***Metamaterials'2007*, Rome**
European Doctoral School on Metamaterials

Engineering Optical Space with Metamaterials: from Metamagnetics to Negative-Index and Cloaking

Vladimir M. Shalaev
School of Electrical & Computer Engineering
Purdue University, USA

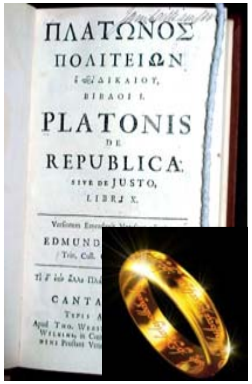
2007.10.26

Invisibility: An Ancient Dream

Perseus' helmet (Greek mythology)

Tarnhelm of invisibility (Norse mythology)

Cloaking devices (Star Trek, USA)



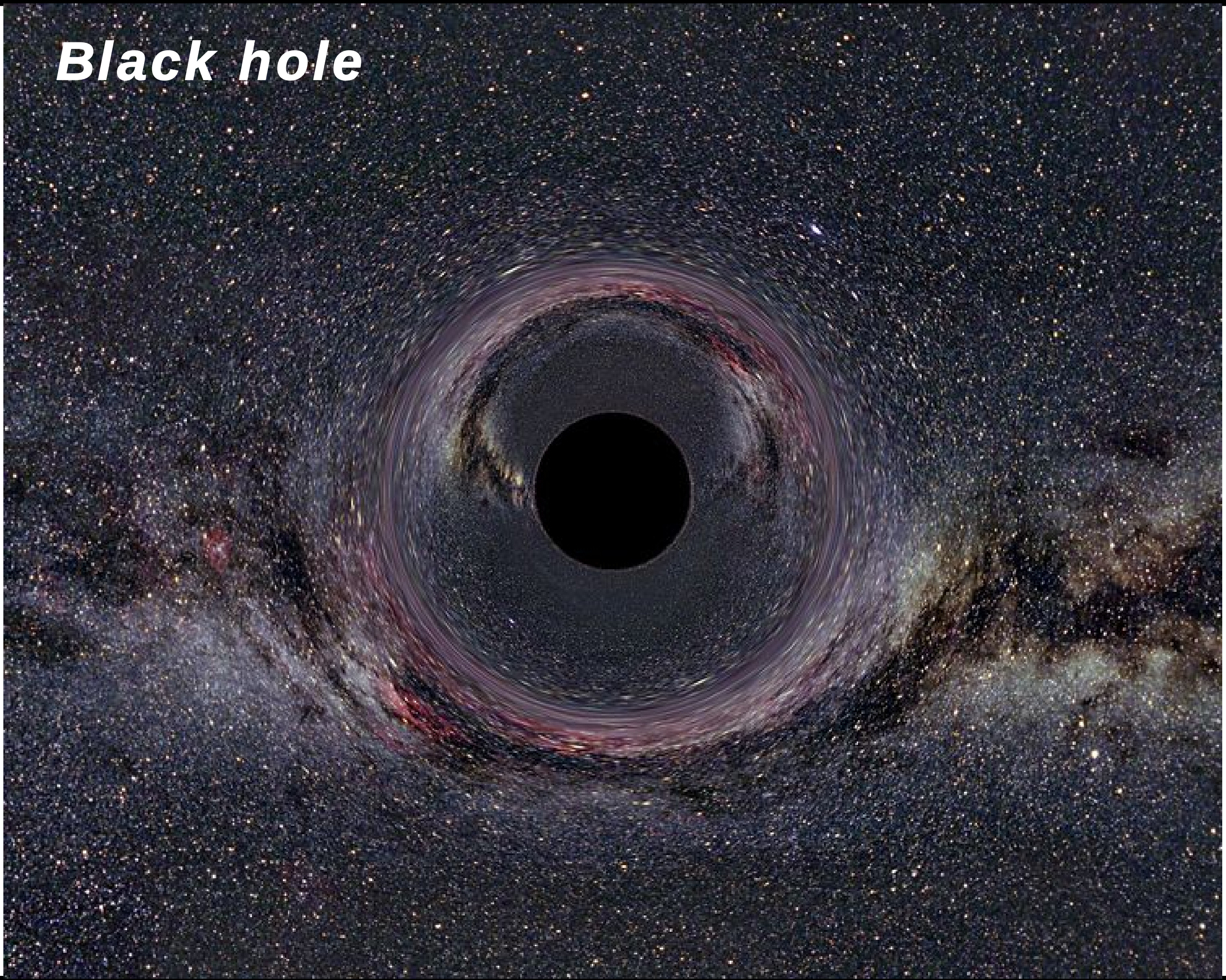
Ring of Gyges ("The Republic", Plato)

The 12 Dancing Princesses (Brothers Grimm, Germany)

Harry Potter's cloak (J. K. Rowling, UK)

Invisibility in Nature: Chameleon Camouflage

Invisibility by Transformation of Time-Space



Invisibility to Radar: Stealth Technology

Stealth technique:

Radar cross-section reductions by absorbing paint / non-metallic frame / shape effect…

Active Camouflage: Real time capture and re-display

Illustrating the concept: active capture and re-display, creates an "illusory transparency",

Optical Camouflage, Tachi Lab, U. of Tokyo, Japan

Cloaking ≠ Invisibility

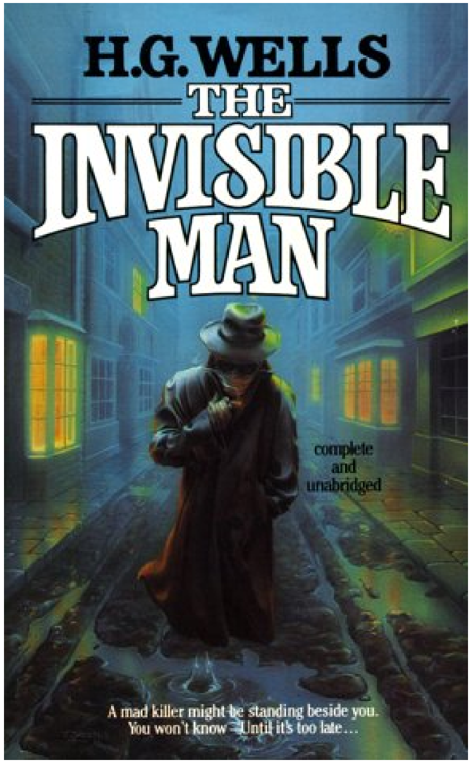
- Cloaking is more than invisibility or camouflage
  - Camouflage: an adaptation to the surrounding environment.
  - Cloaking: No need to adapt to a particular environment, with the ultimate goal of transparency — no scattering; no shade.

- Criteria for an ideal cloak
  - Macroscopic, not limit to subwavelength size or near field region.
  - Independent to the object to be cloaked.
  - Minimized absorption and scattering.
  - Passive
  - Broadband
  - ...

Ideal Cloak: from fiction to fact?

Examples with scientific elements:

- ***The Invisible Man*** by H. G. Wells (1897)



- **"The invisible woman" in *The Fantastic 4*** by Lee & Kirby (1961)

Ideal Cloak: from fiction to fact?

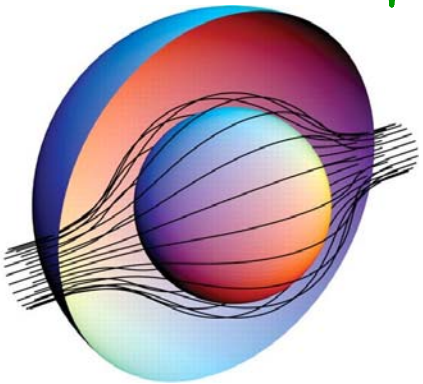
Examples with scientific elements:

- ***The Invisible Man*** by H. G. Wells (1897)

"… it was an idea … to lower the refractive index of a substance, solid or liquid, to that of air — so far as all practical purposes are concerned." -- Chapter 19 "Certain First Principles"

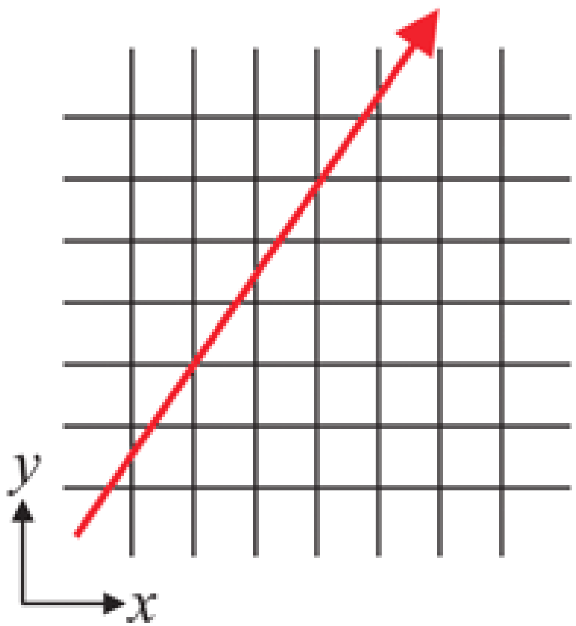
- **"The invisible woman" in *The Fantastic 4*** by Lee & Kirby (1961)

"… she achieves these feats by bending all wavelengths of light in the vicinity around herself … without causing any visible distortion." -- Introduction from Wikipedia

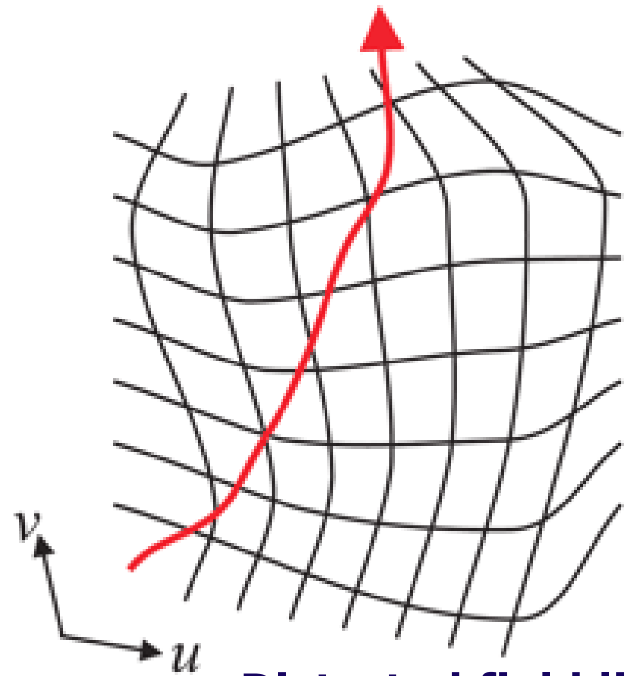


Pendry et al.; Leonhard, Science, 2006
(Earlier work: cloak of thermal conductivity by Greenleaf et al., 2003)

Transformation of Maxwell's equations



Straight field line in Cartesian coordinate



Distorted field line in distorted coordinate

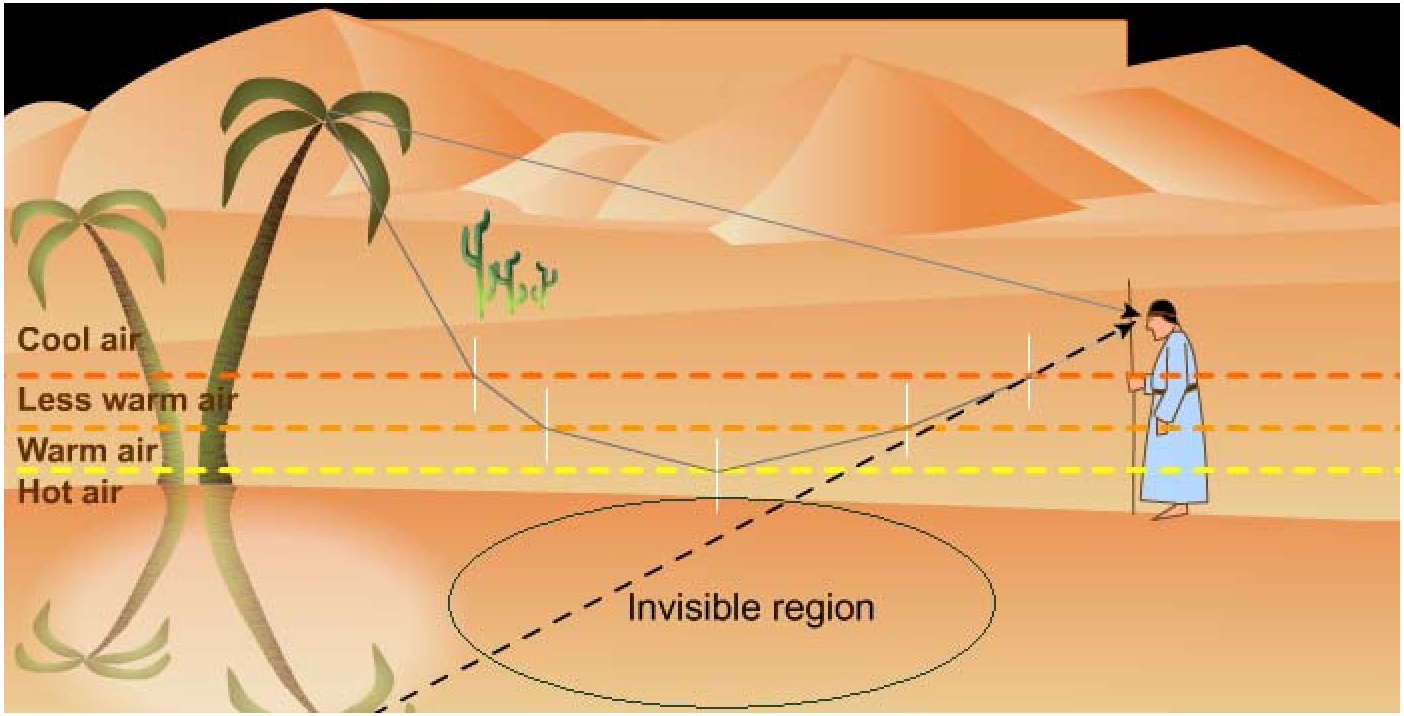
Spatial profile of ε & μ tensors determines the distortion of coordinate

Seeking for profile of ε & μ to make light avoid particular region in space — optical cloaking

Pendry et al., Science, 2006

A similarity in Mother Nature

The bending of light due to the gradient in refractive index in a desert mirage



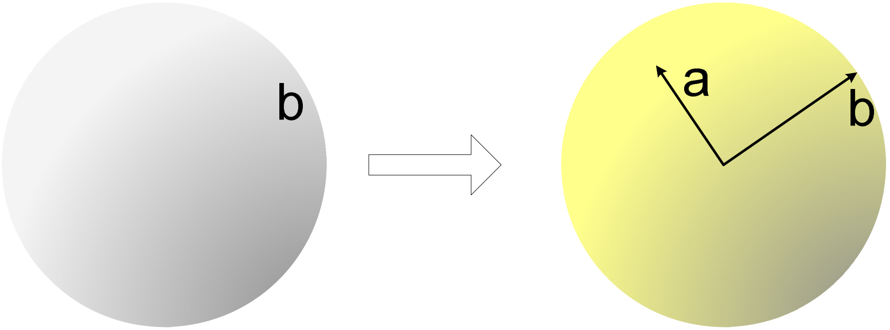
Cloaking in spherical system

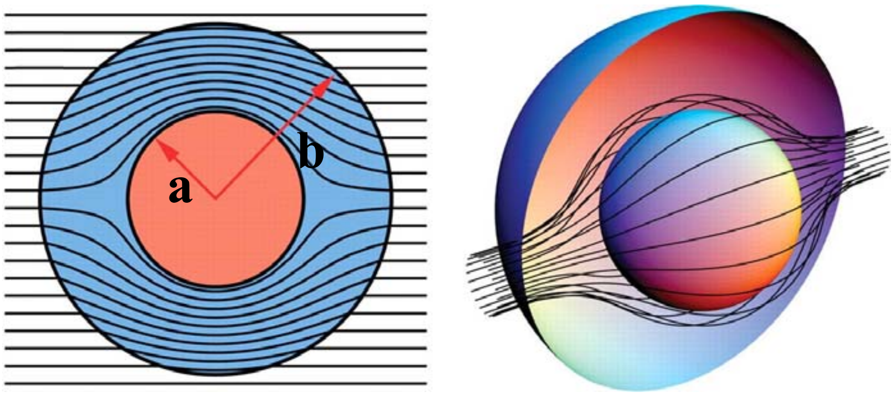
The transformation in spherical system:

$$0 < r < b \longrightarrow a < r' < b$$

$$r' = \frac{b-a}{b} r + a \qquad \theta' = \theta \qquad \phi' = \phi$$

$$\begin{cases} \varepsilon_r = \mu_r = \dfrac{b}{b-a} \dfrac{(r-a)^2}{r^2} \\ \varepsilon_\theta = \mu_\theta = \dfrac{b}{b-a} \\ \varepsilon_\phi = \mu_\phi = \dfrac{b}{b-a} \end{cases}$$



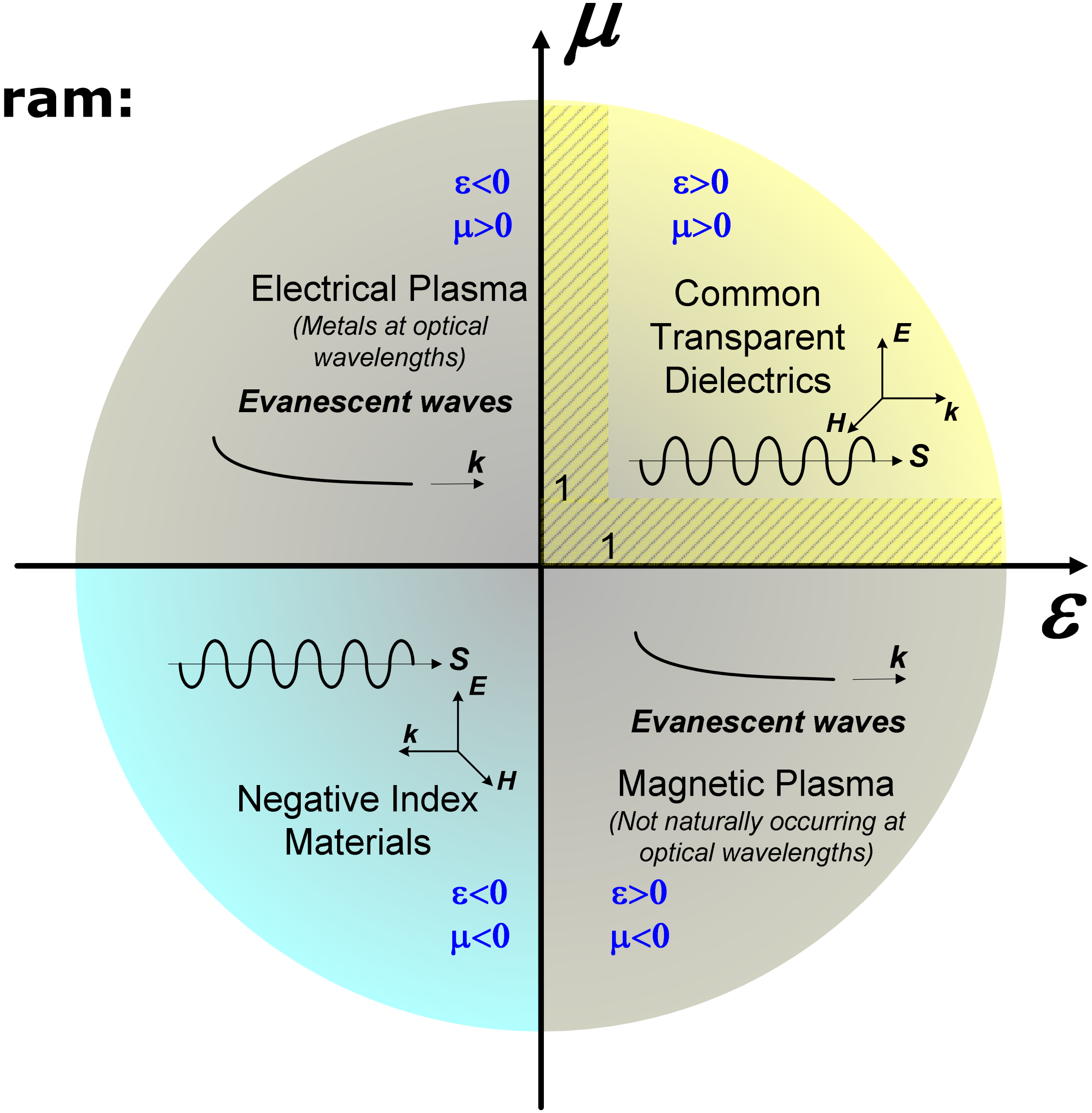


Pendry et al., Science, 2006

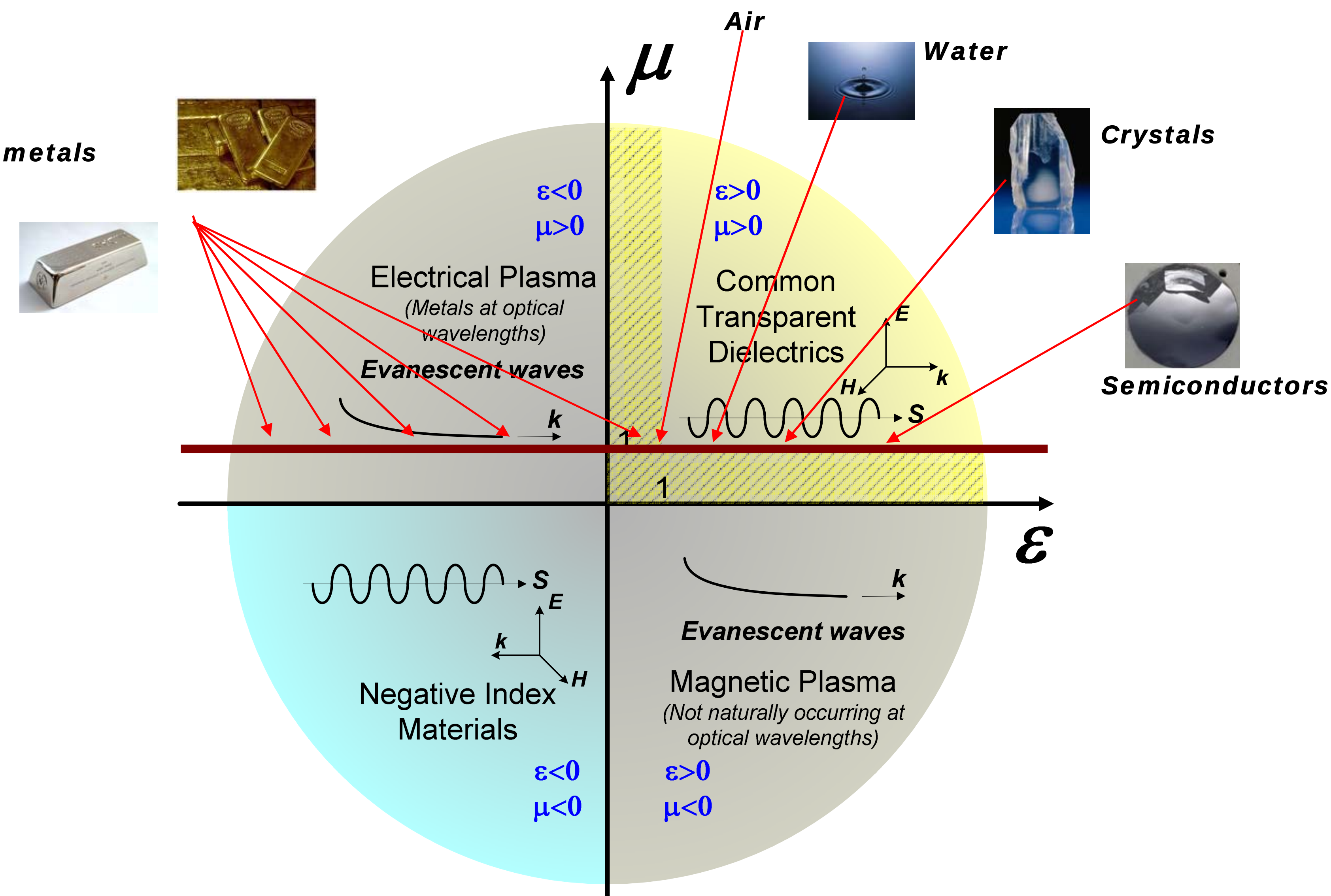
Similar idea was proposed by Leonhardt, Science, 2006

