Engineering Optical Space with Metamaterials: from Metamagnetics to Negative-Index and Cloaking

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Invisibility: An Ancient Dream

Perseus’ helmet (Greek mythology)  Tarnhelm of invisibility (Norse mythology)  Cloaking devices (Star Trek, USA)

Ring of Gyges (“The Republic”, Plato)  The 12 Dancing Princesses (Brothers Grimm, Germany)  Harry Potter’s cloak (J. K. Rowling, UK)
Invisibility in Nature: Chameleon Camouflage
Invisibility by Transformation of Time-Space

Black hole
Invisibility to Radar: Stealth Technology

**Stealth technique:**
Radar cross-section reductions by absorbing paint / non-metallic frame / shape effect...
Active Camouflage: Real time capture and re-display

Illustrating the concept: active capture and re-display, creates an "illusory transparency",

Optical Camouflage, Tachi Lab, U. of Tokyo, Japan
Cloaking ≠ Invisibility

• Cloaking is more than invisibility or camouflage
  – Camouflage: an adaptation to the surrounding environment.
  – Cloaking: No need to adapt to a particular environment, with the ultimate goal of transparency — no scattering; no shade.

• Criteria for an ideal cloak
  – Macroscopic, not limit to subwavelength size or near field region.
  – Independent to the object to be cloaked.
  – Minimized absorption and scattering.
  – Passive
  – Broadband
  – ...

Ideal Cloak: from fiction to fact?

Examples with scientific elements:

- *The Invisible Man* by H. G. Wells (1897)
- “The invisible woman” in *The Fantastic 4* by Lee & Kirby (1961)
Examples with scientific elements:

- **The Invisible Man** by H. G. Wells (1897)

  "... it was an idea ... to lower the refractive index of a substance, solid or liquid, to that of air — so far as all practical purposes are concerned." -- Chapter 19

  "Certain First Principles"

- "The invisible woman" in **The Fantastic 4** by Lee & Kirby (1961)

  "... she achieves these feats by bending all wavelengths of light in the vicinity around herself ... without causing any visible distortion." -- Introduction from Wikipedia

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**Pendry et al.; Leonhard, Science, 2006**
(Earlier work: cloak of thermal conductivity by Greenleaf et al., 2003)
Transformation of Maxwell’s equations

Straight field line in Cartesian coordinate

Distorted field line in distorted coordinate

Spatial profile of $\varepsilon$ & $\mu$ tensors determines the distortion of coordinate

Seeking for profile of $\varepsilon$ & $\mu$ to make light avoid particular region in space — optical cloaking

Pendry et al., Science, 2006
A similarity in Mother Nature

The bending of light due to the gradient in refractive index in a desert mirage
Cloaking in spherical system

The transformation in spherical system:

\[ 0 < r < b \rightarrow a < r' < b \]

\[ r' = \frac{b-a}{b} r + a \quad \theta' = \theta \quad \phi' = \phi \]

\[
\begin{align*}
\varepsilon_r = \mu_r &= \frac{b}{b-a} \frac{(r-a)^2}{r^2} \\
\varepsilon_\theta = \mu_\theta &= \frac{b}{b-a} \\
\varepsilon_\phi = \mu_\phi &= \frac{b}{b-a}
\end{align*}
\]

Similar idea was proposed by Leonhardt, Science, 2006

Pendry et al., Science, 2006
Route to Cloaking: Tailor the $(\varepsilon, \mu)$ distribution

$\varepsilon, \mu$ diagram:

- **Electrical Plasma** (Metals at optical wavelengths)
- **Magnetic Plasma** (Not naturally occurring at optical wavelengths)
- **Evanescent waves**
- **Common Transparent Dielectrics**

- $\varepsilon < 0$
- $\mu > 0$
- $\varepsilon > 0$
- $\mu > 0$
- $\varepsilon < 0$
- $\mu < 0$
Natural Optical Materials

