Light Management at Nanoscale

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ABSTRACT

An analytical theory for extraordinary light transmittance through an optically thick metal film with subwavelength holes is developed. It is shown that the film transmittance has sharp peaks that are due to the Maxwell-Garnet resonances in the holes. At resonances electric and magnetic fields are dramatically enhanced in the holes. These resonances are proposed to guide light over a metal film at a nanoscale.

1 Introduction

In this paper we consider light interaction with nanoholes punched in a metal film. We show that the transmittance through a metal film with subwavelength holes (see Fig. 1) has sharp resonances corresponding to the excitation of the localized surface waves that are specific for a metal film with holes and were not discussed in the literature. We propose that nanoholes can guide light over the film at subwavelength scale.

The extraordinary optical transmittance (EOT) was first discovered and then intensively investigated in the works[1] (see also, for example,[2]). A number of various models (with most of them being numerical simulations)

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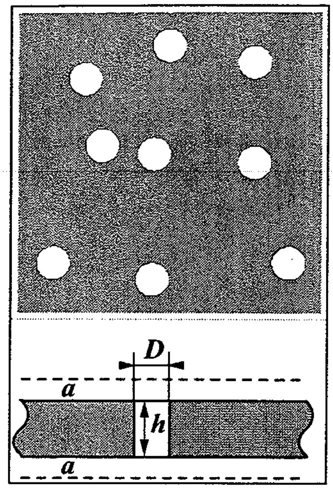

Figure 1: Holes of diameter D in a metal film with thickness h. Electric and magnetic fields are considered in the reference planes placed at distances a from both sides of the film.

were suggested to explain the EOT.[3–8] Despite the very sophisticated simulation codes used, the physical picture of the EOT is not fully understood. In this paper, we use a new analytical approach referred to as the generalized Ohms' law (GOL).[9] This approach allows us to develop a physical model, which provides a simple qualitative picture for the field distributions and the EOT.

2 GOL approximation

A new analytical approach to the calculation of optical properties of metal dielectric films, referred to as the GOL approximation, has recently been proposed.[9] We restrict ourselves, for simplicity, to the case where all the external fields are parallel to the plane of the film (normal incidence). It is supposed that a metal film, with possible holes, voids, or other inhomogeneities, is placed in the xy plane so that z axis is perpendicular to the film, which has thickness h. The external electromagnetic wave is incident onto the $z = -h/2$ interface of the film (front interface) and the transmitted wave is emitted from the $z = h/2$ interface (back interface). A typical spatial scale D of the film defects is supposed to be much smaller than the wavelength λ, i.e., $D \ll \lambda$; for cylindrical holes, D is the cylinder diameter. (In some graphs below we also show results for $D < \lambda$ that should be considered as an extrapolation.)

We consider first the electric and magnetic fields in close vicinity to the film. Namely, the electric and magnetic fields are considered at the distance a in front of the film $\mathbf{E}_1(\mathbf{r}) = \mathbf{E}(\mathbf{r}, -h/2 - a)$, $\mathbf{H}_1(\mathbf{r}) = \mathbf{H}(\mathbf{r}, -h/2 - a)$, and at the distance a behind the film $\mathbf{E}_2(\mathbf{r}) = \mathbf{E}(\mathbf{r}, h/2 + a)$, $\mathbf{H}_2(\mathbf{r}) = \mathbf{H}(\mathbf{r}, h/2 + a)$ (see Fig. 1). All the fields and currents are monochromatic fields, with the usual $\exp(-i\omega t)$ time dependence. The vector $\mathbf{r} = \{x, y\}$ is a two-dimensional vector in the xy plane. In the case of laterally inhomogeneous films, the average electric displacement current $\mathbf{D}(\mathbf{r}) = \int_{-h/2-a}^{h/2+a} \mathbf{D}(\mathbf{r}, z)\, dz = \int_{-h/2-a}^{h/2+a} \varepsilon(\mathbf{r}, z)\, \mathbf{E}(\mathbf{r}, z)\, dz$ and the average magnetic induction $\mathbf{B}(\mathbf{r}) = \int_{-h/2-a}^{h/2+a} \mathbf{B}(\mathbf{r}, z)\, dz = \int_{-h/2-a}^{h/2+a} \mu(\mathbf{r}, z)\mathbf{H}(\mathbf{r}, z)\, dz$ are functions of the vector $\mathbf{r}$. We assume hereafter, for simplicity, that permittivity ε is a scalar and magnetic permeability $\mu = 1$. In the GOL approximation it is supposed that the local electromagnetic field is a superposition of two plane waves propagating in $+z$ and $-z$ directions. This superposition of two waves is, indeed, different in different regions of the film. We neglect scattered and evanescent waves that propagate in the xy plane since their amplitudes proportional to $(\lambda/D)^2$. Thus we use the two-wave approximation when electric $\mathbf{E}(\mathbf{r}, z)$ and magnetic $\mathbf{H}(\mathbf{r}, z)$ fields have their components in the $\{x, y\}$ plane only. Then, the Maxwell equations $\operatorname{curl} \mathbf{E}(\mathbf{r}, z) = ik\mathbf{B}(\mathbf{r}, z)$ and $\operatorname{curl} \mathbf{H}(\mathbf{r}, z) = -ik\mathbf{D}(\mathbf{r}, z)$, when integrated from $z = -h/2 - a$ to $z = h/2 + a$ (see Fig. 1), take the following form :