Route to Cloaking: Tailor the (ε,μ) distribution

ε, μ diagram:



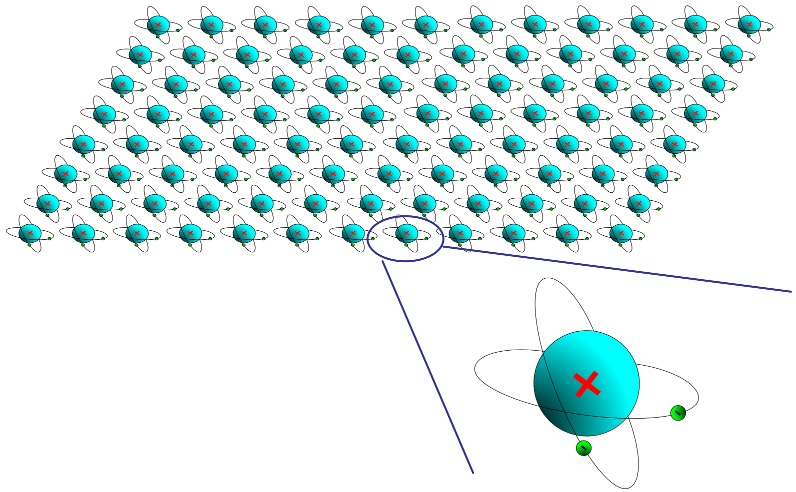
Natural Optical Materials



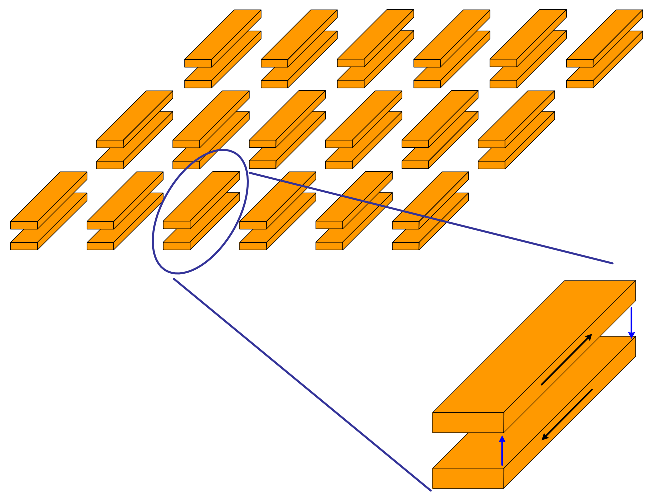
Metamaterials: Artificial media with designed (ε,μ)

Metamaterial is an arrangement of artificial structural elements, designed to achieve advantageous and unusual electromagnetic properties. ---Metamorphose

μετα = meta = beyond (Greek)



A natural material with its atoms



A metamaterial with artificially structured "atoms"

Electromagnetic properties v.s. characteristic sizes

0 — 1 — ∞ — a/λ

$a \ll \lambda$.

Effective medium description using Maxwell equations with μ, ε, n, Z

Example:

Optical crystals

Metamaterials

$a \sim \lambda$.

Structure dominates. Properties determined by diffraction and interference

Example:

Photonics crystals

Phased array radar

X-ray diffraction optics

$a \gg \lambda$.

Properties described using geometrical optics and ray tracing

Example:

Lens system

Shadows

Natural Crystals

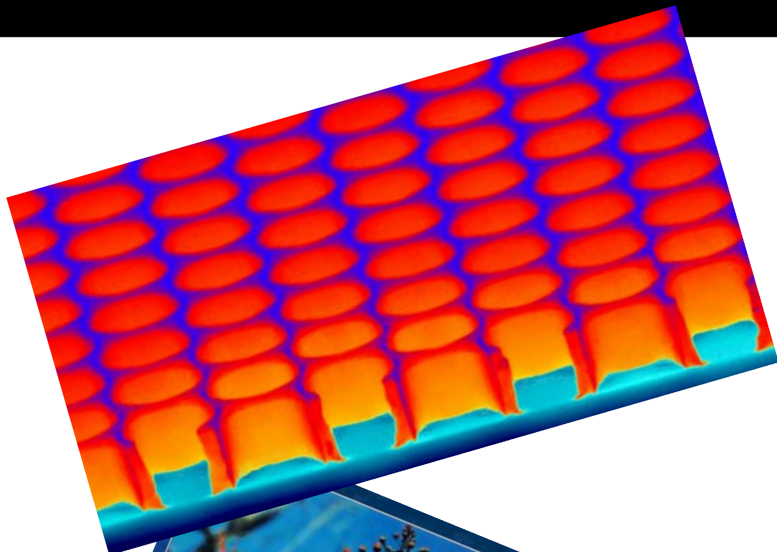
… have lattice constants much smaller than light wavelengths: $a << \lambda$

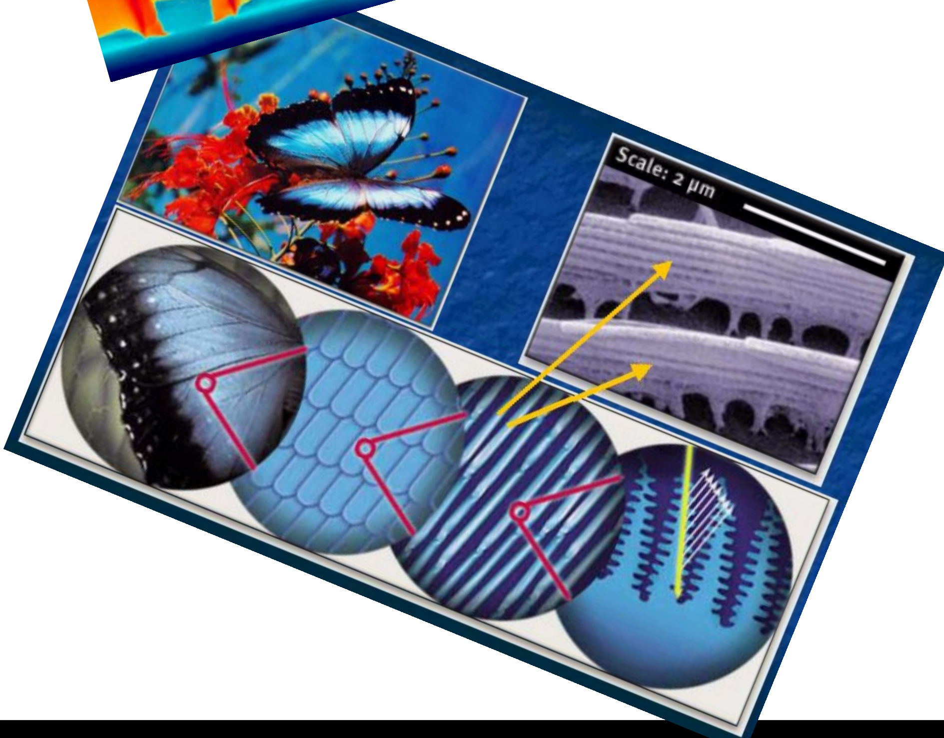
… are treated as homogeneous media with parameters ε, μ, n, Z **(tensors in anisotropic crystals)**

… have a positive refractive index: $n > 1$

… show no magnetic response at optical wavelengths: $\mu = 1$

Photonic crystals





... have lattice constants comparable to light wavelengths: $a \sim \lambda$

... can be artificial or natural

... have properties governed by the diffraction of the periodic structures

... may exhibit a bandgap for photons

... typically are *not* well described using effective parameters ε, μ, n, Z

... often behave like but they are *not* true metamaterials

Noble metal: ε < 1 in nature

Drude model for permittivity:

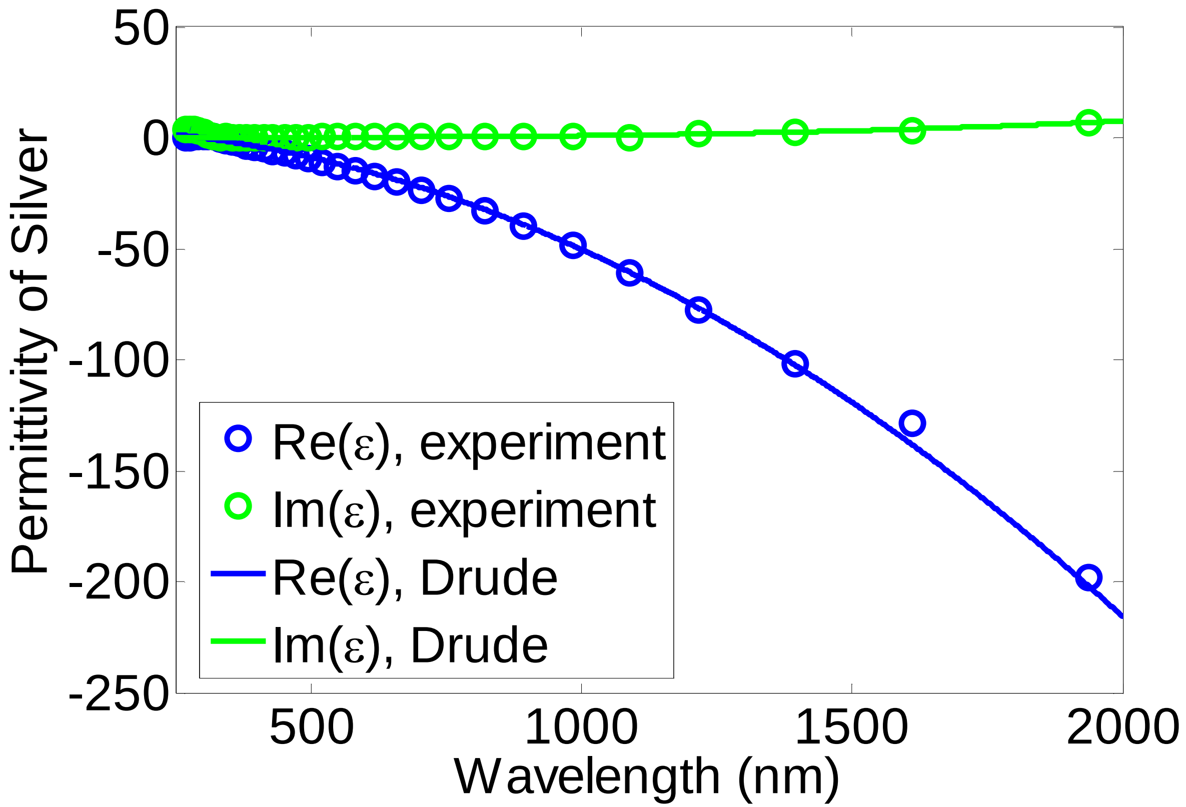
$$\varepsilon(\omega) = \varepsilon_0 - \frac{\omega_p^2}{\omega(\omega + i\Gamma)}$$

Silver parameters:

$\varepsilon_0 = 5.0$

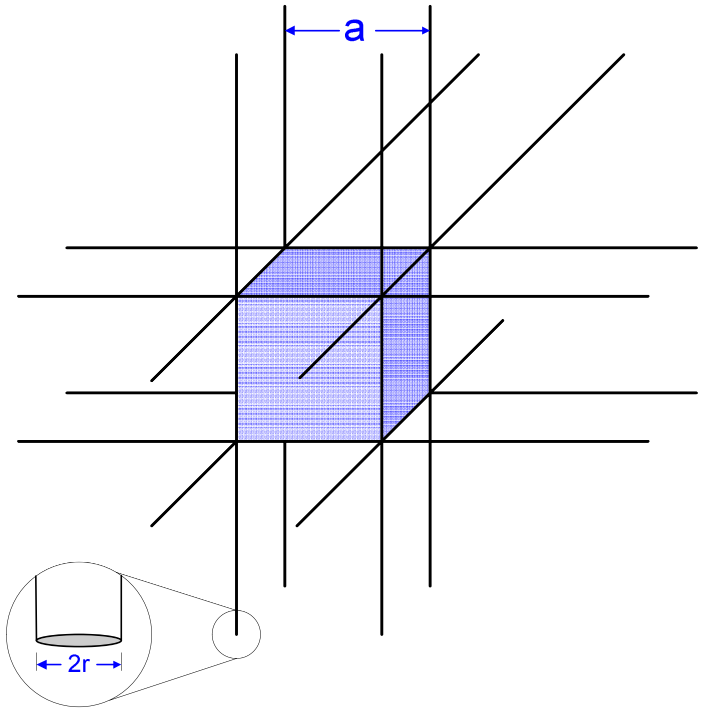
$\omega_p = 9.216 \quad eV$

$\Gamma = 0.0212 \quad eV$



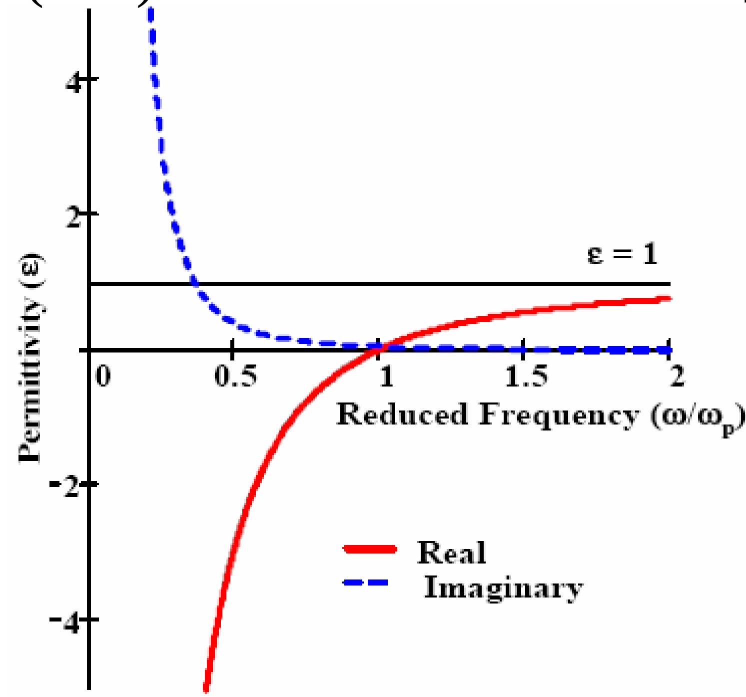
Experimental data from Johnson & Christy, PRB, 1972

Electrical metamaterials:
metal wires arrays with tunable plasma frequency



$$\varepsilon = \varepsilon' + i\varepsilon'' = 1 - \frac{\omega_p^2}{\omega(\omega + i\varepsilon_0 a^2 \omega_p^2 / \pi r^2 \sigma)}$$

$$\omega_p^2 = \frac{2\pi c^2}{a^2 \ln(a/r)}$$



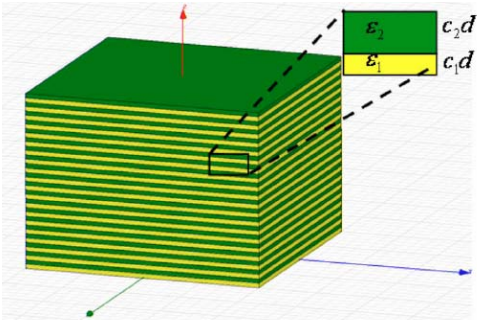
A periodic array of thin metal wires with r<<a<<λ acts as a low frequency plasma

The effective ε is described with modified ω_p

Plasma frequency depends on geometry rather than on material properties

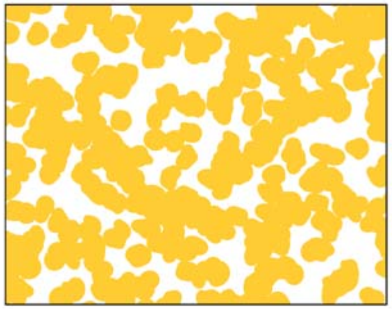
Pendry, PRL (1996)

Metal-Dielectric Composites and Mixing Rules



$$\begin{cases} \varepsilon_{\parallel} = c_1\varepsilon_1 + c_2\varepsilon_2 \\ \varepsilon_{\perp} = \varepsilon_1\varepsilon_2 / (c_1\varepsilon_2 + c_2\varepsilon_1) \end{cases}$$

$$\frac{\varepsilon_{MG}(\omega) - \varepsilon_h(\omega)}{\varepsilon_{MG}(\omega) + 2\varepsilon_h(\omega)} = f\,\frac{\varepsilon_i(\omega) - \varepsilon_h(\omega)}{\varepsilon_i(\omega) + 2\varepsilon_h(\omega)}$$



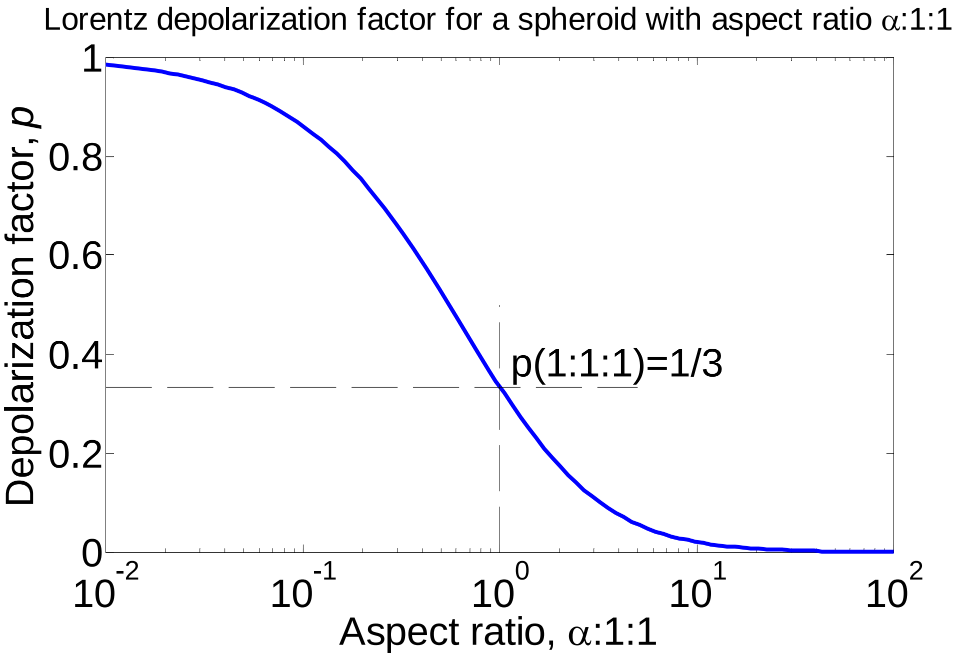
$$\varepsilon_e = \varepsilon'_e + i\varepsilon''_e$$

$$= \frac{1}{2(d-1)} \left\{ \begin{aligned} &(dp-1)\varepsilon_m + (d-1-dp)\varepsilon_d \\ &\pm\sqrt{[(dp-1)\varepsilon_m + (d-1-dp)\varepsilon_d]^2 + 4(d-1)\varepsilon_m\varepsilon_d} \end{aligned} \right\}$$

