Gas drift induced by nonmonochromatic light

A. A. Popov, A. M. Shalagin, V. M. Shalaev, and V. Z. Yakhnin

L. V. Kirenskiĭ Institute of Physics, Siberian Division, USSR Academy of Sciences; Institute of Automation and Electrometry, Siberian Division, USSR Academy of Sciences, and Krasnoyarsk State University

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Light-induced drift of gases in fields of nonmonochromatic radiation with different spectral characteristics is investigated. The dependence of the drift velocity on the form of the spectrum, on the width, intensity, and detuning of the radiation frequency from atomic resonance are analyzed. It is ascertained that at a fixed integrated intensity of the radiation, the drift velocity of the gas is larger in those cases when the spectral width of the radiation agrees in order of magnitude with the Doppler width of the atomic transition, while the wings of the spectrum fall off more rapidly. It is shown that, at equal intensity, nonmonochromatic radiation makes it possible to obtain larger drift velocities than monochromatic radiation. The feasibility in principle of obtaining supersonic light-induced gas streams is investigated.

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INTRODUCTION

It was previously shown[1-4] theoretically and experimentally that a traveling light wave is capable, under certain conditions, of producing a macroscopic flux **J** of absorbing partides located in a buffer-gas medium (optically-induced drift phenomenon). The flux velocity can approach the average thermal velocity of the particles. A theory of light-induced drift (LID) in the field of monochromatic radiation (MR) was developed in Ref. 3. The gist of the LID phenomenon is the following.[1-3] Let the frequency ω of the monochromatic radiation differ little from the frequency ω_0 of the transition between the ground state n and an excited state m of the atom. When the radiation in question interacts with a gas consisting of the described atoms, the velocity distributions $\rho_m(\mathbf{v})$ and $\rho_n(\mathbf{v})$ of the excited and unexcited atoms become asymmetrical. The asymmetry is due to the fact that the radiation excites predominantly those atoms whose velocity is such that the corresponding Doppler shift $\mathbf{k}\cdot\mathbf{v}$ of the radiation frequency ($\mathbf{k}$ is the wave vector) cancels out the frequency detuning $\Omega = \omega - \omega_0$. In this case the asymmetry of the velocity distribution means that the corresponding average velocity differs from zero. Consequently, in each state there exist directed atom motions characterized by the fluxes

$$\mathbf{j}_m = \int \mathbf{v}\rho_m(\mathbf{v})\,d\mathbf{v}, \quad \mathbf{j}_n = \int \mathbf{v}\rho_n(\mathbf{v})\,d\mathbf{v},$$

which are collinear with the wave vector **k**, directed opposite to each other, and equal in magnitude, so that $\mathbf{J} = 0$.

A buffer gas offers resistance to these fluxes. Since the dimensions of the excited and unexcited atoms are generally speaking different, the forces resisting the fluxes of the excited and unexcited atoms are also different. A net force is therefore produced and is exerted by the buffer gas on the absorbing gas as a whole. This leads to a directed macroscopic motion of the latter, characterized by the flux

$$\mathbf{J} = \mathbf{j}_n + \mathbf{j}_m \neq 0.$$

The present paper is devoted to a theoretical investigation of the LID phenomenon in a field of nonmonochromatic radiation with different spectral characteristics. The prospect of using nonmonochromatic radiation to enhance the LID effect was demonstrated by us earlier[5] on the basis of a very simple model of independent radiation modes. In Sec. 2 of the present paper we obtain and integrate expressions that describe the fluxes **J** due to LID in the field of nonmonochromatic radiation, as well as expressions for the velocity distribution functions of the populations of the ground and excited states of the absorbing particles, $\rho_n(\mathbf{v})$ and $\rho_m(\mathbf{v})$, respectively, as function of the velocity distribution $\rho_n(\mathbf{v}) + \rho_m(\mathbf{v})$ of the total number of absorbing particles. Sections 3–5 are devoted to study of the course of the LID of gases in fields with spectra of concrete forms. We analyze the cases of a Lorentz, Gaussian, and rectangular emission spectrum. We investigate the feasibility in principle of the onset of light-induced supersonic streams of an absorbing gas, i.e., of streams with velocity higher than thermal.