$$[\mathbf{n} \times (\mathbf{E}_2(\mathbf{r}) - \mathbf{E}_1(\mathbf{r}))] = ik\mathbf{B}(\mathbf{r}), \qquad [\mathbf{n} \times (\mathbf{H}_2(\mathbf{r}) - \mathbf{H}_1(\mathbf{r}))] = -ik\mathbf{D}(\mathbf{r}), \tag{1}$$

where k is the wavevector, $\mathbf{n} = \{0, 0, 1\}$ is the unite vector normal to the plane of the film, and

$$\mathbf{E}_1(\mathbf{r}) \equiv \mathbf{E}(\mathbf{r}, -h/2 - a), \quad \mathbf{H}_1(\mathbf{r}) \equiv \mathbf{H}(\mathbf{r}, -h/2 - a), \quad \mathbf{E}_2(\mathbf{r}) \equiv \mathbf{E}(\mathbf{r}, h/2 + a), \quad \mathbf{H}_2(\mathbf{r}) \equiv \mathbf{H}(\mathbf{r}, h/2 + a)$$

are two-dimensional vectors defined in the $\{x, y\}$ plane. The vectors $\mathbf{E}_1(\mathbf{r})$, $\mathbf{E}_2(\mathbf{r})$, $\mathbf{H}_1(\mathbf{r})$, and $\mathbf{H}_2(\mathbf{r})$ are curl-free since z components of $\operatorname{curl} \mathbf{E}(\mathbf{r}, z)$ and $\operatorname{curl} \mathbf{H}(\mathbf{r}, z)$ are supposed to vanish in two-wave approximation. It is convenient to introduce the fields $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$ and $\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2$ that are also curl-free: $\operatorname{curl} \mathbf{E}(\mathbf{r}) = 0$, $\operatorname{curl} \mathbf{H}(\mathbf{r}) = 0$. The conservation laws give $\operatorname{div} \mathbf{D}(\mathbf{r}) = 0$ and $\operatorname{div} \mathbf{B}(\mathbf{r}) = 0$. For simplicity, we consider films having the mirror symmetry with respect to reflection in the $z = 0$ plane. For such films, the displacement $\mathbf{D}(\mathbf{r})$ and magnetic induction $\mathbf{B}(\mathbf{r})$ are symmetric functions of the fields $\mathbf{E}_1(\mathbf{r})$, $\mathbf{E}_2(\mathbf{r})$ and $\mathbf{H}_1(\mathbf{r})$, $\mathbf{H}_2(\mathbf{r})$ correspondingly. Therefore, we can write that

$$\mathbf{D}(\mathbf{r}) = u(\mathbf{r})\mathbf{E}(\mathbf{r})/k, \qquad \mathbf{B}(\mathbf{r}) = v(\mathbf{r})\mathbf{H}(\mathbf{r})/k, \tag{2}$$

where $u(\mathbf{r})$ and $v(\mathbf{r})$ are dimensionless "Ohmic" parameter. Equations (2) have the form which is typical for constitutive equations in the electrodynamics, but include parameters u and v that incorporate local geometry of the film.

