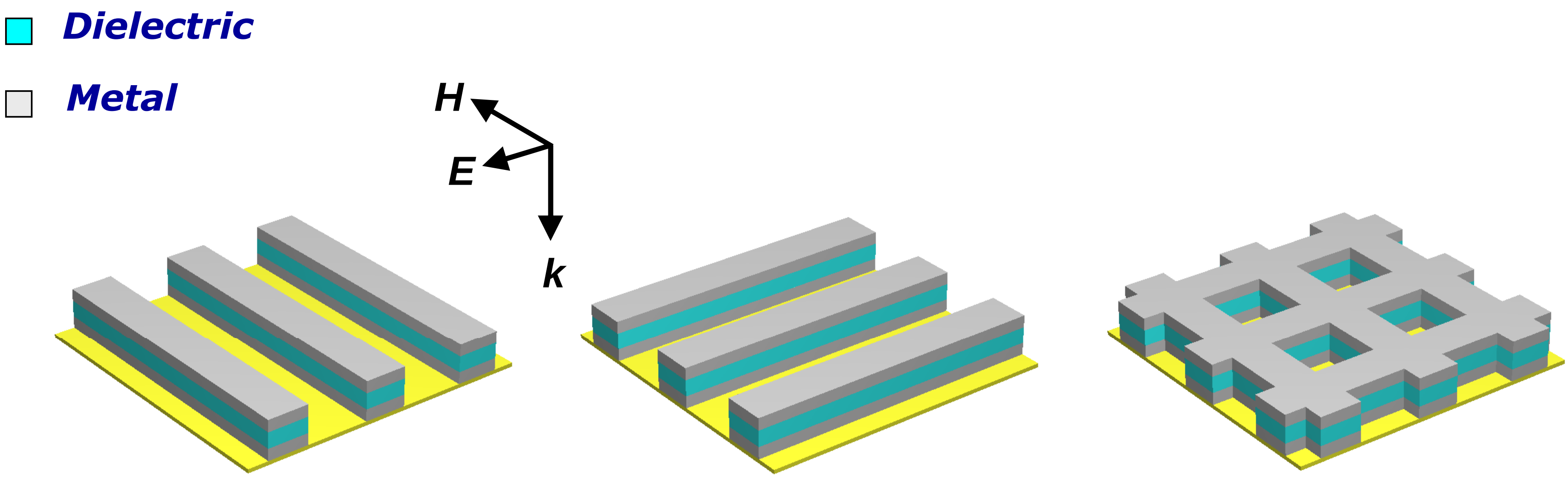
n<0 ***with both ε′ < 0 and μ′ < 0 (F can be large)***

Negative Refractive Index in Optics: State of the Art

Year and Research group	*1st time posted and publication*	*Refractive index, n'*	*Wavelength λ*	*Figure of Merit $F=\|n'\|/n''$*	*Structure used*
<u>*2005:*</u>					
Purdue	**April 13 (2005) arXiv:physics/0504091 Opt. Lett. (2005)**	**–0.3**	**1.5 µm**	**0.1**	**Paired nanorods**
UNM & Columbia	**April 28 (2005) arXiv:physics/0504208 Phys. Rev. Lett. (2005)**	**–2**	**2.0 µm**	**0.5**	**Nano-fishnet with round voids**
<u>*2006:*</u>					
UNM & Columbia	**J. of OSA B (2006)**	**–4**	**1.8 µm**	**2.0**	**Nano-fishnet with round voids**
Karlsruhe & ISU	**OL. (2006)**	**–1**	**1.4 µm**	**3.0**	**Nano-fishnet**
Karlsruhe & ISU	**OL (2006)**	**–0.6**	**780 nm**	**0.5**	**Nano-fishnet**
Purdue	**MRS Bulletin (2008)**	**-0.8 -0.6**	**725nm 710nm**	**1.1 0.6**	**Nano-fishnet**
Purdue	**In preparation (2009)**	**-0.25**	**580nm**	**0.3**	**Nano-fishnet**

CalTech: negative refraction in the visible for MIM waveguide SPPs (2007)

Negative permeability and negative permittivity



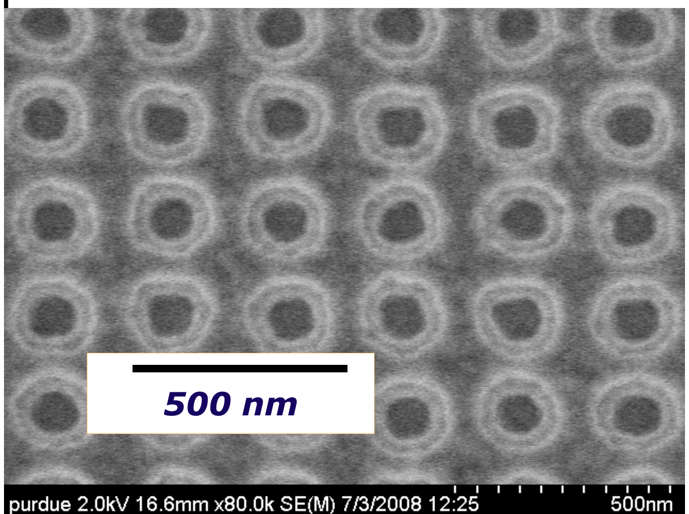
Nanostrip pair (TM)	Nanostrip pair (TE)	Fishnet
$\mu < 0$ (resonant)	$\varepsilon < 0$ (non-resonant)	ε and $\mu < 0$

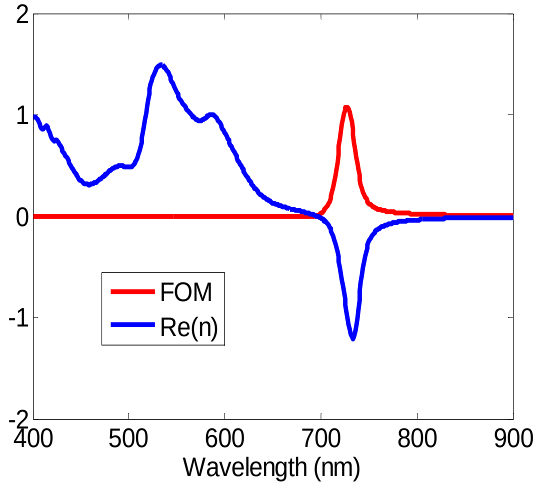
S. Zhang, et al., PRL (2005)

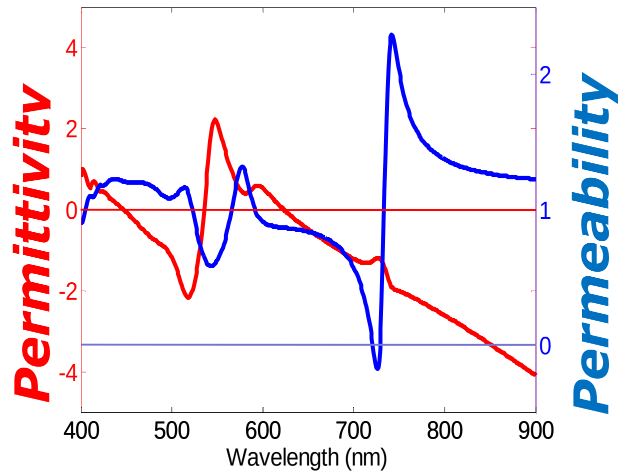
Sample A: Double Negative NIM (n′=-0.8, FOM=1.1, at 725 nm)
Sample B: Single Negative NIM (n′=-0.25, FOM=0.3, at 580 nm)

MRS Bulletin (2008)

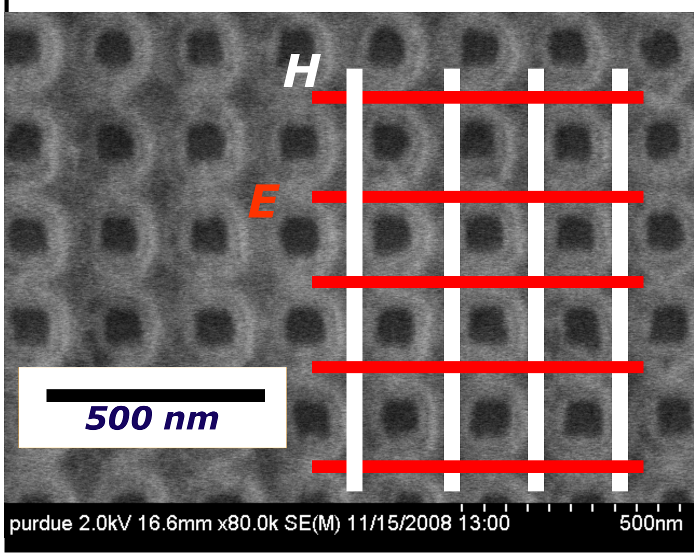
Sample A. period- E: 250 nm; H: 280 nm

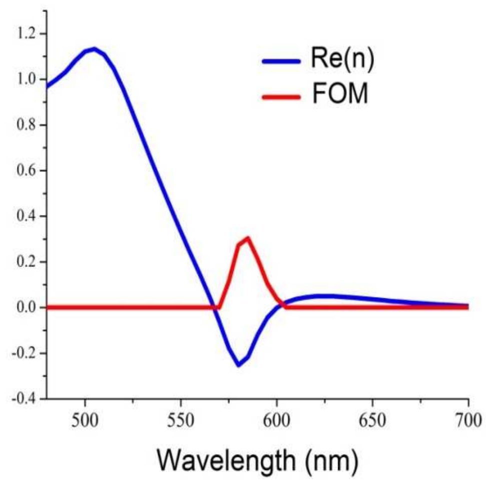


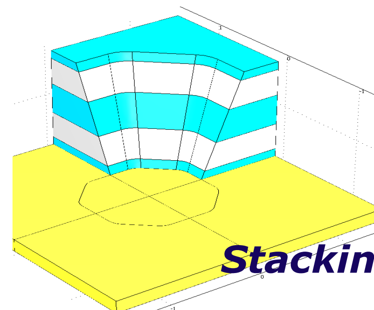




Sample B. period- E:220nm H:220nm







Stacking:

8 nm of Al_2O_3
43 nm of Ag
45 nm of Al_2O_3
43 nm of Ag
8 nm of Al_2O_3

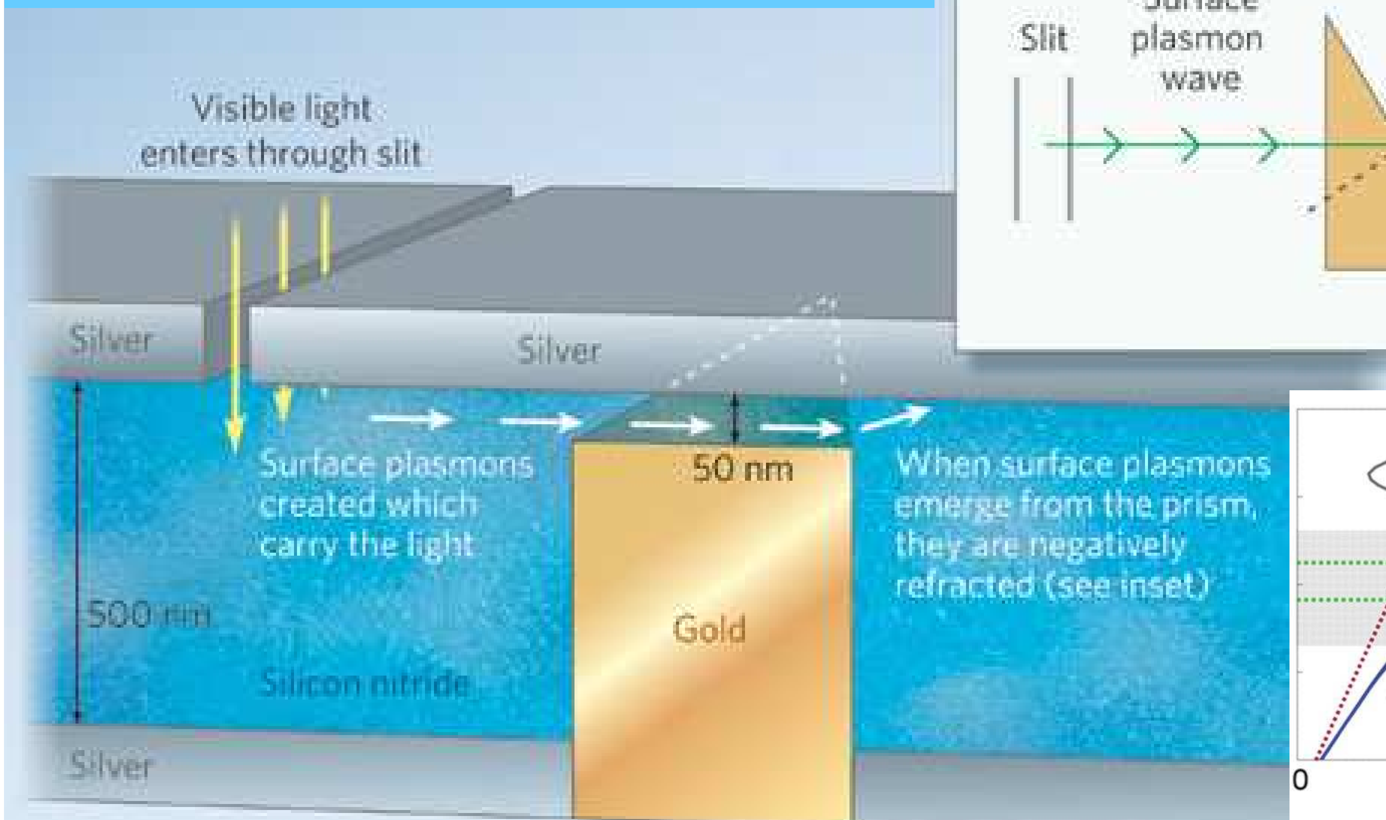
Summary on negative refractive index

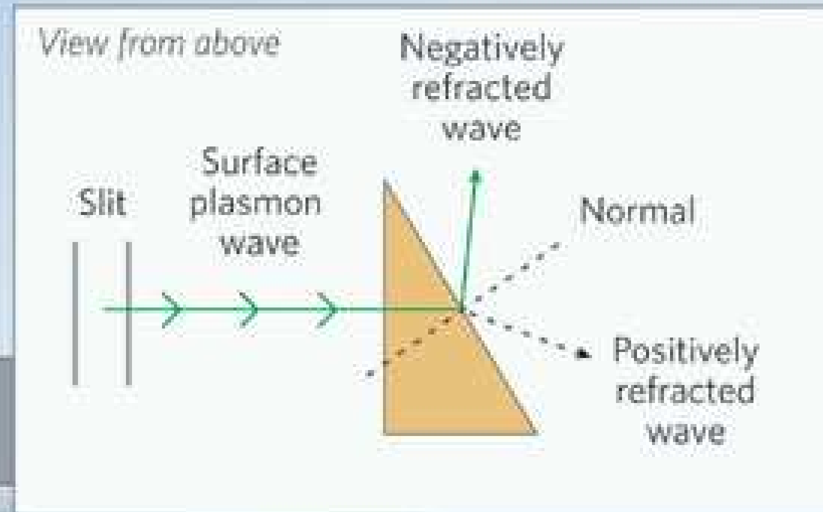
- A Double Negative NIM (Negative index material) is demonstrated at a wavelength of ~725 nm
- A Single Negative NIM behavior is demonstrated at a wavelength of ~580 nm

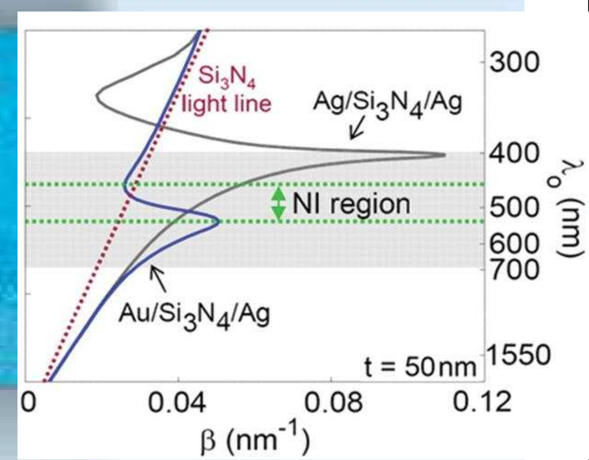
Negative Refraction for Waveguide Modes

n < 0 for 2D SPPs in waveguides

An mode index of ~ -5 is obtained at the green light.







Lezec, Dionne and Atwater, Science, 2007

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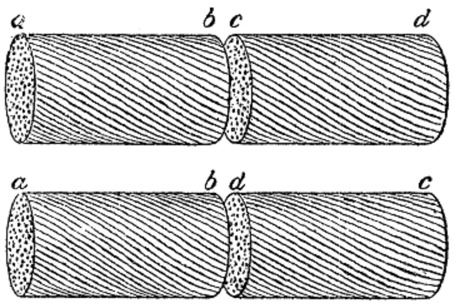
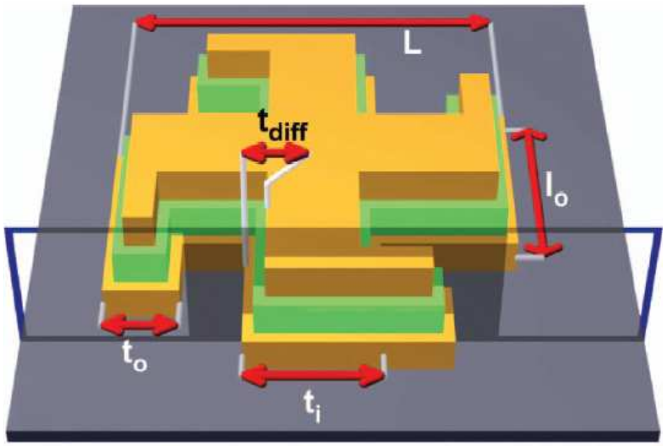
Chiral Optical Elements

Bose's Artificial chiral molecules: Twisted jute elements

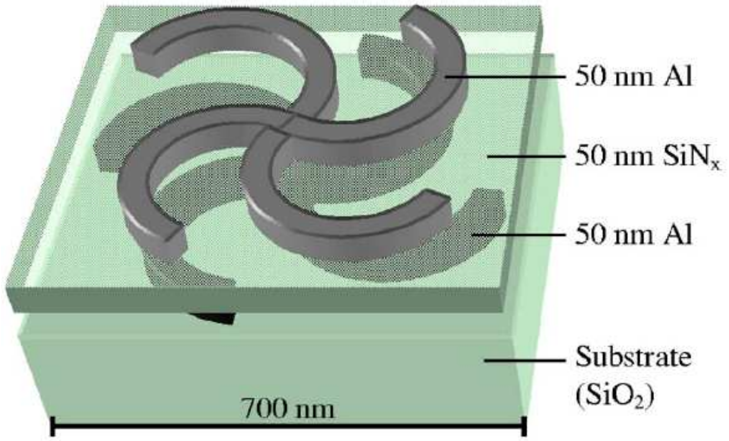


J. C. Bose, *Proceeding of Royal Soc. London*, 1898

Optical counterparts:



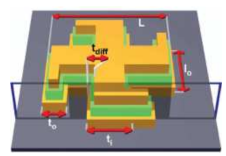
Decher, Klein, Wegener and Linden
Opt. Exp., 2007