Electromagnetic properties of metal wires

Depolarization factor:

$$q_i = \int_0^\infty \frac{a_i a_j a_k ds}{2(s+a_i^2)^{3/2}(s+a_j^2)^{1/2}(s+a_k^2)^{1/2}}$$



Screening factor:

$$\kappa = (1-q)/q$$

Clausius-Mossotti yields shape-dependent EMT:

$$f\frac{\varepsilon_m - \varepsilon_{eff}}{\varepsilon_m + \kappa\varepsilon_{eff}} + (1-f)\frac{\varepsilon_d - \varepsilon_{eff}}{\varepsilon_d + \kappa\varepsilon_{eff}} = 0$$

$$\varepsilon_{eff} = \frac{1}{2\kappa}\left\{\bar{\varepsilon} \pm \sqrt{\bar{\varepsilon}^2 + 4\kappa\varepsilon_m\varepsilon_d}\right\}$$

$$\bar{\varepsilon} = [(\kappa+1)f - 1]\varepsilon_m + [\kappa - (\kappa+1)f]\varepsilon_d$$

Absence of Optical Magnetism in Nature

Magnetic coupling to an atom: ~ $\mu_B = e\hbar / 2m_e c = \alpha e a_0$ ***(Bohr magneton)***

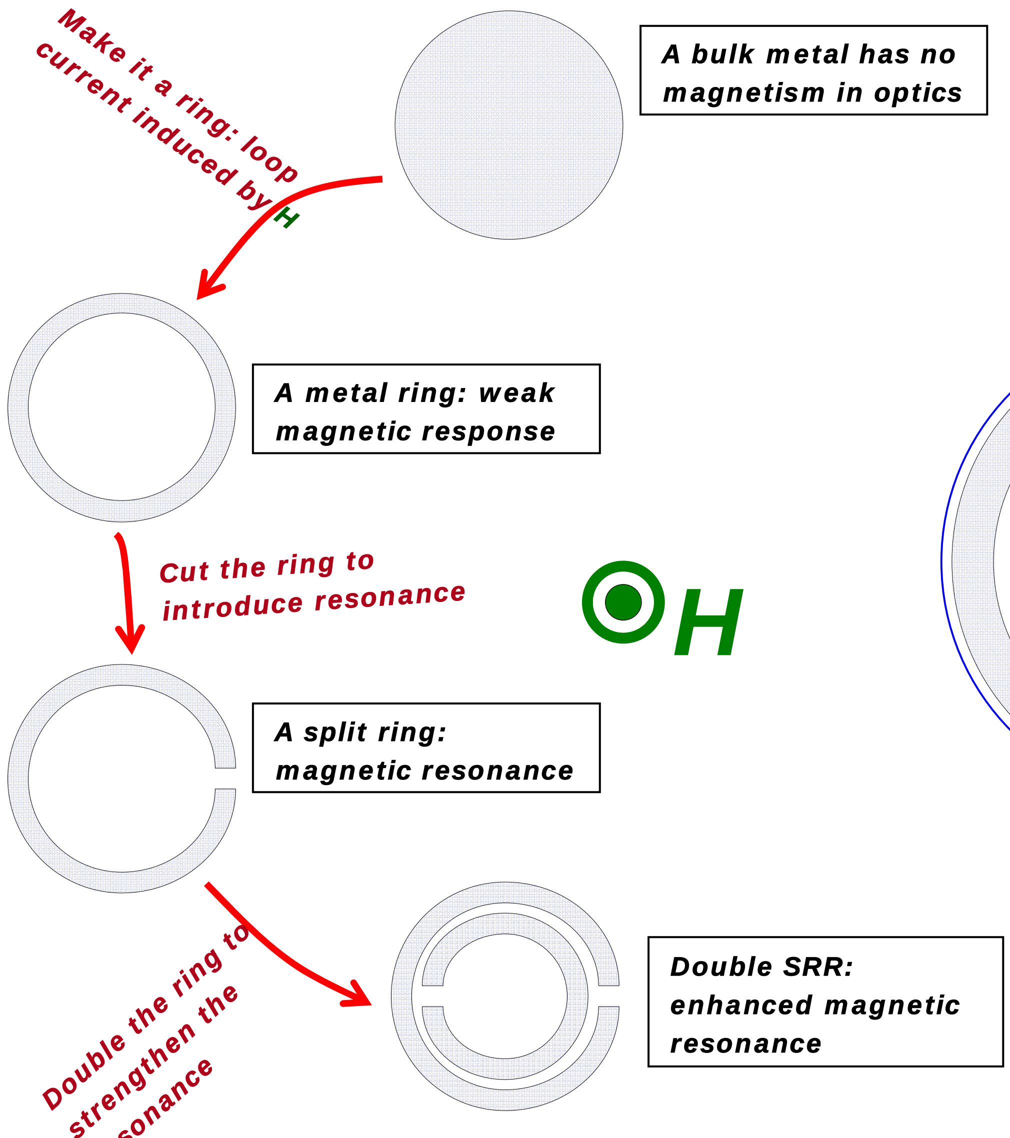
Electric coupling to an atom: ~ $e a_0$

Magnetic effect / electric effect $\approx \alpha^2 \approx (1/137)^2 < 10^{-4}$

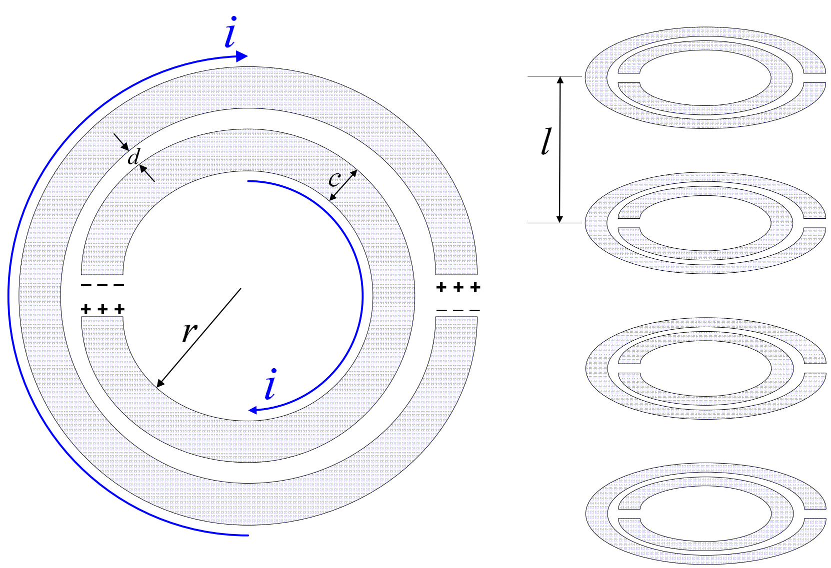
"… the magnetic permeability μ(ω) ceases to have any physical meaning at relatively low frequencies…there is certainly no meaning in using the magnetic susceptibility from optical frequencies onwards, and in discussion of such phenomena we must put μ=1."

Landau and Lifshitz, ECM, Chapter 79.

SRR: the first magnetic metamaterials



Split-ring resonator (SRR)



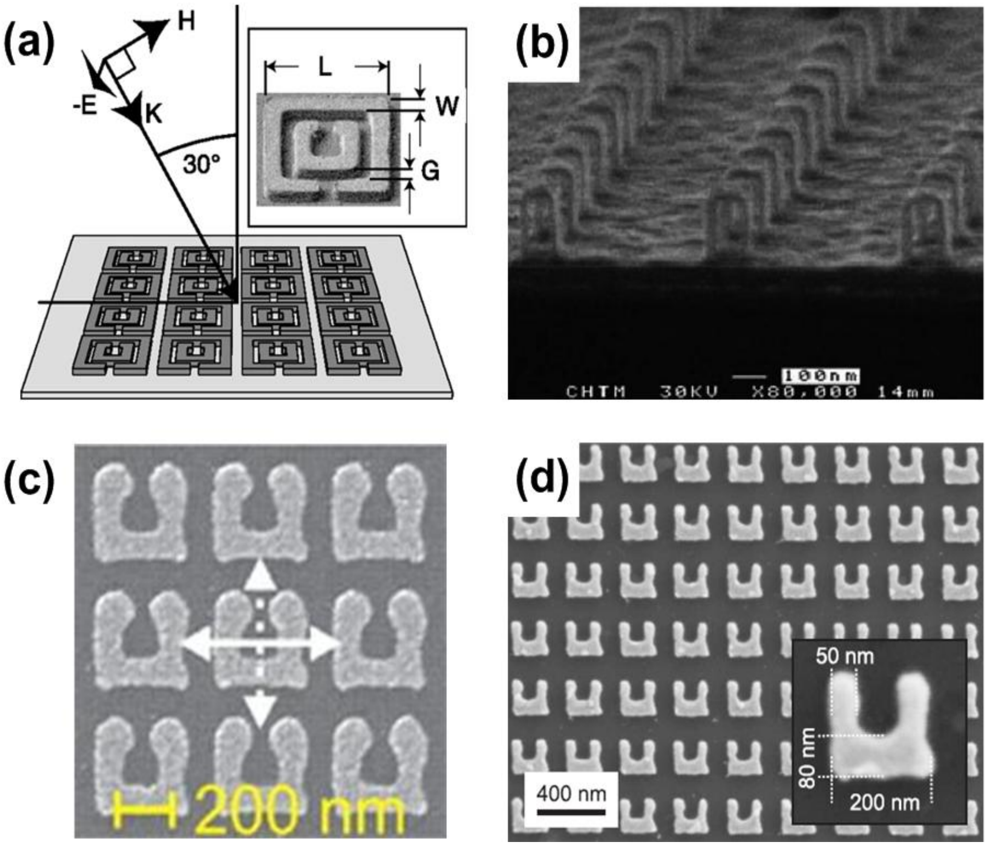
Theory: Pendry et al., 1999.

Experiment: Smith et al., 2000.

Towards Optical Magnetic Metamaterials

Terahertz magnetism

A) Yen, et al. ~ 1THz (2-SRR) – 2004
Katsarakis, et al (SRR – 5 layers) - 2005
b) Zhang et al ~50THz (SRR+mirror) - 2005
c) Linden, et al. 100THz (1-SRR) -2004
d) Enkrich, et al. 200THz (u-shaped)-2005



2004-2007 years:
from 10 GHz to 500 THZ

Limits of size scaling in SRRs

Direct scaling-down the SRR dimensions doesn't help much…

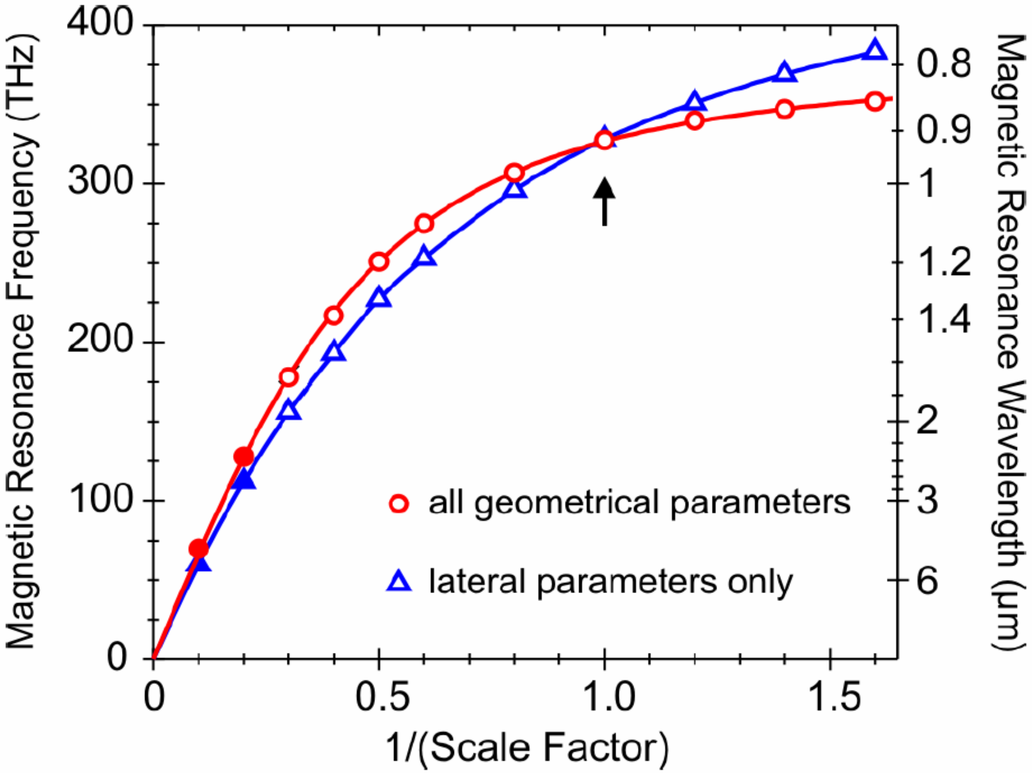
Loss in metal gives kinetic inductance

$L_{coil} \propto size$

$L_{kinetic} \propto \frac{1}{size}$

$$L_{total} = L_{coil} + L_{kinetic}$$

$$C_{total} \propto size$$



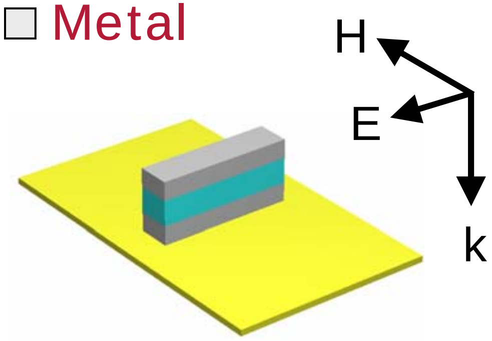
$$\omega_{res} \propto \frac{1}{\sqrt{L_{total} \times C_{total}}} = \frac{1}{\sqrt{(A \cdot size + B / size) \cdot (C \cdot size)}} \propto \frac{1}{\sqrt{size^2 + const.}}$$

Saturation

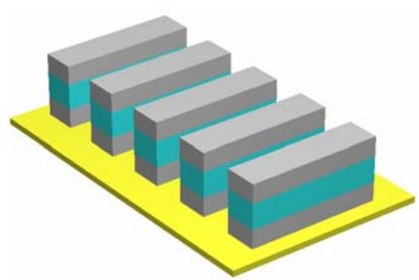
Zhou et al, PRL (2005); Klein, et al., OL (2006)

Magnetic Metamaterial: Nanorod to Nanostrip

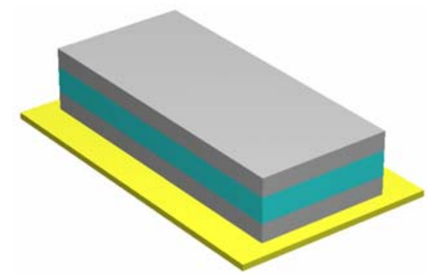
- Dielectric
- Metal



Nanorod pair



Nanorod pair array

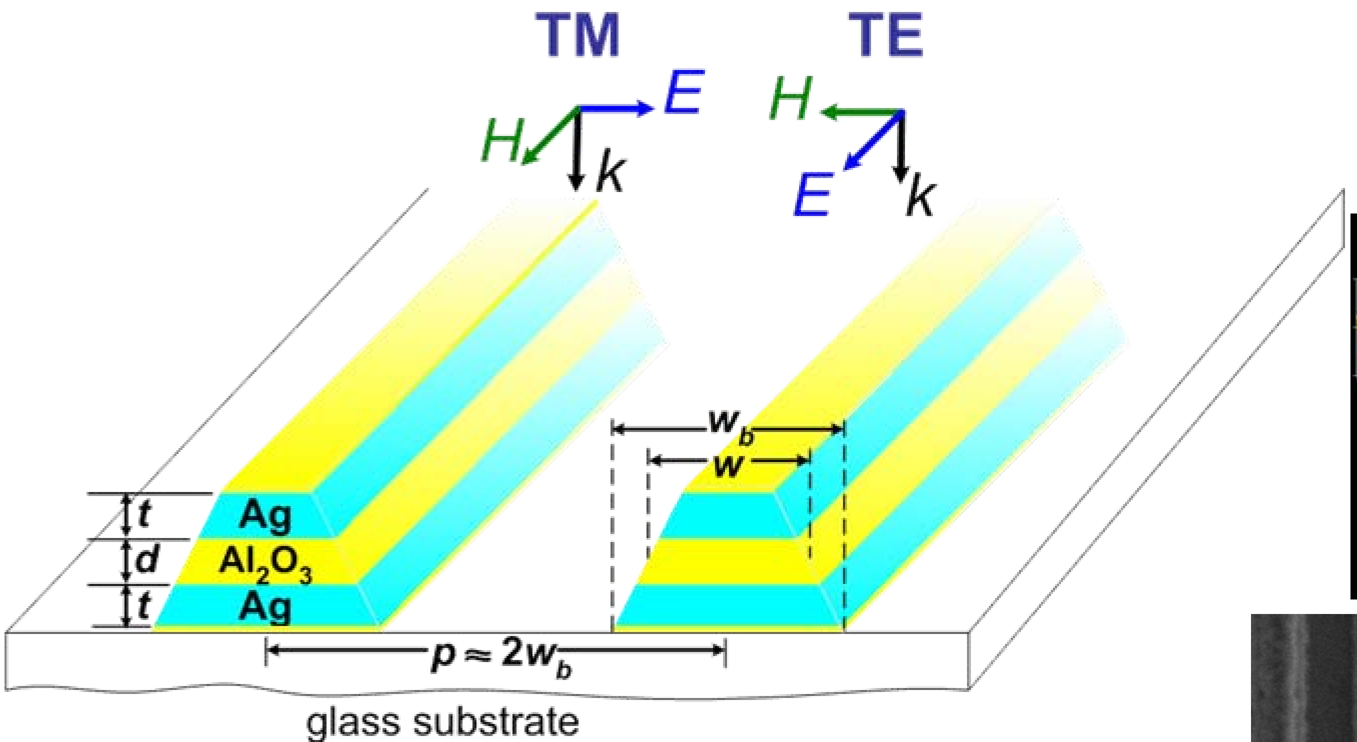


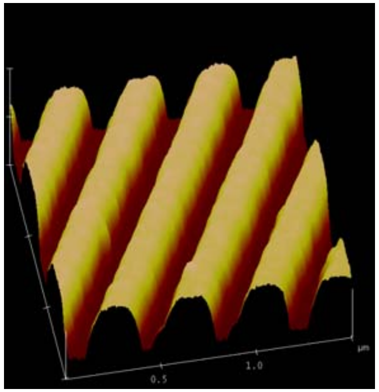
Nanostrip pair

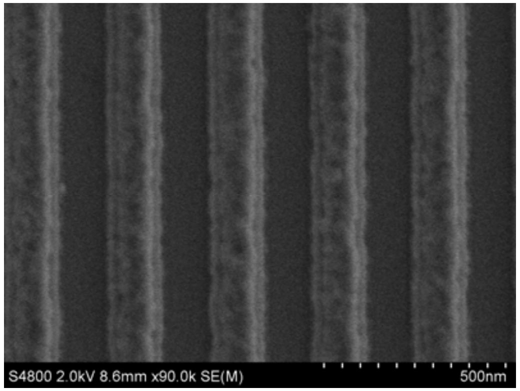
- Nanostrip pair has a much stronger magnetic response

Lagar'kov, Sarychev PRB (1996) - $\mu > 0$
Podolskiy, Sarychev & Shalaev, *JNOPM* (2002) - $\mu < 0$ & $n < 0$
Kildishev et al, *JOSA B* (2006); Shvets et al JOSA (2006) – strip pairs
Svirko, et al, APL (2001) - "crossed" rods for chirality