To find the optical properties of the film, such as transmittance and reflectance, we average Eqs. (1) over the film plane $\{x, y\}$ and introduce the effective film parameters, u_e and v_e, through usual relations $u_e \langle \mathbf{E} \rangle = \langle u\mathbf{E} \rangle$ and $v_e \langle \mathbf{H} \rangle = \langle v\mathbf{H} \rangle$; thus we obtain the "integral" Maxwell equations for the film in the following form:

$$[\mathbf{n}\times(\langle \mathbf{E}_2 \rangle - \langle \mathbf{E}_1 \rangle)] = iv_e \langle \mathbf{H} \rangle\,, \qquad [\mathbf{n}\times(\langle \mathbf{H}_2 \rangle - \langle \mathbf{H}_1 \rangle)] = -iu_e \langle \mathbf{E} \rangle\,, \tag{3}$$

which relate the average fields from both sides of the film. We suppose that the wave enters the film from $z < 0$, so that its amplitude is proportional to e^{ikz}. The incident wave is partially reflected and partially transmitted through the film. The electric field amplitude in the $z < 0$ half-space, away from the film, can be written as $\tilde{E}_1(z) = e^{ikz} + re^{-ikz}$, where r is the reflection amplitude. Well behind the film, the electric component of the electromagnetic wave acquires the form $\tilde{E}_2(z) = te^{ikz}$, where t is the transmission amplitude. In the planes $z = -h/2 - a$ and $z = h/2 + a$, the average electric field equals to $\langle E_1 \rangle$ and $\langle E_2 \rangle$, respectively (see Fig. 1). The electric field in the wave is matched with the average fields in the planes $z = -h/2 - a$ and $z = h/2 + a$, i.e., $\langle E_1 \rangle = \tilde{E}_1(-h/2 - a) = e^{-ik(h/2+a)} + re^{ik(h/2+a)}$ and $\langle E_2 \rangle = \tilde{E}_2(h/2 + a) = te^{ik(h/2+a)}$. The same matching for the magnetic fields gives $\langle H_1 \rangle = e^{-ik(h/2+a)} - re^{ik(h/2+a)}$ and $\langle H_2 \rangle = te^{ik(h/2+a)}$, in the planes $z = -h/2 - a$ and $z = h/2 + a$, respectively. The substitution of these expressions for the fields $\langle E_1 \rangle$, $\langle E_2 \rangle$, $\langle H_1 \rangle$, and $\langle H_2 \rangle$ in Eqs. (3) gives two linear equations for t and r. By solving these equations, we obtain the reflectance and transmittance in the following form

$$R \equiv |r|^2 = \left| \frac{(u_e - v_e)}{(i + u_e)(i + v_e)} \right|^2, \quad T \equiv |t|^2 = \left| \frac{1 + u_e v_e}{(i + u_e)(i + v_e)} \right|^2. \tag{4}$$

Thus, the effective Ohmic parameters u_e and v_e completely determine the optical properties of inhomogeneous films.

3 Transmittance of nanoholes

Now we apply the developed GOL formalism to find the transmittance of a metal film with subwavelength holes. We find the local electric $\mathbf{E}(\mathbf{r})$ and magnetic $\mathbf{H}(\mathbf{r})$ fields and then the effective parameters u_e and v_e for a film with holes from Eqs. (2) and the definition $u_e \langle \mathbf{E} \rangle = \langle u\mathbf{E} \rangle$, $v_e \langle \mathbf{H} \rangle = \langle v\mathbf{H} \rangle$. Since electric $\mathbf{E}(\mathbf{r})$ and magnetic $\mathbf{H}(\mathbf{r})$ fields are curl-free (note also that $\operatorname{div} \mathbf{D}(\mathbf{r}) = 0$ and $\operatorname{div} \mathbf{B}(\mathbf{r}) = 0$) the quasistaic approximation can be used.[4,9] Therefore, a number of efficient analytical and numerical methods, which were developed in quasistaic percolation theory are in our disposal for calculation of the local fields and effective parameters.[9,10] Here we use the simplest approximation, namely, the Maxwell-Garnett (MG) approach that holds when the surface hole concentration is small, $p \ll 1$. In the MG approach the dipole approximation can be used that leads to the following expression for the electric field E_h in a hole