- **Metals**
  - Electrical Plasma (Metals at optical wavelengths)
  - Evanescent waves
- **Common Transparent Dielectrics**
  - Air
  - Water
  - Crystals
  - Semiconductors
- **Negative Index Materials**
  - Magnetic Plasma (Not naturally occurring at optical wavelengths)
  - Evanescent waves

- **Graphical Representation**: The diagram illustrates the properties of different materials based on their permittivity ($\varepsilon$) and permeability ($\mu$). Materials are categorized into regions with specific combinations of $\varepsilon$ and $\mu$ values.
Metamaterials: Artificial media with designed \((\varepsilon, \mu)\)

Metamaterial is an arrangement of artificial structural elements, designed to achieve advantageous and unusual electromagnetic properties. ---Metamorphose

\[\mu\varepsilon\alpha = \text{meta} = \text{beyond} \ (\text{Greek})\]

A natural material with its atoms

A metamaterial with artificially structured “atoms”
Electromagnetic properties v.s. characteristic sizes

\[ \frac{a}{\lambda} \]

- \( a \ll \lambda \):
  - Effective medium description using Maxwell equations with \( \mu, \varepsilon, n, Z \)
  - Example: Optical crystals, Metamaterials

- \( a \approx \lambda \):
  - Structure dominates.
  - Properties determined by diffraction and interference
  - Example: Photonics crystals, Phased array radar, X-ray diffraction optics

- \( a \gg \lambda \):
  - Properties described using geometrical optics and ray tracing
  - Example: Lens system, Shadows
Natural Crystals

... have lattice constants much smaller than light wavelengths: \( a \ll \lambda \)

... are treated as homogeneous media with parameters \( \varepsilon, \mu, n, Z \) (tensors in anisotropic crystals)

... have a positive refractive index: \( n > 1 \)

... show no magnetic response at optical wavelengths: \( \mu = 1 \)
Photonic crystals

... have lattice constants comparable to light wavelengths: $a \sim \lambda$

... can be artificial or natural

... have properties governed by the diffraction of the periodic structures

... may exhibit a bandgap for photons

... typically are not well described using effective parameters $\varepsilon, \mu, n, Z$

... often behave like but they are not true metamaterials
Noble metal: $\varepsilon < 1$ in nature

**Drude model for permittivity:**

$$\varepsilon(\omega) = \varepsilon_0 - \frac{\omega_p^2}{\omega(\omega + i\Gamma)}$$

**Silver parameters:**

- $\varepsilon_0 = 5.0$
- $\omega_p = 9.216 \text{ eV}$
- $\Gamma = 0.0212 \text{ eV}$

Experimental data from Johnson & Christy, PRB, 1972
Electrical metamaterials: 
*metal wires arrays with tunable plasma frequency*

A periodic array of thin metal wires with $r \ll a \ll \lambda$ acts as a low frequency plasma

The effective $\varepsilon$ is described with modified $\omega_p$

Plasma frequency depends on geometry rather than on material properties

$$\varepsilon = \varepsilon' + i\varepsilon'' = 1 - \frac{\omega_p^2}{\omega(\omega + i\varepsilon_0 a^2 \omega_p^2 / \pi r^2 \sigma)}$$

$$\omega_p^2 = \frac{2\pi c^2}{a^2 \ln(a/r)}$$

*Pendry, PRL (1996)*
Metal-Dielectric Composites and Mixing Rules

\[
\begin{align*}
\varepsilon_\parallel &= c_1 \varepsilon_1 + c_2 \varepsilon_2 \\
\varepsilon_\perp &= \varepsilon_1 \varepsilon_2 / (c_1 \varepsilon_2 + c_2 \varepsilon_1)
\end{align*}
\]

\[
\frac{\varepsilon_{MG}(\omega) - \varepsilon_h(\omega)}{\varepsilon_{MG}(\omega) + 2\varepsilon_h(\omega)} = f \frac{\varepsilon_i(\omega) - \varepsilon_h(\omega)}{\varepsilon_i(\omega) + 2\varepsilon_h(\omega)}
\]