Visible magnetism: structure and geometries





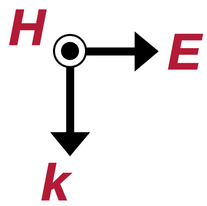


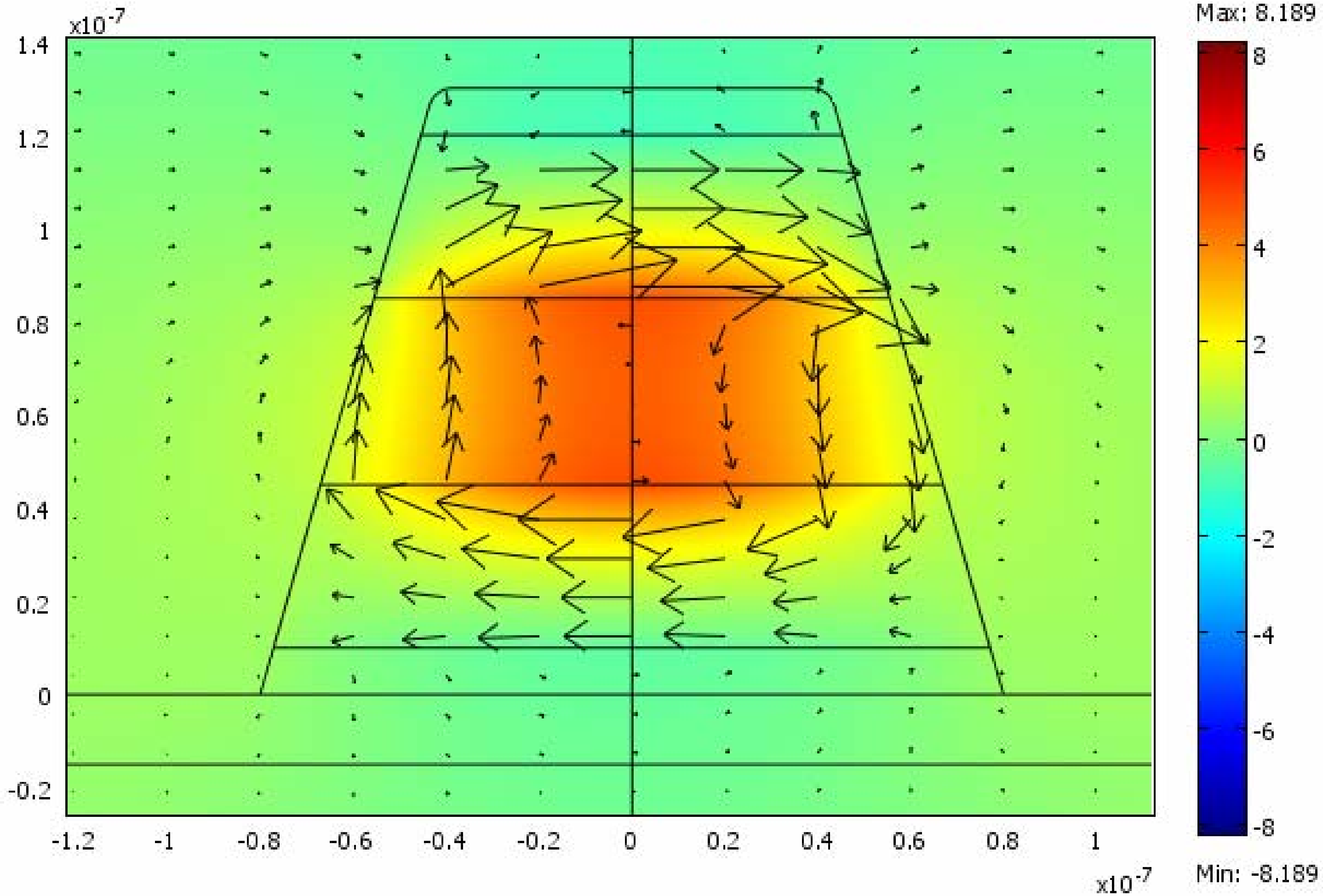
$$t = 35 \quad nm \qquad d = 40 \quad nm \qquad p \approx 2w_b$$

Width varies from 50 nm to 127 nm

Purdue group
Yuan, et al., Opt. Expr., 2007 – red light
Cai, et al., Opt. Expr., 2007 – all the visible

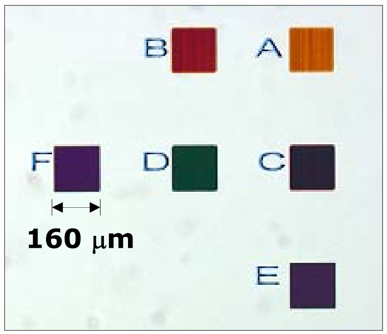
Negative Magnetic Response

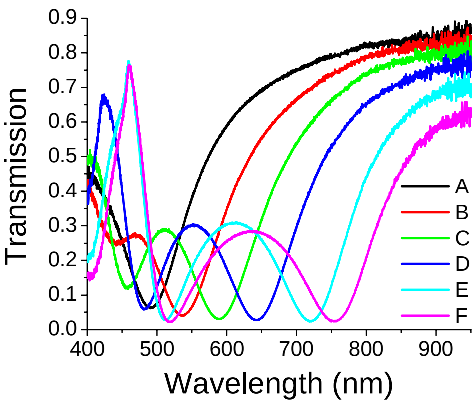




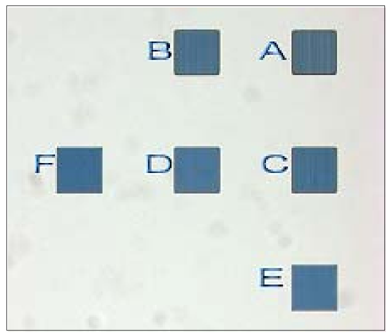
Width tunes resonance

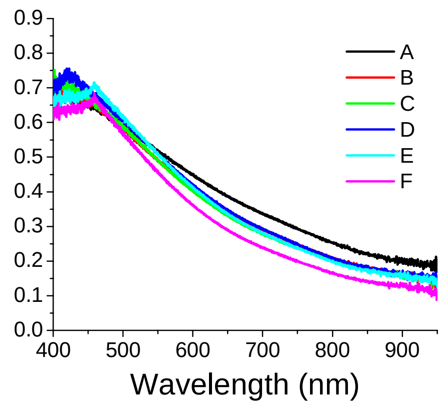
Resonant TM Transmission



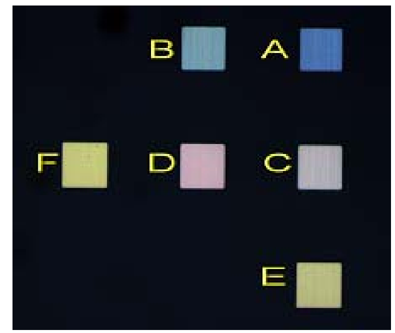


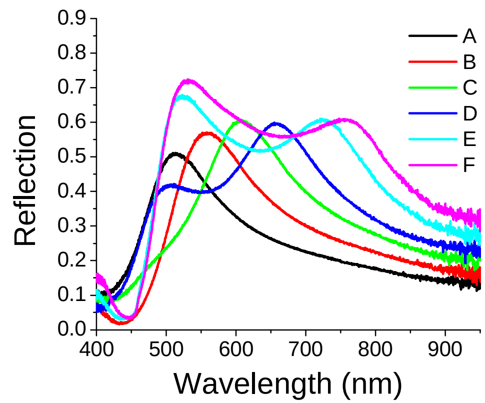
Non-resonant TE Transmission



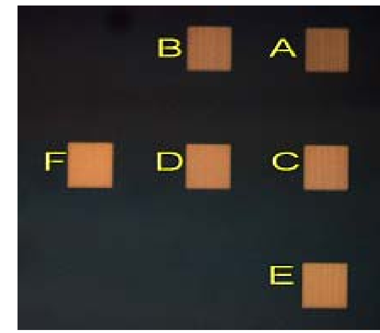


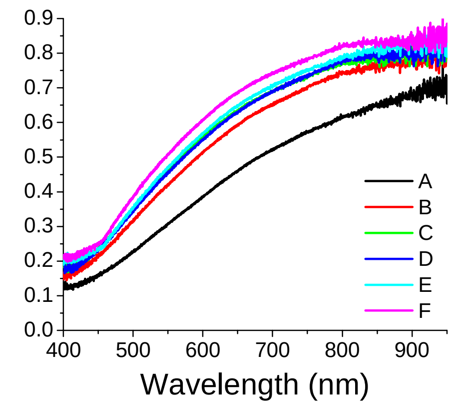
Resonant TM Reflection





Non-resonant TE Reflection

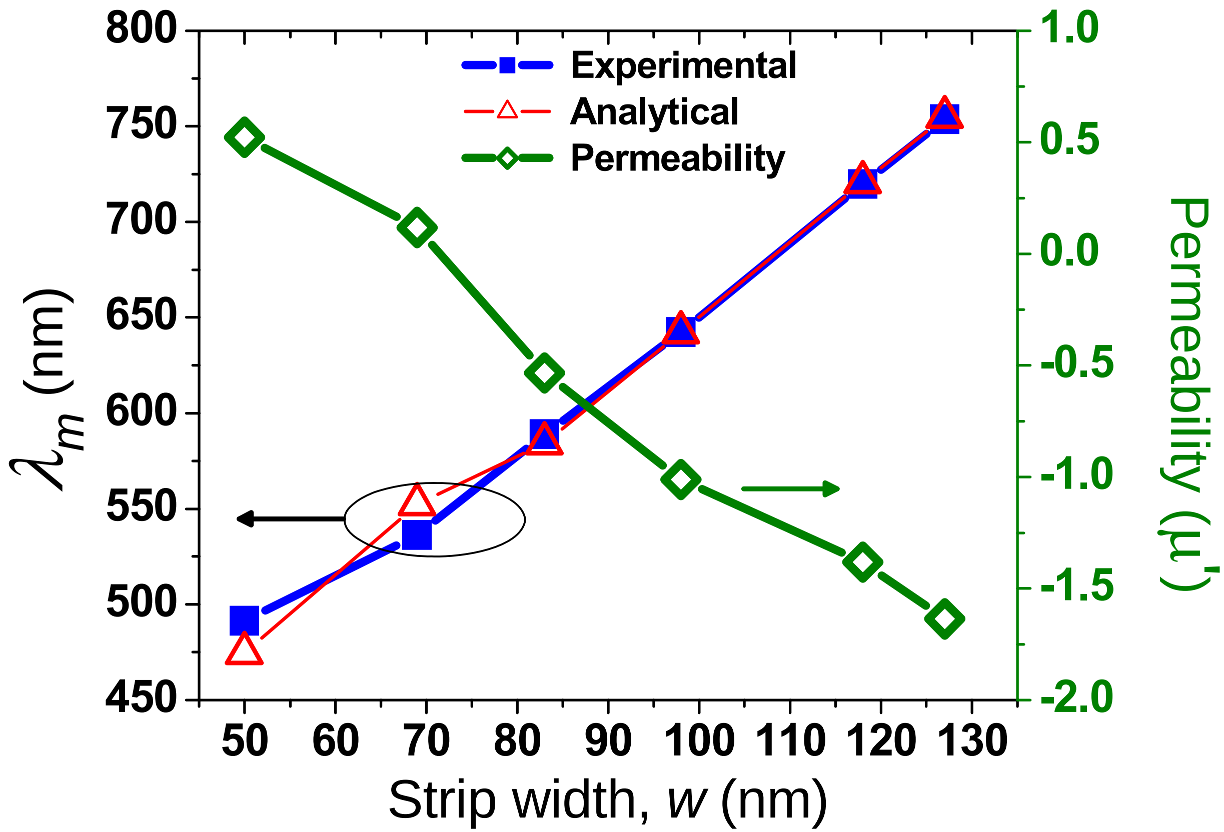




Sample #	A	B	C	D	E	F
Width w (nm)	50	69	83	98	118	127

Cai, et al., Opt. Expr., 15, 3333 (2007)

Dependence of λ_m and μ' on *w*

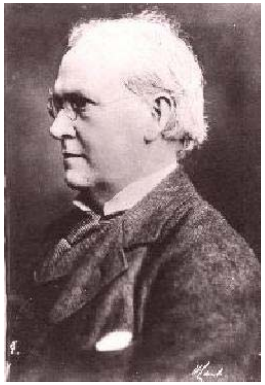


λ_m as a function of strip width "w": experiment vs. theory

<u>Negligible saturation</u> effect on size-scaling (as opposed to SRRs)

Negative refractive index: A historical review


Sir Arthur Schuster


Sir Horace Lamb

... energy can be carried forward at the group velocity but in a direction that is anti-parallel to the phase velocity...
Schuster, 1904


L. I. Mandel'stam

Negative refraction and backward propagation of waves
Mandel'stam, 1945


V. G. Veselago

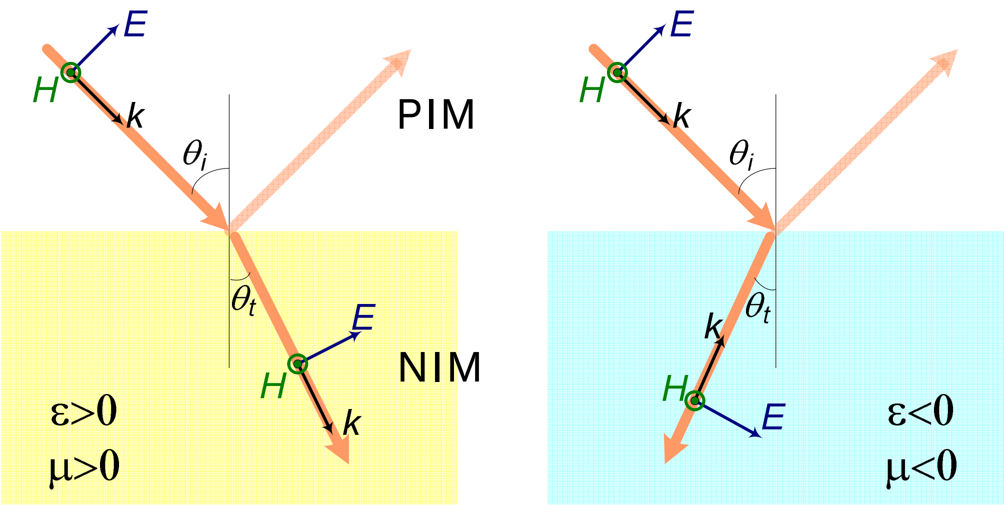
Left-handed materials: the electrodynamics of substances with simultaneously negative values of ε and μ
Veselago, 1968


Sir John Pendry

Pendry, the one who whipped up the recent boom of NIM researches

Perfect lens (2000)
EM cloaking (2006)

Metamaterials with Negative Refraction



Refraction:

$$n^2 = \varepsilon\mu$$

$$n = \pm\sqrt{\varepsilon\mu}$$

n < 0, if **$\varepsilon'|\mu| + \mu'|\varepsilon| < 0$**

Figure of merit

F = |n′|/n″

<u>Single-negative:</u>

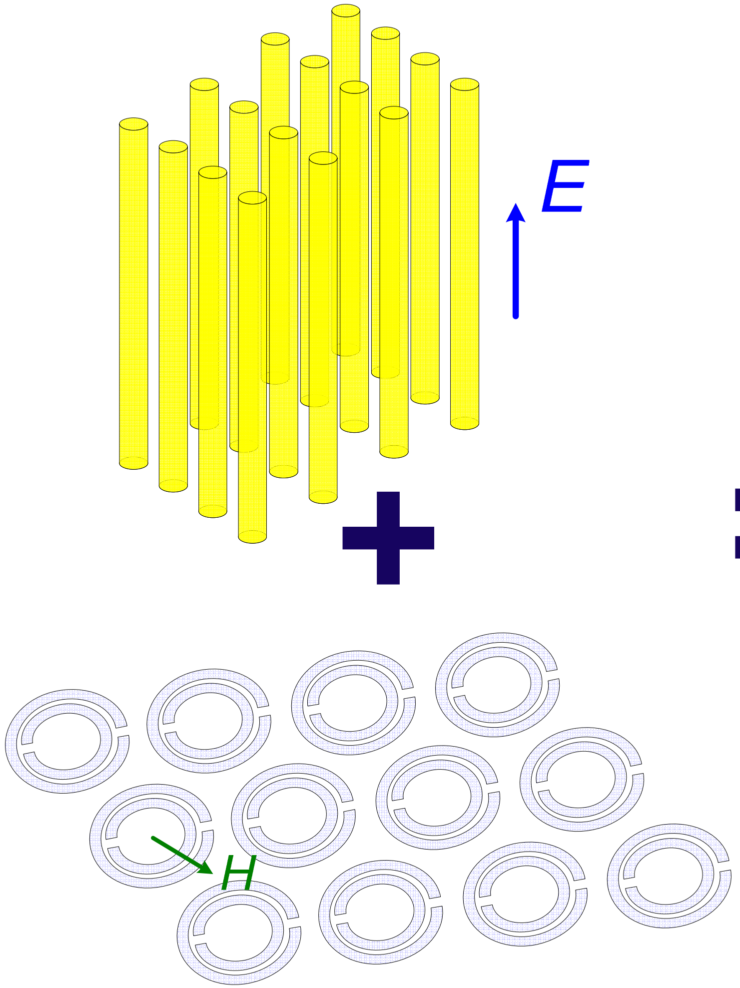
n<0 when ε′ < 0 whereas μ′ > 0
(F is low)

<u>Double-negative:</u>

n<0 with both ε′ < 0 and μ′ < 0
(F can be large)

Negative index metamaterials

Negative electrical metamaterial + Negative magnetic metamaterial = Negative index metamaterial



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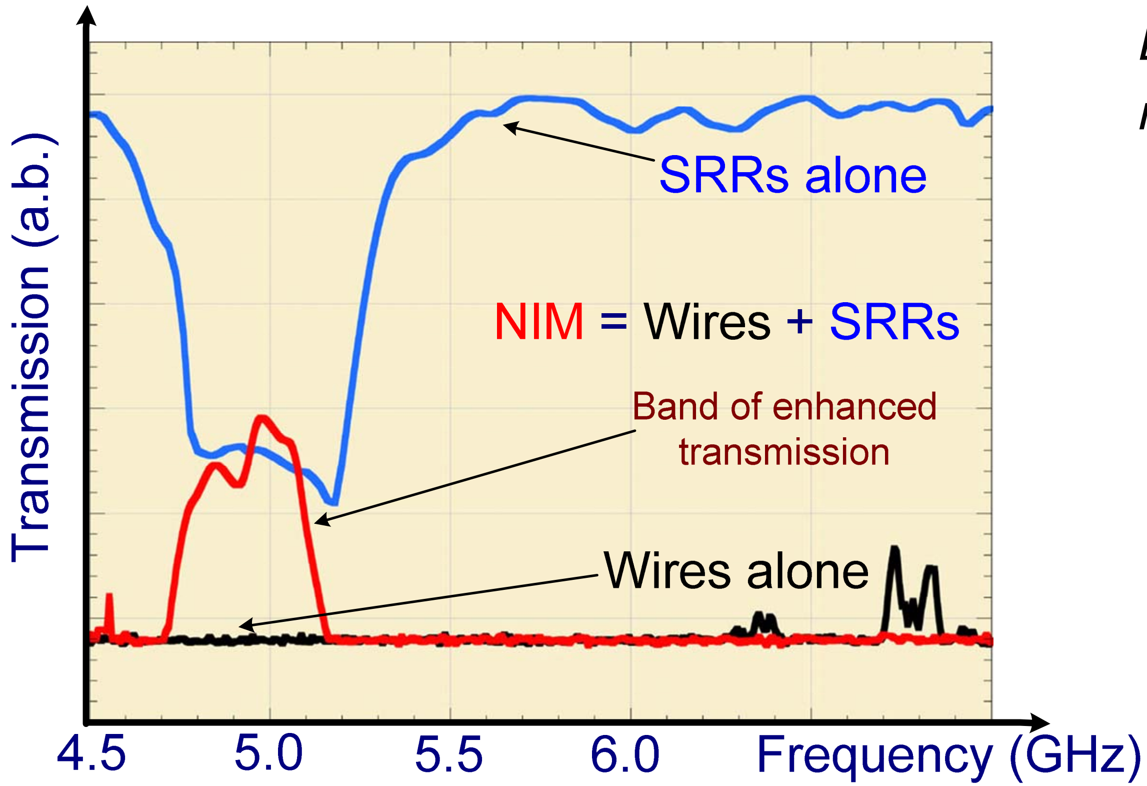
Smith, et al., UCSD, PRL (2000)

The first NIM in microwave

Transmission properties of the structure



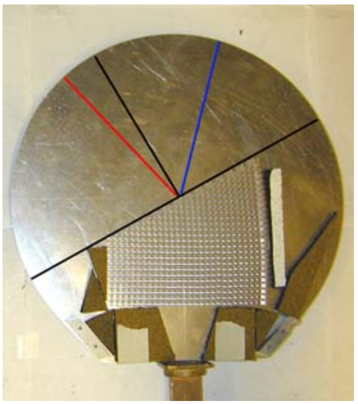
E,H ~ exp[in(ω/c)z]

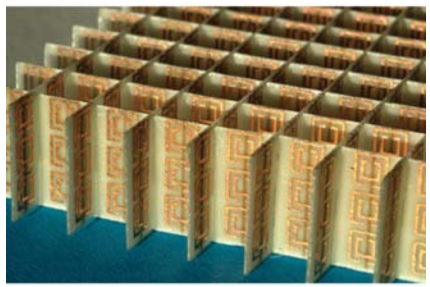
n = ±√(εμ)

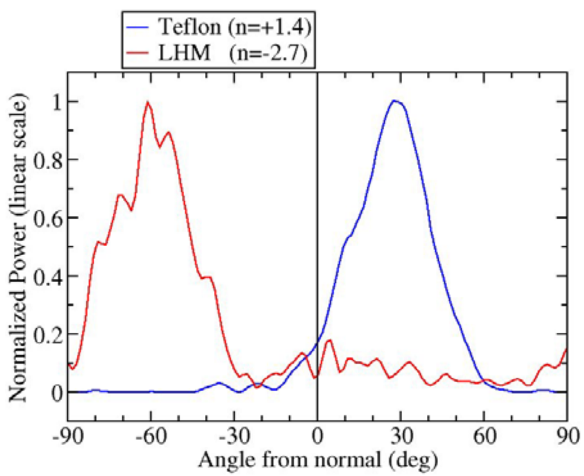
Smith, et al., UCSD, PRL (2000)