$$E_h = \frac{2E_m u_m}{u_m + u_h}, \tag{5}$$

where u_m and u_h are the Ohmic parameters for the metal and holes, and the quantities E_m and E_h are the electric fields averaged over the metal and holes, respectively. From Eq. (5) we obtain the following expression for the "electric" effective parameter u_e:

$$u_e \equiv \frac{\langle uE \rangle}{\langle E \rangle} = \frac{(1-p)\, u_m E_m + p u_h E_h}{(1-p)\, E_m + p E_h}. \tag{6}$$

Repeating the same procedure we find the "magnetic" effective parameter v_e, which is given by Eq. (6), with the following change $u_m \to v_m$ and $u_h \to v_h$.

Now we substitute the parameters u_e and v_e in Eq. (4) and obtain the following expression for the transmittance

$$\begin{aligned} T &= \frac{16p^2 \left| u_m{}^2 \left(1 + u_h v_h\right) \right|^2}{\left| \Sigma_1 \Sigma_2 \right|^2}, \qquad (7) \\ \Sigma_1 &= u_h - p\, u_h + (1+p)\,(1 - i\, u_h)\, u_m - i\,(1-p)\, u_m^2, \\ \Sigma_2 &= (i + u_m)\,(u_m v_h - 1) + p\,(i - u_m)\,(u_m v_h + 1), \end{aligned}$$

where we used the relation $u_m = -1/v_m$ that holds when the film thickness h is much larger than the metal skin depth δ , i.e., when $h \gg \delta$; we also take into account that surface concentration of the holes $p \ll 1$. Hereafter we consider this case of a strong effect, which corresponds to most of experiments with subwavelength holes reported so far.

The electric field E_h in a hole raises formally up to infinity at $u_m \to -u_h$ if there are no losses (see Eq. 5). By substituting the $u_m = -u_h$ in Eq. (7), we obtain the following expression for the resonant transmittance $T = 4\,|u_m| / \left|1 + u_m^2\right|$, which does not depend on the hole concentration p and, therefore, remains finite, even for $p \to 0$. When the magnetic resonance takes place, i.e., $v_m = -1/u_m = -v_h$, the resonant transmittance also remains finite at $p \to 0$. Thus we conclude that the electric and magnetic MG resonances in the holes can result in the extraordinary optical transmittance.

To calculate the transmittance, we find the Ohmic parameters u_m, u_h, v_m, and v_h. Parameters u_m and v_m we can obtain directly from solutions to the Maxwell equations in the GOL approximation:

$$u_m = -\cot(ak), \quad v_m = \tan(ak), \tag{8}$$

so that $u_m = -1/v_m$. (see[9]). To obtain the hole parameters u_h and v_h we have to know the em field distribution inside a hole. The inside field is a superposition of different eigenmodes for this subcritical waveguide. At the hole entrance the internal field is similar to the plane wave, though its amplitude can be different significantly from the amplitude of an incident wave. When we move deeper inside the hole, only the mode with the smallest eigenvalue survives. To simplify further qualitative considerations, we assume that the internal field is a plane wave near the entrance of the hole and it matches with the basic internal mode at the distance a from both ends of the hole. We use for this matching the same distance a as we used before to match local fields with the incident plane wave. As a result of such matching, we obtain

