New Journal of Physics
The open-access journal for physics

Transformation optics: approaching broadband electromagnetic cloaking

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New Journal of Physics **10** (2008) 115029 (13pp)
Received 20 July 2008
Published 27 November 2008
Online at http://www.njp.org/
doi:10.1088/1367-2630/10/11/115029

Abstract. An approach to broadband cloaking of light waves is analysed for a simplified case of a scaling transformation for a general cylindrical coordinate system. The proposed approach requires metamaterials with specifically engineered dispersion. The restriction on the signs of gradients in the dispersion dependencies of the dielectric permittivity and the magnetic permeability for different operational wavelengths is revealed and is shown to cause difficulties unless additional gain-assisted compensation for losses or electromagnetically induced transparency is introduced in the cloaking system.

Contents

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1. Introduction: cloaking devices, invisible metamaterials and transformation optics (TO)

Recently, enormous efforts have been devoted to developing principles, designs and numerical validation schemes for cloaking devices for various physical fields[4]. As a result, cloaking applications range from a broad family of electromagnetic devices across the entire electromagnetic spectrum [1]–[31] to elastodynamics [32], quantum mechanics [33] and acoustics [10, 13] [34]–[36].

During the last two years, electromagnetic cloaking with the TO approach has evolved into one of the hottest topics in metamaterial electromagnetics, with the latest research shown in [19]–[27], [29]–[31], [33]–[35], [37]–[50].

Inspired by Kerker [51], a separate stream of invisibility research abounds with studies of electromagnetics in isotropic bi-layer or multilayer sub-wavelength inclusions of ellipsoidal (spheroidal or spherical) shapes in dielectric host media [8, 52, 53]. On top of general invisibility studies in the proposed composites, special attention has been given to achieving *broadband transparency* by using multiphase spherical inclusions with appropriate layered geometries and materials [54]. These studies are also instrumental for estimating local fields and effective optical properties of heterogeneous media with binary or multi-phase inclusions [55].

There are several early papers relating electromagnetic fields and effective space-time geometries (see, for example the early work by Tamm [56]–[58]). The basics of TO are built on the fundamental results reached by Dolin [59], Post [60] and Lax–Nelson [61]. In his work, Dolin not only showed that Maxwell's equations appear to be form-invariant under a space-deforming transformation, but also demonstrated deeper physical insight. Thus, he obtained the following formulae for the radially anisotropic permeability and permittivity of a spherical material inhomogeneity,

$$\varepsilon = \mu = \begin{pmatrix} \dfrac{R^2}{r^2(R)}\dfrac{\mathrm{d}r(R)}{\mathrm{d}R} & 0 & 0 \\ 0 & \dfrac{1}{\mathrm{d}r(R)/\mathrm{d}R} & 0 \\ 0 & 0 & \dfrac{1}{\mathrm{d}r(R)/\mathrm{d}R} \end{pmatrix}, \qquad (1.1)$$

corresponding to a spatial transformation from the spherical coordinates r, Θ, φ to the coordinates $R(r)$, Θ, φ. Then he noted that *a plane wave incident from infinity on an inhomogeneity with parameters* (1.1) *passes through the inhomogeneity without distortions*.

Unfortunately, the works of Dolin, Post and even of Lax–Nelson have been forgotten, and later seminal and much more insightful works of Pendry and his co-workers literally re-established the field of TO. Their TO studies were built on work by Chew–Weedon [62], Ward–Pendry [63, 64], followed in parallel by Milton *et al* [32] and Leonhard–Philbin [65].

Although most theoretical work on TO deals with hypothetically dispersionless media, the issue of operational bandwidth is certainly of both theoretical value and practical interest for any transformation-based functionality. This paper is focused on the broadband operation of cloaking devices, for which the very first approach has been done by Alu–Engheta [54] for small (sub-wavelength) objects. In this paper, an alternative method

[4] It is worthy of note that, in contrast with science fiction, in industrial journalism the term 'cloak' was perhaps first cast for an acoustic silencing device, see 'Perforated steel "cloak" for jet noise,' in *Industrial Research* **11**, 79 (1969).

is described for broadband cloaking systems made of binary or multiphase metamaterials, where different optical paths are arranged for different wavelengths inside the *macroscopic cloaking structures*. For such systems, we establish an important connection between different frequencies, namely that the cloaking design requirements could be satisfied through appropriate dispersion engineering of optical metamaterials. Finally, we discuss some aspects of the practical realization of realistic dispersion laws. Additionally, the fundamentals of TO for scaling transforms in general orthogonal cylindrical coordinates are shown.

2. Cloaking using scaling transformations

The central concept of an electromagnetic cloak is to create a shell-like structure, whose permittivity and permeability distributions allow the incident waves to be directed around the inner region and recovered afterwards. Among all the possible geometries including spherical, square and elliptical varieties, cloaking in a cylindrical system is perhaps the most mathematically straightforward, and is used as an example in this paper. An all-purpose approach to TO for cases beyond general orthogonal cylindrical coordinates can be treated through the fundamentals shown in the appendix.

2.1. Scaling transformations in general orthogonal cylindrical coordinates

A class of a general *orthogonal cylindrical coordinate system* (OCCS) can be arranged by translating an x–y-plane map ($x = x(\tilde{\nu}, \tilde{\tau})$, $y = y(\tilde{\nu}, \tilde{\tau})$) perpendicular to itself; the resulting physical coordinate system forms families of concentric cylindrical surfaces. Since the unit vectors are orthogonal, $\hat{\mathbf{e}}_{\tilde{\nu}} \times \hat{\mathbf{e}}_{\tilde{\tau}} = \hat{\mathbf{e}}_{\tilde{z}}$, $\hat{\mathbf{e}}_{\tilde{\tau}} \times \hat{\mathbf{e}}_{\tilde{z}} = \hat{\mathbf{e}}_{\tilde{\nu}}$ and $\hat{\mathbf{e}}_{\tilde{z}} \times \hat{\mathbf{e}}_{\tilde{\nu}} = \hat{\mathbf{e}}_{\tilde{\tau}}$, the complexity of TO problems in transverse electric (TE) or transverse magnetic (TM) formulations can be significantly reduced.