2. GENERAL EXPRESSIONS

We consider the interaction of a gas of two-level atom with a nonmonochromatic quasiresonant electromagnetic wave

$$\mathbf{E}(\mathbf{r}, t) = {}^1/_2[\varepsilon(t)e^{-i(\omega t - \mathbf{kr})} + \varepsilon^*(t)e^{i(\omega t - \mathbf{kr})}],$$

where $\varepsilon(t)$ is a random function of the time. In the presence of a large amount of buffer gas, this interaction, using the model of strong collisions that establish a Maxwellian distribution,[6] is described by the following system of equations for the elements of the atomic density matrix:

$$\left[\frac{\partial}{\partial t} - i(\omega_0 + \mathbf{kv}) + \Gamma\right]\rho_{nm}(\mathbf{v}, t) = iV^*[\rho_n(\mathbf{v}, t) - \rho_m(\mathbf{v}, t)]e^{i\omega t},$$

$$\left(\frac{\partial}{\partial t} + \nu_m + \Gamma_m\right)\rho_m(\mathbf{v}, t) = \nu_m W(\mathbf{v})\rho_m(t) - 2\mathrm{Re}[iVe^{-i\omega t}\rho_{nm}(\mathbf{v}, t)], \quad (2.1)$$

$$\left(\frac{\partial}{\partial t} + \nu_n\right)\rho_n(\mathbf{v}, t) = \nu_n W(\mathbf{v})\rho_n(t) + 2\mathrm{Re}[iVe^{-i\omega t}\rho_{nm}(\mathbf{v}, t)] + \Gamma_m\rho_m(\mathbf{v}, t).$$

Here $V = -\varepsilon(t)\mathbf{d}_{mn}/2\hbar$; $\mathbf{d}_{mn}$ is the matrix element of the dipole moment for the transition $m-n$; ω_0 and Γ are the frequency and homogeneous half-width of the given transition; Γ_m is the radiative width of the level m; **v** is the velocity of the atom; ν_m and ν_n are the frequencies at which the absorbing atoms in the excited and the ground states, respectively, collide with the atoms of the buffer gas;

$$W(\mathbf{v})=(\pi^{1/2}\bar{v})^{-3}\exp[-(\mathbf{v}/\bar{v})^2]$$

is the Maxwellian velocity distribution; $\bar{v}$ is the most probable thermal velocity of the absorbing atom; $\rho_j(t)$ $(j=m,n)$ is the population, integrated over the velocities, of the level j. It was assumed that the states m and n are essentially differently perturbed in the collisions, so that in the case of $\rho_{nm}(\mathbf{v},t)$ the collisions lead only to a broadening (Γ) and to a frequency shift. If necessary the latter can be regarded as included in ω_0.

Introducing a new variable $\sigma(\mathbf{v},t)$, such that

$$\rho_{nm}(\mathbf{v},t)=\sigma(\mathbf{v},t)\exp[i(\omega_0+\mathbf{kv})t-\Gamma t],$$

and using the first equation of the system (2.1), we obtain

$$\sigma(\mathbf{v},t)=i\int_{-\infty}^{t}V^*(t')[\rho_n(\mathbf{v},t')-\rho_m(\mathbf{v},t')]\exp(i\Omega't'+\Gamma t')dt',$$

$$\Omega'=\omega-\omega_0-\mathbf{kv}=\Omega-\mathbf{kv}. \qquad (2.2)$$

Using (2.1) and (2.2) and averaging over the random variables (we designate this averaging by angle brackets), we obtain for the diagonal elements of the density matrix the expressions

$$\frac{\partial\langle\rho_m(\mathbf{v},t)\rangle}{\partial t}=-(\Gamma_m+\nu_m)\langle\rho_m(\mathbf{v},t)\rangle+W(\mathbf{v})\nu_m\langle\rho_m(t)\rangle$$
$$+2\mathrm{Re}\int_{-\infty}^{t}\langle V^*(t')V(t)[\rho_n(\mathbf{v},t')-\rho_m(\mathbf{v},t')]\rangle\exp[-i\Omega'(t-t')-\Gamma(t-t')]dt',$$
$$\frac{\partial\langle\rho_n(\mathbf{v},t)\rangle}{\partial t}=-\nu_n\langle\rho_n(\mathbf{v},t)\rangle+W(\mathbf{v})\nu_n\langle\rho_n(t)\rangle+\Gamma_m\langle\rho_m(\mathbf{v},t)\rangle \qquad (2.3)$$
$$-2\mathrm{Re}\int_{-\infty}^{t}\langle V^*(t')V(t)[\rho_n(\mathbf{v},t')-\rho_m(\mathbf{v},t')]\rangle\exp[-i\Omega'(t-t')-\Gamma(t-t')]dt'.$$

Following Burshtein[7] we "uncouple" the field and atomic variables:

$$\langle V^*(t')V(t)[\rho_n(\mathbf{v},t')-\rho_m(\mathbf{v},t')]\rangle$$
$$\to\langle V^*(t')V(t)\rangle\langle[\rho_n(\mathbf{v},t')-\rho_m(\mathbf{v},t')]\rangle.$$