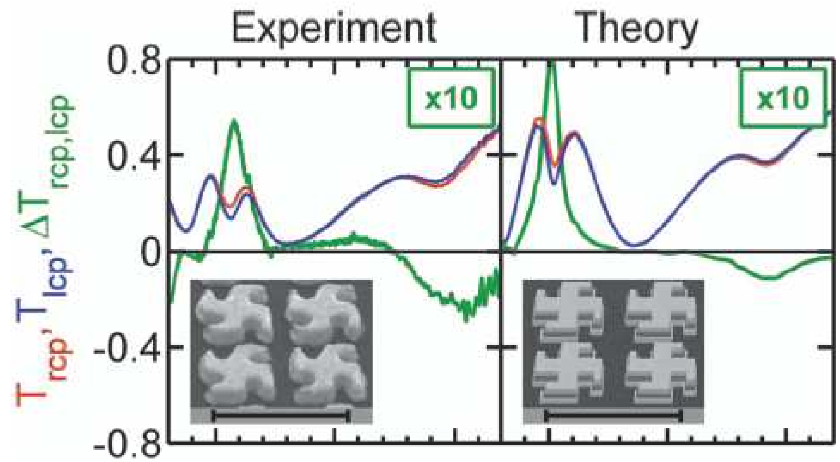


The Zheludev group, U. Southampton
Appl. Phys. Lett., 2007

Chiral Effects in Optical Metamaterials

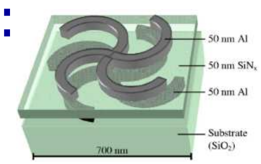
Circular dichroism:

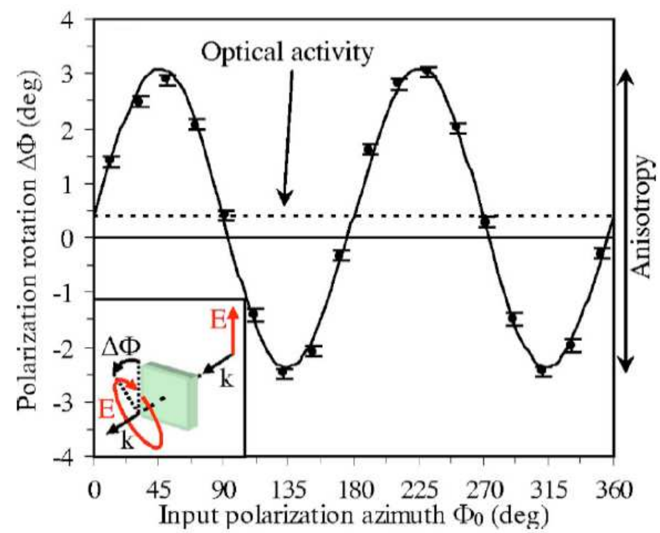




Decher, Klein, Wegener and Linden
Opt. Exp., 2007

Giant optical gyrotropy:





The Zheludev group, U. Southampton
Appl. Phys. Lett., 2007

Chirality can ease obtaining n<0:
Tretyakov, et al (2003), Pendry (Science 2004)

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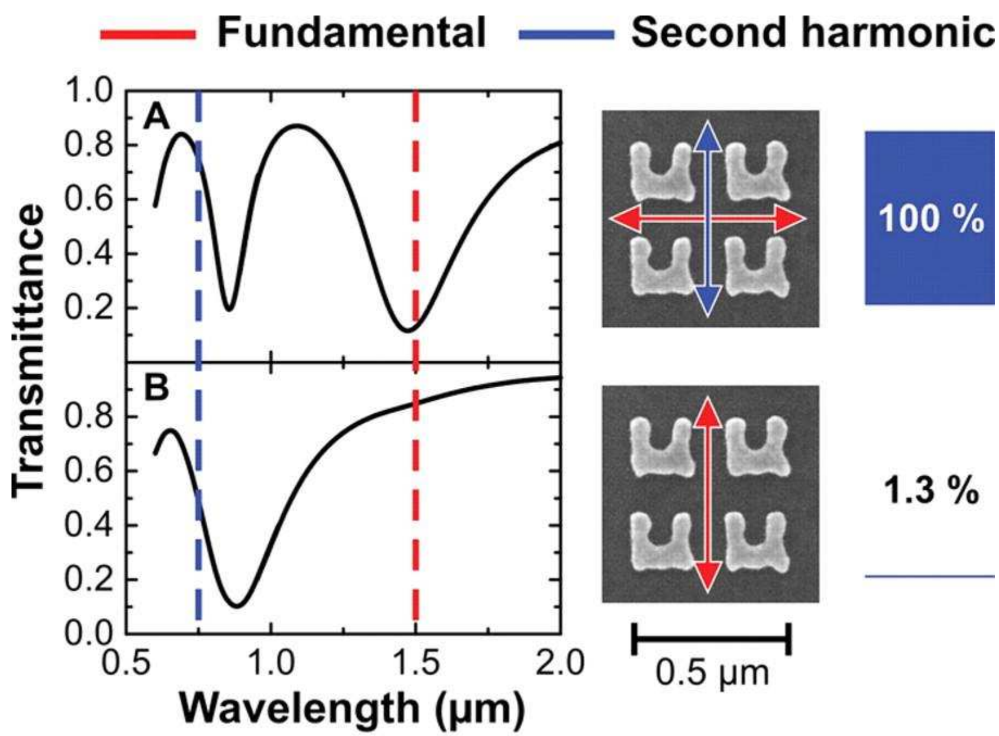
SHG and THG from Magnetic Metamaterial

Excitation when magnetic resonance is excited (1st pol)

Excitation at 2nd pol. (no magnetic resonance)

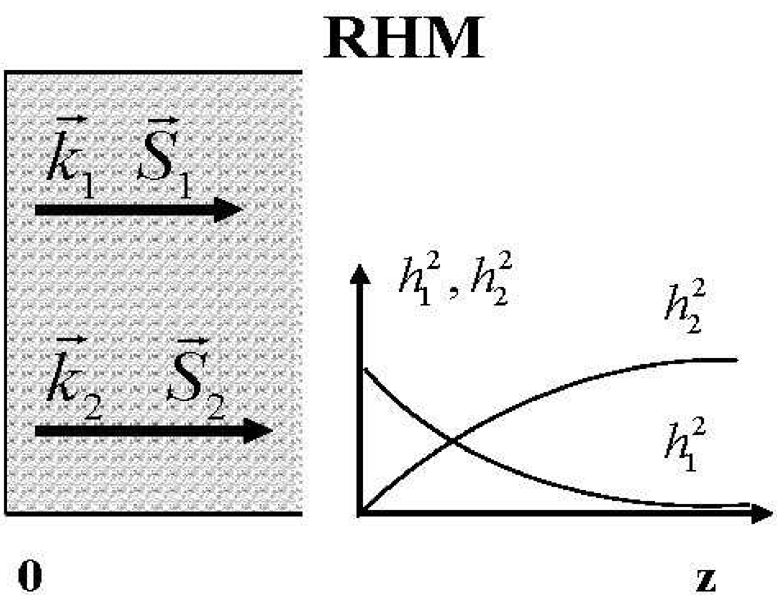


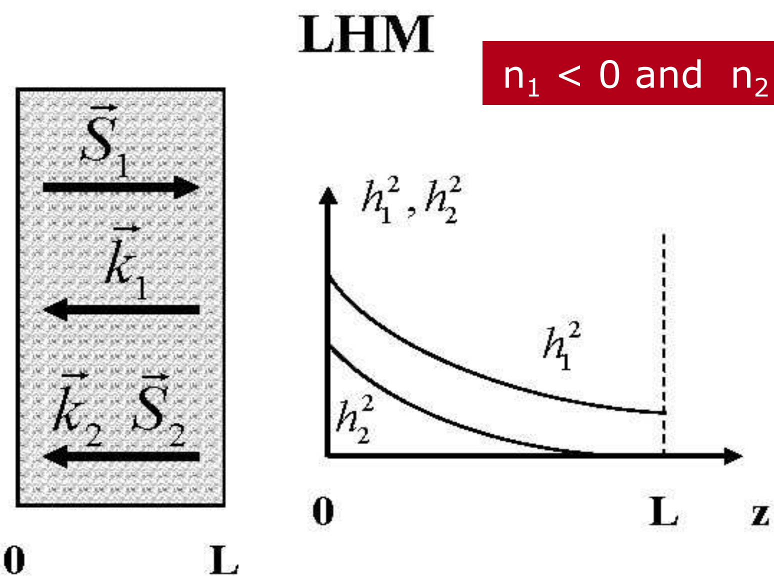
SHG: **Klein, Enkrich, Wegener, and Linden, Science, 2006**
SHG & THG: **Klein, Wegener, Feth and Linden, Opt. Express, 2007**

NLO in NIMs: SHG

Backward Waves in NIMs:

Distributed feedback, cavity-like amplification, etc.





$n_1 < 0$ and $n_2 > 0$

Manley-Rowe Relations

$$\frac{dS_1}{dz} - \frac{dS_2}{dz} = 0,$$

Phase-matching: $\varepsilon_1 = -\varepsilon_2, k_2 = 2k_1 \Rightarrow h_1^2(z) - h_2^2(z) = C$

SHG in NIMs: Nonlinear 100% Mirror

100% reflective SHG Mirror !

$$h_1^2(z) - h_2^2(z) = C$$

Finite Slab:

$$\sqrt{C}\kappa L = \arccos(\sqrt{C} / h_{10})$$

$$h_2(z) = \sqrt{C}\tan[\sqrt{C}\kappa(L-z)]$$

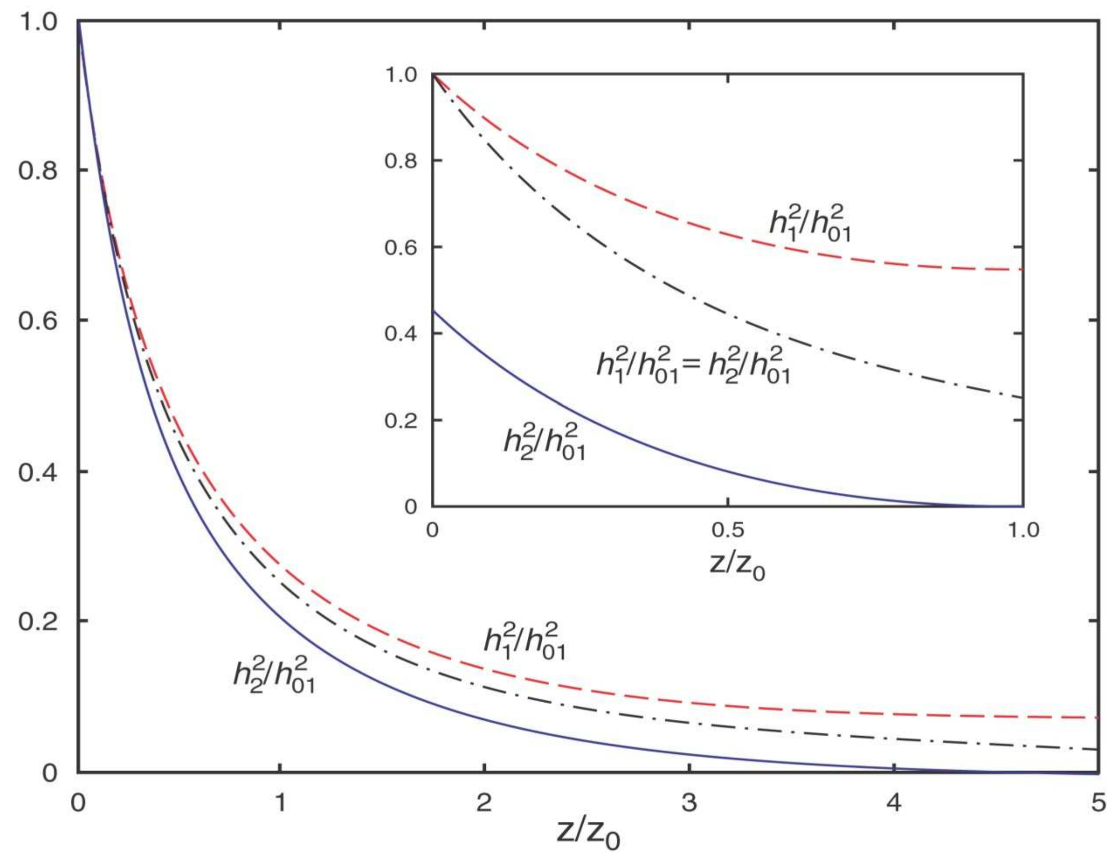
$$h_1(z) = \sqrt{C} / \cos[\sqrt{C}\kappa(L-z)]$$

Semi-Infinite Slab:

$$C = 0, h_2(z) = h_1(z)$$

$$h_2(z) = h_{10} / [1 + (z / z_0)]$$

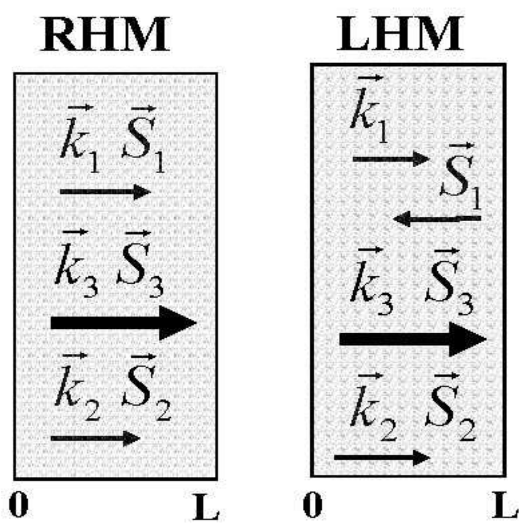
$$\kappa = 4\pi\chi^{(2)}\varepsilon_2\omega_2^2 / k_2c^2 \quad z_0 = [\kappa h_{10}]^{-1}$$

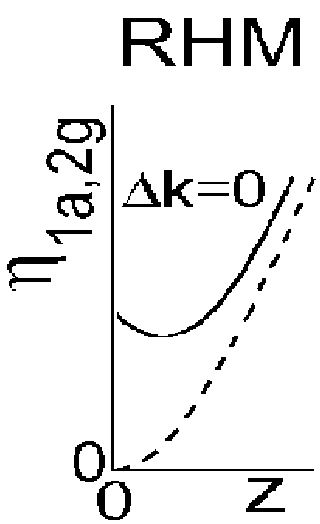


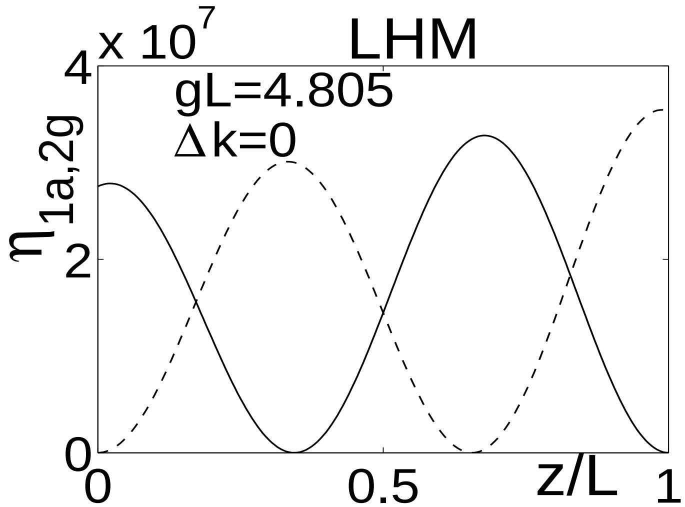
Other work on SHG:
Kivshar et al; Zakhidov et al

Optical Parametric Amplification (OPA) in NIMs

$\omega_3 = \omega_1 + \omega_2$ ($n_1 < 0$, $n_2, n_3 > 0$) S_3 - Control Field (pump)







$$\eta_{1a} = |a_1(z)/a_{1L}|^2, \eta_{1g} = |a_1(z)/a_{20}|^2, \eta_{2g} = |a_2(z)/a_{1L}|^2$$

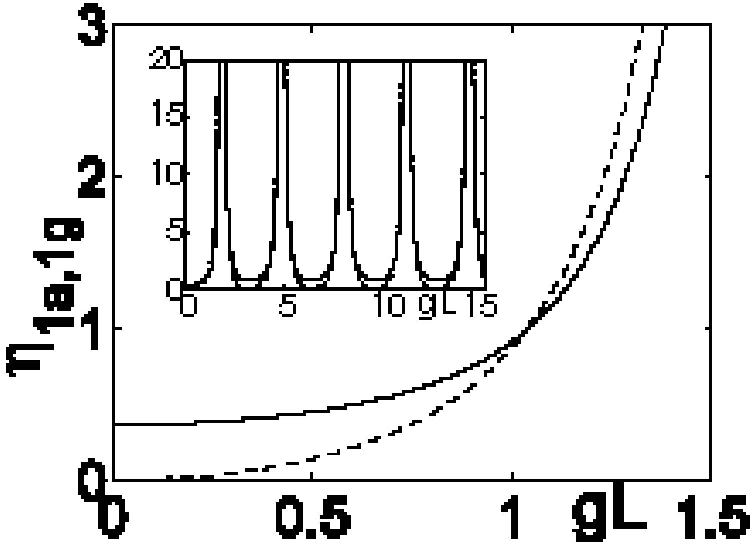
$$g = \left(\sqrt{\omega_1\omega_2}\sqrt[4]{\varepsilon_1\varepsilon_2/\mu_1\mu_2}\right)(8\pi/c)\chi^{(2)}h_3$$

Manley-Rowe Relations:

$$\frac{d}{dz}\left(\frac{S_1}{\hbar\omega_1} - \frac{S_2}{\hbar\omega_2}\right) = 0$$

Popov, VMS, Opt. Lett. (2006)
Appl. Phys. B (2006)
For SHG see also Agranovich et al and Kivshar et al

OPA in NIMs: Loss-Compensator and Cavity-Free Oscillator



Backward waves in NIMs -> Distributed feedback & cavity-like amplification and generation

Popov, VMS, OL (2006)

$$\Delta k = 0$$

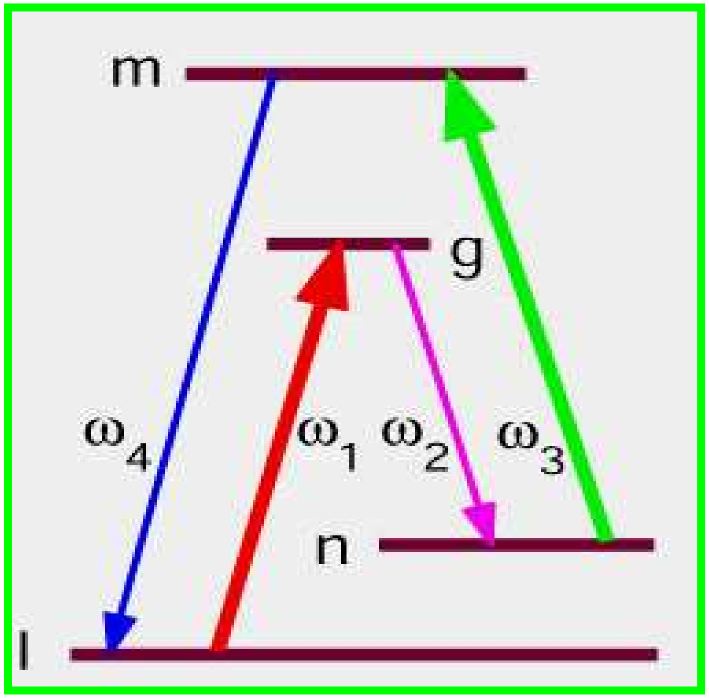
$$g = \left(\sqrt{\omega_1\omega_2}\sqrt[4]{\varepsilon_1\varepsilon_2/\mu_1\mu_2}\right)(8\pi/c)\chi^{(2)}h_3 \qquad \eta_{1a} = \left|a_1(z)/a_{1L}\right|^2, \eta_{1g} = \left|a_1(z)/a_{20}\right|^2$$