First consider the initial OCCS, where a 2D radius-vector is defined by a parametric vector-function $\tilde{\mathbf{r}}(\tilde{\nu}, \tilde{\tau})$, and a 2D vector $\tilde{\mathbf{u}}$ is defined as $\tilde{\mathbf{u}} = \tilde{u}_{\tilde{\nu}}\hat{\mathbf{e}}_{\tilde{\nu}} + \tilde{u}_{\tilde{\tau}}\hat{\mathbf{e}}_{\tilde{\tau}}$. The Jacobian matrix is the diagonal matrix, $\tilde{\mathbf{s}} = \mathrm{diag}(s_{\tilde{\nu}}, s_{\tilde{\tau}})$, with the metric coefficients $s_{\tilde{\nu}} = \sqrt{\tilde{\mathbf{r}}^{(\tilde{\nu})} \cdot \tilde{\mathbf{r}}^{(\tilde{\nu})}}$ and $s_{\tilde{\tau}} = \sqrt{\tilde{\mathbf{r}}^{(\tilde{\tau})} \cdot \tilde{\mathbf{r}}^{(\tilde{\tau})}}$. Then, the following scalar wave equation is straightforwardly obtained from the Maxwell curl equations in an orthogonal cylindrical basis for a general anisotropic medium. Thus, from

$$\tilde{u}_{\tilde{\nu}} = \iota\omega^{-1}\tilde{m}_{\tilde{\nu}}^{-1}\tilde{s}_{\tilde{\tau}}^{-1}\tilde{v}^{(\tilde{\tau})}, \quad \tilde{u}_{\tilde{\tau}} = -\iota\omega^{-1}\tilde{m}_{\tilde{\tau}}^{-1}\tilde{s}_{\tilde{\nu}}^{-1}\tilde{v}^{(\tilde{\nu})}, \quad -\iota\omega\tilde{m}_z\tilde{v} = |\tilde{\mathbf{s}}|^{-1}\left[(\tilde{s}_{\tilde{\tau}}\tilde{u}_{\tilde{\tau}})^{(\tilde{\nu})} - (\tilde{s}_{\tilde{\nu}}\tilde{u}_{\tilde{\nu}})^{(\tilde{\tau})}\right]; \tag{2.1}$$

we arrive at

$$\left(\tilde{s}_{\tilde{\tau}}\tilde{m}_{\tilde{\tau}}^{-1}\tilde{s}_{\tilde{\nu}}^{-1}\tilde{v}^{(\tilde{\nu})}\right)^{(\tilde{\nu})} + \left(\tilde{s}_{\tilde{\nu}}\tilde{m}_{\tilde{\nu}}^{-1}\tilde{s}_{\tilde{\tau}}^{-1}\tilde{v}^{(\tilde{\tau})}\right)^{(\tilde{\tau})} - \omega^2\tilde{m}_z|\mathbf{s}|v = 0. \tag{2.2}$$

where $\tilde{m}_{\tilde{\nu}}$ and $\tilde{m}_{\tilde{\tau}}$ are the only components of a diagonal material property tensor, i.e. anisotropic permeability or anisotropic permittivity (for TM or TE polarization, respectively); the scalar $\tilde{v}$ is the only component of the transverse field, i.e. the magnetic field, $\mathbf{H} = \hat{\mathbf{e}}_z H_z$ (TM), or the electric field, $\mathbf{E} = \hat{\mathbf{e}}_z E_z$ (TE).

Similar to (2.2), yet another wave equation in a new physical OCCS, (ν, τ, z), can be written as

$$\left(s_\tau m_\tau^{-1} s_\nu^{-1} v^{(\nu)}\right)^{(\nu)} + \left(s_\nu m_\nu^{-1} s_\tau^{-1} v^{(\tau)}\right)^{(\tau)} - \omega^2 m_z|\mathbf{s}|v = 0. \tag{2.3}$$

To mimic the behaviour of light waves obeying (2.1), a scaling transformation $\nu = \nu(\tilde{\nu})$ (with $\tau = \tilde{\tau}$, $z = \tilde{z}$ and $\nu' = \nu^{(\tilde{\nu})}$) is introduced. Thus, to get closer to (2.1), equation (2.3) is reshaped as

$$\left(\left[\frac{1}{\nu'}\frac{s_\tau \tilde{m}_{\tilde{\tau}} \tilde{s}_{\tilde{\nu}}}{\tilde{s}_{\tilde{\tau}} m_\tau s_\nu}\right]\frac{\tilde{s}_{\tilde{\tau}}}{\tilde{m}_{\tilde{\tau}}\tilde{s}_{\tilde{\nu}}}\upsilon^{(\tilde{\nu})}\right)^{(\tilde{\nu})} + \left(\left[\nu'\frac{s_\nu \tilde{m}_{\tilde{\nu}} \tilde{s}_\tau}{\tilde{s}_\nu m_\nu s_\tau}\right]\frac{\tilde{s}_\nu}{\tilde{m}_\nu \tilde{s}_\tau}\upsilon^{(\tilde{\tau})}\right)^{(\tilde{\tau})} - \omega^2\left[\nu'\frac{m_z|\mathbf{s}|}{\tilde{m}_z|\tilde{\mathbf{s}}|}\right]\tilde{m}_z|\tilde{\mathbf{s}}|\upsilon = 0. \qquad (2.4)$$

It immediately follows that (2.4) is completely identical to (2.1), provided that the ratios in the square brackets are eliminated. Thus, the TO identities

$$\frac{1}{\nu'}\frac{s_\tau \tilde{m}_{\tilde{\tau}} \tilde{s}_{\tilde{\nu}}}{\tilde{s}_{\tilde{\tau}} m_\tau s_\nu} = 1, \quad \nu'\frac{s_\nu \tilde{m}_{\tilde{\nu}} \tilde{s}_\tau}{\tilde{s}_\nu m_\nu s_\tau} = 1, \quad \nu'\frac{m_z|\mathbf{s}|}{\tilde{m}_z|\tilde{\mathbf{s}}|} = 1, \qquad (2.5)$$

are required to be valid in a new material space (m_ν, m_τ and m_z) in order to mimic the behaviour of light in the initial material space ($\tilde{m}_{\tilde{\nu}}$, $\tilde{m}_{\tilde{\tau}}$ and $\tilde{m}_z$) [1].

The above identities define the material transformation laws, which are valid far beyond cloaking applications. Equations (2.5) solve the critical problem of *designing a new anisotropic continuous material space supporting the required light wave behaviour that is completely equivalent to the behaviour of the light waves mapped back onto the initial space*.

Thus, scaling transformations that expand the initially small domain onto a larger physical domain are pertinent to imaging or light concentration [66, 67], while typical cloaking applications require scaling transforms that shrink the initially larger space to arrange for voids excluded from the initial domain and are therefore inaccessible for light. In both applications the initial virtual spaces share a common input with the rest of the transformed physical world. Examples of different scaling transformations are shown in figure 1.