$$u_h = v_h = \tan\left[(a + h/2)\,k\right], \quad for\ h < 2a; \tag{9}$$

$$\begin{aligned} u_h &= \frac{k\tan(2\,a\,k) - \sqrt{\kappa^2 - k^2}\,\tanh\left[(h/2 - a)\sqrt{\kappa^2 - k^2}\right]}{k + \sqrt{\kappa^2 - k^2}\,\tan(2\,a\,k)\,\tanh\left[(h/2 - a)\sqrt{\kappa^2 - k^2}\right]}, \\ v_h &= \frac{\sqrt{\kappa^2 - k^2}\,\tan(2\,a\,k) + k\tanh\left[(h/2 - a)\sqrt{\kappa^2 - k^2}\right]}{\sqrt{\kappa^2 - k^2} - k\tan(2\,a\,k)\,\tanh\left[(h/2 - a)\sqrt{\kappa^2 - k^2}\right]}, \quad for\ h > 2a; \end{aligned} \tag{10}$$

where $\kappa = 3.68/D$ is the eigenvalue for the basic mode in a cylindrical waveguide,[11] Ch. 91, and D is the diameter of the hole.

Consider first shallow holes with depth $h < 2a$. By substituting Eqs. (8) and (9) in Eq. (7) and considering the limit $p \ll 1$, we obtain the following simple expression for the transmittance

$$T(k) = \sum_j \frac{A_j(k)}{A_j(k) + (4a+h)^2 (k-k_j)^2}, \quad A_j(k) = 4p^2 \sin^4\left(\frac{2aj\pi}{4a+h}\right), \quad k_j = \frac{j\pi}{4a+h} - \frac{p}{4a+h}\sin\left(\frac{4aj\pi}{4a+h}\right), \quad (11)$$

which is sum of the resonances located at $k = k_j$ as it is shown in Fig. 2.

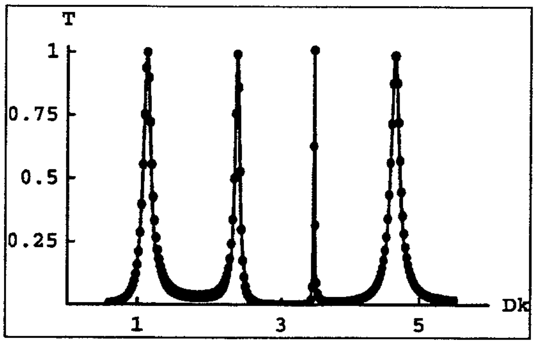

Figure: 2 Transmittance through "shallow" holes ($h < 2a$); $a/D = 0.6$, $h/D = 0.8$, $p = 0.1$. The solid line is the resonance approximation (Eq. 11); the points represent calculations with Eq. (7).

Transmittance T is almost periodical function of k for $p \ll 1$, though the peak width depends on k. Some maxima can disappear when numerators $A_j(k_j)$ in Eq. (11) vanish. The odd resonances in Eq. (11) correspond to the maxima in the electric field in the holes, whereas the even resonances are due to the maxima in the magnetic field in the holes, as shown in Fig. 3.

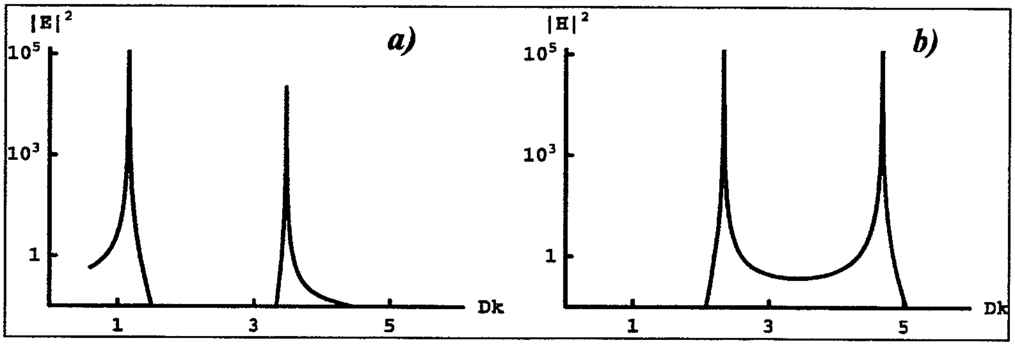

Figure 3: Electric (a) and magnetic (b) field in a hole for the system with the same parameters as in Fig. 2. The incident field amplitude is set to be equal to one.