Then the stationary solution of the system (2.3) satisfies the following equations $(\langle\rho_j(\mathbf{v},\infty)\rangle\equiv\rho_j(\mathbf{v}))$:

$$(\Gamma_m+\nu_m)\rho_m(\mathbf{v})=W(\mathbf{v})\nu_m\rho_m+2|G|^2[\rho_n(\mathbf{v})$$
$$-\rho_m(\mathbf{v})]\mathrm{Re}\int_0^{\infty}\Phi(\tau)\exp[-i\Omega'\tau-\Gamma\tau]d\tau,$$
$$\nu_n\rho_n(\mathbf{v})=W(\mathbf{v})\nu_n\rho_n$$
$$-2|G|^2[\rho_n(\mathbf{v})-\rho_m(\mathbf{v})]\mathrm{Re}\int_0^{\infty}\Phi(\tau)\exp[-i\Omega'\tau-\Gamma\tau]d\tau+\Gamma_m\rho_m(\mathbf{v}), \qquad (2.4)$$
$$|G|^2=\langle|\varepsilon(t)d_{mn}|^2/4\hbar^2\rangle;\quad \Phi(\tau)=\langle\varepsilon^*(t)\varepsilon(t+\tau)\rangle/\langle|\varepsilon(t)|^2\rangle,$$

where $\Phi(\tau)$ is a correlation function connected with the emission line shape $g(\omega'-\omega)$ by the relation

$$g(\omega'-\omega)=\frac{1}{\pi}\mathrm{Re}\int_0^{\infty}d\tau\Phi(\tau)\exp[-i(\omega'-\omega)\tau];\quad \int_{-\infty}^{\infty}g(\omega'-\omega)d\omega'=1.$$

The solution of Eqs. (2.4) is of the form

$$\rho_m(\mathbf{v})=\frac{W(\mathbf{v})/\tau_\alpha}{1+(\tau_\beta/\tau_\alpha)\langle Y\rangle_v}[\tau_{2m}\langle Y\rangle_v+\tau_{1m}Y],$$
$$\rho_n(\mathbf{v})=W(\mathbf{v})-\frac{W(\mathbf{v})/\tau_\alpha}{1+(\tau_\beta/\tau_\alpha)\langle Y\rangle_v}[\tau_{2n}\langle Y\rangle_v+\tau_{1n}Y]. \qquad (2.5)$$

Here

$$Y(\mathbf{v})=\frac{\tau_\alpha A}{1+\tau_\alpha A};\quad A(\mathbf{v})=2|G|^2\mathrm{Re}\int_0^{\infty}\Phi(\tau)\exp[-i\Omega'\tau-\Gamma\tau]d\tau$$
$$=2|G|^2\int_{-\infty}^{\infty}dx\frac{g(x)\Gamma}{\Gamma^2+(\Omega'-x)^2};\quad \tau_{1m}=\frac{1}{\Gamma_m+\nu_m};$$
$$\tau_{2m}=\frac{1}{\Gamma_m}-\tau_{1m}=\frac{\nu_m/\Gamma_m}{\Gamma_m+\nu_m};\quad \tau_{1n}=\frac{1}{\nu_n}(1-\Gamma_m\tau_{1m})=\frac{\nu_m/\nu_n}{\Gamma_m+\nu_m}; \qquad (2.6)$$
$$\tau_{2n}=\frac{1}{\Gamma_m}-\tau_{1n};\quad \tau_\alpha=\tau_{1m}+\tau_{1n}=\frac{\nu_m+\nu_n}{\nu_n}\frac{1}{\Gamma_m+\nu_m};\quad \tau_\beta=\tau_{2n}+\tau_{2m}=\frac{2}{\Gamma_m}-\tau_\alpha.$$

The angle brackets $\langle\cdots\rangle_v$ denote averaging over the velocities with a Maxwellian distribution. Formulas (2.5) generalize the expressions obtained earlier[8] for the case of monochromatic radiation, to include the case of nonmonochromatic radiation with arbitrary spectral composition.

The expressions obtained above are applicable, within the framework of the validity of the employed approximation, namely the splitting of the field and atomic variables. This approximation is valid if during the radiation correlation time τ_c (which is of the order of magnitude of δ^{-1}, where δ is the half-width of the spectrum) no noticeable change takes place in the level populations. The corresponding criterion takes the form $\delta^{-1}g(\Omega)|G|^2\ll1$, or, at $\Omega=0$, the form $\sim|G|^2\ll\delta^2$. We note that at $|G|^2\ll\delta^2$ the saturation parameter $\varkappa\sim|G|^2\tau_\alpha/\delta$ can assume values $\varkappa\gg1$. Thus, the employed approximation enables us to consider also cases of strong saturation of the populations, which are determined by the probability of transition after a time of the order of $\tau_\alpha\gg\delta^{-1}$. A more detailed discussion of this question, is contained, e.g., in Refs. 7 and 9.