In the circular cylindrical coordinates ($\nu = \rho$, $\tau = \phi$) and $s_\rho = 1$, $s_\phi = \rho$, (2.5) give

$$m_\phi = \frac{\rho}{\rho'\tilde{\rho}}\tilde{m}_\phi, \quad m_\rho = \frac{\rho'\tilde{\rho}}{\rho}\tilde{m}_\rho, \quad m_z = \frac{\tilde{\rho}}{\rho\rho'}\tilde{m}_z, \qquad (2.6)$$

i.e. the required material space for an exact cloak, which is an imitation of a cylindrical free-space domain, is defined by the following inhomogeneous and anisotropic material properties (also see for example, [10, 59]):

$$\varepsilon_\rho = \mu_\rho = \tilde{\rho}\rho'/\rho; \quad \varepsilon_\phi = \mu_\phi = \varepsilon_\rho^{-1}; \quad \varepsilon_z = \mu_z = \tilde{\rho}/(\rho'\rho). \qquad (2.7)$$

The material properties can be relaxed for some special modes. Specifically, for TM polarization with the magnetic field polarized along the z-axis, we multiply ε_r and ε_ϕ by μ_z in (2.7) and obtain the following reduced set of non-magnetic cloak parameters:

$$\varepsilon_\rho = (\tilde{\rho}/\rho)^2; \quad \varepsilon_\phi = (\rho')^{-2}; \quad \mu_z = 1. \qquad (2.8)$$

Similarly, for the TE polarization, the required parameters for a general transformation are:

$$\mu_\rho = (\tilde{\rho}/\rho)^2(\rho')^2, \quad \mu_\phi = 1, \quad \varepsilon_z = (\rho')^{-2}. \qquad (2.9)$$

In (2.7)–(2.9), $\tilde{\rho}$ should be replaced by $\tilde{\rho} = \tilde{\rho}(\rho)$ to obtain closed-form expressions.

2.2. Cloaking a finite bandwidth of light waves

Below, we consider the possibility of cloaking a finite bandwidth of electromagnetic waves using a single device. The proposed broadband cloak functions in a wavelength multiplexing manner. Since the anisotropic constituent materials of a cloak for one wavelength cannot be

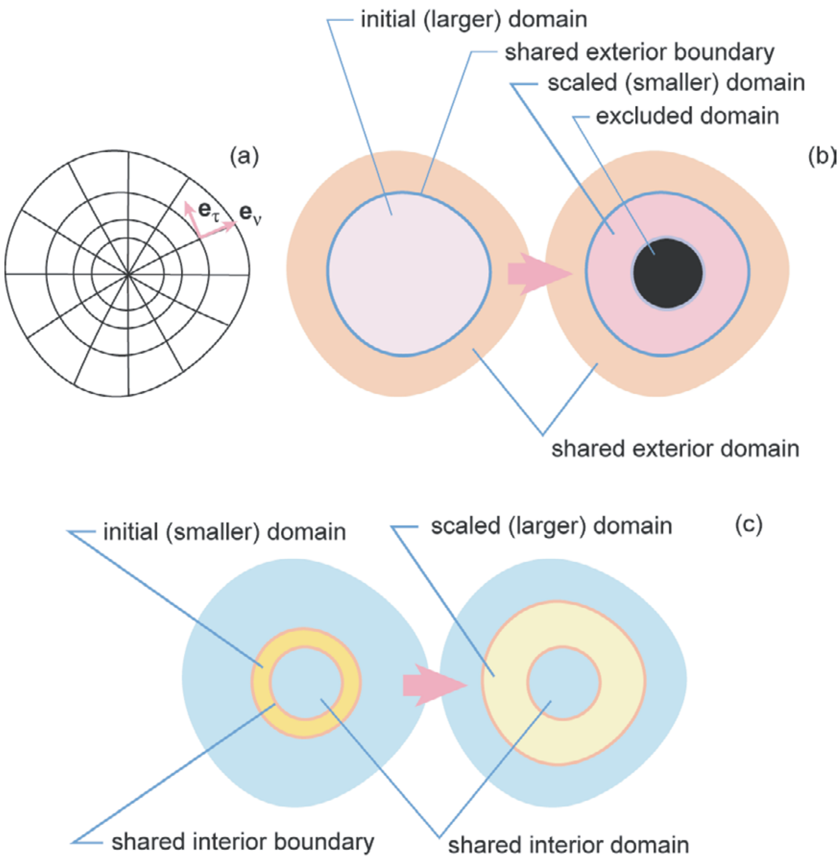

Figure 1. (a) Example of a general orthogonal cylindrical coordinate system. (b) Domain transformation for a cylindrical cloaking device. A larger initial (pink) domain in the left panel is mapped onto a scaled smaller annular domain shown in the right panel, leaving the central (black) domain inaccessible to light. The initial and scaled domains share the same exterior boundary and the common space beyond. (c) Domain transformation for a hyper-lens. A smaller initial (yellow) domain in the left panel is mapped onto a scaled larger domain of the right panel. The initial and scaled domains share the same internal boundary and a common space inside it.

transparent at other frequencies, cloaks for all the wavelengths being considered have to share the same outer boundary—the physical boundary of the device. The inner boundary and the transformation for each operating wavelength, however, are unique. This means that *a number of different inner boundaries and different transformations have to be used to provide the broadband capability*.

A schematic of the operational principle is illustrated in figure 2. Since the wave components at different frequencies go through the system following different physical paths, the proposed system allows the cloaking parameters to be exactly fulfilled over a finite bandwidth without violating basic physical laws or giving rise to a superluminal group velocity. As a result, a 'colourful' (multi-frequency) stationary image appears transparent through the cloaking device.

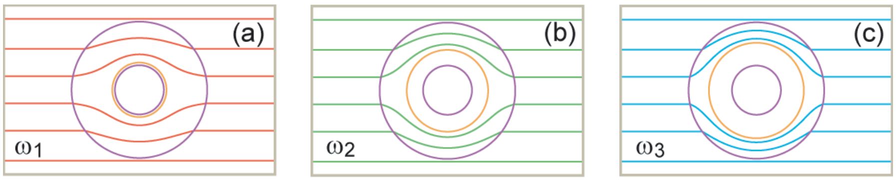

Figure 2. Simulated schematic of a cloaking system for multiple wavelengths or a finite bandwidth, with $\omega_1 > \omega_2 > \omega_3$, shown in (a), (b) and (c), respectively. The two purple circles represent the physical boundaries the cloaking device, and the orange circle between the two refers to the inner boundary for each operational wavelength.