Resonances in output amplification and DFG $\alpha_1 L = 1,\ \alpha_2 L = 1/2$

- **OPA-Compensated Losses**
- **Cavity-free (no mirrors) Parametric Oscillations**
- **Generation of Entangled Counter-propagating LH and RH photons**

OPA with 4WM

Four-level $\chi^{(3)}$ centers embedded in NIM



$\chi^{(3)}$ -OPA assisted by the Raman Gain:

ω_4 – signal; ω_1, ω_3 – control fields

$\omega_2 = \omega_1 + \omega_3 - \omega_4$ – idler

(Raman-enhanced; contributes back to OPA at ω_4)

- **$\chi^{(3)}$ -OPA: compensation of losses: transparency and amplification at ω_4**
- **Cavity-free generation of counter-propagating entangled right- and left-handed photons**
- **Control of local optical parameters through quantum interference**

See talk tomorrow by Popov et al on NLO in MMs

Popov, et al OL (2007)

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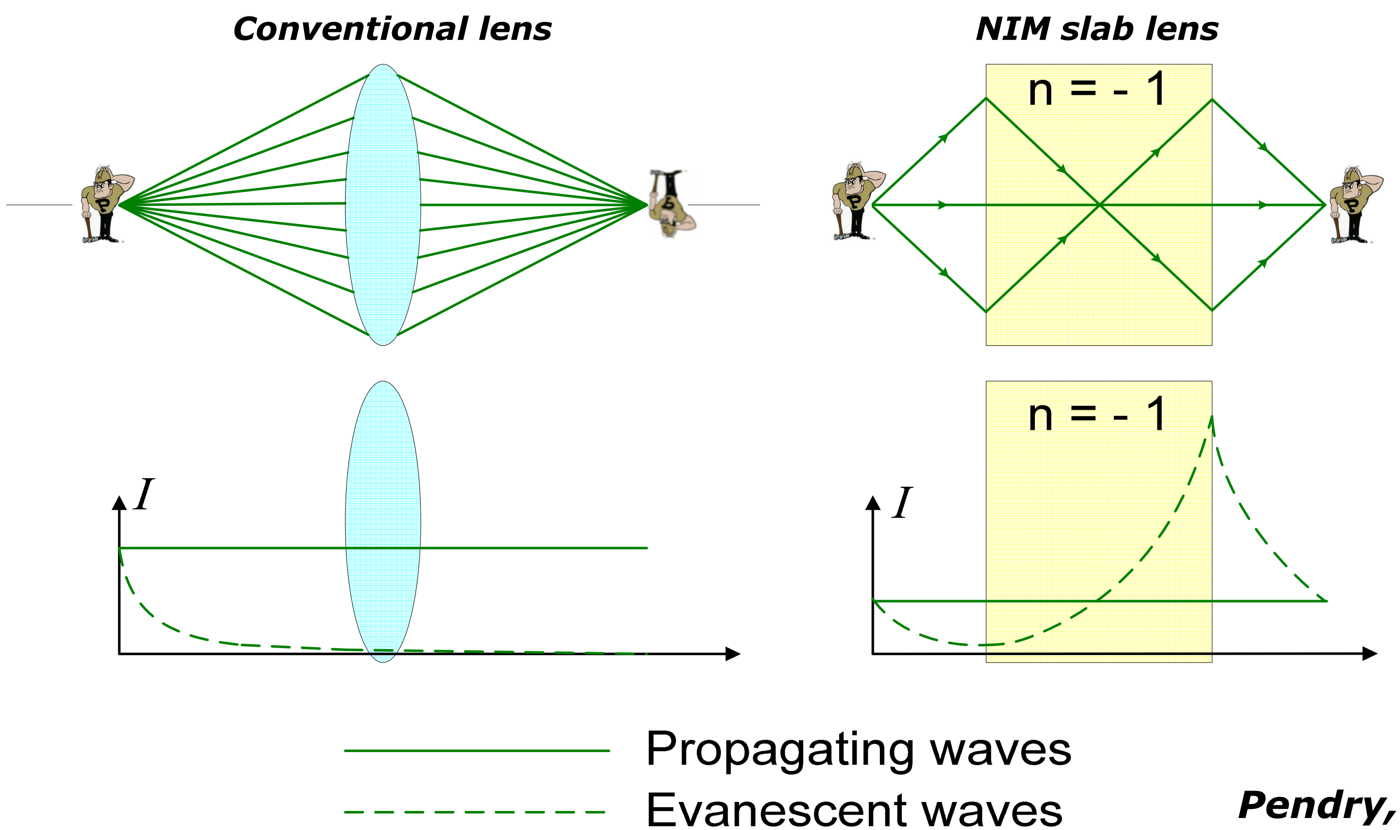
Super-resolution:
Amplification of Evanescent Waves Enables sub-λ Image!

Waves scattered by an object have all the Fourier components $k_z = \sqrt{k_0^2 - k_x^2 - k_y^2}$

The propagating waves are limited to: $k_t = \sqrt{k_x^2 + k_y^2} < k_0$

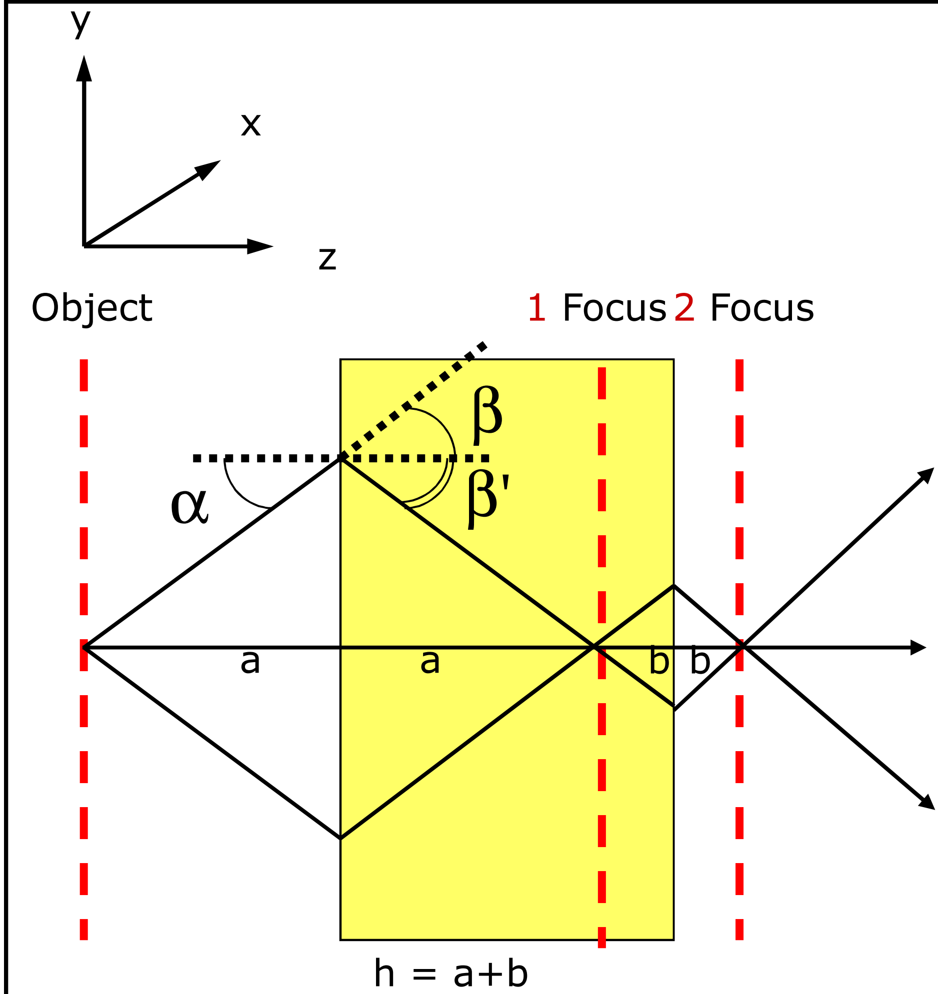
To resolve features Δ, we must have $\lambda_t = 2\pi / k_t < \Delta, \;\; \Delta < \lambda \Rightarrow \;\; k_t = \sqrt{k_x^2 + k_y^2} > k_0, \;\; k_z^2 < 0$

The evanescent waves are “re-grown” in a NIM slab and fully recovered at the image plane



Pendry, PRL, 2000

Perfect Lens



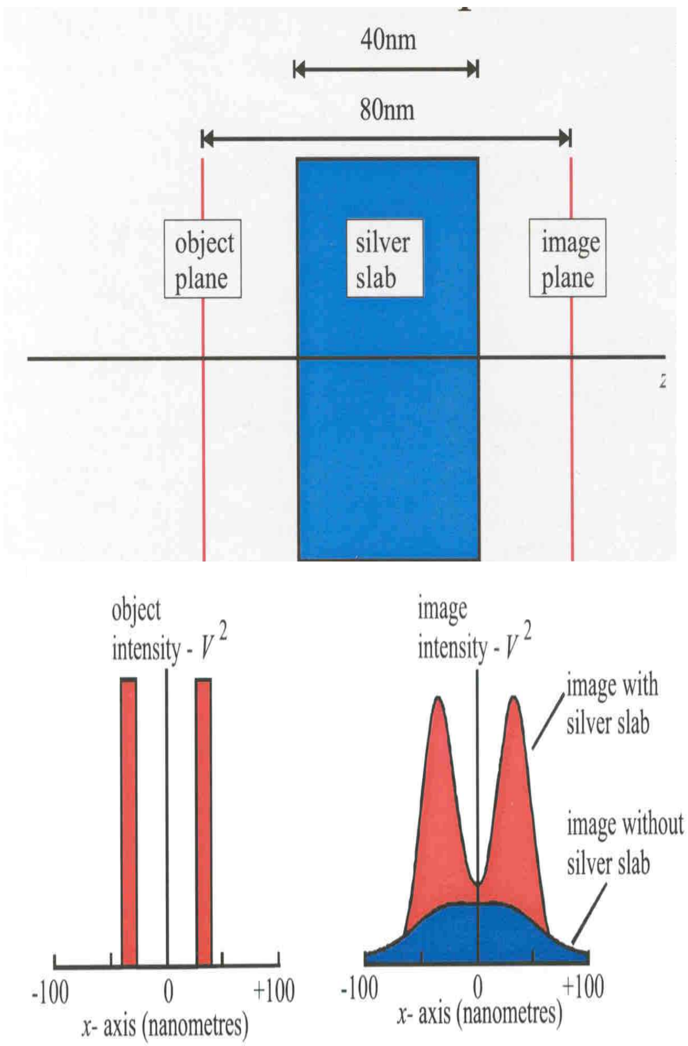
$n = -1$ (ε = -1; μ = -1)

$$\sin(\alpha) = n\sin(\beta') = -\sin(\beta') = \sin(\beta)$$

$$E(y,z) = \sum_q A_q \exp\left(iqy + i\sqrt{(nk)^2 - q^2}\; z \right)$$

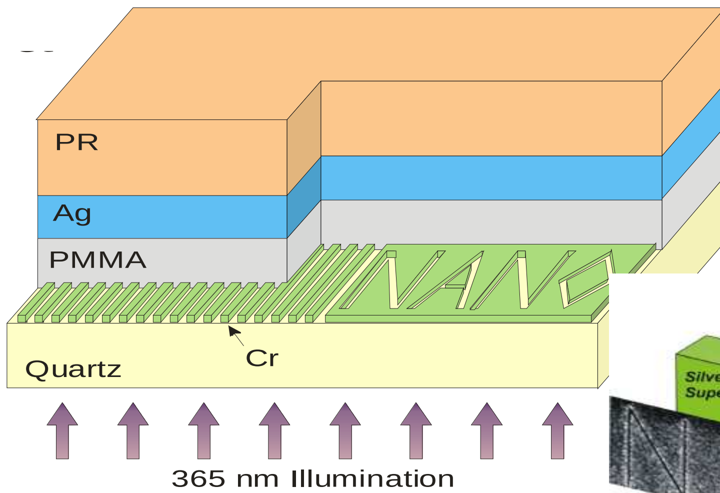
$$Phase\,shift =$$
$$\left(iqy + i\sqrt{k^2 - q^2}\; z\right) + \left(-iqy - i\sqrt{(-k)^2 - q^2}\; z\right) = 0$$

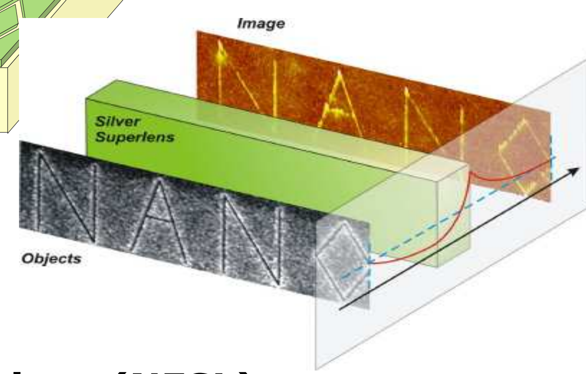
The Poor Man's (Near-Field) Superlens ($\varepsilon < 0, \mu = 1$)



Original implementation by Pendry: use a plasmonic material (silver film) to image 10 nm features with hw = 3.48 eV;

$$\varepsilon = 5.7 - 9^2/\omega^2 + 0.4i \quad (= -\varepsilon_h)$$





Near-field super-lens (NFSL)

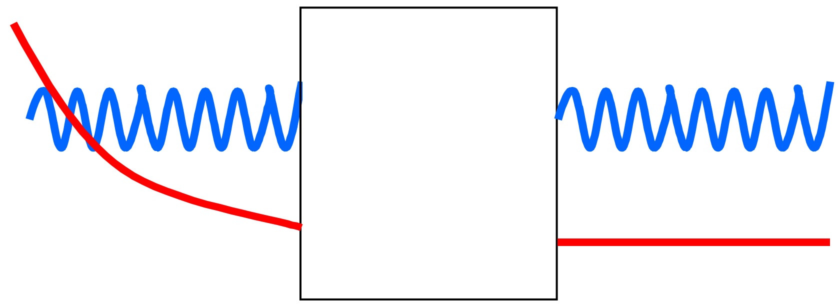
super-resolution with superlens: Zhang et al. (2005); Blaikie, et al (2005)

Mid-IR: Shvets et al. (2006)

Superlens High and Low

Ordinary Lens:

evanescent field lost

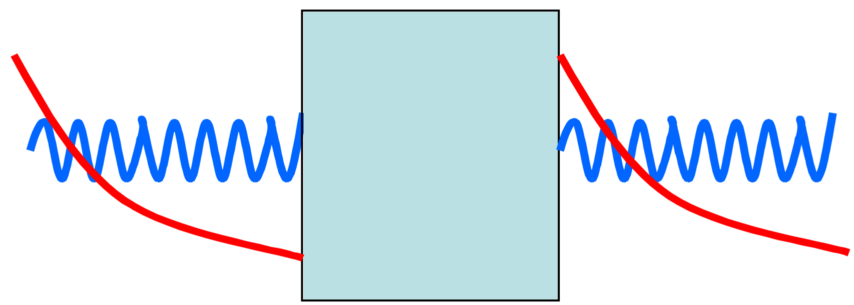


Super Lens:

evanescent field enhanced

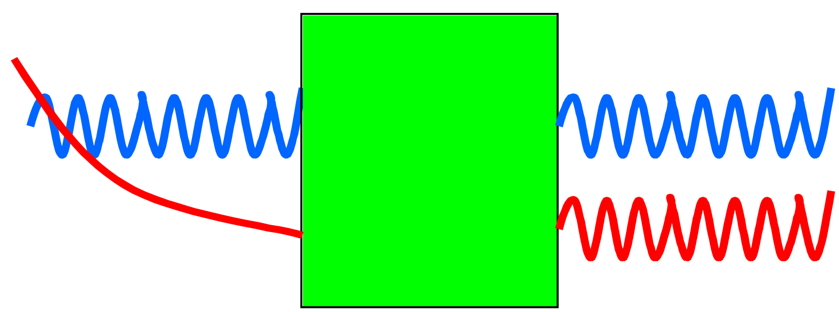
but decays away from the lens

* LIMITED TO NEAR FIELD
* EXPONENTIALLY SENSITIVE TO DISORDER, LOSSES,...



Hyper Lens:

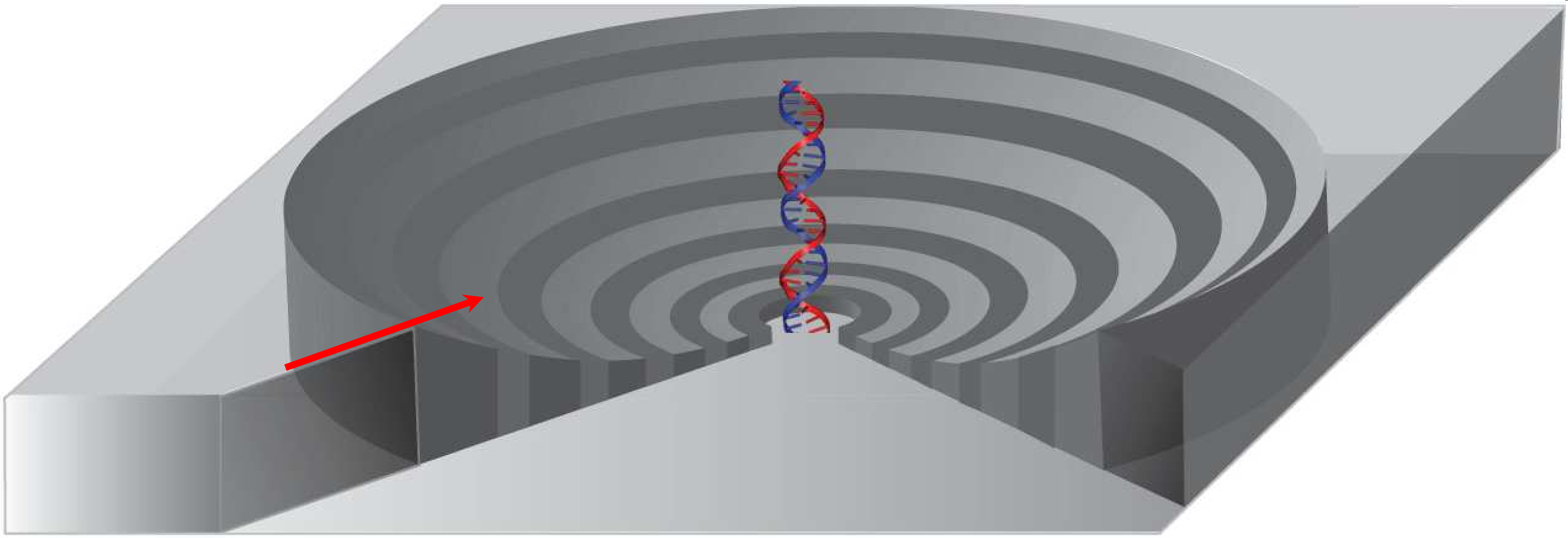
evanescent field converted to propagating waves (that do not mix with the others)



Hyperlens:

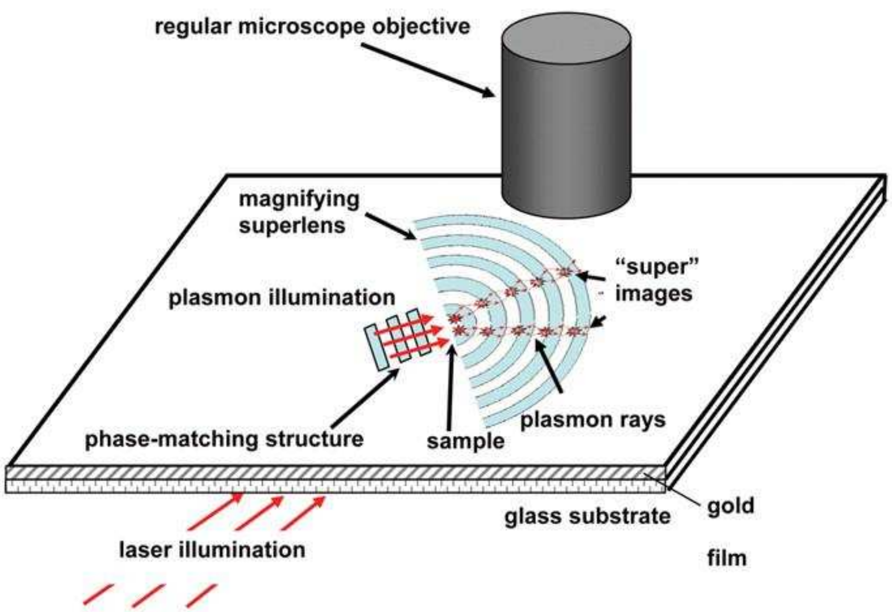
Converting evanescent components to propagating waves

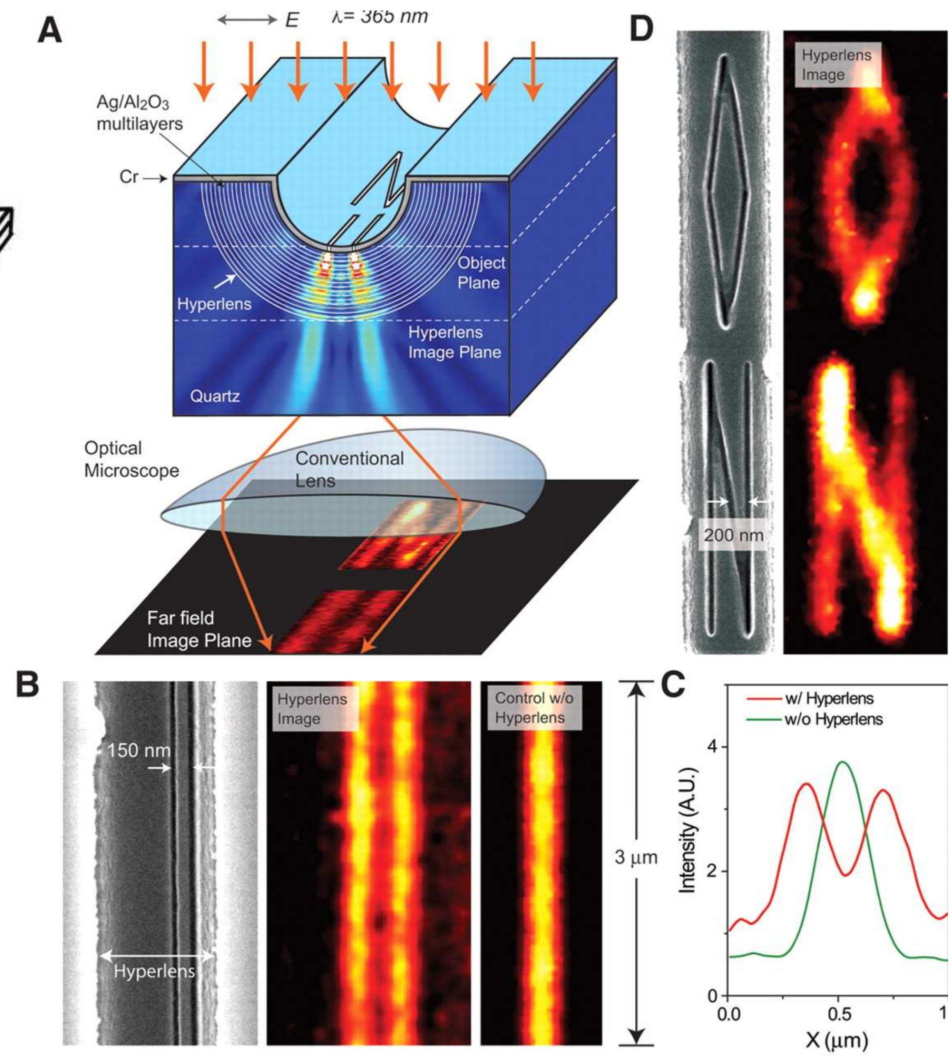
(Narimanov eta al; Engheta et al)



Far-field sub-λ imaging

Optical Hyperlens





Theory:

Jacob, Narimanov, OL, 2006

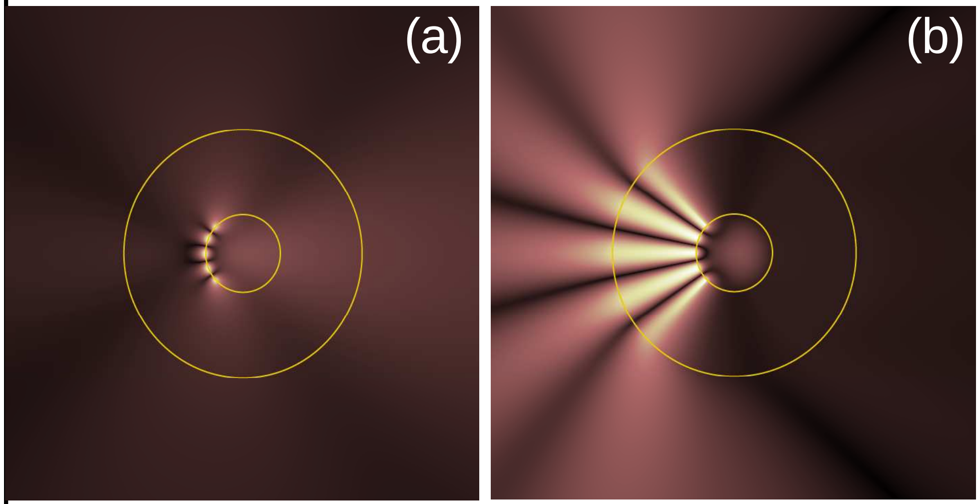
Salandrino, Engehta, PRB, 2006

Experiments:

Z. Liu et al., Science, 2007

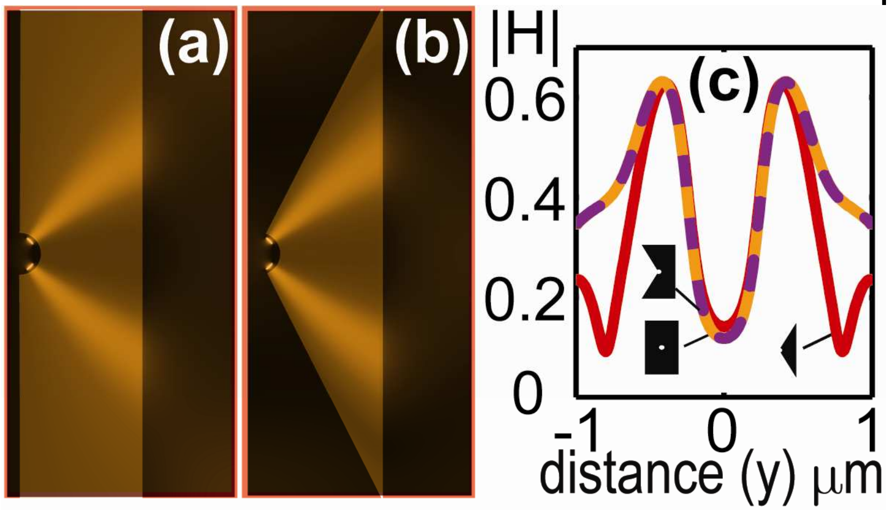
Smolyaninov et al., Science, 2007

Advanced Optical Hyperlens



Impedance-matched hyperlens

Kildishev, Narimanov
(Opt. Lett., 2007)



Flat hyperlenses:
½- & '¼-body lenses

Kildishev, Shalaev
(Opt. Lett., 2008)

Transforming Light with Metamaterials

Part 1: Electrical and Magnetic Metamaterials

Part 2: Negative-Index Metamaterials, NLO, and super/hyper-lens

Part 3: Cloaking and Transformation Optics

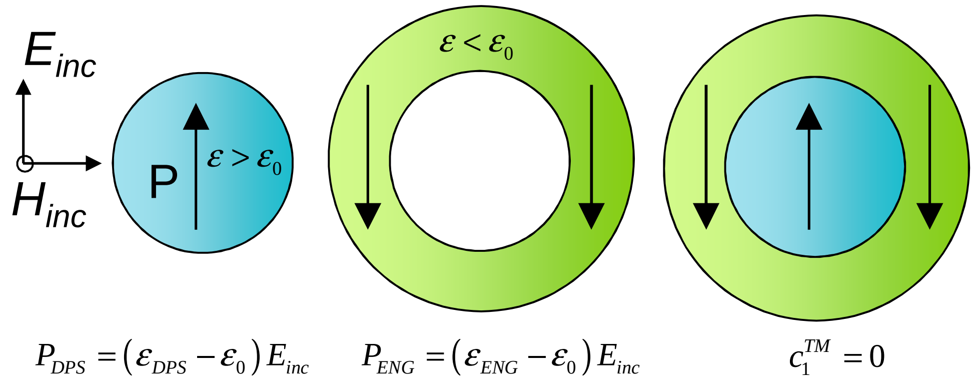
Outline

- What are metamaterials?
- Early electrical metamaterials
- Magnetic metamaterials
- Negative-index metamaterials
- Chiral metamaterials
- Nonlinear optics with metamaterials
- Super-resolution
- Optical cloaking and Transformation Optics

Other versions of cloak/invisibility/transparency

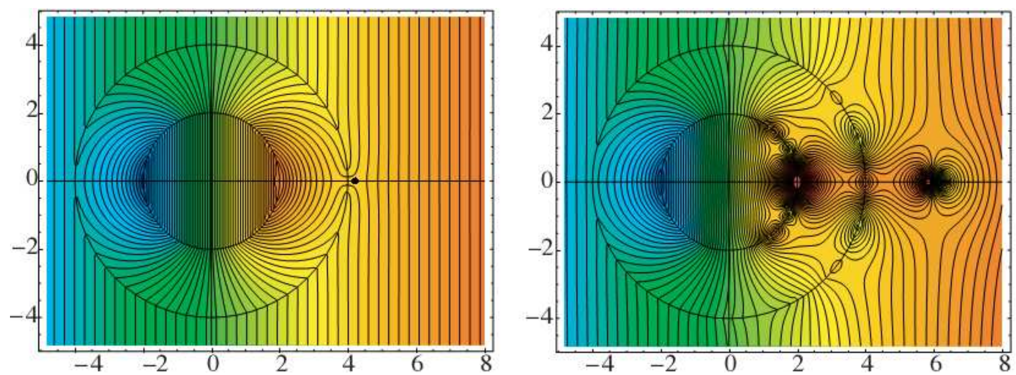
Plasmonic scattering ancellation

Alu and Engheta, PRE, 72, 016623, 2005



Anomalous localized resonance

Nicorovici, McPhedran and Milton, PRB, 1994
Milton & Nicorovici, Proc. R. Soc. A, 2006



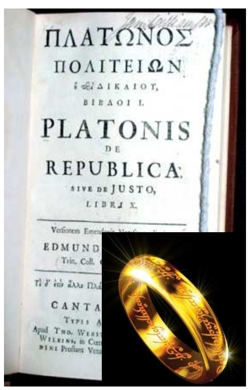
Other schemes include tunneling light transmissions (de Abajo) , active sources (Miller),invisible fish-scale structure (Zheludev et al)

Invisibility: An Ancient Dream

Perseus' helmet (Greek mythology)

Tarnhelm of invisibility (Norse mythology)

Cloaking devices (Star Trek, USA)



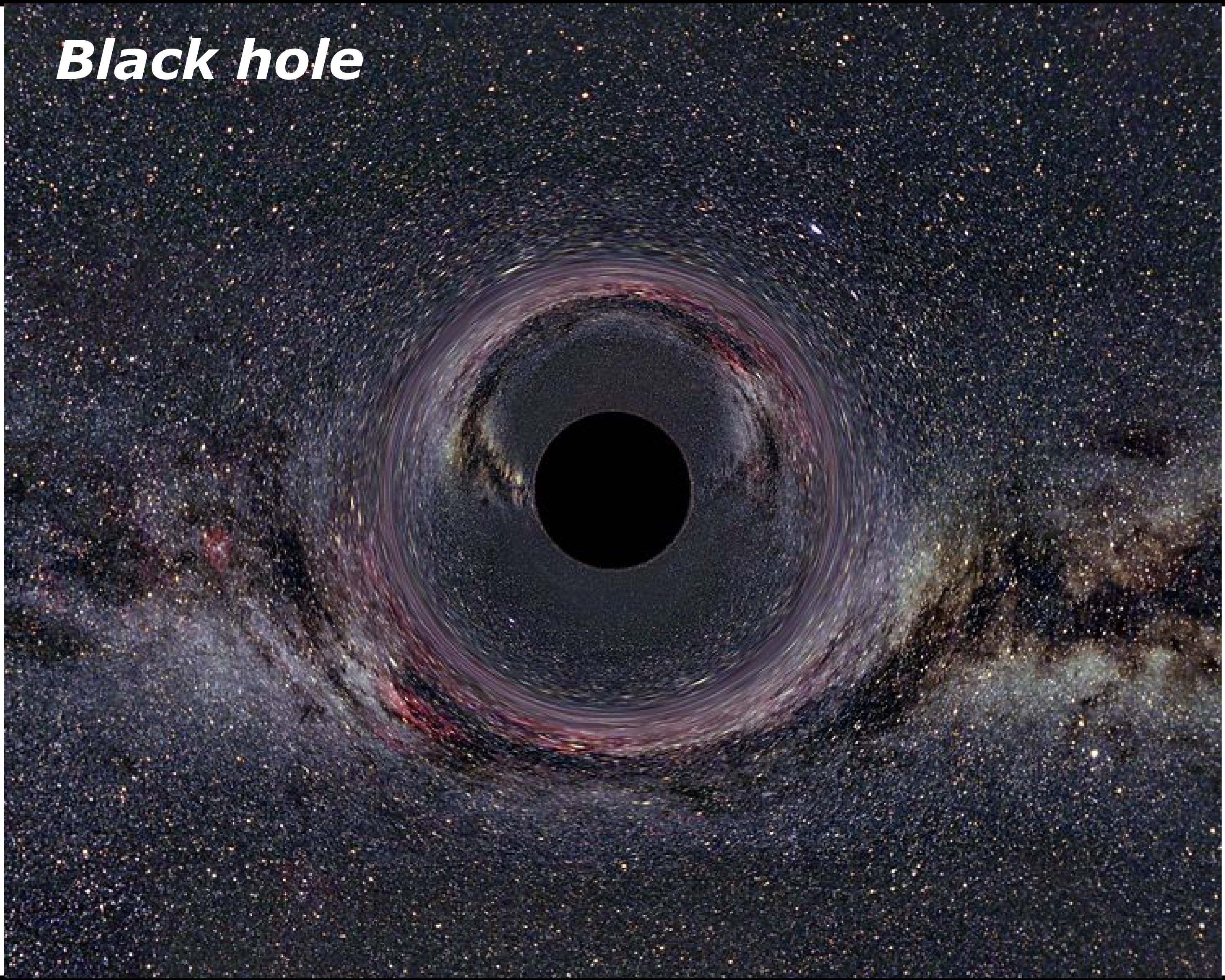
Ring of Gyges ("The Republic", Plato)

The 12 Dancing Princesses (Brothers Grimm, Germany)

Harry Potter's cloak (J. K. Rowling, UK)

Invisibility in Nature: Chameleon Camouflage

Invisibility by Transformation of Time-Space



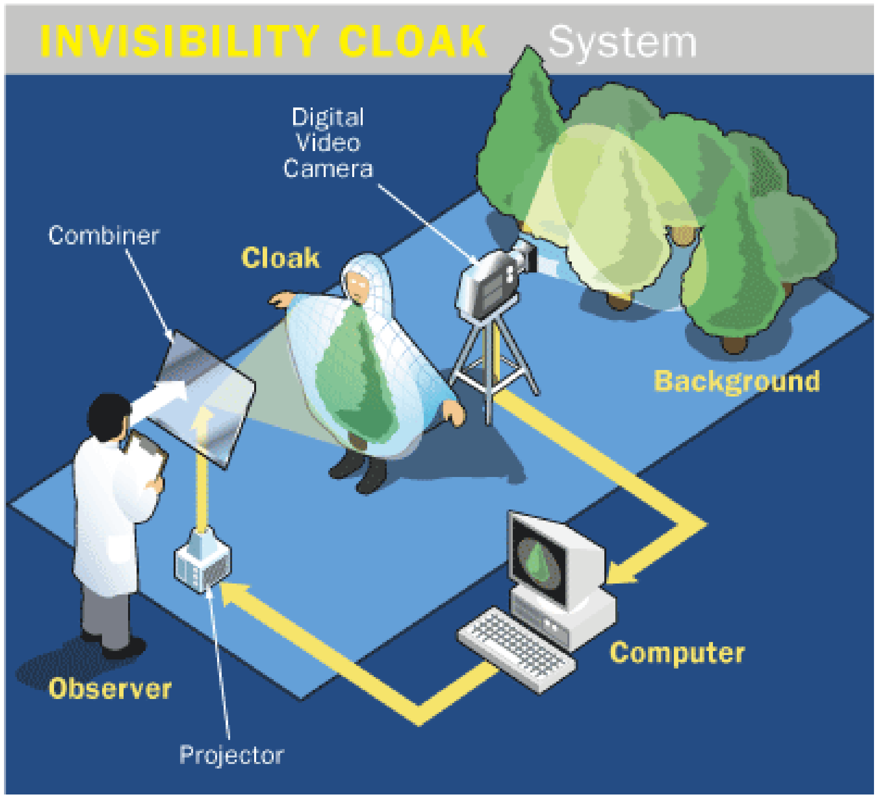
Invisibility to Radar: Stealth Technology

Stealth technique:

Radar cross-section reductions by absorbing paint / non-metallic frame / shape effect…

Optical camouflage (Tachi lab, U. Tokyo)

The camera + projector approach



From: http://www.star.t.u-tokyo.ac.jp

Invisibility: from fiction to fact?