The spatial distribution of the fields near the resonance is presented in Fig. 4

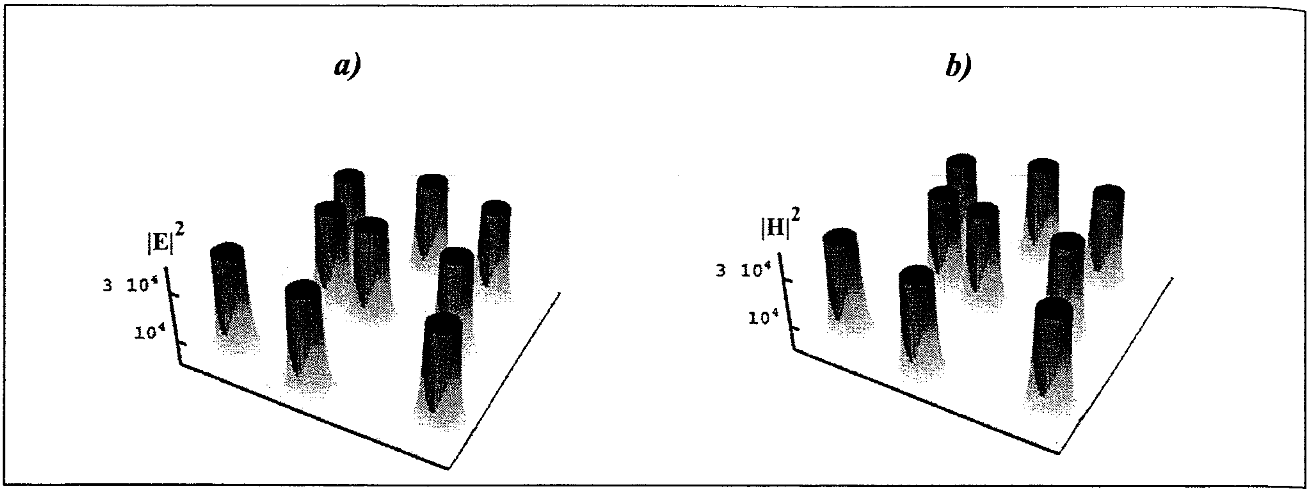

Figure: 4 Spatial distribution of electric (a) and magnetic (b) fields near the MG resonance for the system with the same parameters as in Fig.2; (a) $kD = 0.992$, (b) $kD = 1.96$.

which shows that fields are well localized in the holes. We name these resonances Localized Surface Plasmon-Polaritons (LSPP).

For deep holes that depth $h > 2a$ we obtain transmittance $T(k)$, shown in Fig. 5, by substituting Ohmic parameters from Eqs. (8) and (10) in Eq. (7).

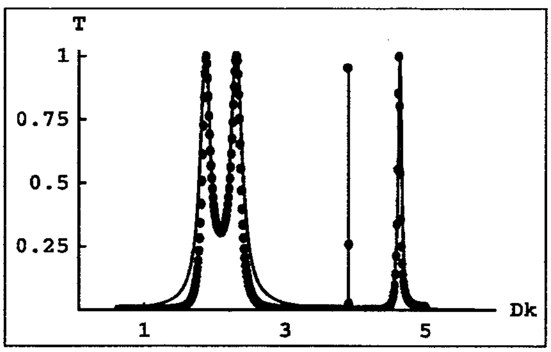

Figure: 5 Transmittance through "deep" holes ($h > 2a$); $a/D = 0.6$, $h/D = 1.45$, $p = 0.1$.

We can see that the k behavior of the transmittance can be rather peculiar when the thickness of the film increases: the peaks corresponding to maxima electric and magnetic fields can move and merge together.

For the considered lossless system, the electric and magnetic fields tend to infinity in the resonance. In any real metal film, the resonant fields acquire some finite values limited by losses. We take losses into account using impedance boundary conditions[11] Ch. 87. Transmittance thus obtained for the square array of nanoholes in a silver film is in qualitative agreement with the well-known experiments[1] as illustrated in Figs. 6. Most maxima in the transmittance are due to excitation of LSSP in the holes.