For a concise mathematical treatment, in this work we focus on the TE mode with material properties given in (2.9), which allows for flexible parameters at the outer boundary of $\rho = b$. We assume that at frequency ω_0, the material properties required by a TE cloak are exactly satisfied based on the transformation $\rho = \rho(\tilde{\rho})$ within the range, $a \leqslant \rho \leqslant b$:

$$\mu_\rho(\omega_0, \rho) = (\tilde{\rho}/\rho)^2 (\rho')^2, \quad \mu_\phi(\omega_0, \rho) = 1, \quad \varepsilon_z(\omega_0, \rho) = (\rho')^{-2}. \tag{2.10}$$

Dispersion is the fundamental limitation to the broadband performance of a cloaking system. In the following analysis, we use the first-order Taylor expansion of the dispersion function; that is, we assume that the cloaking materials exhibit a linear dispersion around the initial frequency ω_0:

$$\mu_\rho(\omega, \rho) = \mu_\rho(\omega_0, \rho) + \mu_\rho^{(\omega)}(\omega_0, \rho)(\omega - \omega_0) \tag{2.11}$$

and

$$\varepsilon_z(\omega, \rho) = \varepsilon_z(\omega_0, \rho) + \varepsilon_z^{(\omega)}(\omega_0, \rho)(\omega - \omega_0). \tag{2.12}$$

In (2.11) and (2.12) the two frequency derivatives $\mu_\rho^{(\omega)}(\omega_0, \rho) = \partial\mu_\rho(\omega, \rho)/\partial\omega|_{\omega=\omega_0}$ and $\varepsilon_z^{(\omega)}(\omega_0, \rho) = \partial\varepsilon_z(\omega, \rho)/\partial\omega|_{\omega=\omega_0}$ are continuous functions of ρ. Since there is no magnetic response along the ϕ-direction at ω_0, it is reasonable to assume that $\mu_\phi(\omega, \rho) = \mu_\phi(\omega_0, \rho) = 1$.

The initial formulation of the problem is then to find, at a frequency $\omega_1 = \omega_0 + \delta\omega$, a combination of the transformation $\rho_1 = \rho_1(\tilde{\rho})$ along with yet another inner radius a_1 such that the function $\rho_1(\tilde{\rho})$ maps $[0, b]$ onto $[a_1, b]$ with $a < a_1 < b$, while satisfying the boundary conditions

$$\rho_1(0) = a_1, \quad \rho_1(b) = b \tag{2.13}$$

along with the monotonicity condition

$$\rho_1' > 0 \tag{2.14}$$

and the material transforms of the reduced TE cloak

$$\mu_\rho(\omega_1, \rho_1) = (\tilde{\rho}/\rho_1)^2 (\rho_1')^2, \quad \varepsilon_z(\omega_1, \rho_1) = (\rho_1')^{-2}, \tag{2.15}$$

where $\tilde{\rho} = g_1^{-1}(\rho)$, $a_1 \leqslant \rho \leqslant b$.

The transformation $\rho_1(\tilde{\rho})$ for $\omega_1 = \omega_0 + \delta\omega$ is related to the original transformation at ω_0 and the dispersion functions in the following way:

$$(\tilde{\rho}(\rho_1)/\rho_1)^2 (\rho_1')^2 = (\tilde{\rho}(\rho)/\rho)^2 (\rho')^2 + \mu_\rho^{(\omega)}(\omega_0, \rho)(\omega - \omega_0) \tag{2.16}$$

and

$$(\rho_1')^{-2} = (\rho')^{-2} + \varepsilon_z^{(\omega)}(\omega_0, \rho)(\omega - \omega_0), \tag{2.17}$$

within the range of $a_1 \leqslant \rho_1 \leqslant b$ with the boundary conditions mentioned above.

3. Results and discussions

Apparently, (2.16) and (2.17) cannot be fulfilled for arbitrary gradients of dispersion functions $\mu_\rho^{(\omega)}(\omega_0, \rho)$ and $\varepsilon_z^{(\omega)}(\omega_0, \rho)$. Therefore, the problem of broadband cloaking now converges to the task of dispersion engineering and management. This additional task can then be described as follows: what physically-possible functions $\mu_\rho^{(\omega)}(\omega_0, \rho)$ and $\varepsilon_z^{(\omega)}(\omega_0, \rho)$ should be engineered to make the cloaking effect possible at a given frequency $\omega_1 = \omega_0 + \delta\omega$ in addition to cloaking at ω_0?

After some algebra, we can show that (2.13)–(2.17) require that

$$\mu_\rho^{(\omega)}(\omega_0, \rho)\, \varepsilon_z^{(\omega)}(\omega_0, \rho) < 0. \tag{3.1}$$

Thus, (3.1) indicates that the dispersion of the radial permeability $\mu_\rho(\omega, \rho)$ and the axial permittivity $\varepsilon_z(\omega, \rho)$ must have opposite slopes as functions of the frequency.

The bandwidth of a transformation-based cloaking device is determined by the frequency range over which the material properties in (2.7)–(2.9) are exactly satisfied. Unfortunately, the curved trajectory of light within the cloak implies a refractive index n of less than 1 in order to satisfy the minimal optical path requirement of the Fermat principle, but any metamaterial with $n < 1$ must be dispersive to fulfil causality and must work at only a single target wavelength. Because of this, it has been considered so far that broadband cloaks are intrinsically impossible.

Indeed, recent theoretical analyses indicate that the anisotropic cloaking parameters required by a first-order transformation can be precisely fulfilled only for a single frequency value [12]. Since the material parameters for a cloak cannot be exactly satisfied over a finite bandwidth, regardless of how narrow it is, in previous designs one had to sacrifice cloaking performance by introducing additional cross sections for quasi-broadband operation [68].

In our approach, specifically engineered strong anomalous dispersion is required because (3.1) cannot be satisfied with normal dispersion, where $\partial\varepsilon/\partial\omega > 0$ and $\partial\mu/\partial\omega > 0$. However, anomalous dispersion is normally associated with substantial loss. Therefore, a broadband electromagnetic cloak using our approach necessitates additional loss-compensation.

In summary, we conclude that since passive materials always exhibit normal dispersion with $\partial\varepsilon/\partial\omega > 0$ and $\partial\mu/\partial\omega > 0$ away from the resonance bands, and because anomalous dispersion usually occurs only around the absorption bands, the proposed wavelength multiplexing cloak with broadband capability is only achievable when gain materials or electromagnetically induced transparency or chirality (see for example, [69, 70]) are introduced to make low-loss anomalous dispersion possible. For instance, in an active medium where the optical gain is indicated by a negative imaginary part of permittivity over a finite bandwidth, the real part of permittivity around the active band will exhibit an anti-Lorentz line shape, as governed by the Kramers–Kronig relations. As a result, anomalous dispersion with relatively low loss occurs in the wings of the gain spectrum. Incorporating gain materials into plasmonics and metamaterials has been proposed and demonstrated for various applications, including near-field superlensing [71], perfect tunnelling transmittance [72], enhanced surface plasmons [73] and lossless negative-index materials [74].

Before we conclude, it is worth mentioning the possibility of a pseudo-broadband cloak based on the fast switching of the cloaking wavelength over the entire visible range. With a switching period shorter than the response time of a detector (such as an eye), such a system would cause the slower detector to observe a broadband cloaking phenomenon. One could mimic such broadband cloaking provided that there is amplification at the 'invisibility wavelength' whereas at all other wavelengths the signal is suppressed.