In the population velocity distribution functions $\rho_j(\mathbf{v})$ we can separate the equilibrium (Maxwellian) and nonequilibrium (selective) parts proportional to $Y(\mathbf{v})$. The function $Y(\mathbf{v})$ describes the dips and the peaks in the distribution function, called in the case of MR the Bennett dips and peaks.

The nonequilibrium and equilibrium parts of $\rho_j(\mathbf{v})$ are proportional respectively to the quantities τ_{1j} and τ_{2j}, which have the dimension of time. For the level m these quantities are interpreted in the following manner: τ_{1m} is the lifetime of the state with given velocity $\rho_m(\mathbf{v})$; τ_{2m} is the lifetime of an excited atom in a state with Maxwellian velocity distribution. We note that $\tau_{1m}+\tau_{2m}=\Gamma_m^{-1}$, where Γ_m^{-1} is the total lifetime on the level m. In accordance with their physical meanings, τ_{1m} and τ_{2m} govern the weights of the selective and equilibrium parts of the velocity distribution of the excited atoms. The field part of the distribution $\rho_n(\mathbf{v})$ has the same structure as $\rho_m(\mathbf{v})$. For the level n, however, besides the induced transitions and the change of the velocities on account of the collisions, an important role is played by arrival from the level m via the spontaneous relaxation channel. Therefore the interpretation of τ_{1n} and τ_{2n} is more complicated.

Using (2.5), we easily obtain the total velocity distribution of the absorbing atoms

$$N[\rho_m(\mathbf{v})+\rho_n(\mathbf{v})]=W(\mathbf{v})\left[1+\frac{\nu_n-\nu_m}{\nu_n+\nu_m}\frac{Y(\mathbf{v})-\langle Y\rangle_v}{1+(\tau_\beta/\tau_\alpha)\langle Y\rangle_v}\right]N. \quad (2.7)$$

Here N is the density of the absorbing particles. From (2.7) it is easily seen that the deviation from the Maxwellian distribution is due to the difference between the collision frequencies ν_m and ν_n, in full agreement with the qualitative reasoning presented above.

Multiplying (2.7) by v and integrating over the velocities, we obtain an expression for the total flux of the absorbing atoms J and for the drift velocity u:

$$\mathbf{J}=\frac{\mathbf{k}}{k}uN=\frac{\nu_n-\nu_m}{\nu_n+\nu_m}\frac{\langle \mathbf{v}Y\rangle_v}{1+(\tau_\beta/\tau_\alpha)\langle Y\rangle_v}N. \quad (2.8)$$

We proceed now to analyze the influence exerted on the LID by the spectral characteristics of the radiation. We first demonstrate the advantages of using nonmonochromatic radiation, using as the simplest example a Lorentz radiation spectrum.

3. LID OF GASES IN THE FIELD OF NONMONOCHROMATIC RADIATION WITH A LORENTZ SPECTRUM

Using for $g(\omega'-\omega)$ the expression

$$g(\omega'-\omega)=\frac{\delta/\pi}{\delta^2+(\omega'-\omega)^2},$$

where δ is the half-width of the emission spectrum, we obtain

$$Y(\mathbf{v})=\frac{\varkappa_L(\delta+\Gamma)^2}{\Omega'^2+(\delta+\Gamma)^2(1+\varkappa_L)};\ \varkappa_L=2\frac{\tau_\alpha|G|^2}{\delta+\Gamma}.$$

According to (2.8), the expression for the drift velocity takes the form

$$u=\frac{\nu_n-\nu_m}{\nu_n+\nu_m}\frac{\pi^{1/2}y\,\mathrm{Re}[zW(z)]\bar{v}}{1+1/\varkappa_L+(\tau_\beta/\tau_\alpha)\pi^{1/2}y\,\mathrm{Re}[W(z)]}, \quad (3.1)$$

$$W(z)=e^{-z^2}\left(1+2i\pi^{-1/2}\int_0^z e^{t^2}dt\right);\ z=x+iy;\ x=\Omega/k\bar{v};\quad y=(\delta+\Gamma)(1+\varkappa_L)^{1/2}/k\bar{v}.$$

Formula (3.1) generalizes, by means of the substitution $\Gamma\to\delta+\Gamma$, formula (9) of Ref. 8, obtained for the case of monochromatic radiation, to the case of nonmonochromatic radiation with a Lorentz spectrum.