Negative-index Material in Microwave

2D waveguide

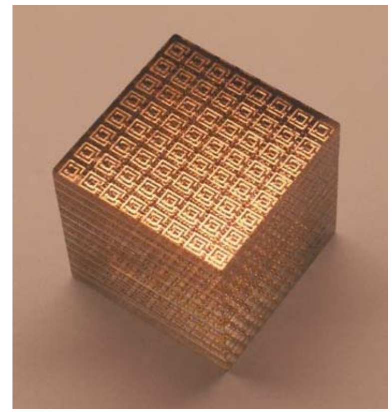




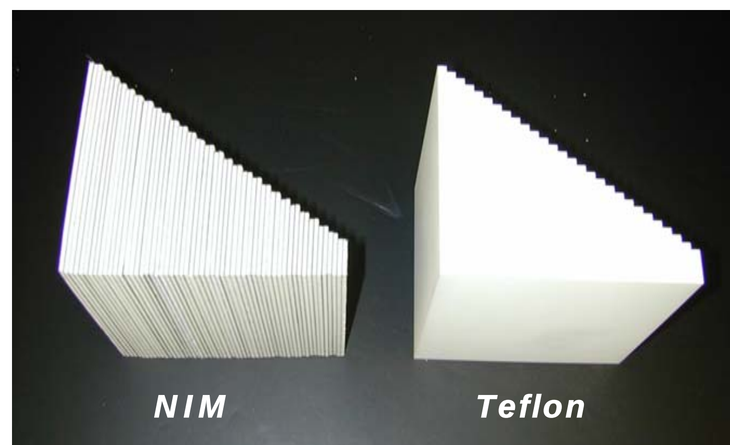


Smith, et al., UCSD, Science (2001)

3D free space



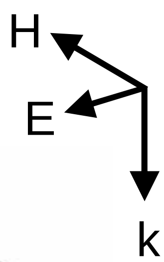
The Boeing Cube

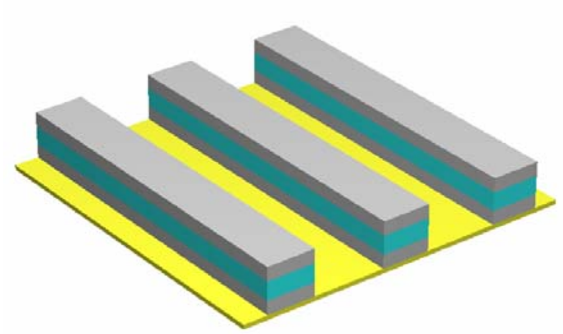


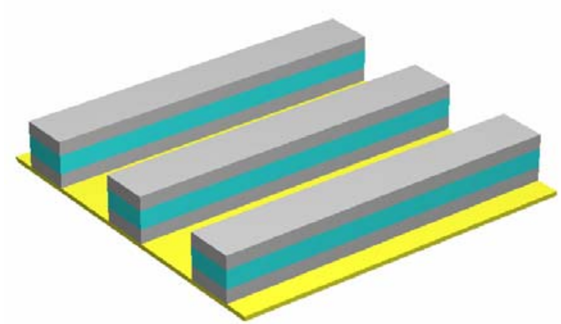
Parazzoli, et al., Boeing, PRL (2003)

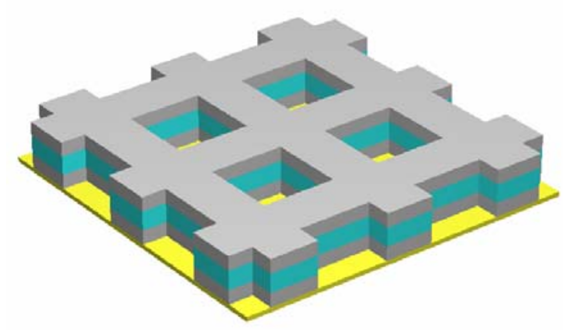
Negative Index Design for Optical Frequencies

- Dielectric
- Metal









Nanostrip pair (TM)

$\mu < 0$ (resonant)

Nanostrip pair (TE)

$\varepsilon < 0$ (non-resonant)

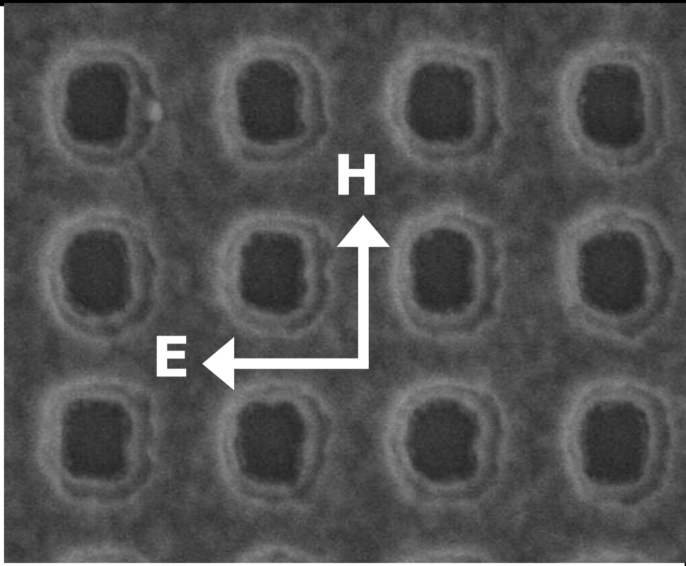
Fishnet

ε and $\mu < 0$

S. Zhang, et al., PRL (2005)

A Negative-index Material close to the Visible

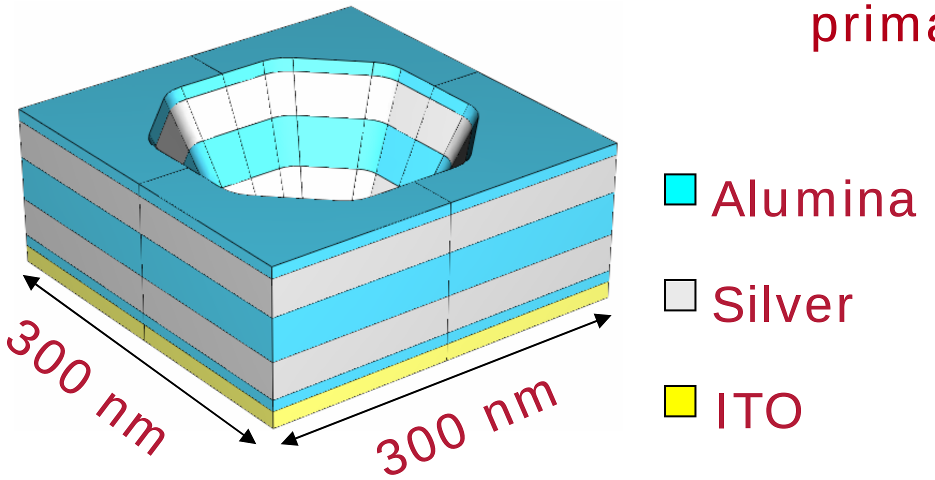
- E-beam lithography
- Period = 300 nm along both axis
- Average width of strips along H = 130 nm
  Average width of strips along E = 95 nm



SEM image and primary polarization

Stacking:

10 nm of Al_2O_3
33 nm of Ag
38 nm of Al_2O_3
33 nm of Ag
10 nm of Al_2O_3

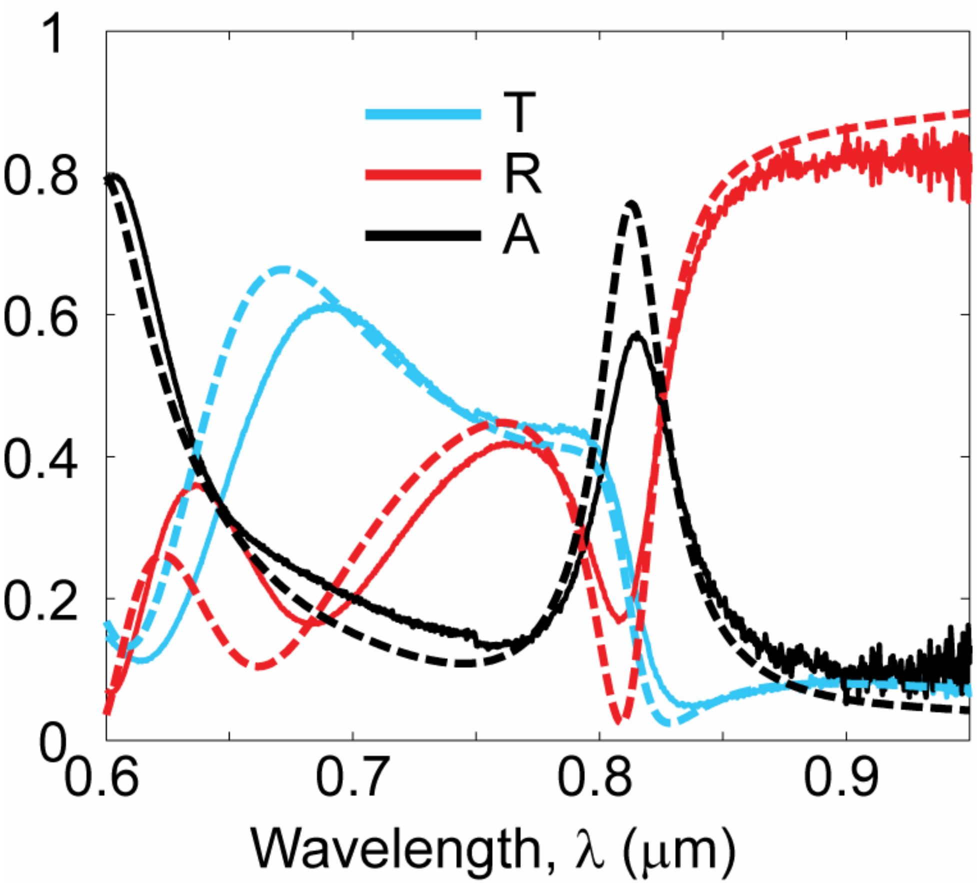


Spectra for Primary Polarization

- Magnetic resonance around λ = 800 nm
- Electric resonance around λ = 600 nm
- Finite Elements

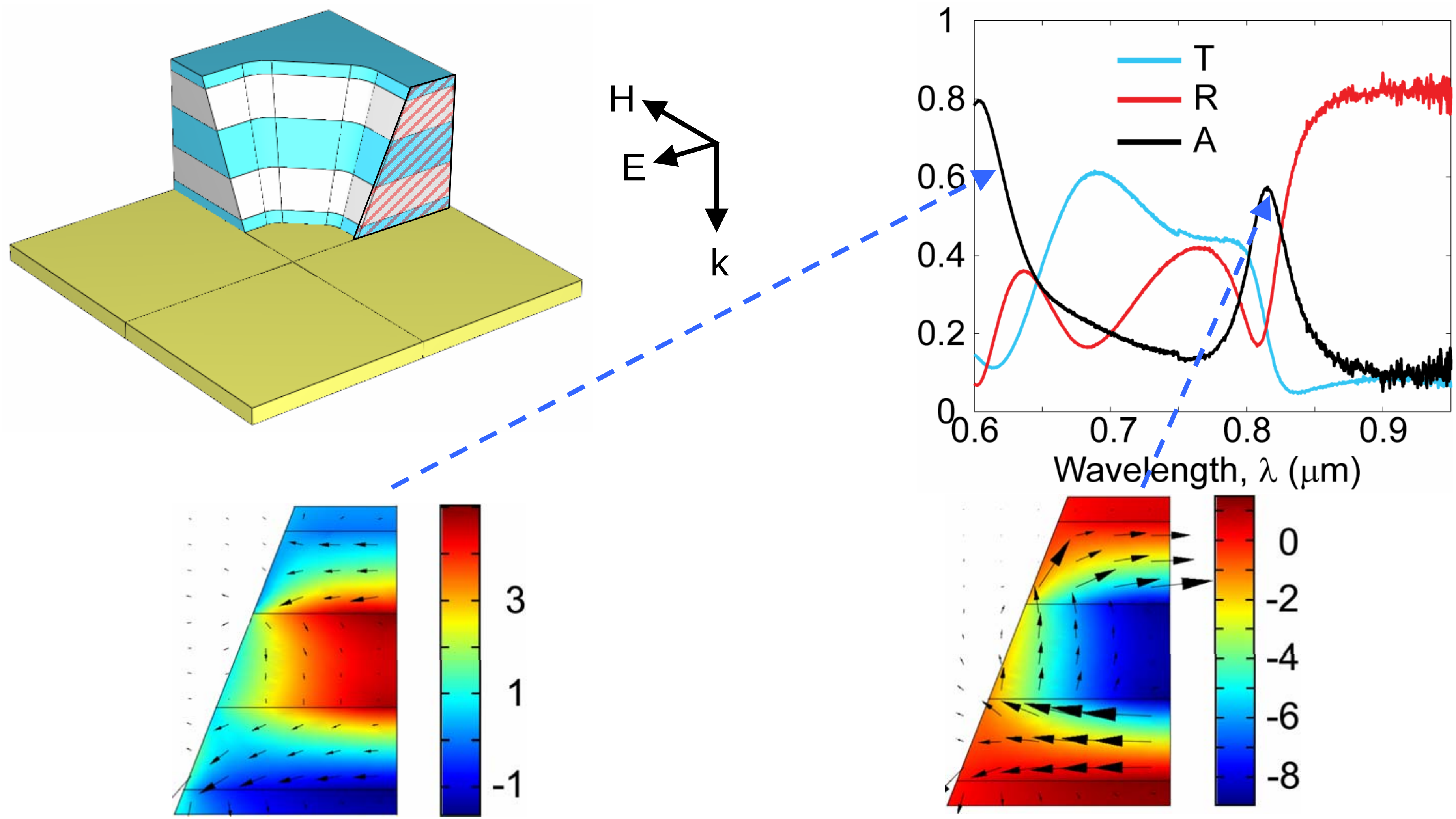
Solid line : Experimental

Dashed line: Simulated



U. K. Chettiar, *et al*., Optics Letters **32**, 1671 (2007)

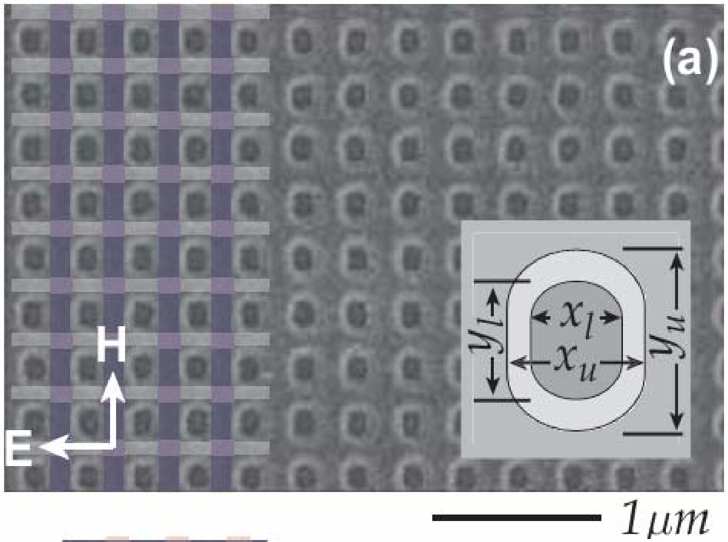
Field Maps for Primary Polarization

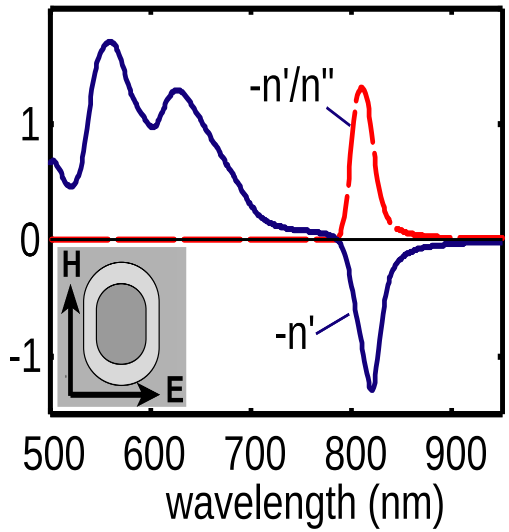


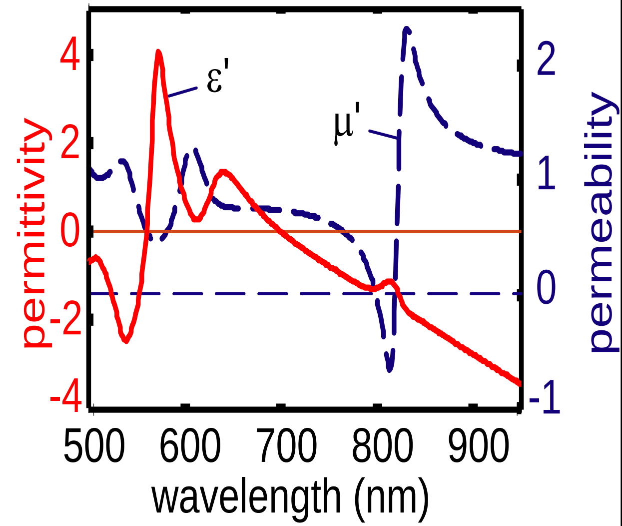
Electrical Resonance, λ = 625 nm

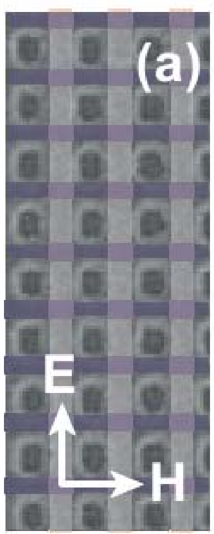
Magnetic Resonance, λ = 815 nm

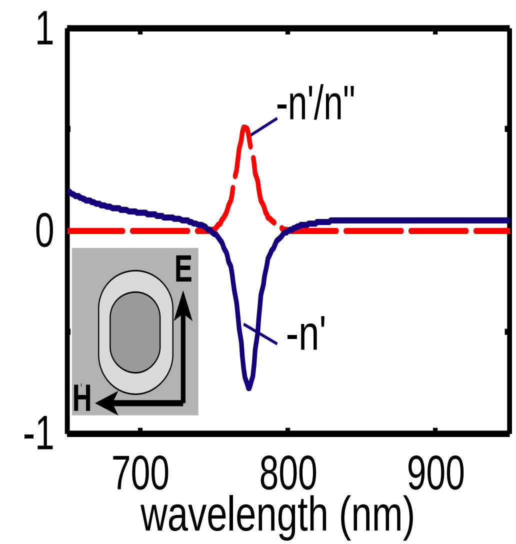
Double Negative NIM (n’=-1.0, FOM=1.3, at 810 nm)
Single Negative NIM (n’=-0.9, FOM=0.7, at 770 nm)

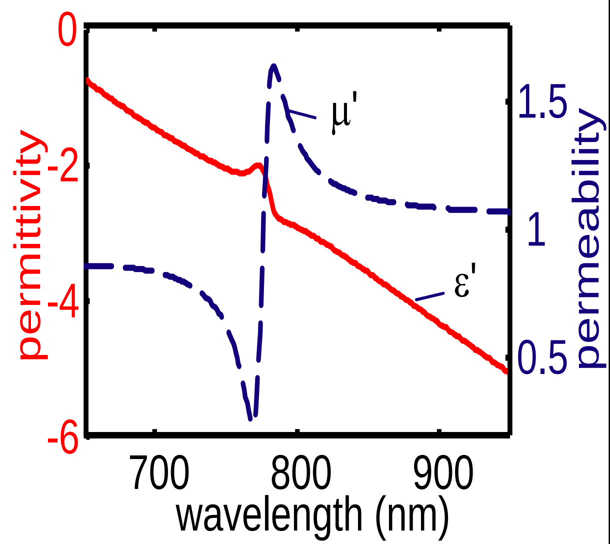








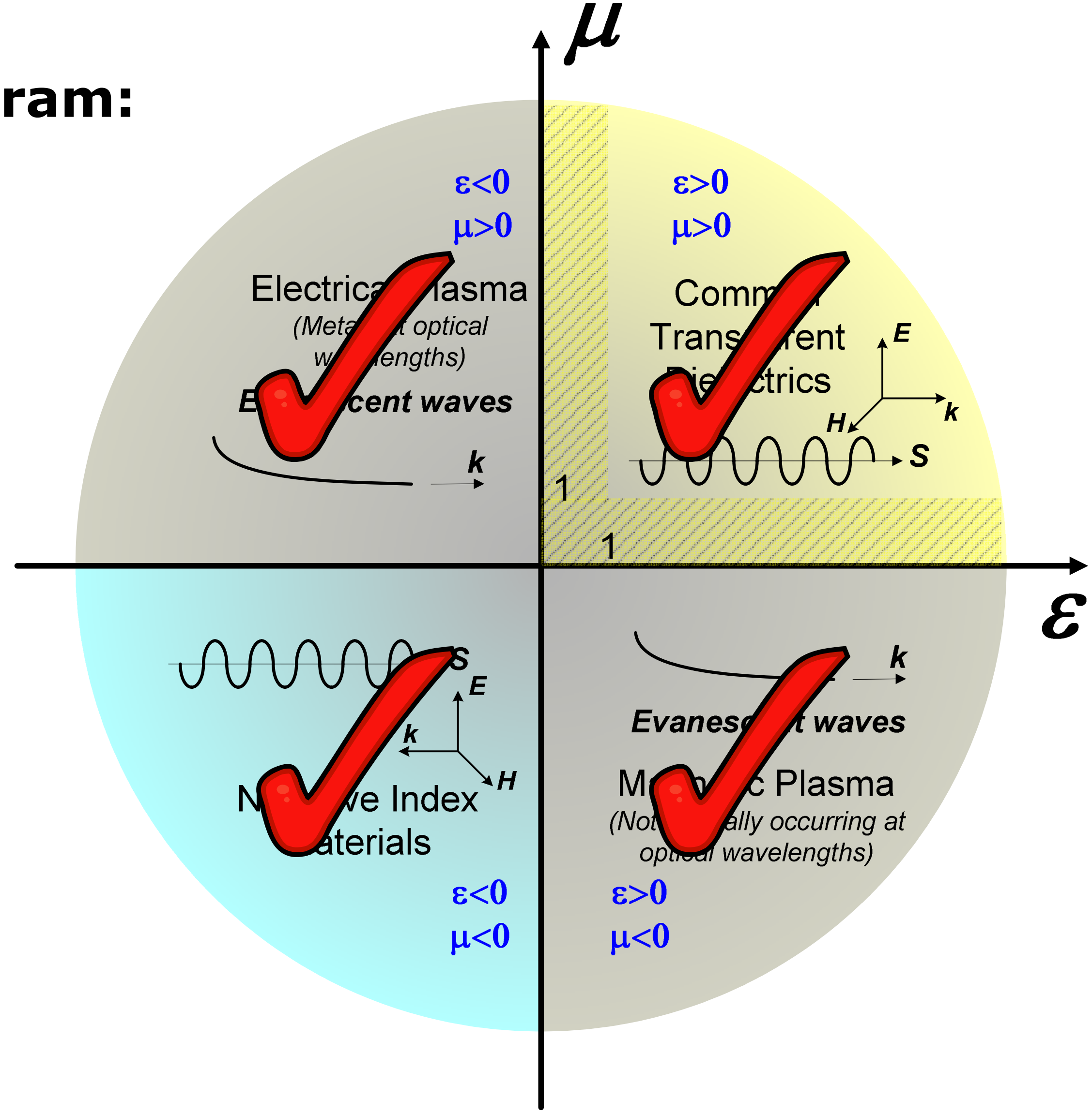




Chettiar et al
OL (2007)