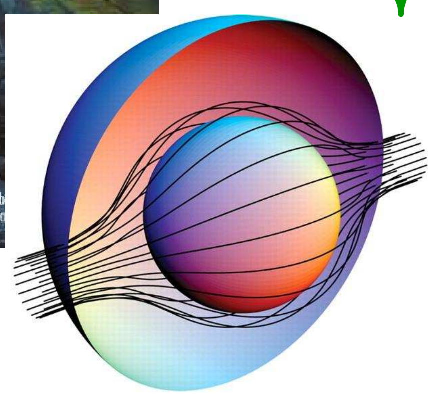
Examples with scientific elements:

- ***The Invisible Man*** by H. G. Wells (1897)

"... it was an idea ... to lower the refractive index of a substance, solid or liquid, to that of air — so far as all practical purposes are concerned." -- Chapter 19 "Certain First Principles"

- **"The invisible woman" in *The Fantastic 4*** by Lee & Kirby (1961)

"... she achieves these feats by bending all wavelengths of light in the vicinity around herself ... without causing any visible distortion." -- Introduction from Wikipedia



Pendry et al.; Leonhard, Science, 2006
(Earlier work: cloak of thermal conductivity by Greenleaf et al., 2003)

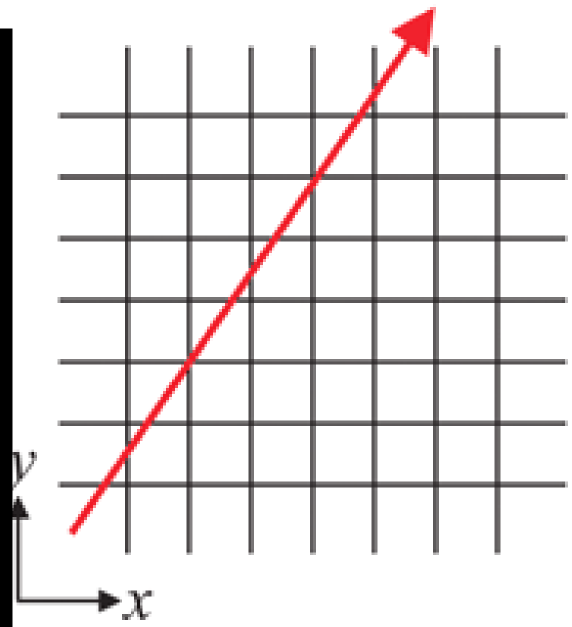
Designing Space for Light with Transformation Optics

Fermat:

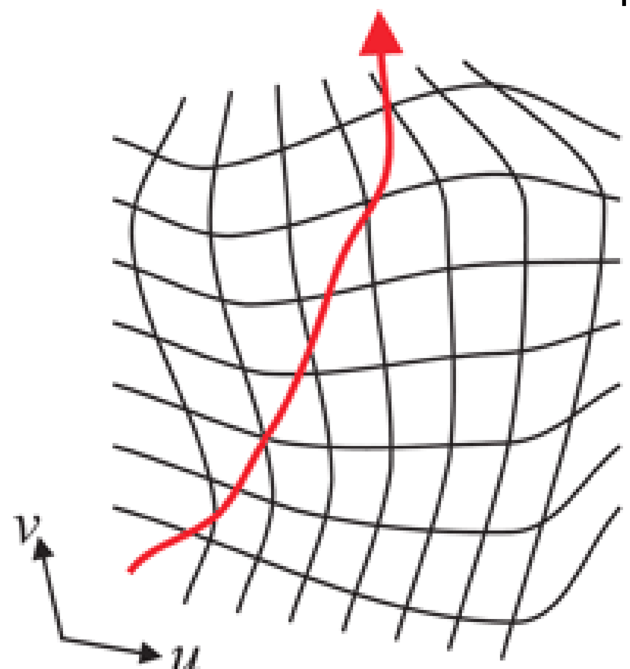
$$\delta\int n\,dl = 0$$

$$n = \sqrt{\varepsilon(r)\mu(r)}$$

"curving" optical space



Straight field line in Cartesian coordinate



Distorted field line in distorted coordinate

Spatial profile of ε & μ tensors determines the distortion of coordinate

Seeking for profile of ε & μ to make light avoid particular region in space — optical cloaking

Pendry et al., Science, 2006
Leonhard, Science, 2006

Form-invariance of Maxwell's equations

Coordinate transformation from x to coordinate x′ is described using the Jacobian matrix G: $g_{ij} = \partial x'_i / \partial x_j$

Maxwell's equation in x

$$\nabla \cdot (\varepsilon \vec{E}) = \rho$$

$$\nabla \cdot (\mu \vec{H}) = 0$$

$$\nabla \times \vec{E} = -\mu \, \partial \vec{H} / \partial t$$

$$\nabla \times \vec{H} = \varepsilon \, \partial \vec{E} / \partial t + \vec{J}$$

Transformation of variables

$$\varepsilon' = \frac{G \varepsilon G^T}{|G|}; \; \mu' = \frac{G \mu G^T}{|G|}$$

$$\vec{E}' = (G^T)^{-1} \vec{E}; \; \vec{H}' = (G^T)^{-1} \vec{H}$$

$$\vec{J}' = \frac{G \vec{J}}{|G|}; \; \rho' = \frac{\rho}{|G|}$$

$$\nabla \to \nabla'$$

Ward and Pendry, J. Mod.Opt. 43, 777 (1996)

Transformation Optics and Cloaking

ИЗВЕСТИЯ ВЫСШИХ УЧЕБНЫХ ЗАВЕДЕНИЙ

Том IV, № 5 РАДИОФИЗИКА

О ВОЗМОЖНОСТИ СОПОСТАВЛЕНИЯ ТРЕХМЕРНЫХ ЭЛЕ[…] МАГНИТНЫХ СИСТЕМ С НЕОДНОРОДНЫМ АНИЗОТРОП[…] ЗАПОЛНЕНИЕМ

Л. С. Долин

Показано, что, основываясь на инвариантности уравнений Макс[…] относительно определенного вида преобразований метрики простра[…] проницаемостей среды, можно исследовать трехмерные системы с […] родным анизотропным заполнением путем их сопоставления с други[…] лее простыми трехмерными системами.

$$\| \varepsilon_{ik} \| = \| \mu_{ik} \| = \begin{vmatrix} \frac{R^2}{r^2(R)} \frac{dr(R)}{dR} & 0 & 0 \\ 0 & \frac{1}{dr(R)/dR} & 0 \\ 0 & 0 & \frac{1}{dr(R)/dR} \end{vmatrix}.$$

Как видно, при условии (4) $\varepsilon_{ik} = \mu_{ik} \xrightarrow[r \to \infty]{} 1$. Плоская вол[…] щая из бесконечности на неоднородность с параметрами (5), через нее без искажений.

[…]едовательский радиофизический институт при Горьковском университете

Поступила в редакцию 11 марта 1961 г.

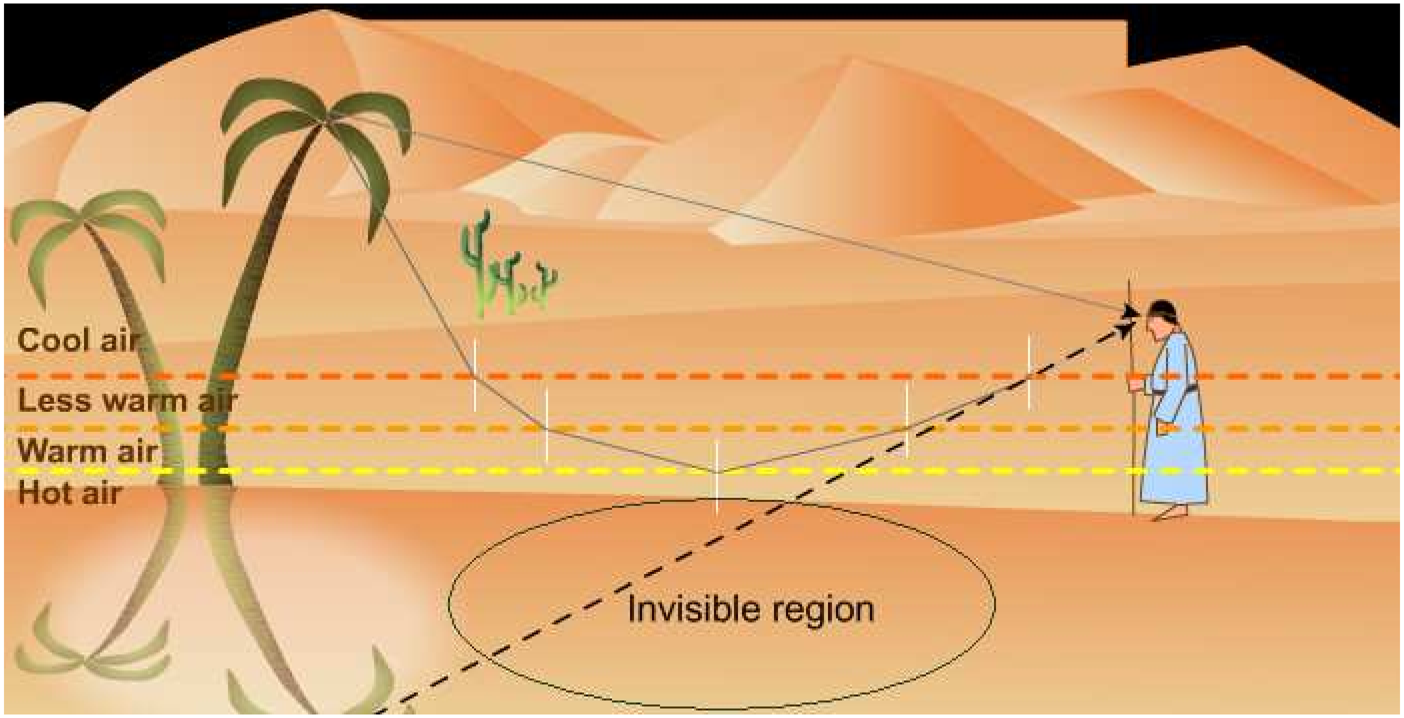
[…] POSSIBILITY OF COMPARISON OF THREE-DIMENSIONAL […]LECTROMAGNETIC SYSTEMS WITH NONUNIFORM ANISOTROPIC FILLING

L. S. Dolin

[…]s shown that it is possible to investigate three-dimensional systems with […] anisotropic filling by comparison them with other, more simple three-dimen[…] […]ems. The examination is made basing on an invariance of Maxwels equations […] the certain type of transformation of space metric and medium permeability […]ivity.

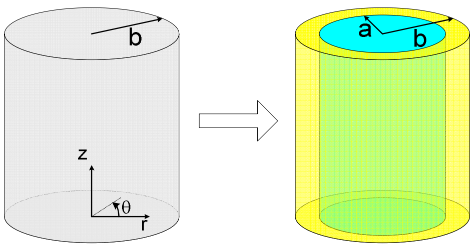
A similarity in Mother Nature

The bending of light due to the gradient in refractive index in a desert mirage



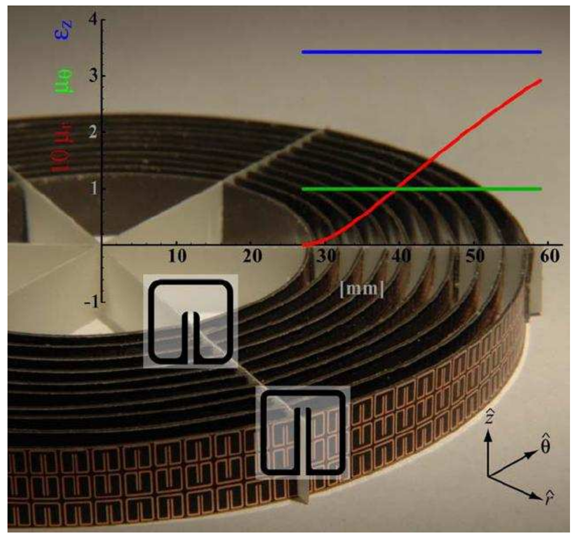
Cloaking based on coordinate transformation

General math. requirements and microwave demonstrations

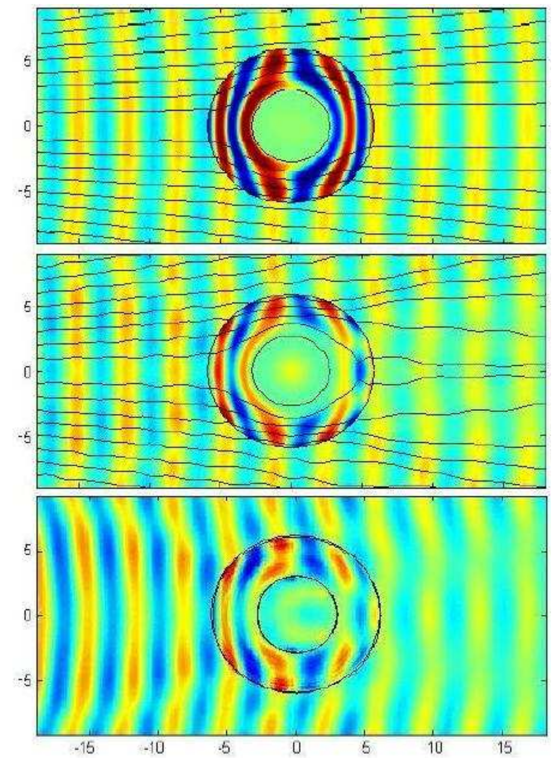


$$r \;\Rightarrow\; \frac{b-a}{b}r' + a$$

$$\begin{cases} \varepsilon_r = \mu_r = \dfrac{r-a}{r} \\ \varepsilon_\theta = \mu_\theta = \dfrac{r}{r-a} \\ \varepsilon_z = \mu_z = \left(\dfrac{b}{b-a}\right)^2 \dfrac{r-a}{r} \end{cases}$$



Structure of the cloak



Ideal case

Reduced parameter

Experimental data

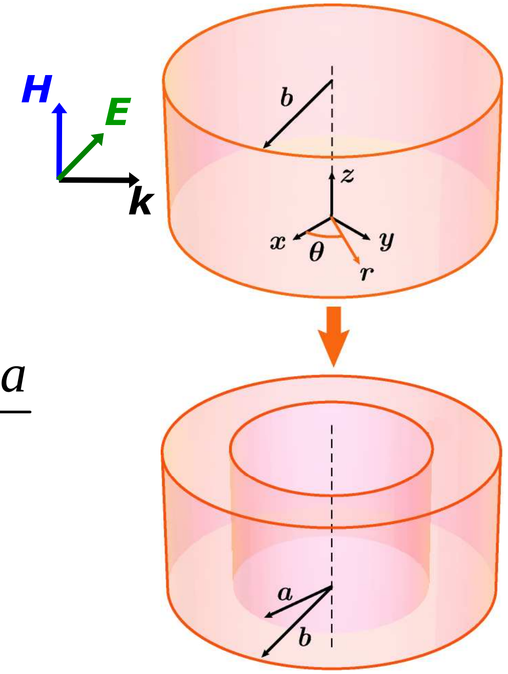
Schurig et al., Science, 2006

How about optical frequencies?

Scaling the microwave cloak design?

☹ ***Intrinsic limits to the scaling of SRR size***

☹ ***High loss in resonant structures***



$$\varepsilon_r = \mu_r = \frac{r-a}{r}, \quad \varepsilon_\theta = \mu_\theta = \frac{r}{r-a}, \quad \varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r}$$

TM incidence

$$\begin{cases} \mu_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r} \\ \varepsilon_\theta = \frac{r}{r-a} \\ \varepsilon_r = \frac{r-a}{r} \end{cases}$$

To maintain the dispersion relation

$$\begin{cases} \mu_z \varepsilon_\theta = constant \\ \mu_z \varepsilon_r = constant \end{cases}$$

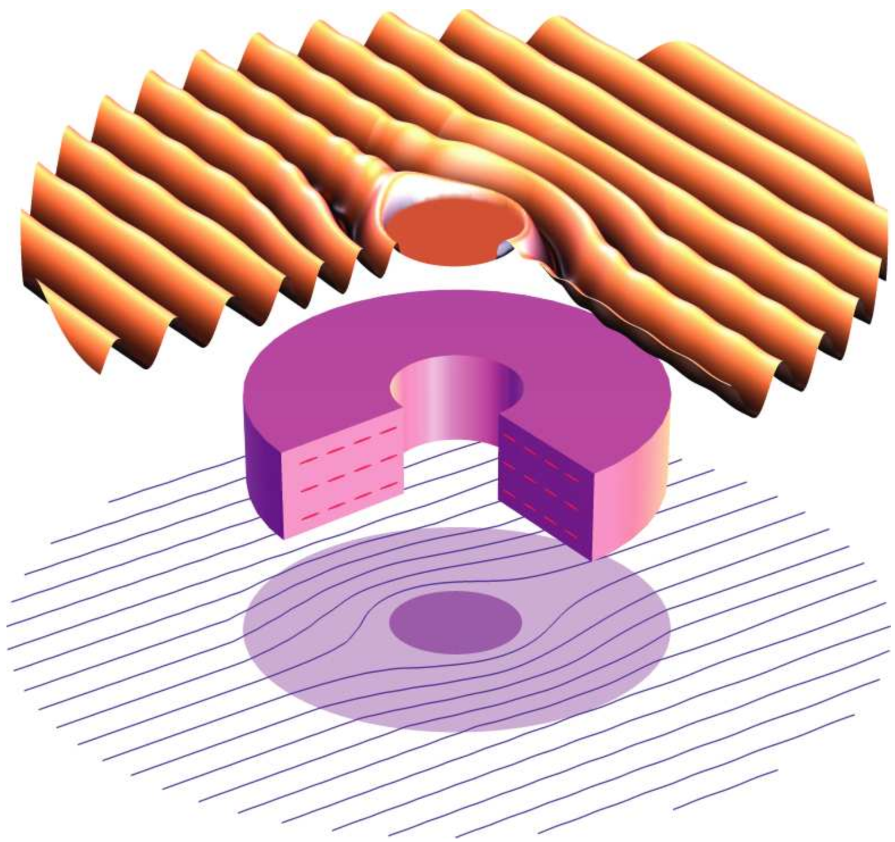
(for in-plane k)

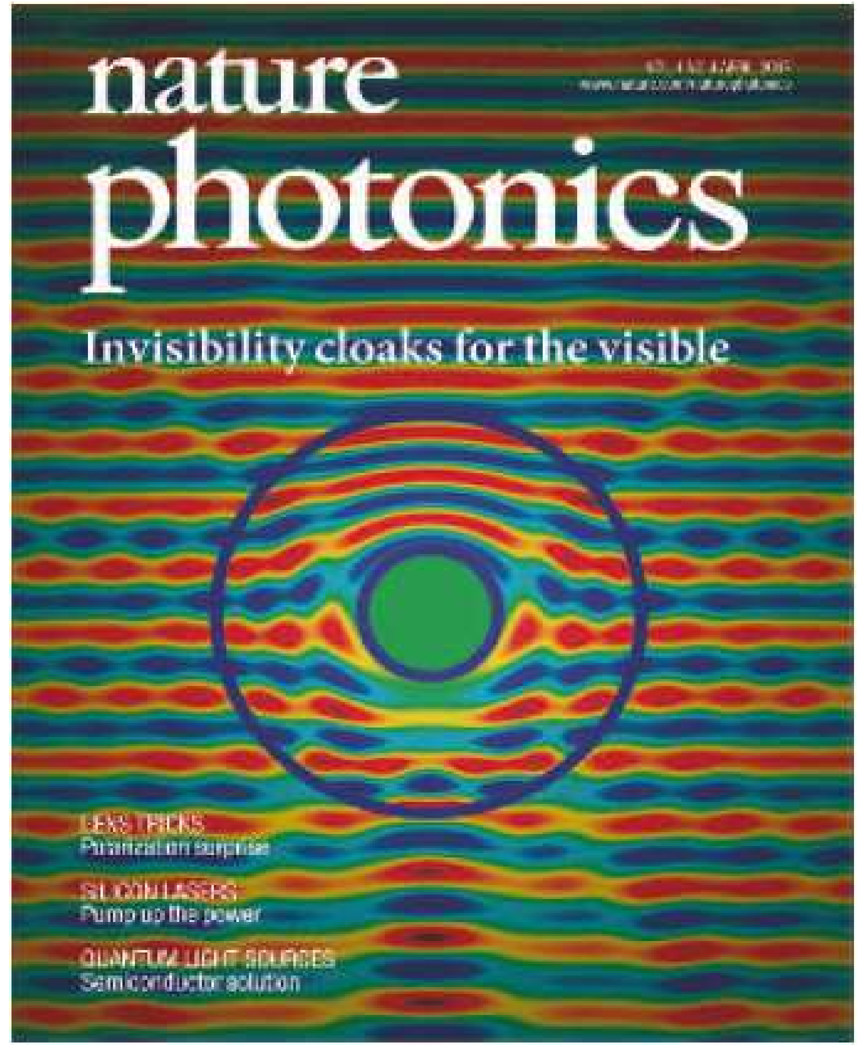
$$\begin{cases} \mu_z = 1 \\ \varepsilon_\theta = \left(\frac{b}{b-a}\right)^2 \\ \varepsilon_r = \left(\frac{b}{b-a}\right)^2 \left(\frac{r-a}{r}\right)^2 \end{cases}$$