\[
\varepsilon_e = \varepsilon'_e + i\varepsilon''_e = \frac{1}{2(d-1)} \left\{ (dp-1)\varepsilon_m + (d-1-dp)\varepsilon_d \pm \sqrt{[(dp-1)\varepsilon_m + (d-1-dp)\varepsilon_d]^2 + 4(d-1)\varepsilon_m\varepsilon_d} \right\}
\]
Electromagnetic properties of metal wires

**Depolarization factor:**

\[ q_i = \int_0^\infty \frac{a_i a_j a_k ds}{2(s + a_i^2)^{3/2} (s + a_j^2)^{1/2} (s + a_k^2)^{1/2}} \]

**Screening factor:**

\[ \kappa = \frac{(1 - q)}{q} \]

**Clausius-Mossotti yields shape-dependent EMT:**

\[ f \frac{\epsilon_m - \epsilon_{\text{eff}}}{\epsilon_m + \kappa \epsilon_{\text{eff}}} + (1 - f) \frac{\epsilon_d - \epsilon_{\text{eff}}}{\epsilon_d + \kappa \epsilon_{\text{eff}}} = 0 \]

\[ \epsilon_{\text{eff}} = \frac{1}{2\kappa} \left\{ \bar{\epsilon} \pm \sqrt{\bar{\epsilon}^2 + 4\kappa \epsilon_m \epsilon_d} \right\} \]

\[ \bar{\epsilon} = [(\kappa + 1)f - 1]\epsilon_m + [\kappa - (\kappa + 1)f]\epsilon_d \]
Absence of Optical Magnetism in Nature

**Magnetic coupling to an atom:** \( \sim \mu_B = \frac{e\hbar}{2m_e c} = \alpha e a_0 \) (Bohr magneton)

**Electric coupling to an atom:** \( \sim ea_0 \)

**Magnetic effect / electric effect** \( \approx \alpha^2 \approx (1/137)^2 < 10^{-4} \)

“...the magnetic permeability \( \mu(\omega) \) ceases to have any physical meaning at relatively low frequencies...there is certainly no meaning in using the magnetic susceptibility from optical frequencies onwards, and in discussion of such phenomena we must put \( \mu=1 \).”

*Landau and Lifshitz, ECM, Chapter 79.*
SRR: the first magnetic metamaterials

- **A bulk metal has no magnetism in optics**

- **A metal ring: weak magnetic response**

- **Cut the ring to introduce resonance**

- **A split ring: magnetic resonance**

- **Double the ring to strengthen the resonance**

- **Double SRR: enhanced magnetic resonance**

**Split-ring resonator (SRR)**

**Theory:** Pendry et al., 1999.
**Experiment:** Smith et al., 2000.
Towards Optical Magnetic Metamaterials

Terahertz magnetism

A) Yen, et al. ~ 1THz (2-SRR) – 2004
Katsarakis, et al (SRR – 5 layers) - 2005
b) Zhang et al ~50THz (SRR+mirror) - 2005
c) Linden, et al. 100THz (1-SRR) -2004
d) Enkrich, et al. 200THz (u-shaped)-2005

2004-2007 years:
from 10 GHz to 500 THZ
Limits of size scaling in SRRs

Direct scaling-down the SRR dimensions doesn’t help much...