Acknowledgments

Useful discussions with Mark Smith are appreciated. This work was supported in part by ARO Award 50342-PH-MUR and Lockheed Martin FMC.

Appendix A. Fundamentals of transformation optics

A.1. Vectors, tensors and the curl

Consider an initial material space defined by its radius-vector $\tilde{\mathbf{r}}(\tilde{x}, \tilde{y}, \tilde{z})$ and an inhomogeneous distribution of an anisotropic material property (e.g. either anisotropic permittivity, ε, or anisotropic permeability, μ), given by a tensor, $\tilde{\mathbf{m}} = \tilde{\mathbf{m}}(\mathbf{r})$. Suppose that we want to modify the initial distribution of coupled vector fields, $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}(\tilde{\mathbf{r}})$ and $\tilde{\mathbf{u}} = \tilde{\mathbf{u}}(\tilde{\mathbf{r}})$, using a tensor $\mathbf{j}$. The transformation could be formally achieved by mapping the initial space, using a coordinate transformation ($\mathbf{r} = \mathbf{r}(\tilde{\mathbf{r}})$, i.e. $x = x(\tilde{x}, \tilde{y}, \tilde{z})$, $y = y(\tilde{x}, \tilde{y}, \tilde{z})$, $z = z(\tilde{x}, \tilde{y}, \tilde{z})$) with a non-singular Jacobian matrix $\mathbf{j}$, ($|\mathbf{j}| \neq 0$), so that it is a one-to-one transformation in a neighbourhood of each point. The Jacobian matrix $\mathbf{j}$ is arranged from the columns of base vectors,

$$\mathbf{j} = \begin{pmatrix} \mathbf{r}^{(\tilde{x})} & \mathbf{r}^{(\tilde{y})} & \mathbf{r}^{(\tilde{z})} \end{pmatrix}, \tag{A.1}$$

or its transposition can be arranged from the columns of gradients

$$\mathbf{j}^{\mathrm{T}} = \begin{pmatrix} \tilde{\nabla} x & \tilde{\nabla} y & \tilde{\nabla} z \end{pmatrix}. \tag{A.2}$$

In (A.1) and (A.2), $f^{(\cdot)}$ and $\tilde{\nabla} f = f^{(\tilde{x})}\hat{\mathbf{x}} + f^{(\tilde{y})}\hat{\mathbf{y}} + f^{(\tilde{z})}\hat{\mathbf{z}}$ denote a partial derivative and a gradient, respectively. The Jacobian determinant $|\mathbf{j}|$ is equal to the triple vector product [75], $\mathbf{r}^{(x)}\mathbf{r}^{(y)}\mathbf{r}^{(z)}$. Vectors $\tilde{\mathbf{v}}$ and $\mathbf{v}$ are made of scalar components, as $\tilde{\mathbf{v}} = \tilde{v}_{\tilde{x}}\hat{\mathbf{x}} + \tilde{v}_{\tilde{y}}\hat{\mathbf{y}} + \tilde{v}_{\tilde{z}}\hat{\mathbf{z}}$ and $\mathbf{v} = v_x\hat{\mathbf{x}} + v_y\hat{\mathbf{y}} + v_z\hat{\mathbf{z}}$, respectively.

Thus, a general invertible field-deforming transformation,

$$\tilde{\mathbf{v}} = \mathbf{j}^{\mathrm{T}}\mathbf{v}, \quad \tilde{\mathbf{u}} = \mathbf{j}^{\mathrm{T}}\mathbf{u}, \tag{A.3}$$

links the vectors of the initial vector-space $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}(\tilde{\mathbf{r}})$ and $\tilde{\mathbf{u}} = \tilde{\mathbf{u}}(\tilde{\mathbf{r}})$ with the new vectors of a deformed vector-space $\mathbf{v} = \mathbf{v}(\mathbf{r})$ and $\mathbf{u} = \mathbf{u}(\mathbf{r})$ obtained at the corresponding points of the new material domain.

In the art of manipulating light at the nanoscale, we are always concerned with obtaining a material solution that can be prototyped in order to achieve a given transformation of the fields (A.3). This brings up the question, then, of how to modify the initial material properties, $\tilde{\mathbf{m}} = \tilde{\mathbf{m}}(\mathbf{r})$, in order to obtain the required transformation of the vector fields as determined by (A.3). This question is the core problem in transformation optics.

First, we consider a formal connection between the expressions for gradients before and after the change of variables, $x = x(\tilde{x}, \tilde{y}, \tilde{z})$, $y = y(\tilde{x}, \tilde{y}, \tilde{z})$, $z = z(\tilde{x}, \tilde{y}, \tilde{z})$. As follows [75], we have $\tilde{\nabla} f(x, y, z) = f^{(x)}\tilde{\nabla}x + f^{(y)}\tilde{\nabla}y + f^{(z)}\tilde{\nabla}z$, which yields a general result that is completely analogous to (A.3)

$$\tilde{\nabla} = \mathbf{j}^{\mathrm{T}}\nabla. \tag{A.4}$$

The transformation identity for the curl can be straightforwardly derived first for *pseudo-vectors* $\mathbf{p} = \mathbf{u} \times \mathbf{v}$ and $\tilde{\mathbf{p}} = \tilde{\mathbf{u}} \times \tilde{\mathbf{v}}$. The standard vector algebra [75], gives

$$\tilde{\mathbf{u}} \times \tilde{\mathbf{v}} = (\mathbf{j}^{\mathrm{T}}\mathbf{u}) \times (\mathbf{j}^{\mathrm{T}}\mathbf{v}) = |\mathbf{j}|\mathbf{j}^{-1}(\mathbf{u} \times \mathbf{v}), \tag{A.5}$$

consequently connecting pseudo-vectors $\mathbf{p}$ and $\tilde{\mathbf{p}}$ through

$$\mathbf{p} = |\mathbf{j}|^{-1}\mathbf{j}\,\tilde{\mathbf{p}}. \tag{A.6}$$

A.2. Maxwell's curl equations

To get closer to Maxwell's curl equations, yet another product of the material tensor $\mathbf{m}$ and a vector $\mathbf{u}$ can be defined as $(\mathbf{mu})^{(t)} = \nabla \times \mathbf{v}$, such that for time-independent material properties, $(\mathbf{mu})^{(t)} = \mathbf{mu}^{(t)}$, this yields

$$\mathbf{mu}^{(t)} = \nabla \times \mathbf{v}. \tag{A.7}$$

The right side of (A.7) is identical to $\mathbf{p} = \mathbf{u} \times \mathbf{v}$, provided that vector $\mathbf{u}$ is replaced with ∇, following the result shown in (A.4). The use of the same sets of vector components, i.e. $\nabla \times \tilde{\mathbf{v}} = (\mathbf{j}^{\mathrm{T}}\mathbf{u}) \times (\mathbf{j}^{\mathrm{T}}\mathbf{v}) = |\mathbf{j}|\mathbf{j}^{-1}(\nabla \times \mathbf{v})$ gives $\mathbf{mu}^{(t)} = \nabla \times \mathbf{v}$. Finally, using $\tilde{\mathbf{m}} = |\mathbf{j}|\mathbf{j}^{-1}\mathbf{m}(\mathbf{j}^{\mathrm{T}})^{-1}$, (A.7) can be rewritten as,

$$\tilde{\mathbf{m}}\tilde{\mathbf{u}}^{(t)} = \hat{\nabla} \times \tilde{\mathbf{v}}. \tag{A.8}$$