Materials ready…

ε, μ diagram:

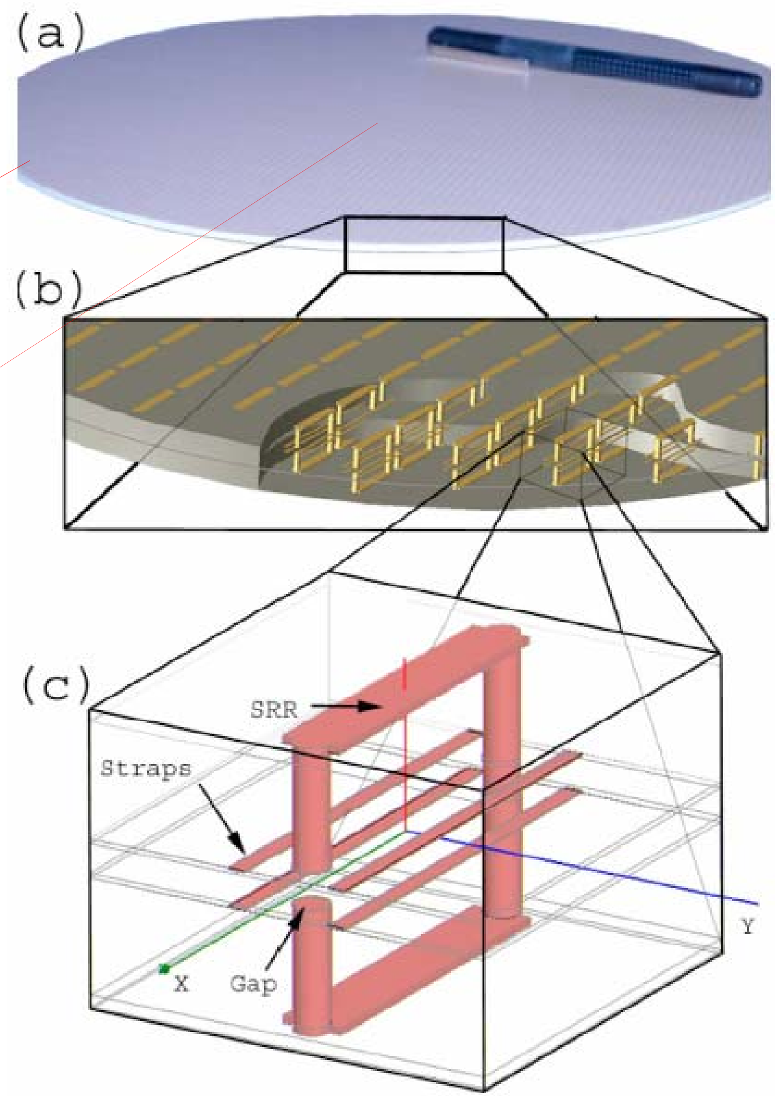


Gradient Metamaterial: A critical step

Negative index gradient lens

n = -2.7 on edge

n = -1.0 on center



D. R. Smith Group, PRE 2005; APL 2006

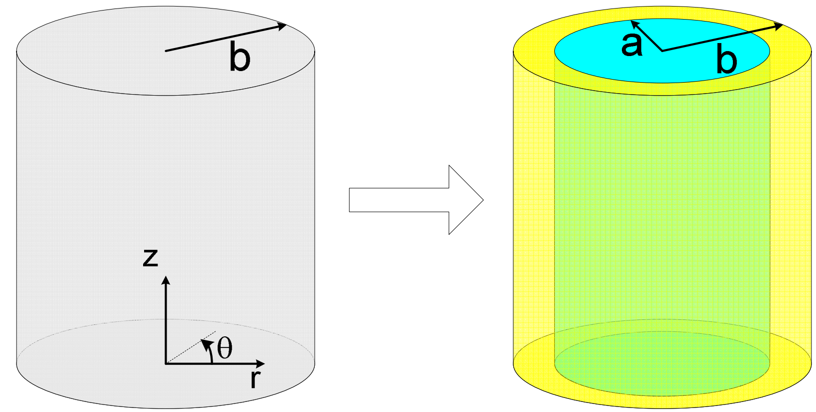
Back to Cloak: cylindrical system

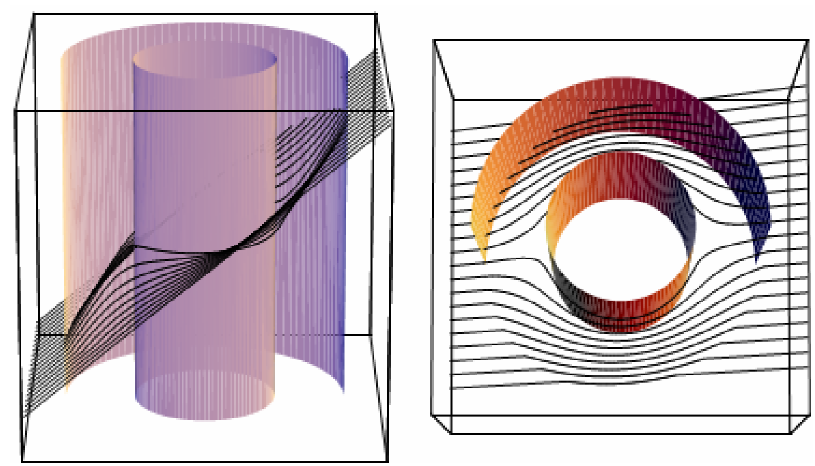
The transformation in cylindrical system:

$$0 < r < b \longrightarrow a < r' < b$$

$$r' = \frac{b-a}{b} r + a \qquad \theta' = \theta \qquad z' = z$$

$$\begin{cases} \varepsilon_r = \mu_r = \dfrac{r-a}{r} \\ \varepsilon_\theta = \mu_\theta = \dfrac{r}{r-a} \\ \varepsilon_z = \mu_z = \left(\dfrac{b}{b-a}\right)^2 \dfrac{r-a}{r} \end{cases}$$





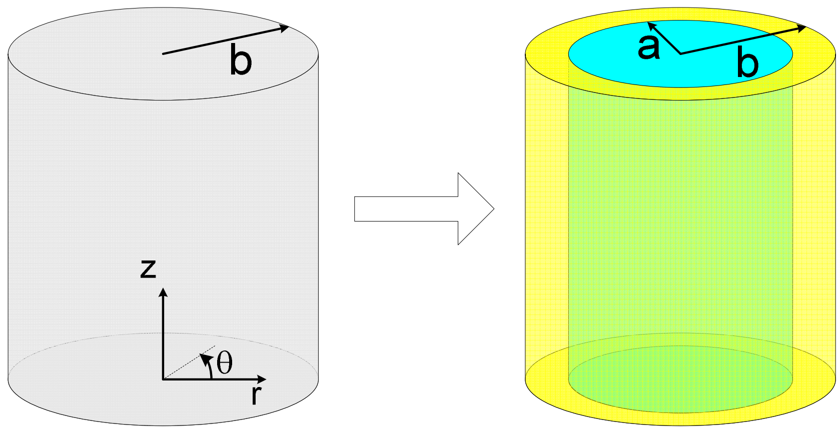
Smith et al., arXiv, 2006

Towards an experimental demonstration

The transformation in cylindrical system:

$$0 < r < b \longrightarrow a < r' < b$$

$$r' = \frac{b-a}{a} r + a \qquad \theta' = \theta \qquad z' = z$$



$$\begin{cases} \varepsilon_r = \mu_r = \dfrac{r-a}{r} \\ \varepsilon_\theta = \mu_\theta = \dfrac{r}{r-a} \\ \varepsilon_z = \mu_z = \left(\dfrac{b}{b-a}\right)^2 \dfrac{r-a}{r} \end{cases}$$

TE incidence

$$\begin{cases} \varepsilon_z = \left(\dfrac{b}{b-a}\right)^2 \dfrac{r-a}{r} \\ \mu_\theta = \dfrac{r}{r-a} \\ \mu_r = \dfrac{r-a}{r} \end{cases}$$

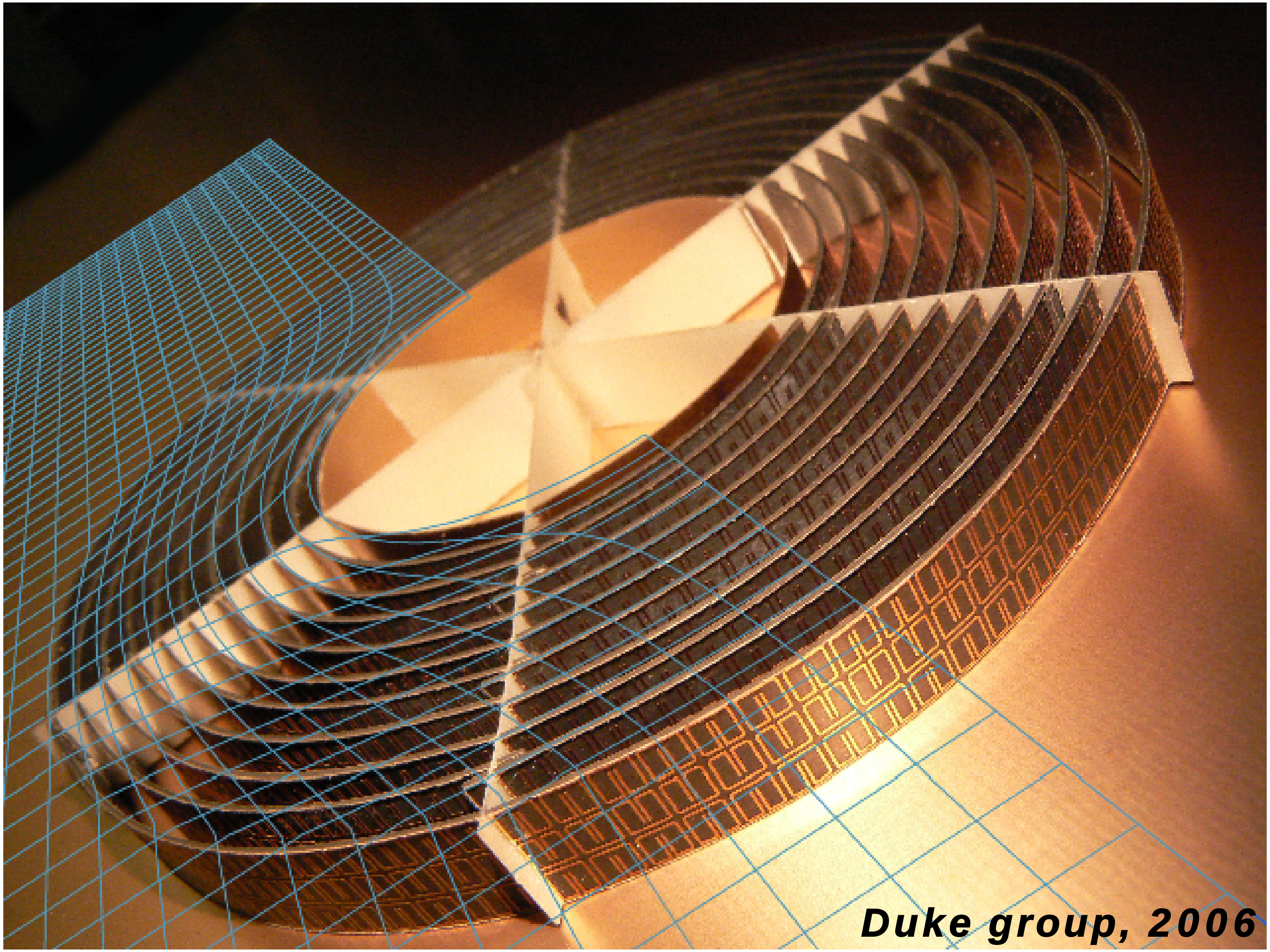
To maintain the dispersion relation only

$$\begin{cases} \mu_\theta \varepsilon_z = constant \\ \mu_r \varepsilon_z = constant \end{cases}$$

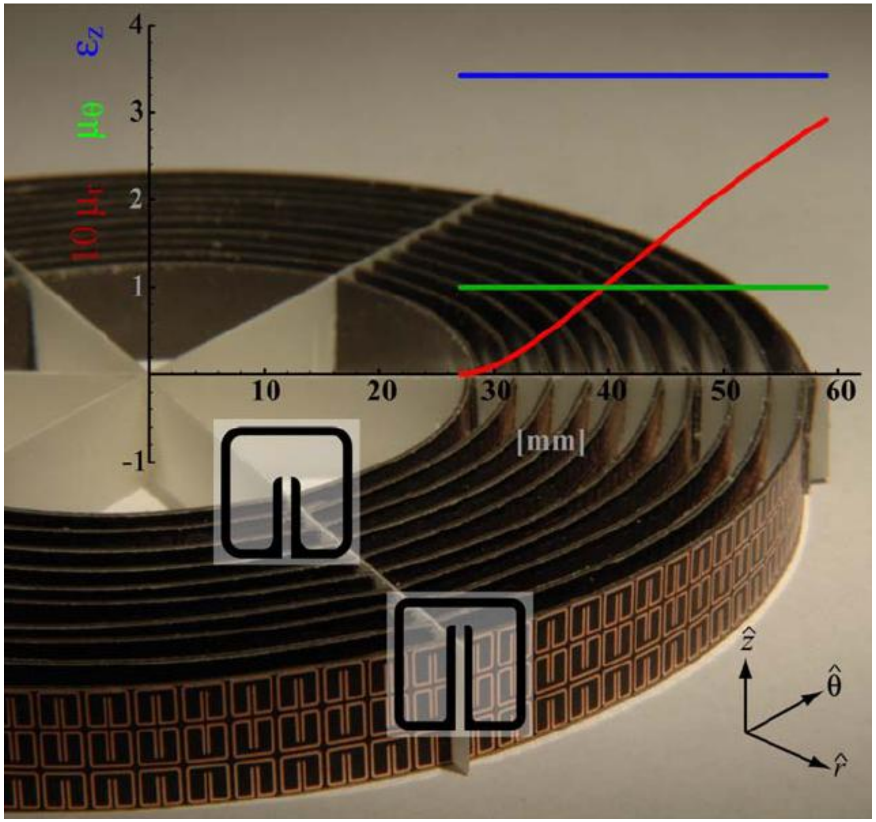
$$\begin{cases} \varepsilon_z = \left(\dfrac{b}{b-a}\right)^2 \\ \mu_\theta = 1 \\ \mu_r = \left(\dfrac{r-a}{r}\right)^2 \end{cases}$$

Schurig et al., Science, 2006

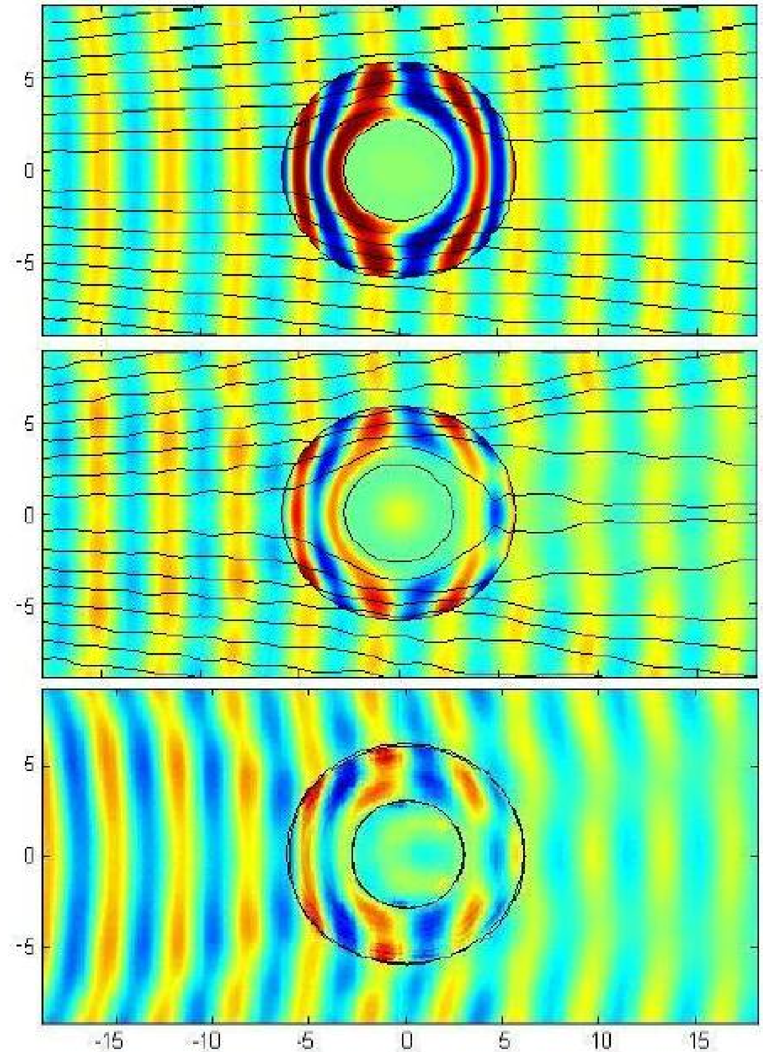
The First Metamaterial Cloak



Experimental demonstration at microwave frequency



Structure of the cloak



Ideal case

Reduced parameter

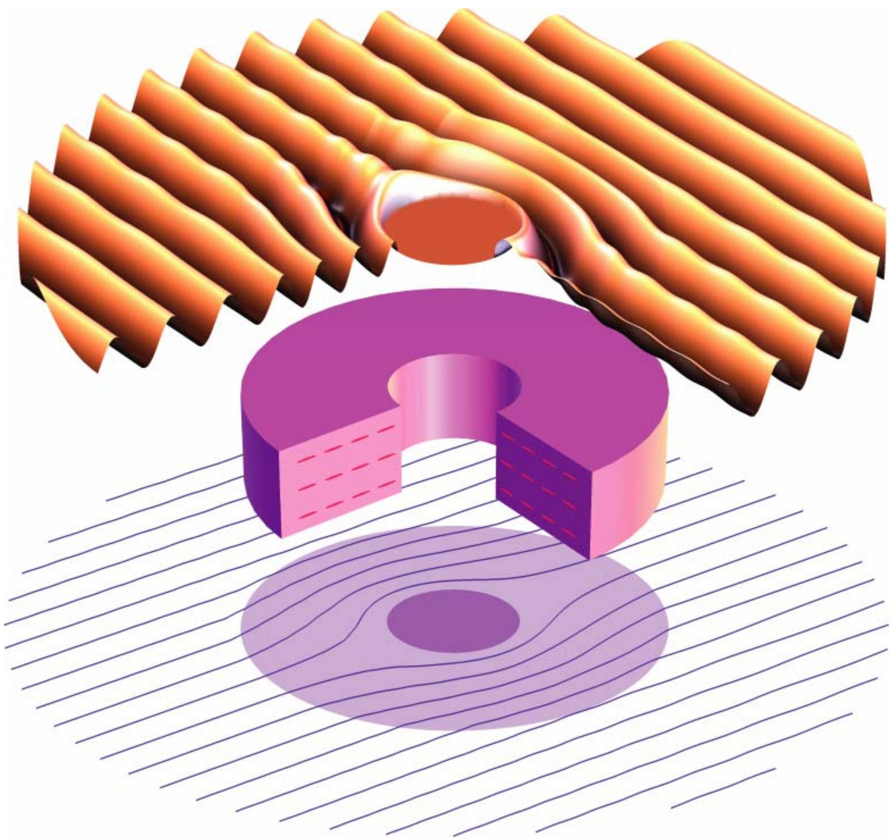
Experimental data

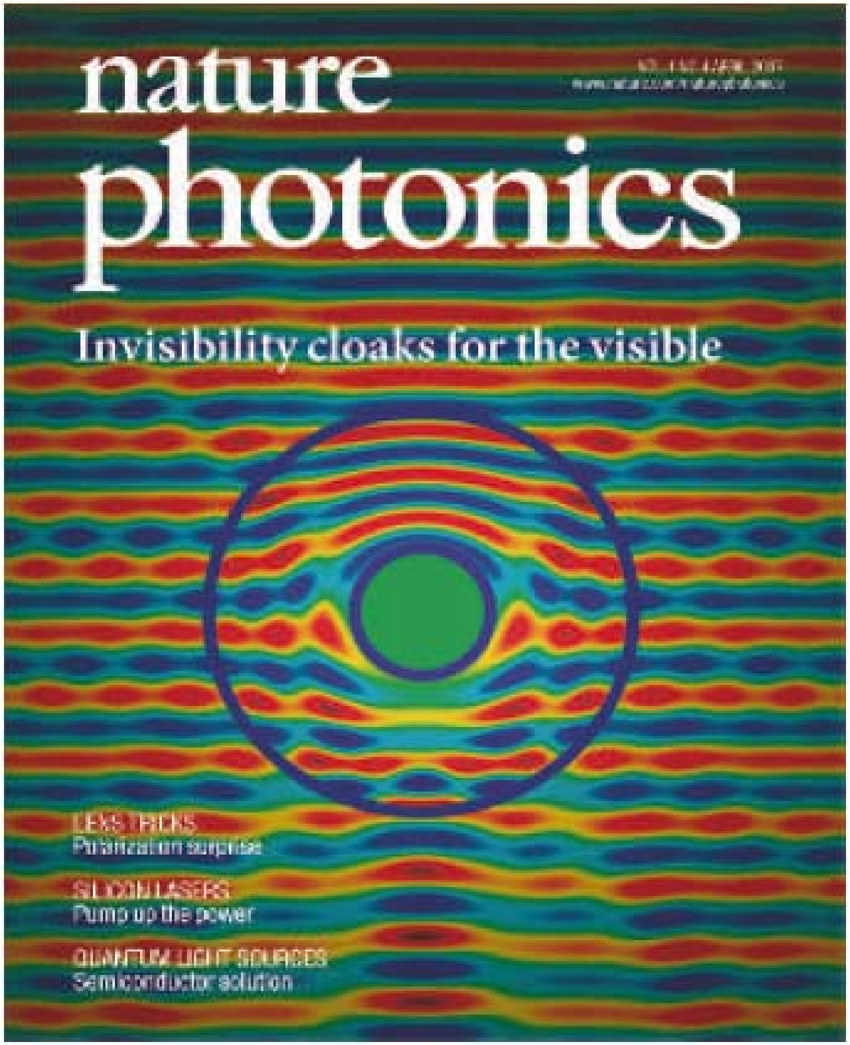
Schurig et al., Science, 2006

Movie: How it works

News release, Duke Univ., Oct. 2006

Optical Cloaking with Metamaterials: Can Objects be Invisible in the Visible?