- $\mu_z = 1$ → **No magnetism required!**
- ε_θ → **A constant permittivity of a dielectric;** $\varepsilon_\theta > 1$
- ε_r → **Gradient in *r* direction only;** ε_r **changing from 0 to 1.**

Cai, et al., Nature Photonics, 1, 224 (2007)

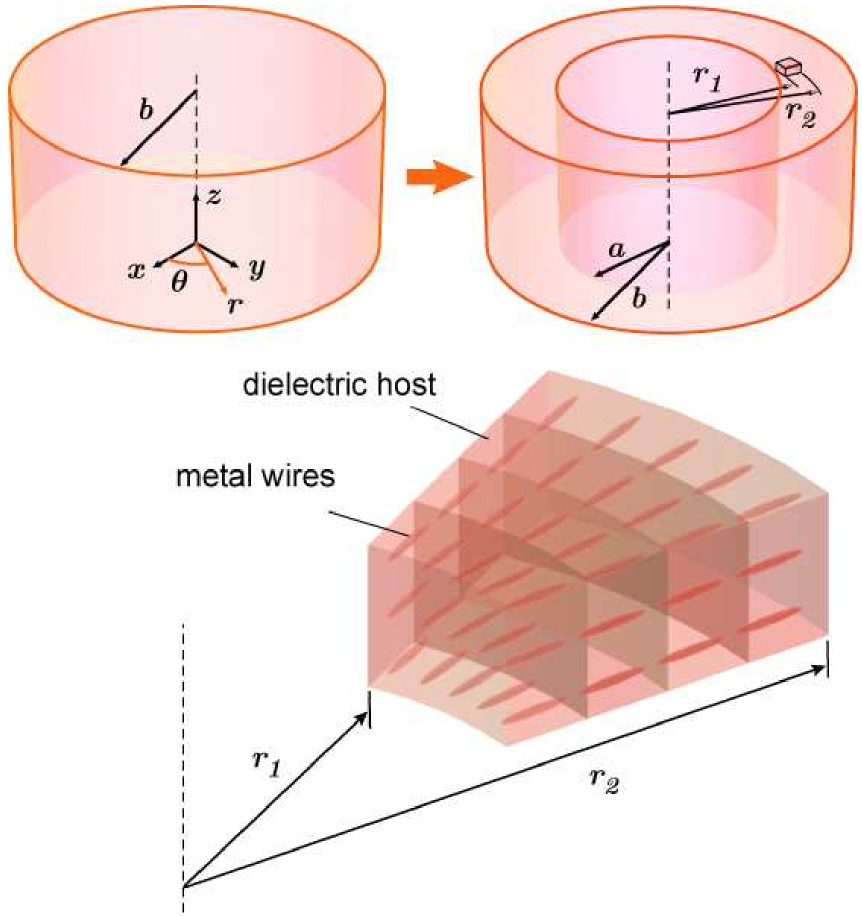
Optical Cloaking with Metamaterials: Can Objects be Invisible in the Visible?



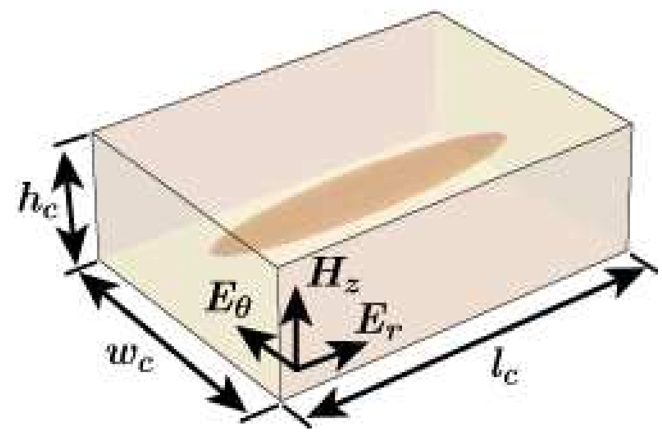


Cover article of Nature Photonics (April, 2007)

Structure of the cloak: "Round brush"



Unit cell:



Flexible control of ε_r ;
Negligible perturbation in ε_θ

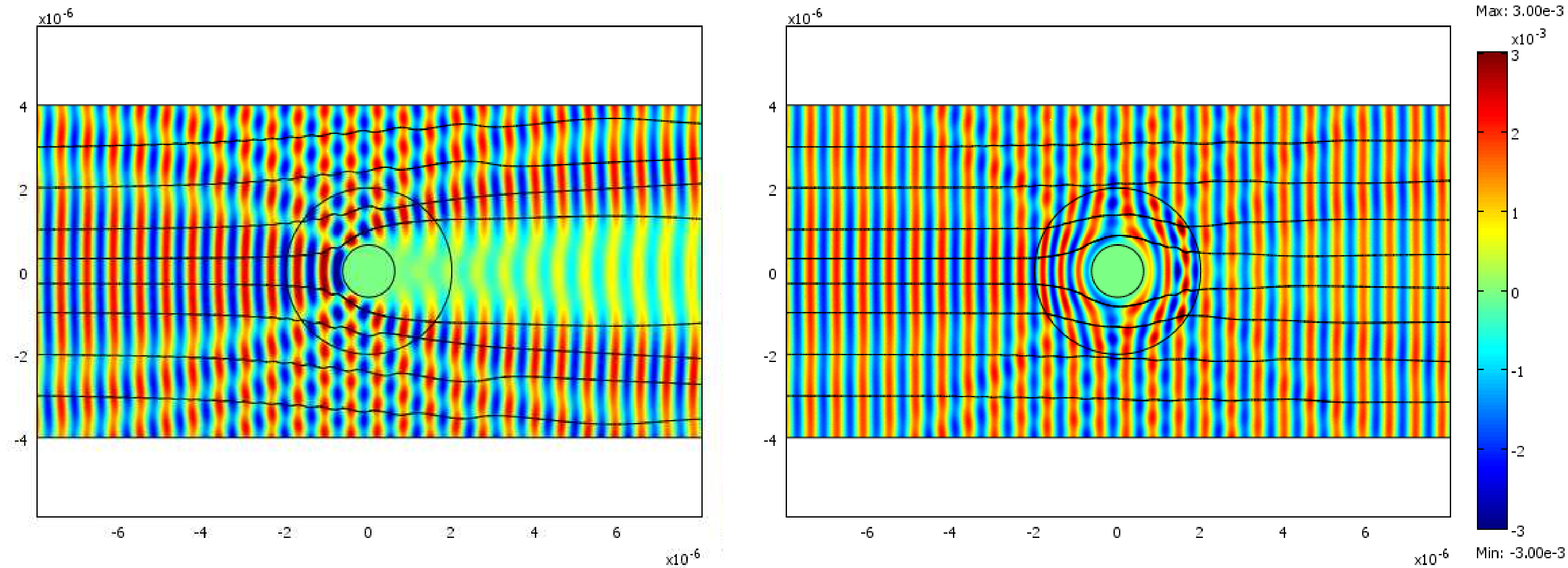
metal needles embedded in dielectric host

Cai, et al., Nature Photonics, 1, 224 (2007)

Cloaking performance: Field mapping movies

Example:

Non-magnetic cloak @ 632.8nm with silver wires in silica

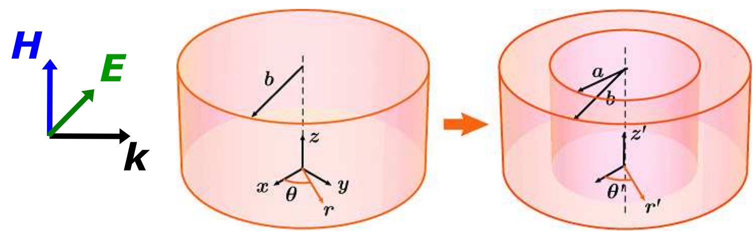


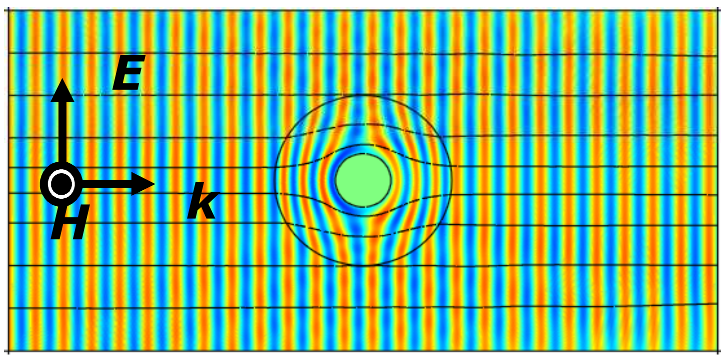
Cloak OFF ***Cloak ON***

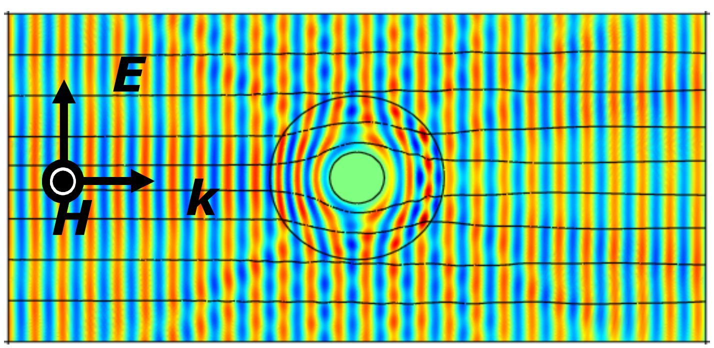
Scattering issue in a linear non-magnetic cloak

Linear transformation

$$r = \frac{b-a}{b} r' + a$$







Ideal cloak:

$$Z\big|_{r=b} = \sqrt{\mu_z / \varepsilon_\theta}\Big|_{r=b} = 1$$

Perfectly matched impedance results in zero scattering

Linear non-magnetic cloak:

$$Z\big|_{r=b} = \sqrt{\mu_z / \varepsilon_\theta}\Big|_{r=b} = 1 - a/b$$

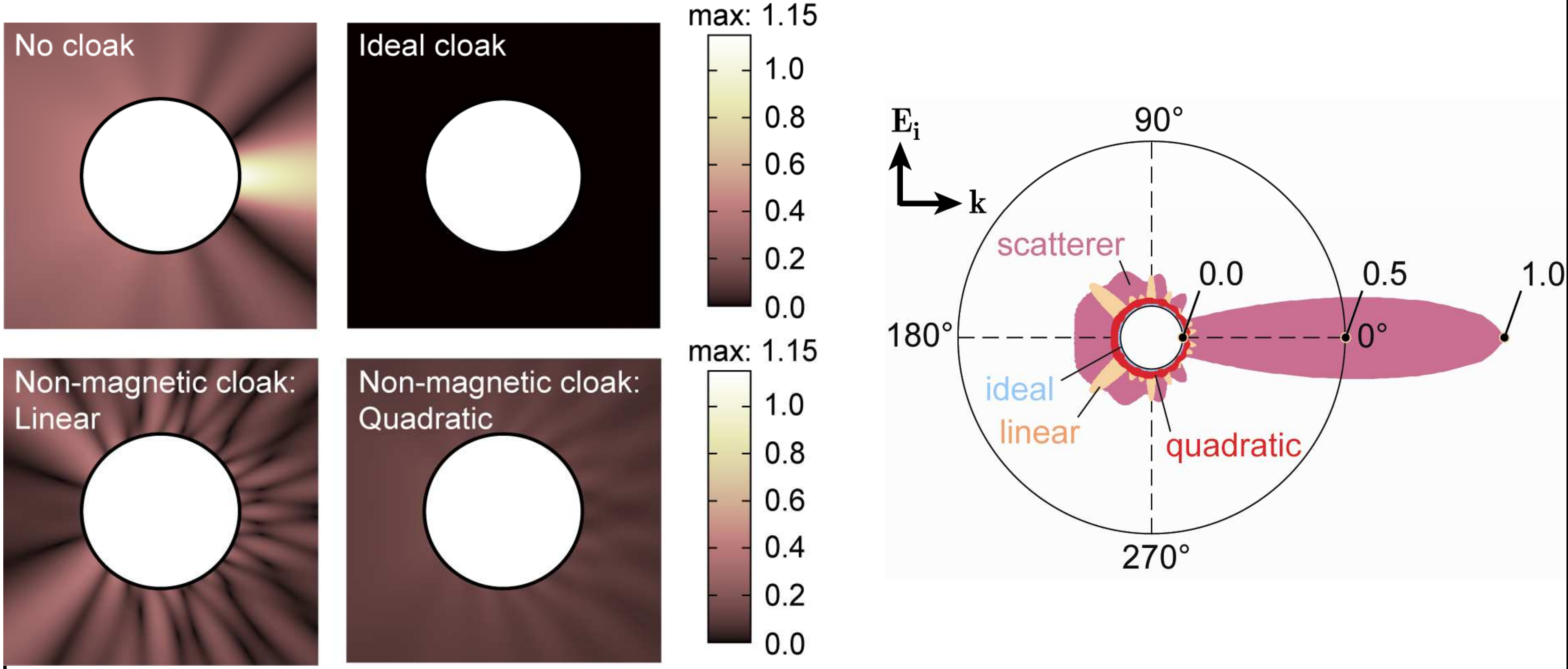
Detrimental scattering due to impedance mismatch

Nonlinear transformation –> no scattering

High-order transformation for cloaking

2-nd order transformation for non-magnetic cloak:

$$r = g(r') = \left[1 - a/b + p\left(r' - b\right)\right] r' + a \qquad with \qquad p = a/b^2$$



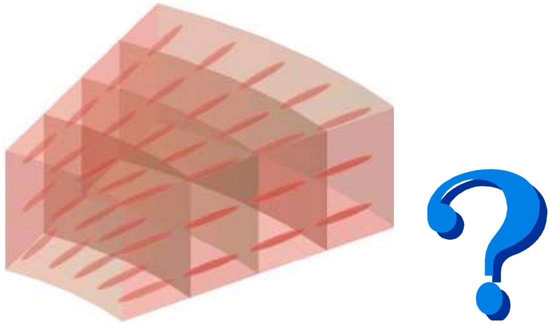
W. Cai et al (APL 91, 111105, 2007); with G. Milton

Towards experimental realization

We need a design that is …

Less complicated in fabrication

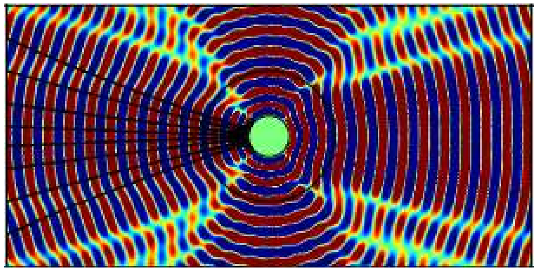
Compatibility with mature fabrication technique like direct deposition and direct etching

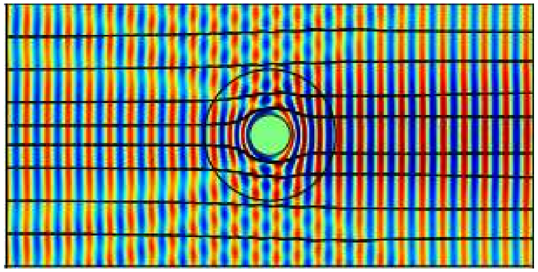


Better loss features

Loss might be ultimate limiting issue for cloaking

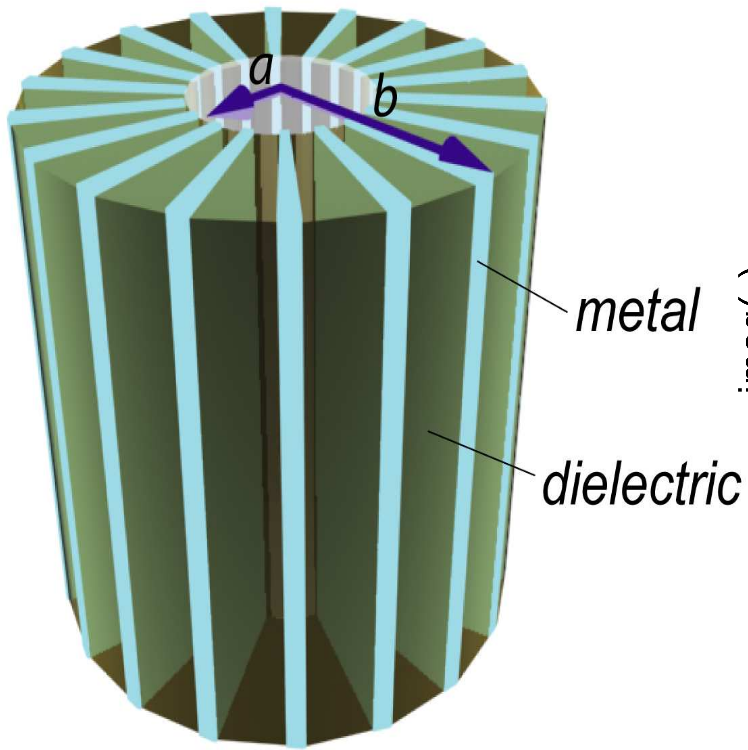
$\varepsilon_r'' = 0.1$

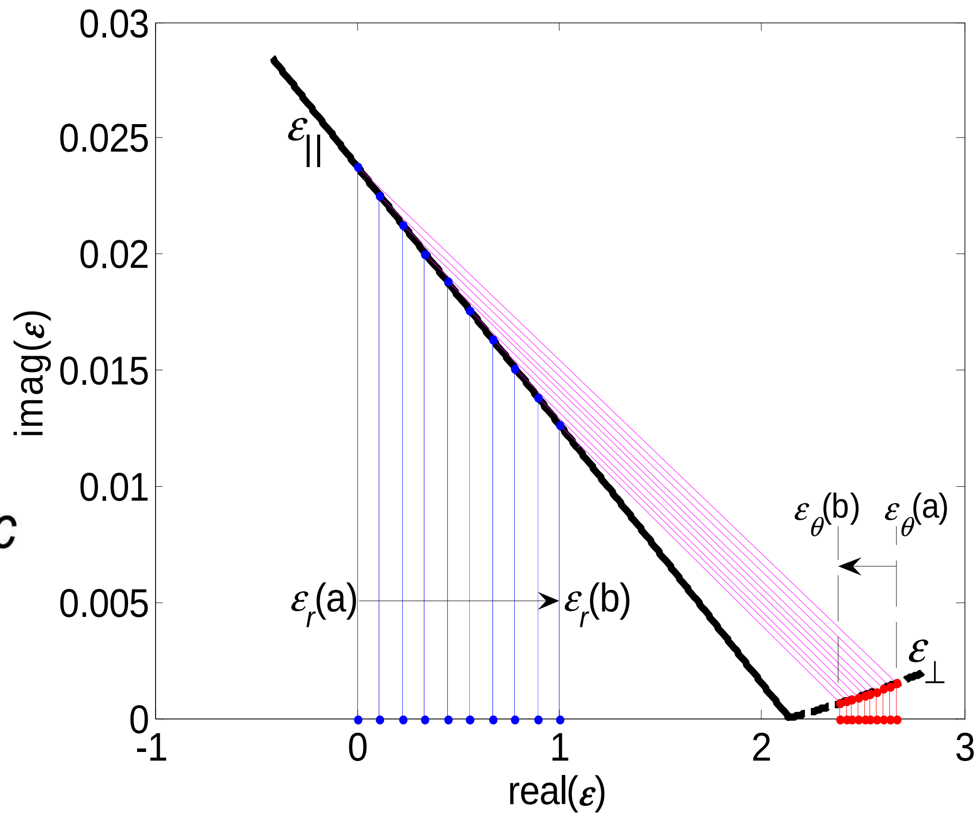




$\varepsilon_r'' = 0.03$

Structures of realistic high-order TO cloaks





Cai, et al, (OE, 2008)