Then, the required transform for tensors $\tilde{\mathbf{m}}$ and $\mathbf{m}$ is given by the following equation [60, 76]

$$\mathbf{m} = |\mathbf{j}|^{-1}\mathbf{j}\tilde{\mathbf{m}}\mathbf{j}^{\mathrm{T}}. \tag{A.9}$$

In summary, as depicted in figure A.1 the required spatial transformation of the vector fields performed by tensor $\mathbf{j}$ through (A.3) is treated as a spatial transformation $\mathbf{r} = \mathbf{r}(\tilde{x}, \tilde{y}, \tilde{z})$, see figure A.1, with $\mathbf{j}$ being its Jacobian matrix, $\mathbf{j} = (\ \mathbf{r}^{(\tilde{x})} \quad \mathbf{r}^{(\tilde{y})} \quad \mathbf{r}^{(\tilde{z})}\ )$.

For the divergence relationships in Maxwell's equations, the derivation begins with a scalar product of vector $\mathbf{v}$ and pseudo-vector $\mathbf{p}$, which gives a scalar q (i.e. $\mathbf{v} \cdot \mathbf{p} = q$). Then, using (A.6) the scalar products yields $\tilde{\mathbf{v}} \cdot \tilde{\mathbf{p}} = (\mathbf{j}^{\mathrm{T}}\mathbf{v}) \cdot (|\mathbf{j}|\mathbf{j}^{-1}\,\mathbf{p}) = |\mathbf{j}|\,\mathbf{v} \cdot \mathbf{p}$, and an equivalent divergence equation is obtained through substitution of $\mathbf{v}$ and $\tilde{\mathbf{v}}$ with ∇ and $\tilde{\nabla}$, i.e.

$$\tilde{\nabla} \cdot \tilde{\mathbf{p}} = |\mathbf{j}|\ \nabla \cdot \mathbf{p}. \tag{A.10}$$

A.3. Invariance identities for Maxwell's equations

Equations (A.8) cast the Maxwell curl equations $\nabla \times \mathbf{E} = -\mu\mathbf{H}^{(t)}$ and $\nabla \times \mathbf{H} = \varepsilon\mathbf{E}^{(t)}$ into a new set of similar equations, $\tilde{\nabla} \times \tilde{\mathbf{E}} = -\tilde{\mu}\tilde{\mathbf{H}}^{(t)}$ and $\tilde{\nabla} \times \tilde{\mathbf{H}} = \tilde{\varepsilon}\tilde{\mathbf{E}}^{(t)}$, where

$$\mathbf{H} = \left(\mathbf{j}^{\mathrm{T}}\right)^{-1}\tilde{\mathbf{H}}, \quad \mathbf{E} = \left(\mathbf{j}^{\mathrm{T}}\right)^{-1}\tilde{\mathbf{E}} \tag{A.11}$$

and

$$\varepsilon = |\mathbf{j}|^{-1}\mathbf{j}\tilde{\varepsilon}\mathbf{j}^{\mathrm{T}} \quad \mu = |\mathbf{j}|^{-1}\mathbf{j}\tilde{\mu}\mathbf{j}^{\mathrm{T}}. \tag{A.12}$$

Note that $(\mathbf{j}^{\mathrm{T}})^{-1}$in (A.11) is a matrix arranged of the columns of reciprocal vectors $(\mathbf{j}^{\mathrm{T}})^{-1} = (\ \mathbf{r}^{(\tilde{y})} \times \mathbf{r}^{(\tilde{z})} \quad \mathbf{r}^{(\tilde{z})} \times \mathbf{r}^{(\tilde{x})} \quad \mathbf{r}^{(\tilde{x})} \times \mathbf{r}^{(\tilde{y})}\ )|\mathbf{j}|^{-1}$.

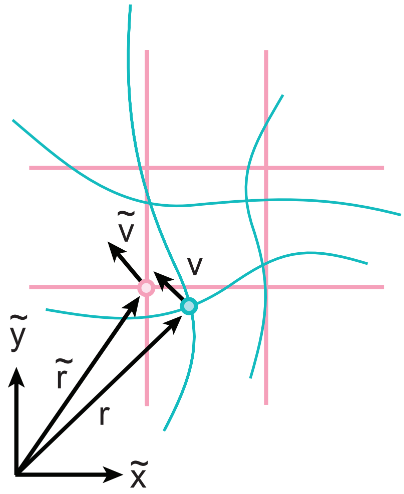

Figure A.1. Transformation of a vector field.

Thus, provided that the electromagnetic properties of the new material space follow (A.12), then the Poynting vector in the new space, $\mathbf{S} = \frac{1}{2}(\mathbf{E} \times \mathbf{H}^*)$, will obey (A.6), satisfying the following transformation of the initial Poynting vector $\tilde{\mathbf{S}} = \frac{1}{2}(\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*)$

$$\mathbf{S} = |\mathbf{j}|^{-1}\mathbf{j}\,\tilde{\mathbf{S}}. \tag{A.13}$$

The above is also valid for other pseudo-vectors, e.g. the time derivatives of magnetic flux densities $\mathbf{B}^{(t)}$ and $\tilde{\mathbf{B}}^{(t)}$, and displacement currents, $\mathbf{D}^{(t)}$ and $\tilde{\mathbf{D}}^{(t)}$ [61].

In a similar way, the divergence equations $\tilde{\nabla} \cdot \tilde{\mathbf{D}} = \tilde{q}$ and $\nabla \cdot \mathbf{D} = q$ link the charge densities through (A.10) as

$$q = |\mathbf{j}|^{-1}\tilde{q}. \tag{A.14}$$

The above conversions solve a fundamental problem of designing a continuous material space for a required spatial transformation of electromagnetic vectors, and therefore, achieving a desired functionality. In other words, for the physical Poynting vector, $\mathbf{S}$, to match the required transformation of the Poynting vector, $\mathbf{S} = |\mathbf{j}|^{-1}\mathbf{j}\,\tilde{\mathbf{S}}$, the material properties in the new space, $\mathbf{r} = \mathbf{r}(\tilde{\mathbf{r}})$, should satisfy $\varepsilon = |\mathbf{j}|^{-1}\mathbf{j}\tilde{\varepsilon}\mathbf{j}^{\mathrm{T}}$ and $\mu = |\mathbf{j}|^{-1}\mathbf{j}\tilde{\mu}\mathbf{j}^{\mathrm{T}}$.