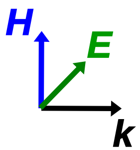


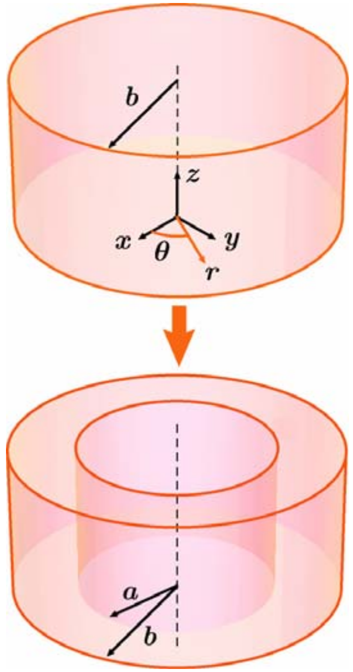
Cover article of Nature Photonics (April, 2007)

How about optical frequencies?

Scaling the microwave cloak design?

- ☹ ***Intrinsic limits to the scaling of SRR size***
- ☹ ***High loss in resonant structures***





$$\varepsilon_r = \mu_r = \frac{r-a}{r}, \quad \varepsilon_\theta = \mu_\theta = \frac{r}{r-a}, \quad \varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r}$$

TM incidence

$$\begin{cases} \mu_z = \left(\dfrac{b}{b-a}\right)^2 \dfrac{r-a}{r} \\ \varepsilon_\theta = \dfrac{r}{r-a} \\ \varepsilon_r = \dfrac{r-a}{r} \end{cases}$$

To maintain the dispersion relation

$$\begin{cases} \mu_z \varepsilon_\theta = constant \\ \mu_z \varepsilon_r = constant \end{cases}$$

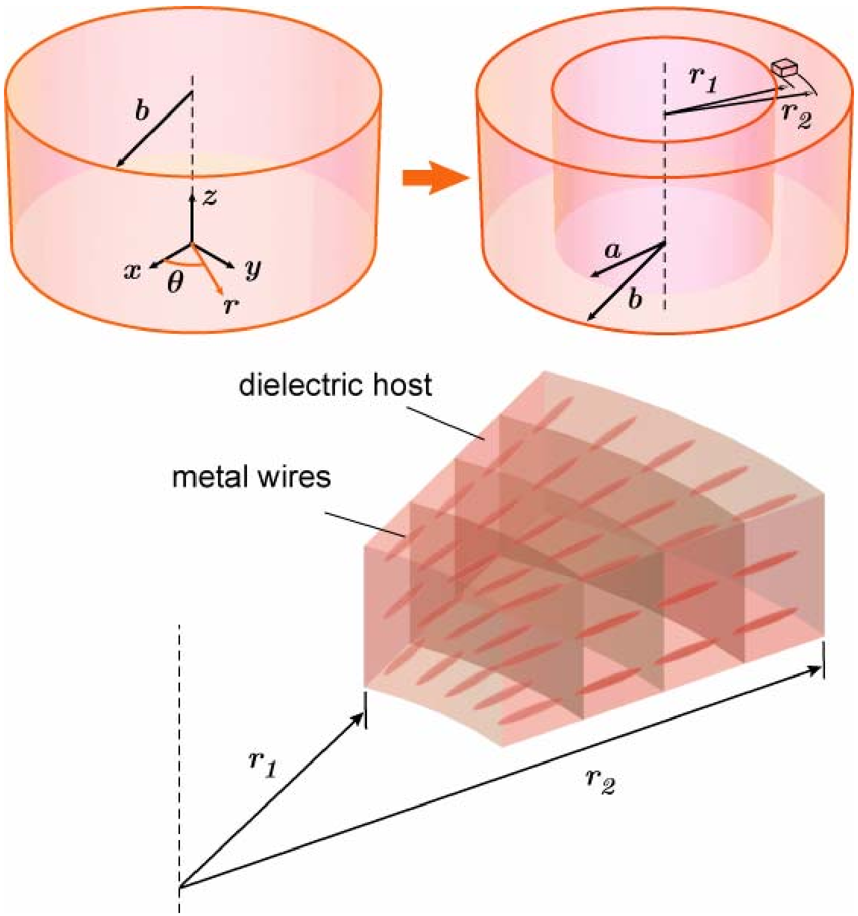
(for in-plane k)

$$\begin{cases} \mu_z = 1 \\ \varepsilon_\theta = \left(\dfrac{b}{b-a}\right)^2 \\ \varepsilon_r = \left(\dfrac{b}{b-a}\right)^2 \left(\dfrac{r-a}{r}\right)^2 \end{cases}$$

- $\mu_z = 1$ → **No magnetism required!**
- ε_θ → **A constant permittivity of a dielectric; $\varepsilon_\theta > 1$**
- ε_r → **Gradient in *r* direction only; ε_r changing from 0 to 1.**

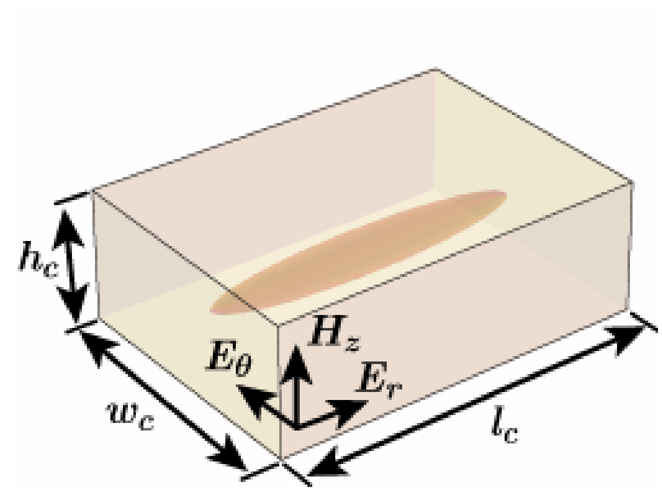
Cai, et al., Nature Photonics, 1, 224 (2007)

Structure of the cloak: "Round brush"



metal needles embedded in dielectric host

Unit cell:



Flexible control of ε_r ;

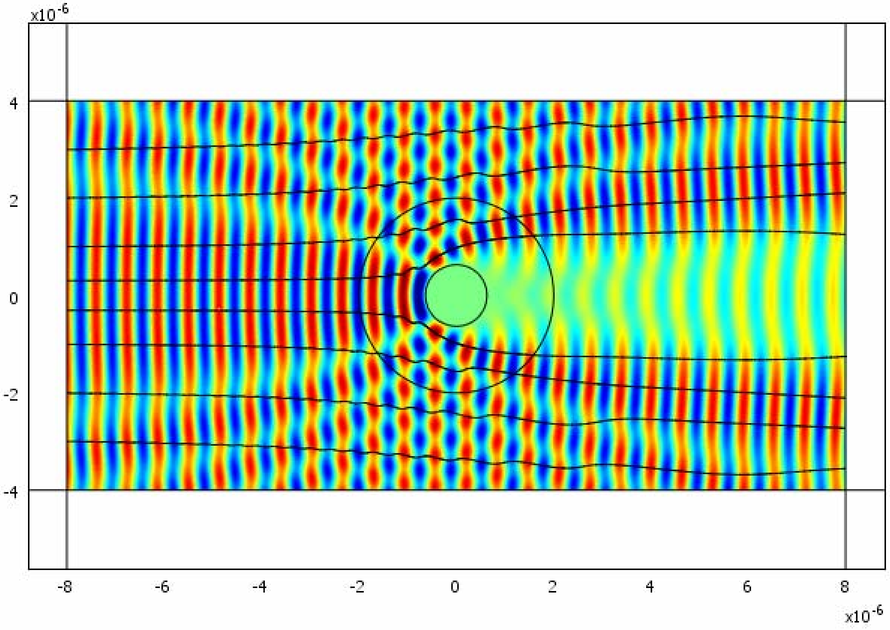
Negligible perturbation in ε_θ

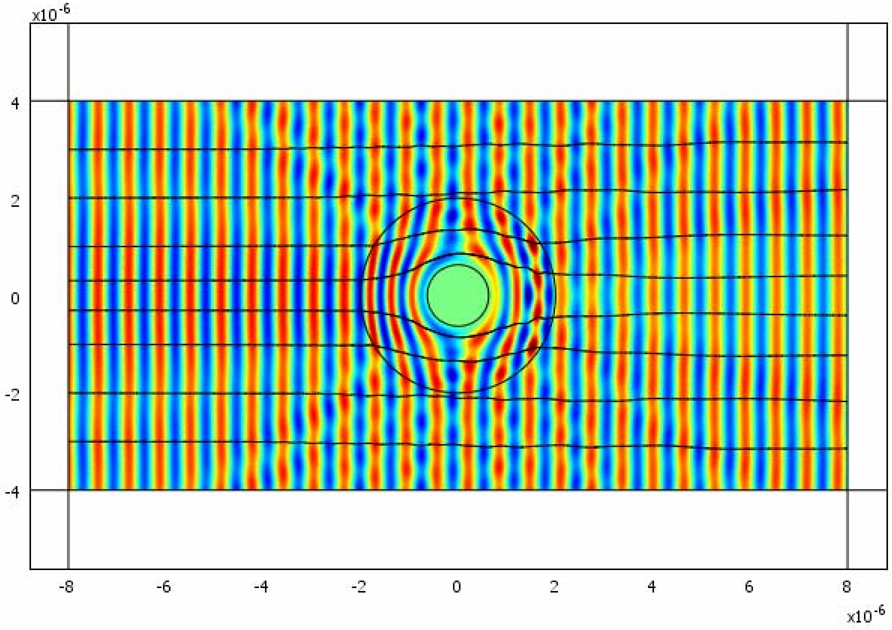
Cai, et al., Nature Photonics, 1, 224 (2007)

Cloaking performance: Field mapping movies

Example:

Non-magnetic cloak @ 632.8nm

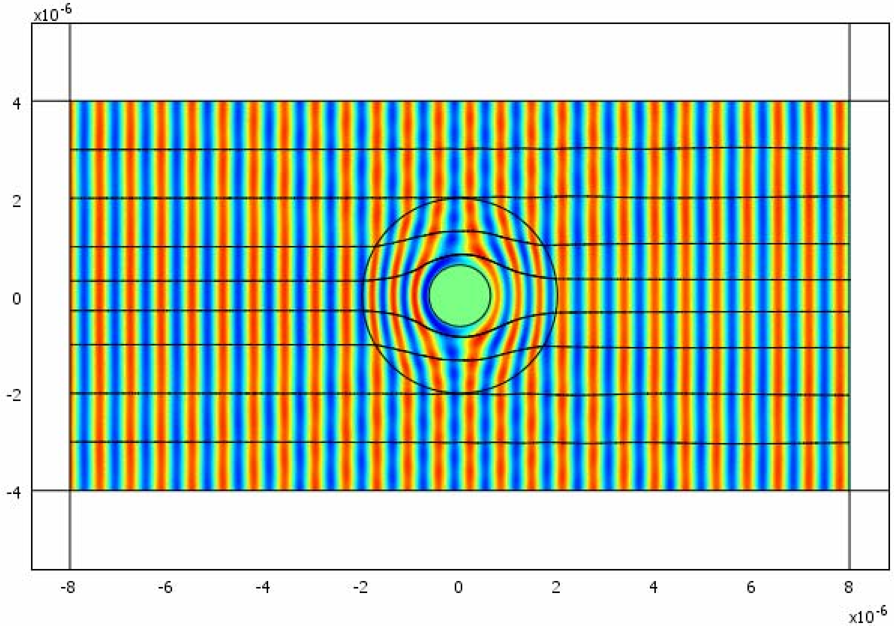




Cloak OFF ***Cloak ON***

Scattering issue in a non-magnetic cloak

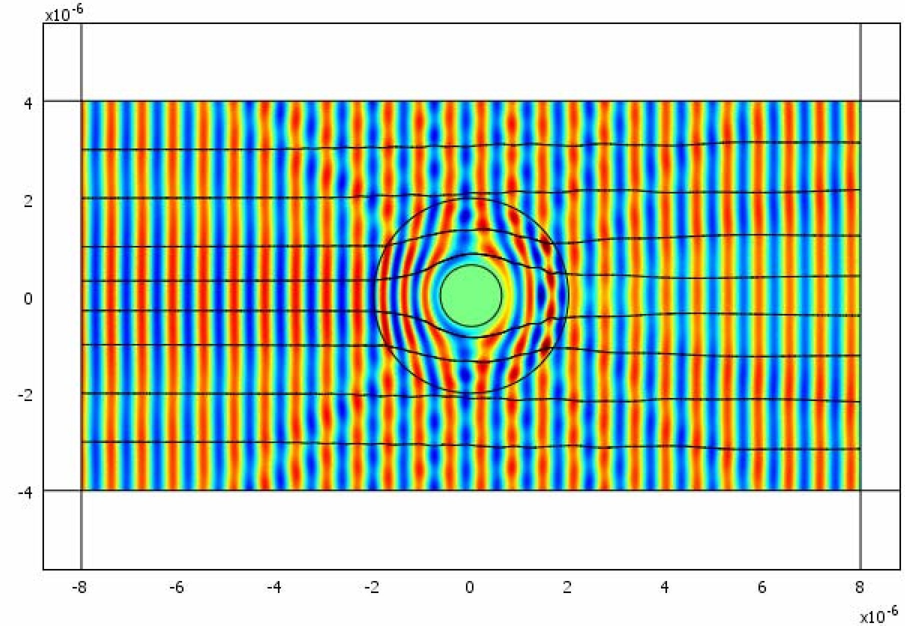
Ideal Cloak



$$Z\big|_{r=b} = \sqrt{\mu_z/\varepsilon_\theta}\,\Big|_{r=b} = 1$$

Perfectly matched impedance results in zero scattering

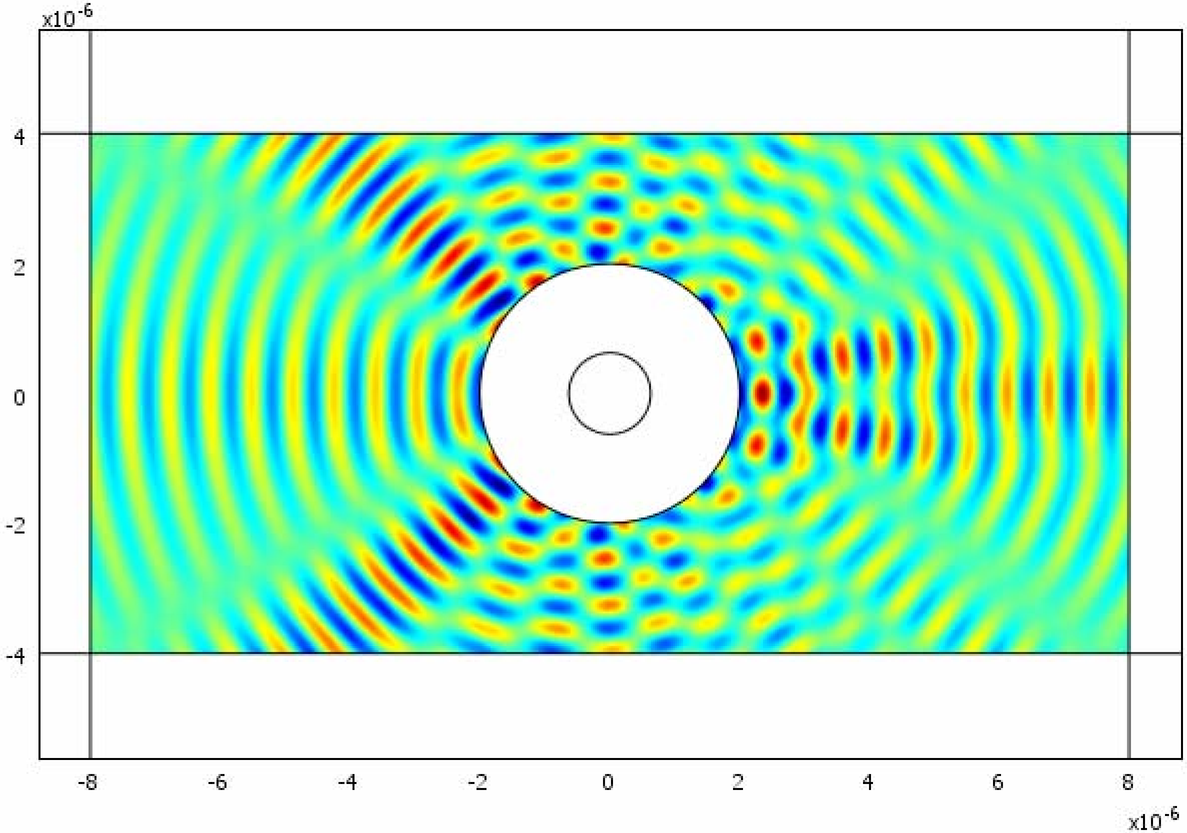
Non-magnetic



$$Z\big|_{r=b} = \sqrt{\mu_z/\varepsilon_\theta}\,\Big|_{r=b} = 1 - a/b$$

Detrimental scattering due to impedance mismatch

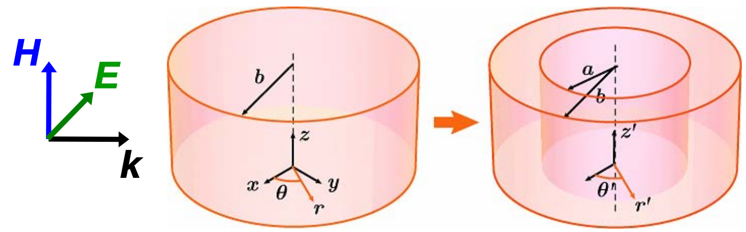
Normalized scattered field



High-order transformations minimize scattering

Linear transformation

$$r = \frac{b-a}{b} r' + a$$



Nonlinear transformation

$r = g(r')$ → $g(0) = a;\ \ g(b) = b;\ \ \partial g(r')/\partial r' > 0$

Jacobian Matrix

ε And μ tensors for ideal cloak →

$$\begin{cases} \varepsilon_r = \mu_r = \left(r'/r\right)\partial g(r')/\partial r' \\ \varepsilon_\theta = \mu_\theta = \left(r/r'\right)\left[\partial g(r')/\partial r'\right]^{-1} \\ \varepsilon_z = \mu_z = \left(r'/r\right)\left[\partial g(r')/\partial r'\right]^{-1} \end{cases}$$

Corresponding non-magnetic parameters →

$$\varepsilon_r = \left(r'/r\right)^2;\ \ \varepsilon_\theta = \left[\partial g(r')/\partial r'\right]^{-2};\ \ \mu_z = 1$$