Loss in metal gives kinetic inductance

\[ L_{\text{coil}} \propto \text{size} \quad \text{and} \quad L_{\text{kinetic}} \propto \frac{1}{\text{size}} \]

\[ L_{\text{total}} = L_{\text{coil}} + L_{\text{kinetic}} \]

\[ C_{\text{total}} \propto \text{size} \]

\[ \omega_{\text{res}} \propto \frac{1}{\sqrt{L_{\text{total}} \times C_{\text{total}}}} = \frac{1}{\sqrt{(A \cdot \text{size} + B / \text{size}) \cdot (C \cdot \text{size})}} \propto \frac{1}{\sqrt{\text{size}^2 + \text{const.}}} \]

Zhou et al., PRL (2005); Klein, et al., OL (2006)
Magnetic Metamaterial: Nanorod to Nanostrip

- Dielectric
- Metal

Nanorod pair
Nanorod pair array
Nanostrip pair

- Nanostrip pair has a much stronger magnetic response

Lagar’kov, Sarychev PRB (1996) - $\mu > 0$
Podolskiy, Sarychev & Shalaev, JNQPM (2002) - $\mu < 0$ & $n < 0$
Svirko, et al, APL (2001) - “crossed” rods for chirality
Visible magnetism: structure and geometries

Purdue group

Width varies from 50 nm to 127 nm
Negative Magnetic Response
Width tunes resonance

**Resonant TM Transmission**

**Non-resonant TE Transmission**

**Resonant TM Reflection**

**Non-resonant TE Reflection**

<table>
<thead>
<tr>
<th>Sample #</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width w (nm)</td>
<td>50</td>
<td>69</td>
<td>83</td>
<td>98</td>
<td>118</td>
<td>127</td>
</tr>
</tbody>
</table>

Dependence of $\lambda_m$ and $\mu'$ on $w$

$\lambda_m$ as a function of strip width “$w$”: experiment vs. theory

Negligible saturation effect on size-scaling (as opposed to SRRs)
Negative refractive index: A historical review

... energy can be carried forward at the group velocity but in a direction that is anti-parallel to the phase velocity...

Schuster, 1904

Negative refraction and backward propagation of waves

Mandel’stam, 1945

Left-handed materials: the electrodynamics of substances with simultaneously negative values of $\varepsilon$ and $\mu$

Veselago, 1968

Pendry, the one who whipped up the recent boom of NIM researches

Perfect lens (2000)
EM cloaking (2006)
Metamaterials with Negative Refraction

Single-negative:
\[ n < 0, \quad \text{if } \varepsilon' |\mu| + \mu' |\varepsilon| < 0 \]

Double-negative:
\[ n < 0, \quad \text{with both } \varepsilon' < 0 \text{ and } \mu' < 0 \]

(F can be large)

Figure of merit
\[ F = |n'|/n'' \]
Negative index metamaterials

*Negative electrical metamaterial + Negative magnetic metamaterial = Negative index metamaterial*

Smith, et al., UCSD, PRL (2000)
The first NIM in microwave

Transmission properties of the structure

\[ E, H \sim \exp[in(\omega/c)z] \]
\[ n = \pm \sqrt{\varepsilon \mu} \]

Smith, et al., UCSD, PRL (2000)
Negative-index Material in Microwave

2D waveguide

3D free space

Smith, et al., UCSD, Science (2001)

Negative Index Design for Optical Frequencies

- **Dielectric**
- **Metal**

Nanostrip pair (TM) $\mu < 0$ (resonant)
Nanostrip pair (TE) $\varepsilon < 0$ (non-resonant)
Fishnet $\varepsilon$ and $\mu < 0$

*S. Zhang, et al., PRL (2005)*
A Negative-index Material close to the Visible

- E-beam lithography
- Period = 300 nm along both axis
- Average width of strips along H = 130 nm
  Average width of strips along E = 95 nm

Stacking:
10 nm of Al₂O₃
33 nm of Ag
38 nm of Al₂O₃
33 nm of Ag
10 nm of Al₂O₃
Spectra for Primary Polarization

- Magnetic resonance around $\lambda = 800$ nm
- Electric resonance around $\lambda = 600$ nm
- Finite Elements

Solid line: Experimental
Dashed line: Simulated

Field Maps for Primary Polarization

Electrical Resonance, $\lambda = 625$ nm

Magnetic Resonance, $\lambda = 815$ nm
Double Negative NIM (n’=-1.0, FOM=1.3, at 810 nm)
Single Negative NIM (n’=-0.9, FOM=0.7, at 770 nm)

Chettiar et al
OL (2007)
Materials ready...