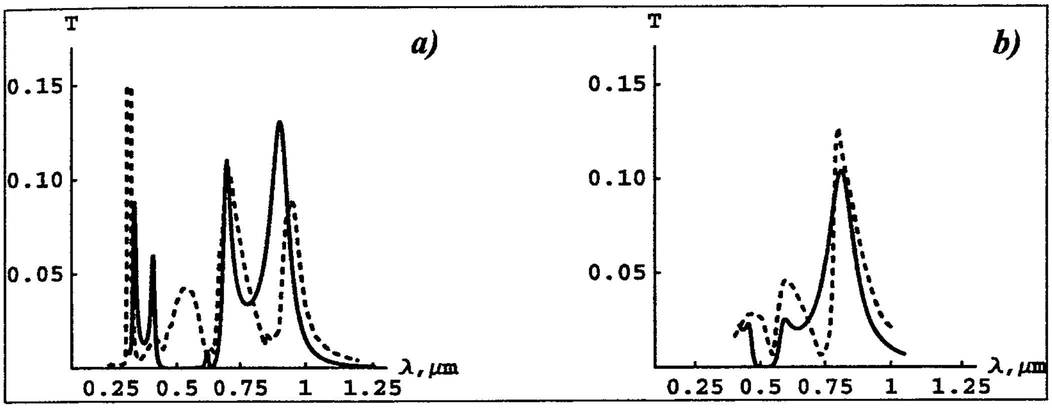

Figure: 6 Extraordinary optical transmittance through a regular array of holes in two different silver films, a free-standing silver film (a) and a silver film on a quartz substrate (b). The dashed lines represent experimental data[1] (a) Apll. Phys. Lett **77** and (b) PRB **58**. The solid lines show results of the theory. The parameters used are as follows: a) $a = 0.17\ \mu m$, $D = 0.28\ \mu m$, $h = 0.32\ \mu m$, $b = 0.75\ \mu m$, and b): $a = 0.09\ \mu m$, $D = 0.15\ \mu m$, $h = 0.2\ \mu m$, $b = 0.6\ \mu m$, $p = 0.049$.

In calculating transmittance presented in Fig. 6 we also take into account the usual, propagated surface plasmon-polaritons that are excited when distance between holes coincides with their wavelength. This results in the maximum at $\lambda \simeq 0.6\,\mu m$ in Fig. 5a and small change in amplitude of the peak at $\lambda \simeq 0.8\,\mu m$ in Fig. 5b. Note, that our theory does predict that the long wavelength peaks in the transmittance are not sensitive to the periodicity, which corresponds to the experiments[1] show that the extraordinary transmittance can occur even for a set of seven holes only. Finally, in recent near-field experiments[2] strong enhancement of the local field has been observed for a single hole and a pair of holes; thus the local field enhancement, which is needed for the extraordinary transmittance, does not require, in general, the periodicity.

4 Light circuiting in nanoholes

Above, we considered the case when a metal film is irradiated homogeneously by a plane electromagnetic wave.