ε >>1 - problem

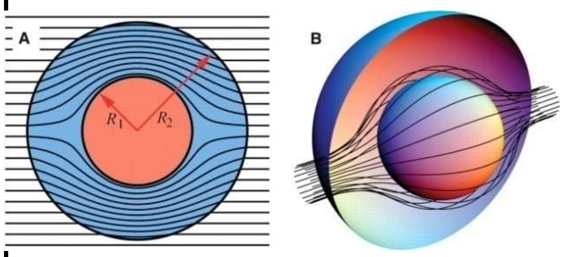
ε found from Wiener's bounds

cloak @ 530 nm with alternating silver- silica slices based on nonlinear transformations

Bandwidth problem in electromagnetic cloak

Fermat's principle → *$v_p = c/n$* → *$v_s \leq c$*

Curved wave trajectory → **Refractive index n<1** → **Phase velocity $v_p > c$** → **$v_p \neq v_s$ Dispersive material**

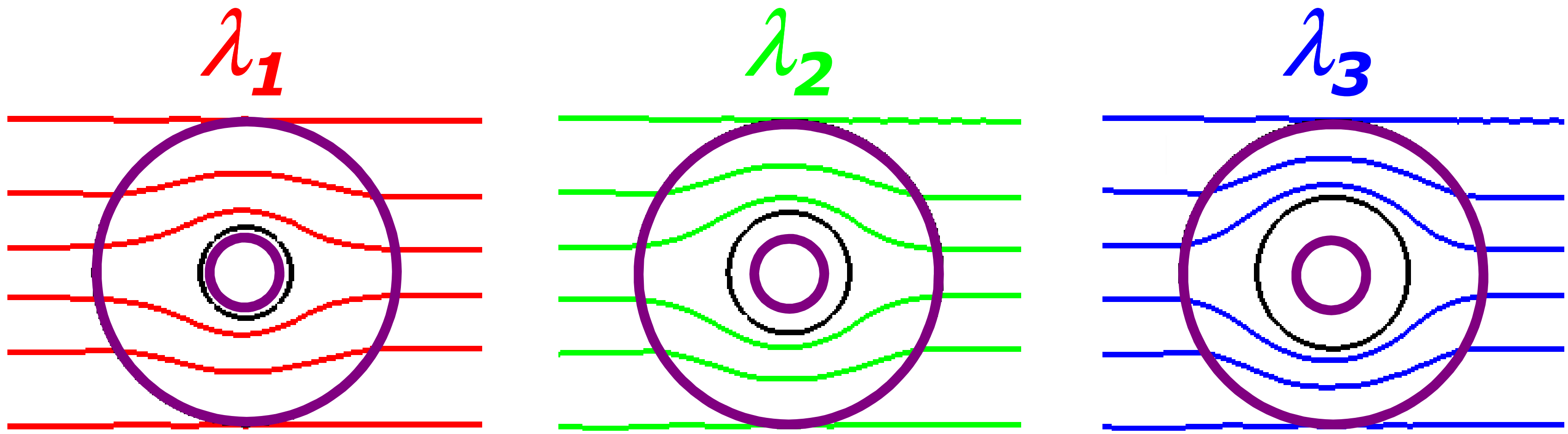


$$\frac{\Delta\omega}{\omega} \leq \frac{\Delta s}{s}$$

- ω - **operating frequency**
- $\Delta\omega$ - **operating bandwidth**
- s – **geometrical cross-section**
- Δs - **scattering cross-section**

Chen, et al., PRB, 76, 241104 (2007)

Wavelength Multiplexing Cloak

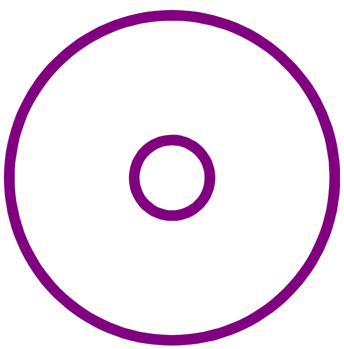


Combination of techniques:

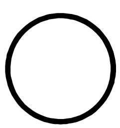
Virtual inner boundary

Dispersion control

Active medium or EIT?



Physical boundaries the cloaking device



Virtual inner boundary for different wavelengths

$$\left(\frac{\partial \varepsilon_z}{\partial \omega}\right)\left(\frac{\partial \mu_r}{\partial \omega}\right) < 0$$

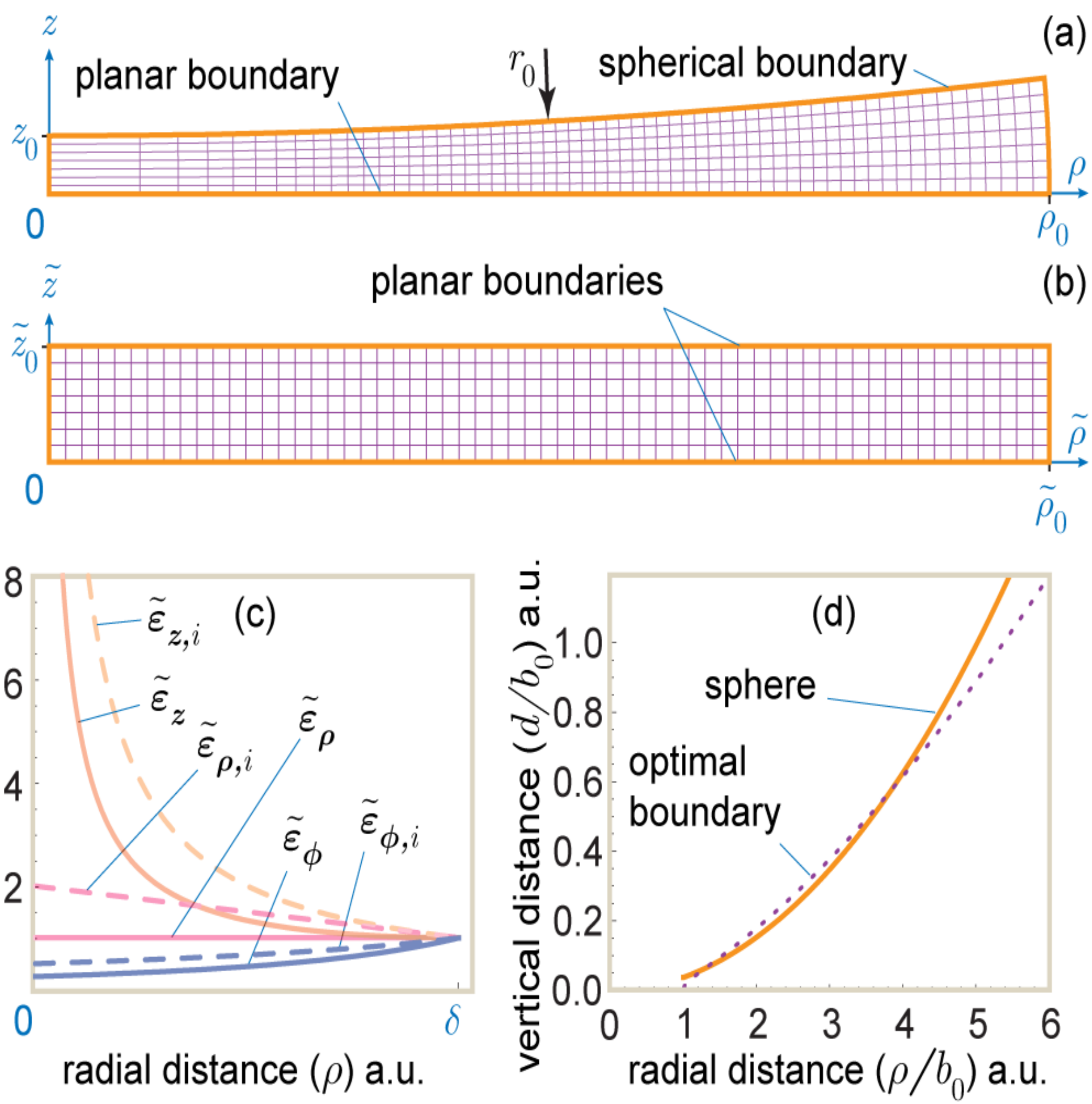
Kildishev, et al (NJP, 2008)

Broadband Optical Cloaking in Tapered Waveguides

I.I. Smolayninov, V.N. Smolyaninova, A.V. Kildishev and V.M. Shalaev

(PRL , May 29, 2009)

Emulating Anisotropic Metamaterials with Tapered Waveguides

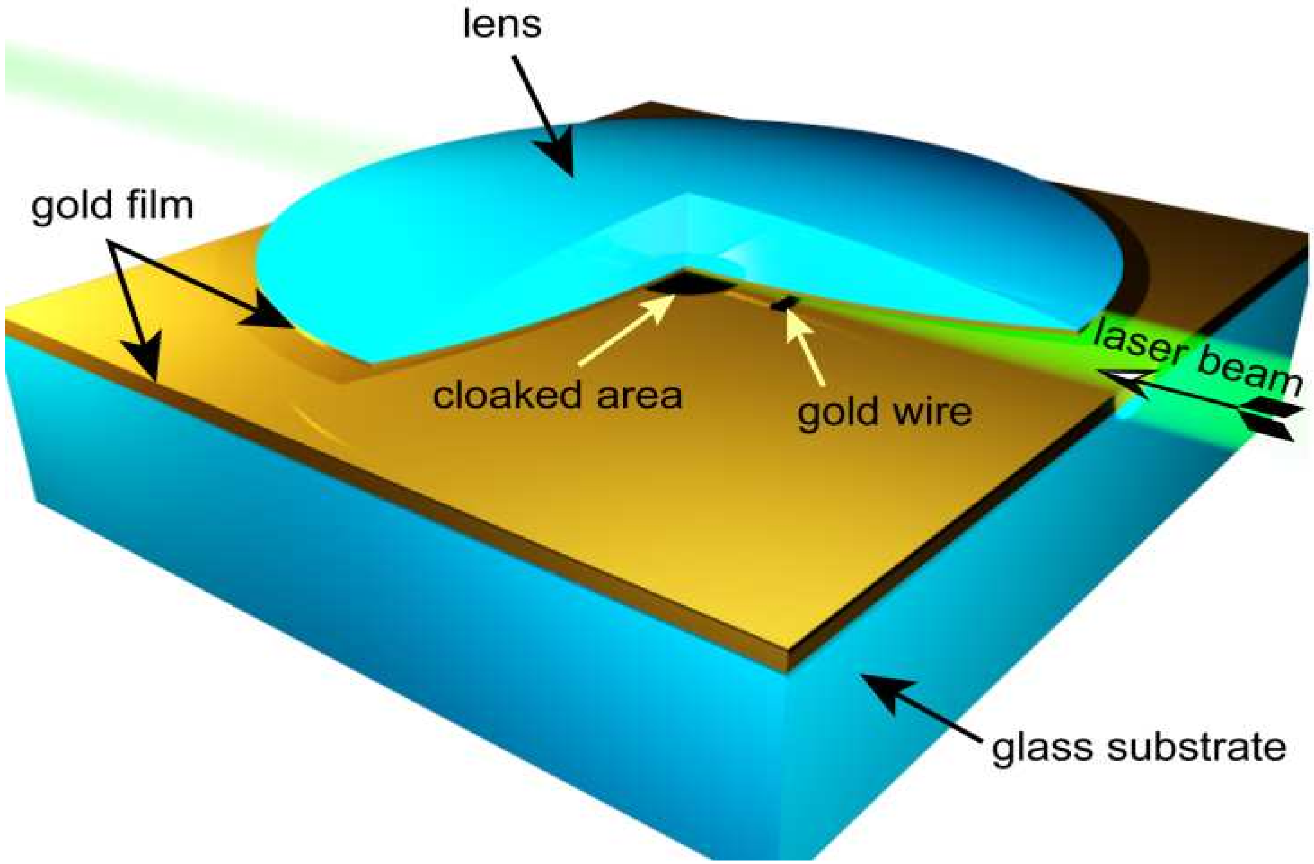


A space between a spherical and a planar surface (a) mapped onto a planar anisotropic MM (b)

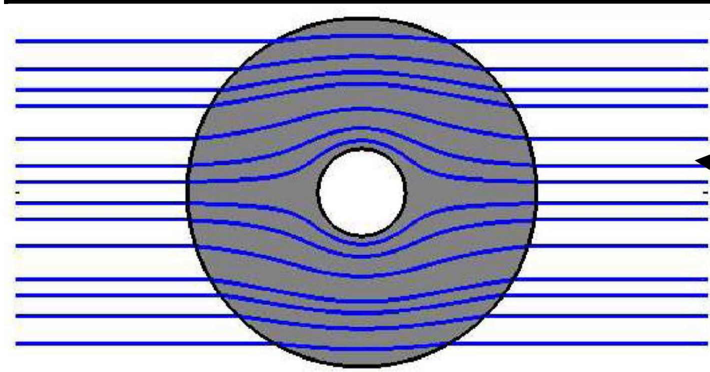
(c) Distribution of radial (top), azimuthal (middle), and axial components of ε = μ in equivalent planar MM. Dashed lines show same components in the ideal cloak.

(d) Normalized profile of optimal and "plane-sphere" waveguides for a cloak with radius of b_0 = 172 μm.

Broadband Optical Cloak



Fermat Principle and Waveguide Cloak



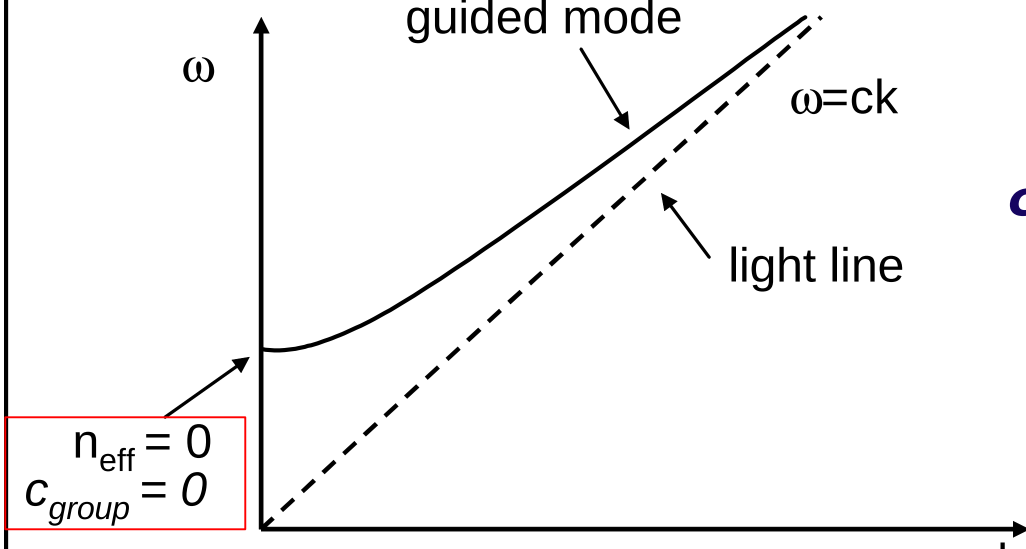
$n_{eff} L$ = const (Fermat)

Cloaking Hamiltonian: (Narimanov, OE, 2008)

$$\left(\omega/c\right)^2 = k_\rho^2 + k_\phi^2\left(\rho - b\right)^{-2}$$

Dispersion law of a guided mode:

$$\left(\omega/c\right)^2 = k_\rho^2 + \left(k_\phi/\rho\right)^2 + \left[\pi l/d(\rho)\right]^2$$