Set Z=1 at r=b to fix g(r′) →

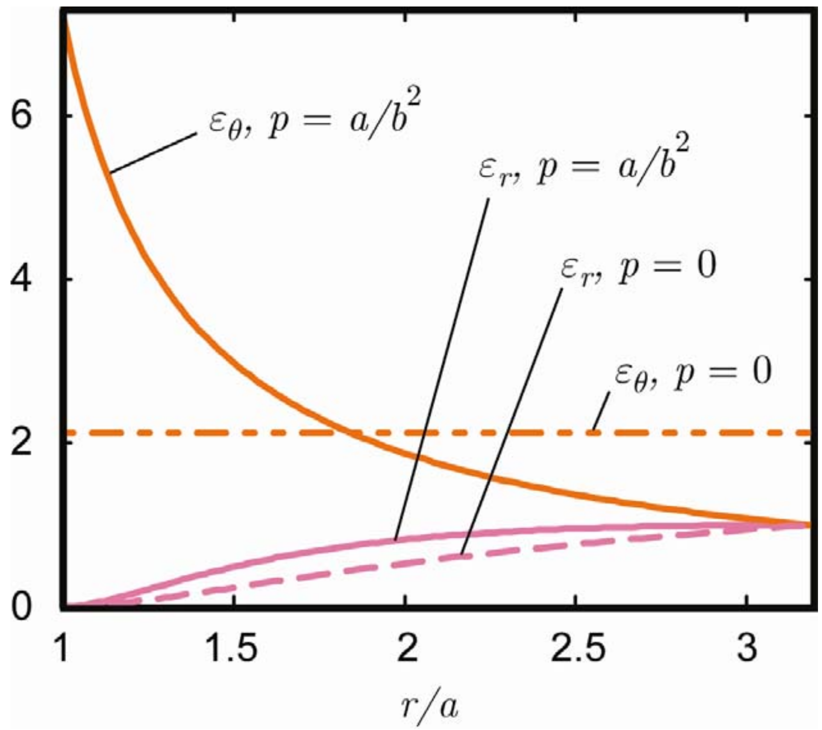
$$Z\big|_{r=b} = \sqrt{\mu_z/\varepsilon_\theta}\Big|_{r=b} = \partial g(r')/\partial r'\Big|_{r=b} = 1$$

Cai, et al., App. Phys. Lett, 91, 111105 (2007)

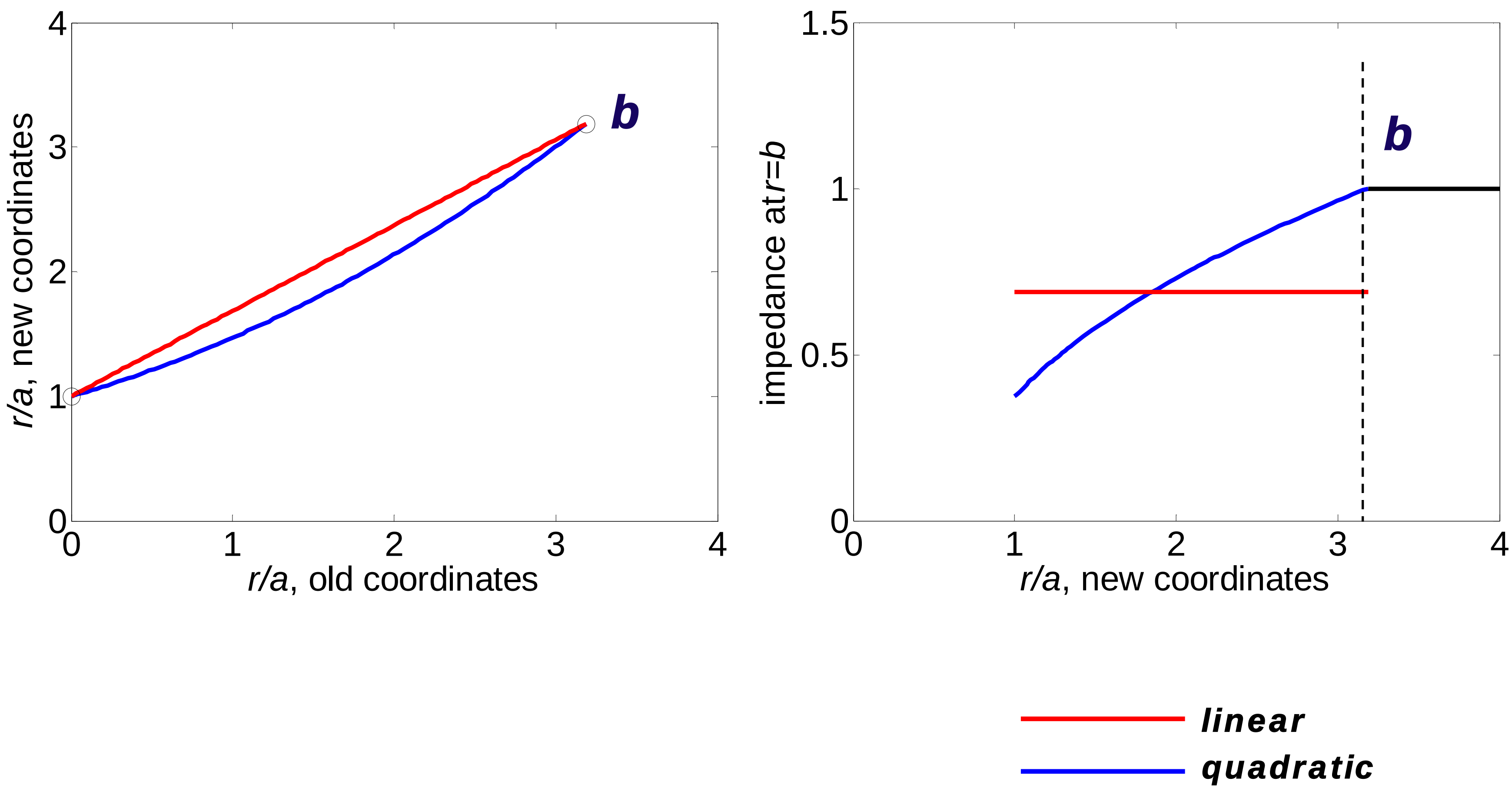
Example: Optimized quadratic transformation

A second-order transformation for non-magnetic cloak with minimized scattering

$$r = g(r') = \left[1 - a/b + p\left(r' - b\right)\right]r' + a \qquad with \qquad p = a/b^2$$

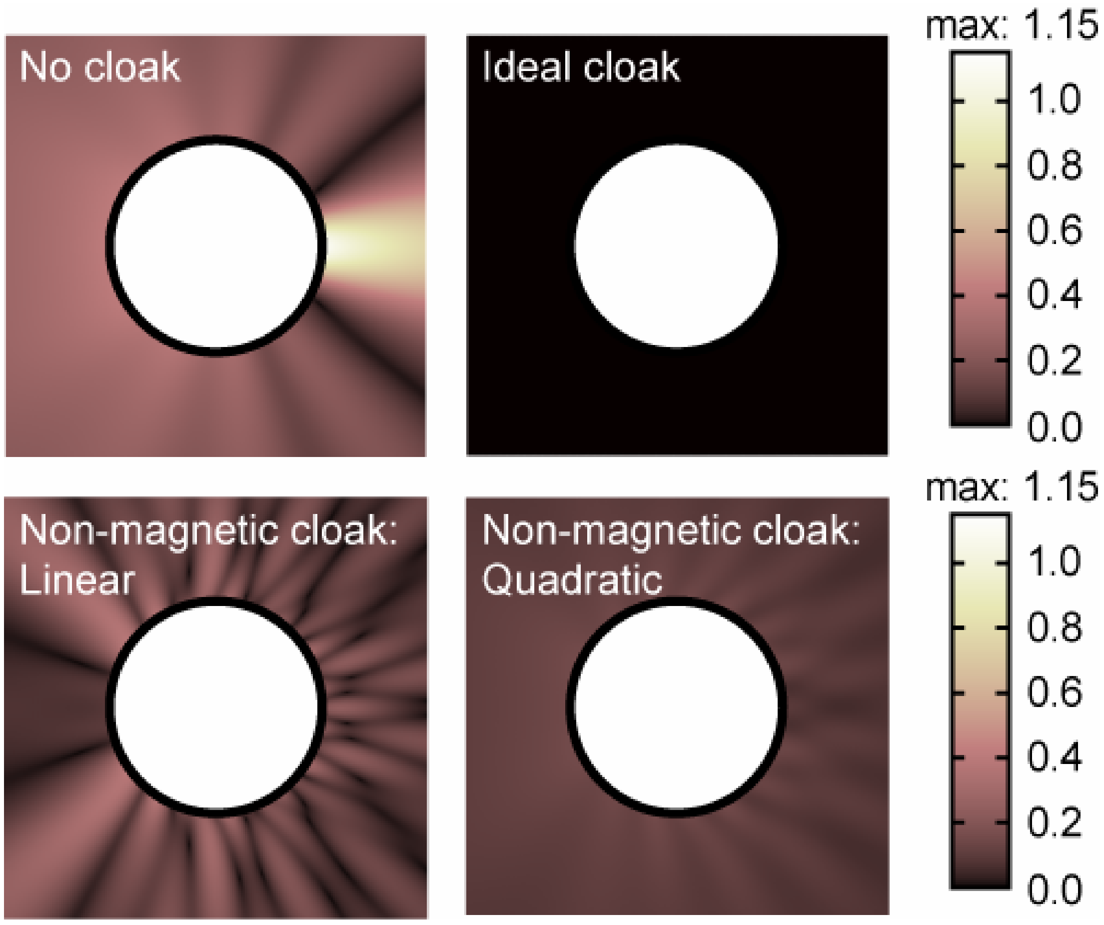


Transformation and impedance



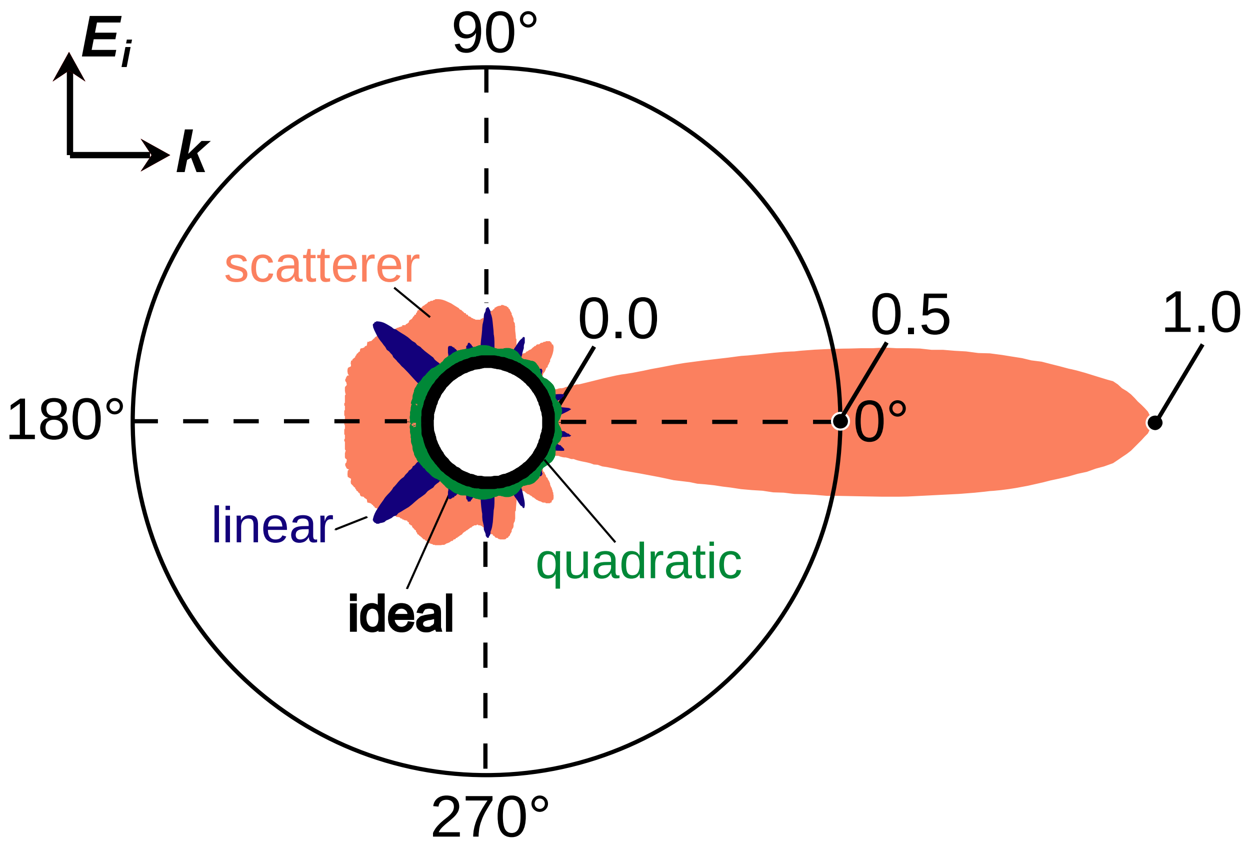
Reduced scattering from nonlinear cloak

Normalized scattered field

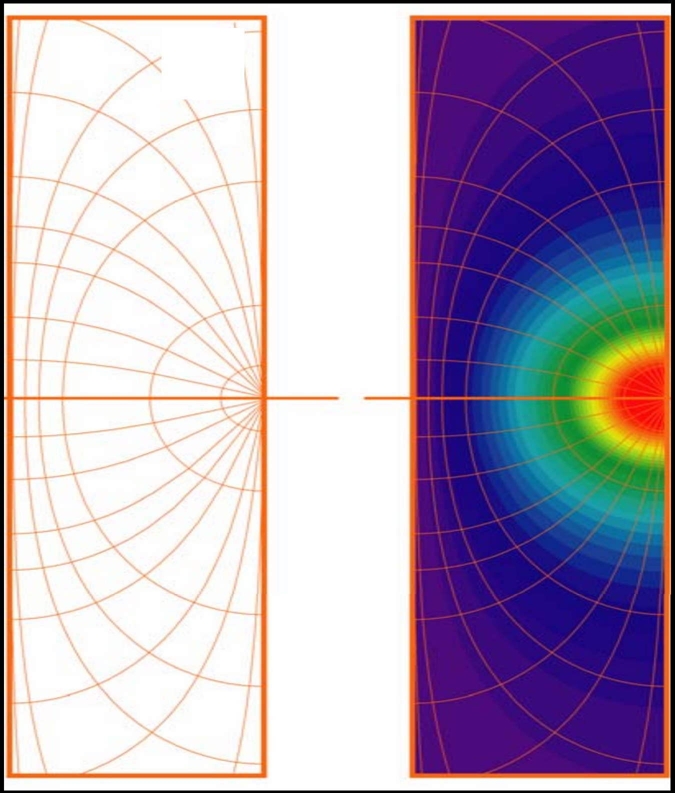


Suppression in both magnitude and directivity

Scattering radiation pattern

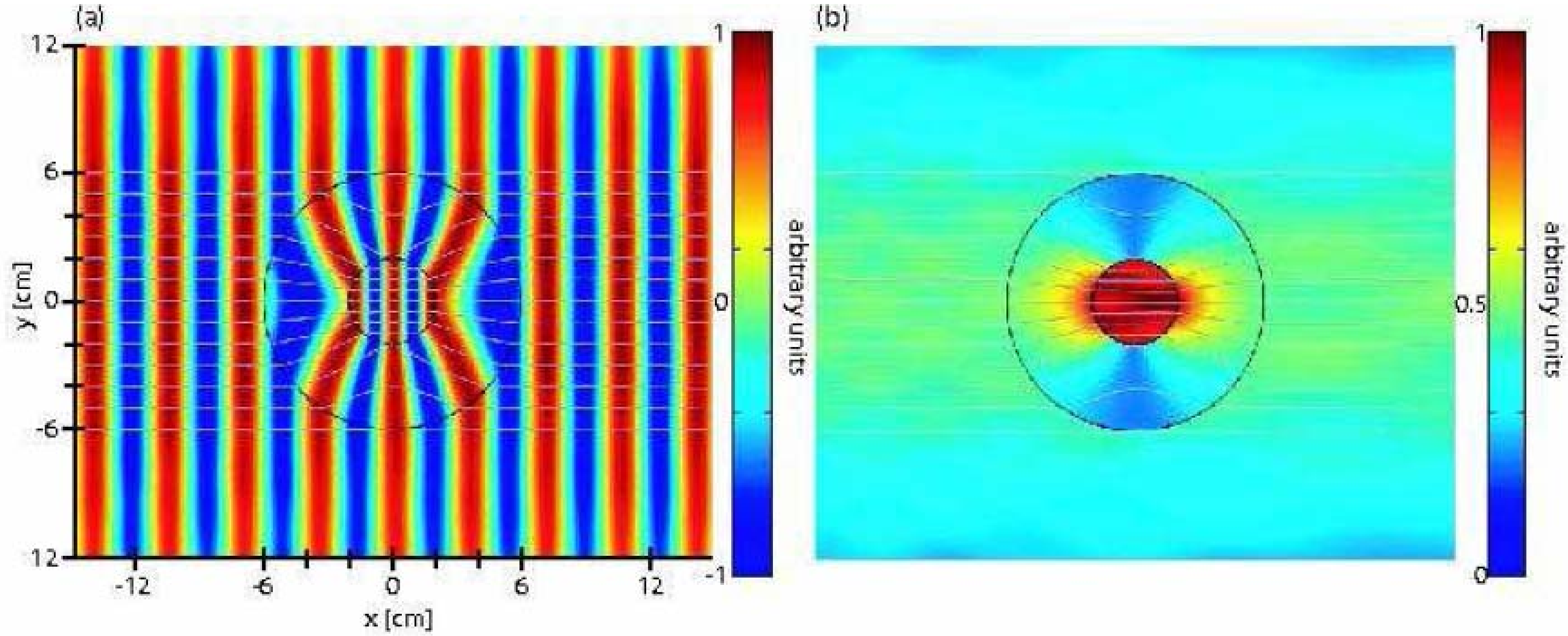


Engineering Optical Meta-Space:
Controlling & Manipulating Light via design and fabrication of (ε,μ)-distribution



Light-concentrating slab (as an example):

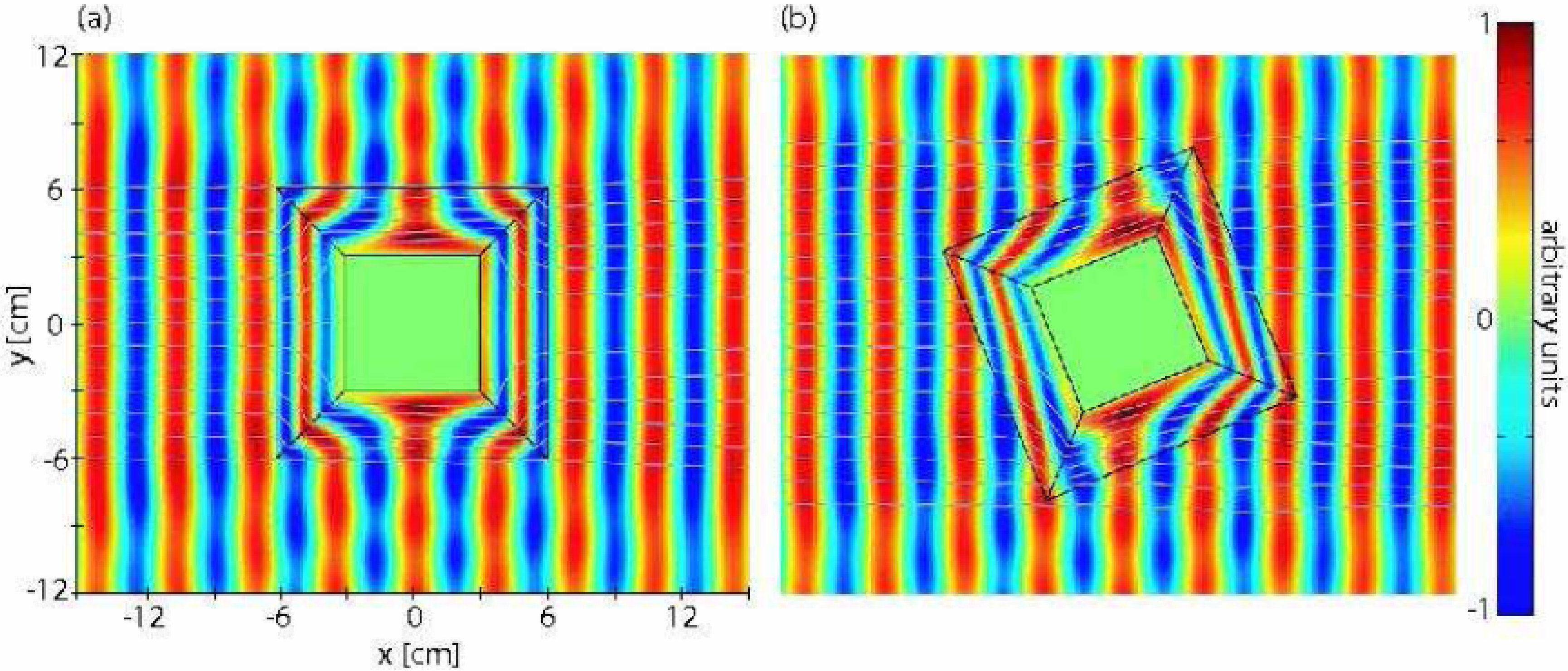
Example of Engineering Optical Meta-Space: Optical concentrator



Electrical field distribution ***Power flow distribution***

Rahm et al. (Duke U.), arXiv 0706.2452

Example of Engineering Optical Meta-Space: Square cloak



Rahm et al. (Duke U.), arXiv 0706.2452