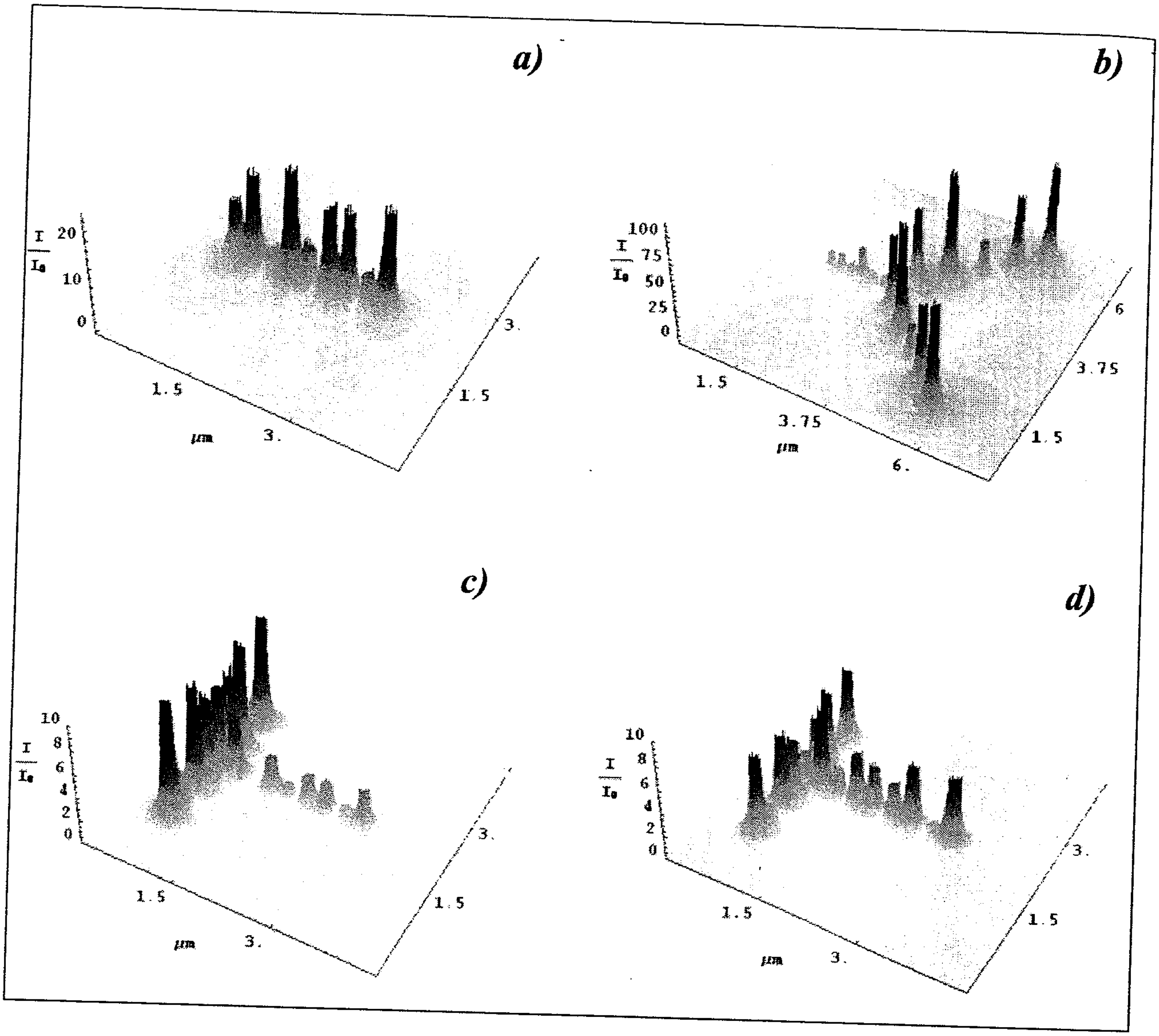

Figure: 7 Nano-circuiting with metal-holes systems; *a*) line waveguide, first left hole is excited, *b*) fork waveguide, first left hole is excited, *c*), *d*) – switch (center hole in top "T" is excited). Holes parameters: diameter: $0.15\,\mu m$, film thickness: $0.2\,\mu m$, distance between the holes centers: $0.3\,\mu m$. Wavelength of excitation light $0.78\ \mu m$.

It is interesting to consider another possibility when only one of the holes is illuminated by light source. This can be accomplished, for example, using a nanometer-size probe of near-field scanning optical microscope. At the resonance, electric (or/and magnetic) fields spread out from the illuminated hole toward other holes because of interactions between the holes via plasmons. Such holes can be arranged into any desired structures that can localize light and guide the propagation of the electromagnetic energy along the structures as it is shown in Fig. 7. Thus in Figs. 7*c* and *d* we show how light circuiting depends on the polarization of the source. When electric field, which excites the central hole, changes its polarization from parallel to stem of "*T*" (Fig. 7*c*) to perpendicular (Fig. 7*d*) the light changes direction of its propagation at nanoscale. The discussed nanoengineered structures can be used as integrated elements in various optoelectronic and photonic devices, including most sophisticated ones, such as optical computers

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