References

[1] Pendry J B, Schurig D and Smith D R 2006 Controlling electromagnetic fields *Science* **312** 1780–2

[2] Leonhardt U 2006 Optical conformal mapping *Science* **312** 1777–80

[3] Schurig D, Mock J J, Justice B J, Cummer S A, Pendry J B, Starr A F and Smith D R 2006 Metamaterial electromagnetic cloak at microwave frequencies *Science* **314** 977–80

[4] Cummer S A, Popa B I, Schurig D, Smith D R and Pendry J 2006 Full-wave simulations of electromagnetic cloaking structures *Phys. Rev.* E **74** 036621

[5] Leonhardt U 2006 Notes on conformal invisibility devices *New J. Phys.* **8** 118

[6] Miller D A B 2006 On perfect cloaking *Opt. Express* **14** 12457–66

[7] Milton G W and Nicorovici N A P 2006 On the cloaking effects associated with anomalous localized resonance *Proc. R. Soc.* A **462** 3027–59

[8] Alu A and Engheta N 2007 Cloaking and transparency for collections of particles with metamaterial and plasmonic covers *Opt. Express* **15** 7578–90

[9] Bruno O P and Lintner S 2007 Superlens-cloaking of small dielectric bodies in the quasistatic regime *J. Appl. Phys.* **102** 124502

[10] Cai L W and Sanchez-Dehesa J 2007 Analysis of Cummer–Schurig acoustic cloaking *New J. Phys.* **9** 450

[11] Cai W, Chettiar U K, Kildishev A V and Shalaev V M 2007 Optical cloaking with metamaterials *Nat. Photonics* **1** 224–7

[12] Chen H S, Wu B I, Zhang B and Kong J A 2007 Electromagnetic wave interactions with a metamaterial cloak *Phys. Rev. Lett.* **99** 063903

[13] Chen H Y and Chan C T 2007 Acoustic cloaking in three dimensions using acoustic metamaterials *Appl. Phys. Lett.* **91** 183518

[14] Greenleaf A, Kurylev Y, Lassas M and Uhlmann G 2007 Improvement of cylindrical cloaking with the SHS lining *Opt. Express* **15** 12717–34

[15] Guenneau S, Ramakrishna S A, Enoch S, Chakrabarti S, Tayeb G and Gralak B 2007 Cloaking and imaging effects in plasmonic checkerboards of negative epsilon and mu and dielectric photonic crystal checkerboards *Photonics Nanostruct. Fundam. Appl.* **5** 63–72

[16] Nicorovici N A P, Milton G W, McPhedran R C and Botten L C 2007 Quasistatic cloaking of two-dimensional polarizable discrete systems by anomalous resonance *Opt. Express* **15** 6314–23

[17] Silveirinha M G, Alu A and Engheta N 2007 Parallel-plate metamaterials for cloaking structures *Phys. Rev.* E **75** 036603

[18] Teixeira F L 2007 Differential form approach to the analysis of electromagnetic cloaking and masking *Microw. Opt. Technol. Lett.* **49** 2051–3

[19] Alitalo P, Luukkonen O, Jylha L, Venermo J and Tretyakov S A 2008 Transmission-line networks cloaking objects from electromagnetic fields *IEEE Trans. Antennas Propag.* **56** 416–24

[20] Blanchard C, Porti J, Wu B I, Morente J A, Salinas A and Kong J A 2008 Time domain simulation of electromagnetic cloaking structures with TLM method *Opt. Express* **16** 6461–70

[21] Cai W S, Chettiar U K, Kildishev A V and Shalaev V M 2008 Designs for optical cloaking with high-order transformations *Opt. Express* **16** 5444–52

[22] Gaillot D P, Croenne C and Lippens D 2008 An all-dielectric route for terahertz cloaking *Opt. Express* **16** 3986–92

[23] Jacob Z and Narimanov E E 2008 Semiclassical description of non magnetic cloaking *Opt. Express* **16** 4597–604

[24] Kohn R V, Shen H, Vogelius M S and Weinstein M I 2008 Cloaking via change of variables in electric impedance tomography *Inverse Probl.* **24** 015016

[25] Kwon D H and Werner D H 2008 Restoration of antenna parameters in scattering environments using electromagnetic cloaking *Appl. Phys. Lett.* **92** 113507

[26] Liang Z X, Yao P J, Sun X W and Jiang X Y 2008 The physical picture and the essential elements of the dynamical process for dispersive cloaking structures *Appl. Phys. Lett.* **92** 131118

[27] Vanbesien O, Fabre N, Melique X and Lippens D 2008 Photonic-crystal-based cloaking device at optical wavelengths *Appl. Opt.* **47** 1358–62

[28] Weder R 2008 A rigorous analysis of high-order electromagnetic invisibility cloaks *J. Phys. A: Math. Theor.* **41** 065207

[29] Xiao D and Johnson H T 2008 Approximate optical cloaking in an axisymmetric silicon photonic crystal structure *Opt. Lett.* **33** 860–2

[30] Zhao Y, Argyropoulos C and Hao Y 2008 Full-wave finite-difference time-domain simulation of electromagnetic cloaking structures *Opt. Express* **16** 6717–30

[31] Zharova N A, Shadrivov I V and Kivshar Y S 2008 Inside-out electromagnetic cloaking *Opt. Express* **16** 4615–20

[32] Milton G W, Briane M and Willis J R 2006 On cloaking for elasticity and physical equations with a transformation invariant form *New J. Phys.* **8** 248

[33] Zhang S, Genov D A, Sun C and Zhang X 2008 Cloaking of matter waves *Phys. Rev. Lett.* **100** 123002

[34] Cheng Y, Yang F, Xu J Y and Liu X J 2008 A multilayer structured acoustic cloak with homogeneous isotropic materials *Appl. Phys. Lett.* **92** 151913

[35] Cummer S A, Popa B I, Schurig D, Smith D R, Pendry J, Rahm M and Starr A 2008 Scattering theory derivation of a 3D acoustic cloaking shell *Phys. Rev. Lett.* **100** 024301

[36] Cummer S A and Schurig D 2007 One path to acoustic cloaking *New J. Phys.* **9** 45