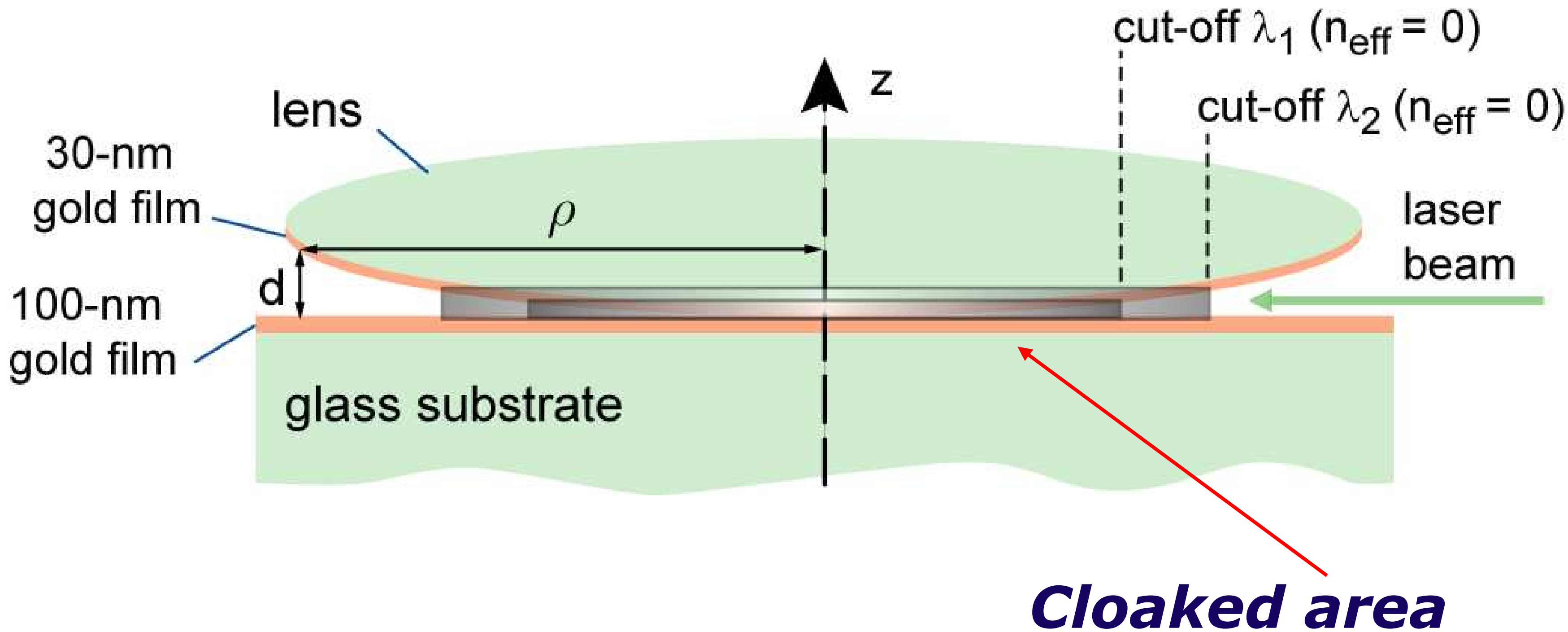


$c_{phase} = \omega / k$ $c_{group} = d\omega / dk$

$n_{eff} = c / c_{phase} = ck / \omega \rightarrow 0$

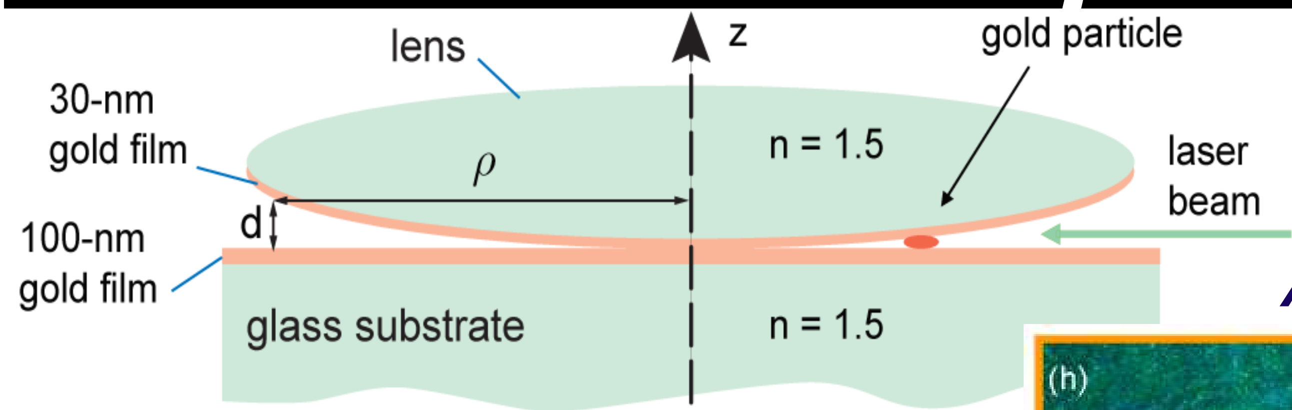
near cutoff

Broadband Cloaking in Tapered Waveguide



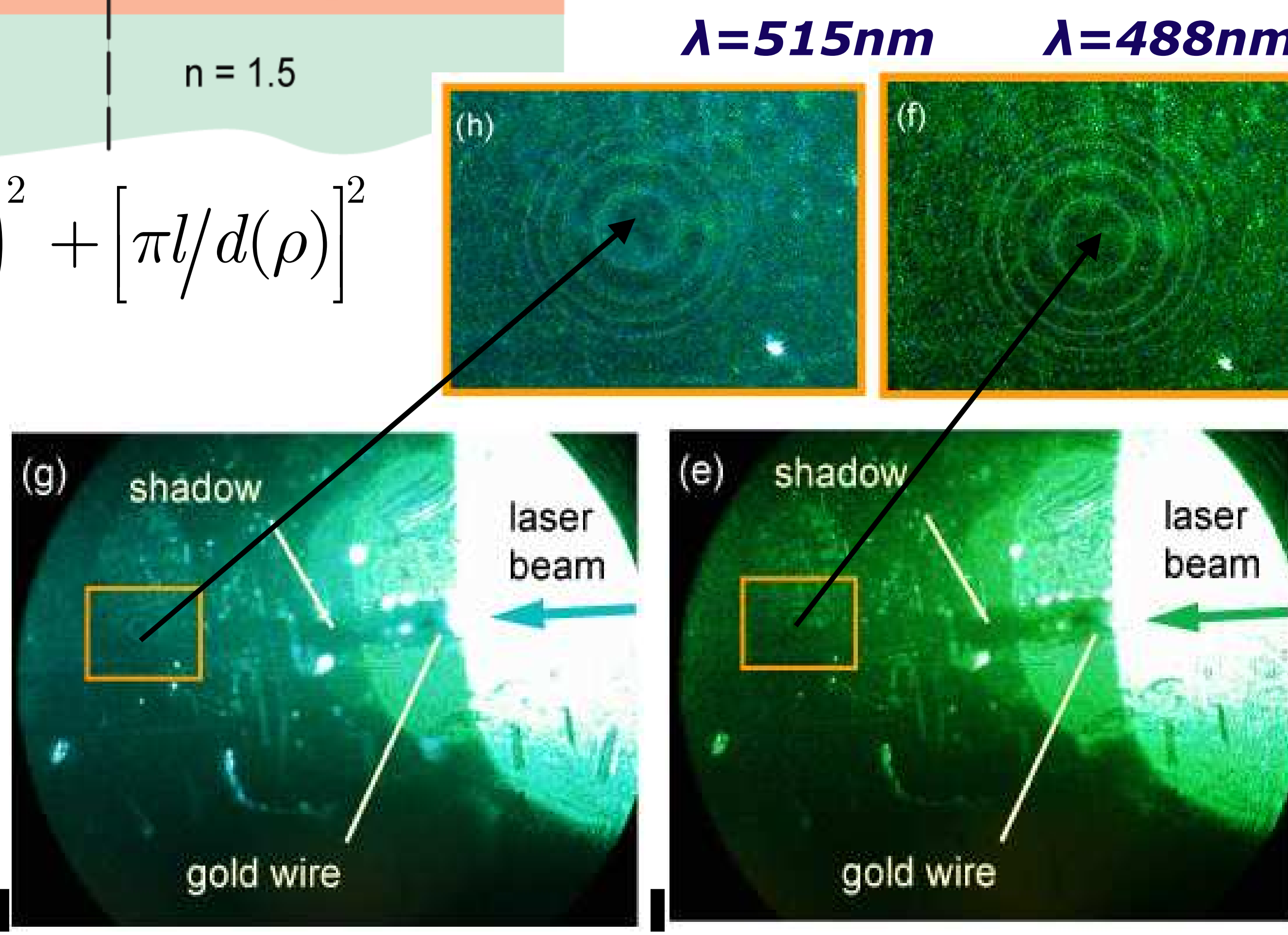
$$\left(\omega/c\right)^2 = k_\rho^2 + \left(m/\rho\right)^2 + \left[\pi l/d(\rho)\right]^2 = k_\rho^2 + k_\phi^2(\rho - b)^{-2}$$

Broadband Optical Cloak



$$\left(\omega/c\right)^2 = k_\rho^2 + \left(m/\rho\right)^2 + \left[\pi l/d(\rho)\right]^2$$

$$= k_\rho^2 + k_\phi^2 (\rho - b)^{-2}$$



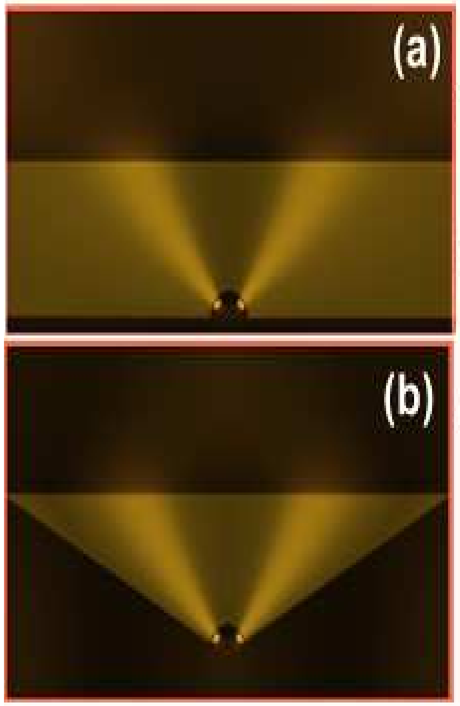
Engineering Meta-Space for Light: via Transformation Optics

Kildishev, VMS (OL, 2008); VMS, Science <u>322</u>, 384 (2008)

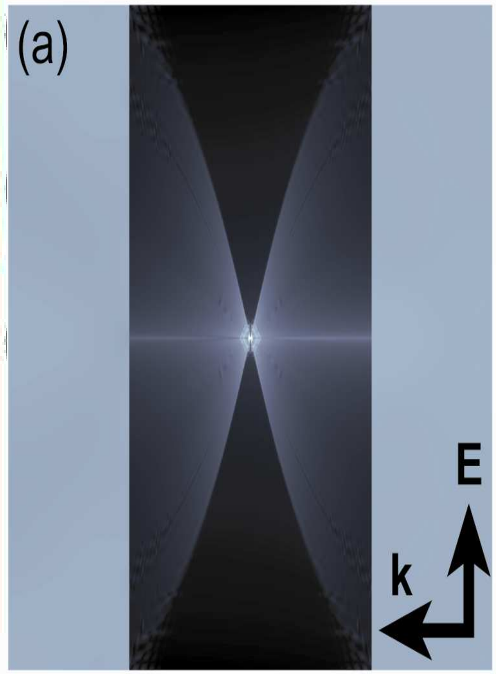
Fermat: δ∫ndl = 0

n = √ε(r)μ(r)

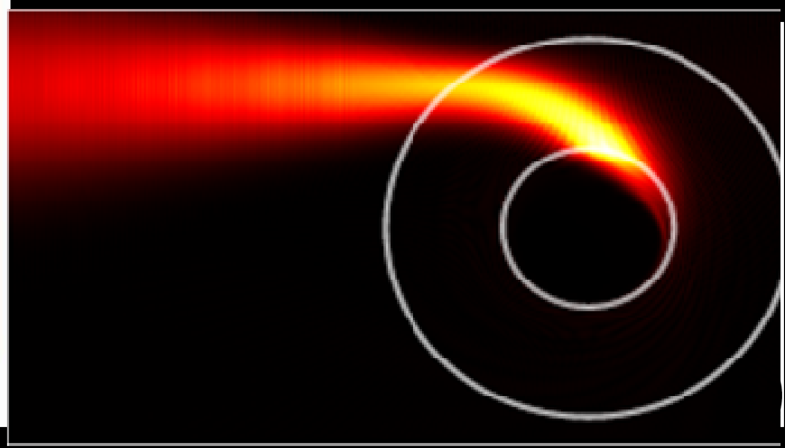
curving & nano"crafting" optical space



Planar hyperlens (Magnifies; no loss problem)



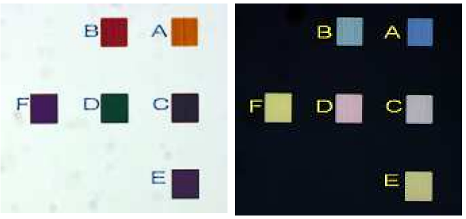
Light concentrator (also, Schurig et al)



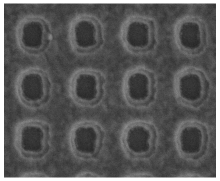
Optical Black Hole (Narimanov, Kildishev)

Take Home Messages:

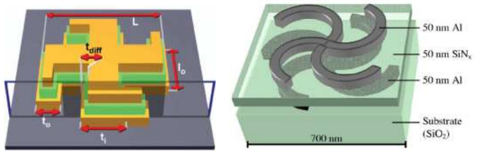
- Metamagnetics with rainbow colors



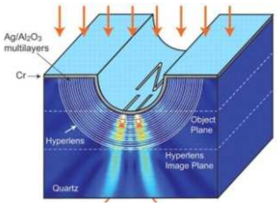
- (single-negative) MM with n = -0.3 at 580nm and (double-negative) MM with n = 0.81 at 725 nm



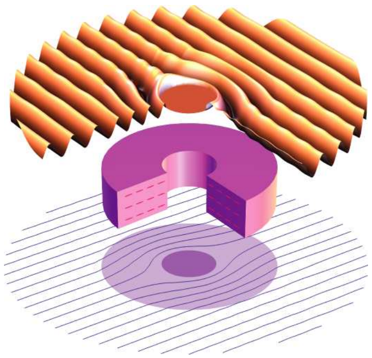
- Chiral metamaterials



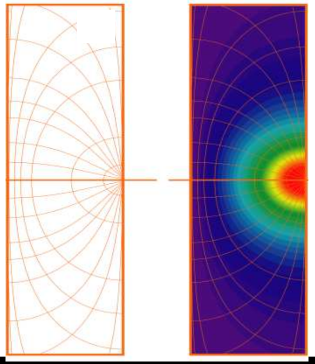
- NLO with NIMs
- Super-resolution



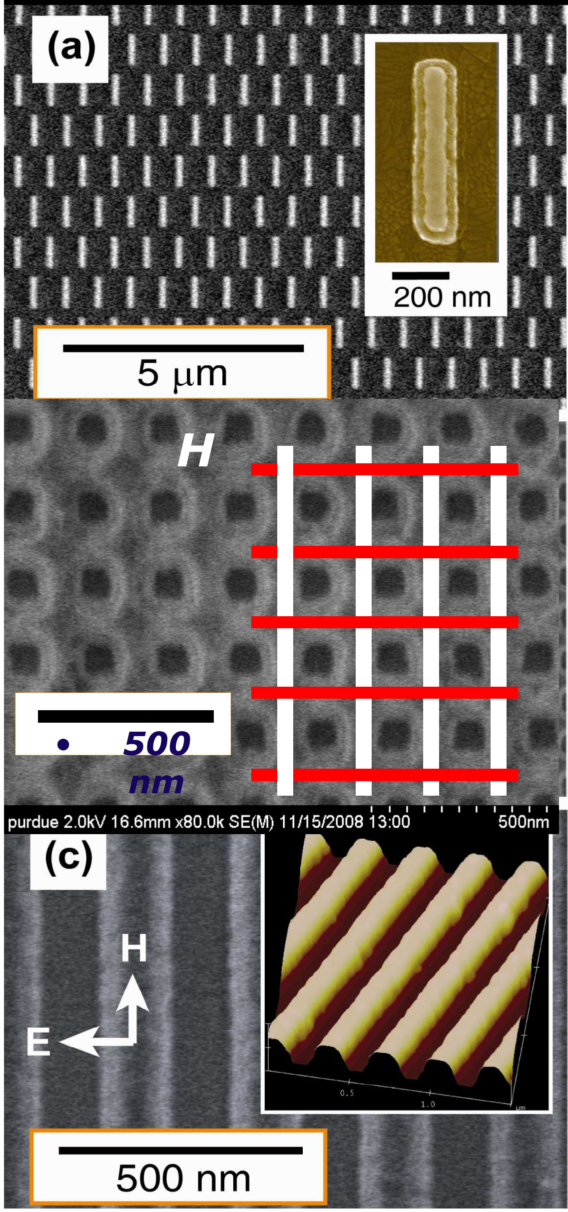
- Optical cloak of invisibility



- Engineered meta-space for light



Highlights of Purdue "Meta-Research"



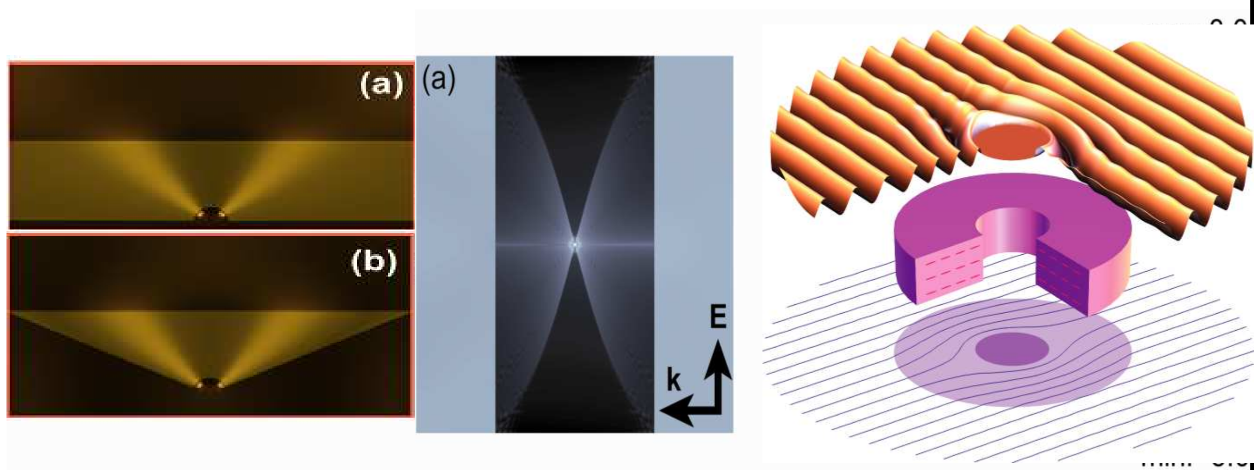
Purdue Photonic Metamaterials

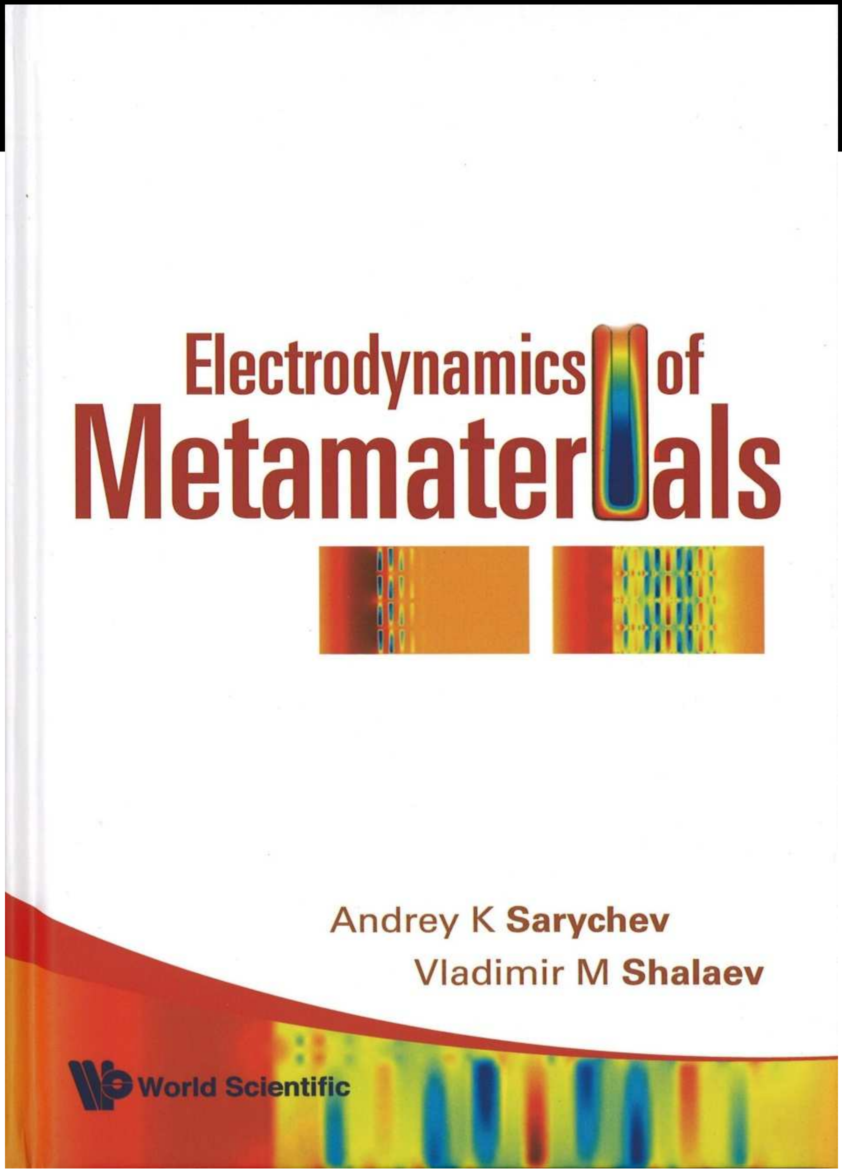
(a) 1-st optical negative-index MM (1.5 μm; 2005)

(b) Negative index MM at shortest λ (~580nm; 2009)

(c) 1-st magnetic MM across entire visible (2007)

Transformation Optics with MMs: Flat hyperlens, concentrator, and cloak




Electrodynamics of Metamaterials
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World Scientific

Cast of Characters