$\epsilon, \mu$ diagram:

- Electric Plasma (Metal, at optical wavelengths)
- Evanescent waves
- Negative Index Materials

$\epsilon < 0, \mu > 0$ for Electric Plasma

$\epsilon > 0, \mu > 0$ for Common Transparent Dielectrics

$\epsilon < 0, \mu < 0$ for Evanescent waves

Gradient Metamaterial: A critical step

Negative index gradient lens

\[ n = -2.7 \text{ on edge} \]

\[ n = -1.0 \text{ on center} \]

D. R. Smith Group, PRE 2005; APL 2006
Back to Cloak: cylindrical system

The transformation in cylindrical system:

\[ 0 < r < b \quad \rightarrow \quad a < r' < b \]

\[ r' = \frac{b - a}{b} r + a \quad \theta' = \theta \quad z' = z \]

\[
\begin{align*}
\varepsilon_r &= \mu_r = \frac{r - a}{r} \\
\varepsilon_\theta &= \mu_\theta = \frac{r}{r - a} \\
\varepsilon_z &= \mu_z = \left( \frac{b}{b - a} \right)^2 \frac{r - a}{r}
\end{align*}
\]

Smith et al., arXiv, 2006
Towards an experimental demonstration

The transformation in cylindrical system:

\[ 0 < r < b \quad \rightarrow \quad a < r' < b \]

\[ r' = \frac{b-a}{a} r + a \quad \theta' = \theta \quad z' = z \]

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\end{align*}
\]

TE incidence

\[
\begin{align*}
\varepsilon_z &= \left( \frac{b}{b-a} \right)^2 \frac{r-a}{r} \\
\mu_\theta &= \frac{r}{r-a} \\
\mu_r &= \frac{r-a}{r}
\end{align*}
\]

To maintain the dispersion relation only

\[
\begin{align*}
\varepsilon_z &= \left( \frac{b}{b-a} \right)^2 \\
\mu_\theta &= 1 \\
\mu_r &= \left( \frac{r-a}{r} \right)^2
\end{align*}
\]

Schurig et al., Science, 2006
The First Metamaterial Cloak
Experimental demonstration at microwave frequency

Structure of the cloak

<table>
<thead>
<tr>
<th>Ideal case</th>
<th>Reduced parameter</th>
<th>Experimental data</th>
</tr>
</thead>
</table>

Schurig et al., Science, 2006
Movie: How it works

Optical Cloaking with Metamaterials: Can Objects be Invisible in the Visible?

Cover article of Nature Photonics (April, 2007)
How about optical frequencies?

**Scaling the microwave cloak design?**

iero Intrinsic limits to the scaling of SRR size
- **High loss in resonant structures**

\[ \varepsilon_r = \mu_r = \frac{r-a}{r}, \quad \varepsilon_\theta = \mu_\theta = \frac{r}{r-a}, \quad \varepsilon_z = \mu_z = \left( \frac{b}{b-a} \right)^2 \frac{r-a}{r} \]

TM incidence

\[
\begin{align*}
\mu_z &= \left( \frac{b}{b-a} \right)^2 \frac{r-a}{r} \\
\varepsilon_\theta &= \frac{r}{r-a} \\
\varepsilon_r &= \frac{r-a}{r}
\end{align*}
\]

To maintain the dispersion relation

\[
\begin{align*}
\mu_z &= 1 \\
\varepsilon_\theta &= \left( \frac{b}{b-a} \right)^2 \\
\varepsilon_r &= \left( \frac{b}{b-a} \right)^2 \left( \frac{r-a}{r} \right)^2
\end{align*}
\]

- No magnetism required!
- A constant permittivity of a dielectric; \( \varepsilon_\theta > 1 \)
- Gradient in \( r \) direction only; \( \varepsilon_r \) changing from 0 to 1.