[37] Cai W, Chettiar U K, Kildishev A V, Shalaev V M and Milton G W 2007 Nonmagnetic cloak with minimized scattering *Appl. Phys. Lett.* **91** 111105

[38] Kwon D H and Werner D H 2008 Two-dimensional eccentric elliptic electromagnetic cloaks *Appl. Phys. Lett.* **92** 013505

[39] Jenkins A 2008 Optical cloaking —a many-layered solution *Nat. Photonics* **2** 270

[40] Jiang W X, Cui T J, Yu G X, Lin X Q, Cheng Q and Chin J Y 2008 Arbitrarily elliptical–cylindrical invisible cloaking *J. Phys. D: Appl. Phys.* **41** 085504

[41] Kwon D H and Werner D H 2008 Two-dimensional electromagnetic cloak having a uniform thickness for elliptic cylindrical regions *Appl. Phys. Lett.* **92** 113502

[42] Rahm M, Cummer S A, Schurig D, Pendry J B and Smith D R 2008 Optical design of reflectionless complex media by finite embedded coordinate transformations *Phys. Rev. Lett.* **100** 063903

[43] Rahm M, Schurig D, Roberts D A, Cummer S A, Smith D R and Pendry J B 2008 Design of electromagnetic cloaks and concentrators using form-invariant coordinate transformations of Maxwell's equations *Photonics Nanostruct. Fundam. Appl.* **6** 87–95

[44] Sipos M and Thompson B G 2008 Electrodynamics on a grid: the finite-difference time-domain method applied to optics and cloaking *Am. J. Phys.* **76** 464–9

[45] Wood J 2008 New metamaterial may lead to a magnetic cloak *Mater. Today* **11** 8

[46] Yao P J, Liang Z X and Jiang X Y 2008 Limitation of the electromagnetic cloak with dispersive material *Appl. Phys. Lett.* **92** 031111

[47] You Y, Kattawar G W, Zhai P W and Yang P 2008 Zero-backscatter cloak for aspherical particles using a generalized DDA formalism *Opt. Express* **16** 2068–79

[48] Zhang B L, Chen H S, Wu B I and Kong J A 2008 Extraordinary surface voltage effect in the invisibility cloak with an active device inside *Phys. Rev. Lett.* **100** 063904

[49] Zhang J J, Huangfu J T, Luo Y, Chen H S, Kong J A and Wu B I 2008 Cloak for multilayered and gradually changing media *Phys. Rev.* B **77** 035116

[50] Zhang P, Jin Y and He S L 2008 Cloaking an object on a dielectric half-space *Opt. Express* **16** 3161–6

[51] Kerker M 1975 Invisible bodies *J. Opt. Soc. Am.* **65** 376–9

[52] Alu A and Engheta N 2007 Plasmonic materials in transparency and cloaking problems: mechanism, robustness, and physical insights *Opt. Express* **15** 3318–32

[53] Alu A and Engheta N 2008 Robustness in design and background variations in metamaterial/plasmonic cloaking *Radio Sci.* **43** RS4S01

[54] Alu A and Engheta N 2008 Multifrequency optical invisibility cloak with layered plasmonic shells *Phys. Rev. Lett.* **100** 113901

[55] Kwon D H, Wang X, Bayraktar Z, Weiner B and Werner D H 2008 Near-infrared metamaterial films with reconfigurable transmissive/reflective properties *Opt. Lett.* **33** 545–7

[56] Tamm I E 1924 Electrodynamics of an anisotropic medium in the special theory of relativity *J. Russ. Phys. Chem. Soc.* **56** 248

[57] Tamm I E 1925 Crystal-optics of the theory of relativity pertinent to the geometry of a biquadratic form *J. Russ. Phys. Chem. Soc.* **57** 1

[58] Mandelstamm L I and Tamm I E 1925 Elektrodynamik des anisotropen medien und der speziallen relativitatstheorie *Math. Ann.* **95** 154

[59] Dolin L S 1961 On a possibility of comparing three-dimensional electromagnetic systems with inhomogeneous filling *Izv. Vyssh. Uchebn. Zaved. Radiofiz.* **4** 964–7

[60] Post E G 1962 *Formal Structure of Electromagnetics; General Covariance and Electromagnetics* (New York: Interscience)

[61] Lax M and Nelson D F 1976 Maxwell equations in material form *Phys. Rev.* B **13** 1777

[62] Chew W C and Weedon W H 1994 A 3d perfectly matched medium from modified Maxwell's equations with stretched coordinates *Microw. Opt. Technol. Lett.* **7** 599–604

[63] Ward A J and Pendry J B 1996 Refraction and geometry in Maxwell's equations *J. Mod. Opt.* **43** 773–93

[64] Ward A J and Pendry J B 1998 Calculating photonic Green's functions using a nonorthogonal finite-difference time-domain method *Phys. Rev.* B **58** 7252

[65] Leonhardt U and Philbin T G 2006 General relativity in electrical engineering *New J. Phys.* **8** 247

[66] Kildishev A V and Narimanov E E 2007 Impedance-matched hyperlens *Opt. Lett.* **32** 3432–4

[67] Kildishev A V and Shalaev V M 2008 Engineering space for light via transformation optics *Opt. Lett.* **33** 43–5

[68] Chen H Y, Liang Z X, Yao P J, Jiang X Y, Ma H R and Chan C T 2007 Extending the bandwidth of electromagnetic cloaks *Phys. Rev.* B **76** 241104

[69] Harris S E 1997 Electromagnetically induced transparency *Phys. Today* **50** 36–42

[70] Kastel J, Fleischhauer M, Yelin S F and Walsworth R L 2007 Tunable negative refraction without absorption via electromagnetically induced chirality *Phys. Rev. Lett.* **99** 073602

[71] Ramakrishna S A and Pendry J B 2003 Removal of absorption and increase in resolution in a near-field lens via optical gain *Phys. Rev.* B **67** 201101

[72] Garcia de Abajo F J, Gomez-Santos G, Blanco L A, Borisov A G and Shabanov S V 2005 Tunneling mechanism of light transmission through metallic films *Phys. Rev. Lett.* **95** 067403

[73] Noginov M A, Zhu G, Bahoura M, Adegoke J, Small C E, Ritzo B A, Drachev V P and Shalaev V M 2006 Enhancement of surface plasmons in an Ag aggregate by optical gain in a dielectric medium *Opt. Lett.* **31** 3022–4

[74] Klar T A, Kildishev A V, Drachev V P and Shalaev V M 2006 Negative-index metamaterials: going optical *IEEE J. Sel. Top. Quantum Electron.* **12** 1106–15

[75] Lass H 1950 *Vector and Tensor Analysis* (New York: McGraw-Hill)

[76] Schurig D, Pendry J B and Smith D R 2006 Calculation of material properties and ray tracing in transformation media *Opt. Express* **14** 9794–804