**Cai, et al., Nature Photonics, 1, 224 (2007)**
Structure of the cloak: “Round brush”

Unit cell:

Flexible control of $\varepsilon_r$;
Negligible perturbation in $\varepsilon_\theta$


metal needles embedded in dielectric host
Cloaking performance: Field mapping movies

Example:
Non-magnetic cloak @ 632.8nm

Cloak OFF  Cloak ON
Scattering issue in a non-magnetic cloak

**Ideal Cloak**

$$Z\big|_{r=b} = \sqrt{\frac{\mu_z}{\varepsilon_\theta}}_{r=b} = 1$$

Perfectly matched impedance results in zero scattering

**Non-magnetic**

$$Z\big|_{r=b} = \sqrt{\frac{\mu_z}{\varepsilon_\theta}}_{r=b} = 1 - \frac{a}{b}$$

Detrimental scattering due to impedance mismatch
Normalized scattered field
High-order transformations minimize scattering

**Linear transformation**

\[ r = \frac{b-a}{b} r' + a \]

**Nonlinear transformation**

\[ r = g(r') \]

\[ g(0) = a; \ g(b) = b; \ \frac{\partial g(r')}{\partial r'} > 0 \]

\[ \begin{align*}
\varepsilon_r &= \mu_r = \left( \frac{r'}{r} \right) \frac{\partial g(r')}{\partial r'} \\
\varepsilon_\theta &= \mu_\theta = \left( \frac{r'}{r} \right) \left[ \frac{\partial g(r')}{\partial r'} \right]^{-1} \\
\varepsilon_z &= \mu_z = \left( \frac{r'}{r} \right) \left[ \frac{\partial g(r')}{\partial r'} \right]^{-1} 
\end{align*} \]

\[ \varepsilon_r = \left( \frac{r'}{r} \right)^2; \ \varepsilon_\theta = \left[ \frac{\partial g(r')}{\partial r'} \right]^{-2}; \ \mu_z = 1 \]

\[ Z \bigg|_{r=b} = \sqrt{\mu_z / \varepsilon_\theta} \bigg|_{r=b} = \left. \frac{\partial g(r')}{\partial r'} \right|_{r=b} = 1 \]

**Jacobian Matrix**

**\( \varepsilon \) And \( \mu \) tensors for ideal cloak**

**Corresponding non-magnetic parameters**

**Set \( Z=1 \) at \( r=b \) to fix \( g(r') \)**

Example: Optimized quadratic transformation

**A second-order transformation for non-magnetic cloak with minimized scattering**

\[
r = g(r') = \left[1 - a/b + p(r' - b)\right] r' + a \quad \text{with} \quad p = a/b^2
\]
Transformation and impedance

- Transformation and new coordinates
- Impedance at $r=b$

Graphs showing:
- Old coordinates $r/a$
- New coordinates $r/a$
- Linear and quadratic impedance curves

$\text{linear}$

$\text{quadratic}$
Reduced scattering from nonlinear cloak

Normalized scattered field

- No cloak
- Ideal cloak
- Non-magnetic cloak: Linear
- Non-magnetic cloak: Quadratic
Suppression in both magnitude and directivity

Scattering radiation pattern

![Diagram showing scattering radiation pattern with labels for ideal, linear, and quadratic cases.](image)
Engineering Optical Meta-Space:
Controlling & Manipulating Light via design and fabrication of $(\varepsilon, \mu)$-distribution

Light-concentrating slab (as an example):
Example of Engineering Optical Meta-Space: Optical concentrator

Electrical field distribution

Power flow distribution

Rahm et al. (Duke U.), arXiv 0706.2452
Example of Engineering Optical Meta-Space: Square cloak

Rahm et al. (Duke U.), arXiv 0706.2452