At $\nu_n\ll\nu_m$ and $\nu_n\ll\Gamma_m$, the parameter τ_β/τ_α takes on minimum values close to –1. This condition, together with the condition $\varkappa_L\gg1$, ensures minimal values of the denominator of expression (3.1). At the same time, at $\nu_n\ll\nu_m$ the factor $(\nu_n-\nu_m)/(\nu_n+\nu_m)$ takes on the largest absolute values. Thus, it is clear that the maximum of the quantity $|u|$ is realized under the conditions $\varkappa_L\gg1$, $\nu_n\ll\nu_m$, and $\nu_n\ll\Gamma_m$. Using the tables of Ref. 10, we find that the maximum drift velocity is reached at $x\approx0.5$ and $y\approx1-2$, and its value is $|u|\approx0.5\bar{v}$. Thus, the drift velocity amounts to half the thermal velocity of the absorbing atoms.

If ν_n and ν_m are of the same order of magnitude, $\nu_n\sim\nu_m$, then the attainable drift velocity values become smaller than given above, but remain comparable with $\bar{v}$.

If monochromatic radiation is used ($\delta\ll\Gamma$) at $\varkappa_L\sim1$ the condition $y\sim1$, as can be easily seen, takes the form $\Gamma\sim k\bar{v}$. The parameters Γ and ν_j are not independent, since both are determined by the collisions of the absorbing atoms with the buffer-gas atoms. The condition $\Gamma\sim k\bar{v}$ means thus that $\nu_j\gg\Gamma_m$. For typical electronic transitions in atoms we have $\Gamma_m/k\bar{v}\sim10^{-3}$, and the condition $\Gamma/k\bar{v}\sim1$ can be satisfied only if Γ is governed practically entirely by the collisions (impact broadening). Recognizing also that the broadening cross section usually exceeds the transport cross section by not more than an order of magnitude ($\nu_j/\Gamma\sim10^{-1}$), we arrive at the conclusion that the condition $\Gamma/k\bar{v}\sim1$ corresponds to a ratio $\nu_j/\Gamma_m\sim10^2$. The ratio τ_β/τ_α differs in this case from the optimal value. Under conditions when this ratio is minimal we have $\Gamma\sim\Gamma_m\ll k\bar{v}$, and the optimal values $y\sim1$ can be attained only by substantially increasing the field intensity (at $\Gamma_m/k\bar{v}\sim10^{-3}$ and $\nu_j\sim\Gamma_m$ we should have $\varkappa_L\sim10^4$).

If nonmonochromatic radiation is used, the condition $y\sim1$ is of the form

$$(\Gamma+\delta)(1+\varkappa_L)^{1/2}/k\bar{v}\sim1.$$

Since the parameters δ and ν_j are independent, this condition can be satisfied simultaneously with the condition $\nu_j\le\Gamma_m$ for arbitrary $\varkappa_L$, including $\varkappa_L\sim1$, if $\delta\sim k\bar{v}$. The saturation parameter $\varkappa_L$, both in the case of monochromatic and nonmonochromatic radiation, is proportional to the integrated intensities of the radiation, I_{MR} and I_{NMR}, respectively. Starting from the definition of $\varkappa_L$, it can be easily shown that if nonmonochromatic radiation is used, a drift velocity u comparable with $\bar{v}$ can be obtained at integrated intensities I_{NMR} that are smaller by a factor $\delta/\Gamma\sim k\bar{v}/\Gamma\sim10^2-10^3$ than the I_{MR} needed for this purpose. The last conclusion is most important and demonstrates the substantial advantages that the use of nonmonochromatic radiation offers over monochromatic radiation for the study and practical utilization of the LID of gases.

4. INFLUENCE OF THE FORM OF THE SPECTRUM ON THE LID

In the preceding section we have demonstrated the advantages of the use of nonmonochromatic radiation over monochromatic radiation. This raises the question of whether the characteristic form of the spectrum is of importance. We indicate first a case when the form of the spectrum is not a significant factor. We consider a situation in which the homogeneous width of the atomic transitions and the width of the spectrum are much smaller than the Doppler width of the transition, and the radiation intensity is limited by the condition $\tau_\alpha A\ll1$ [see formula (2.6)], meaning smallness of the fraction of the selectively excited particles. Then $Y\approx\tau_\alpha A$. Recognizing that $A(\mathbf{v})$ is according to (2.6) a more rapidly varying function than $W(\mathbf{v})$ and $\mathbf{v}W(\mathbf{v})$, when integrating with respect to v in (2.8) we can take the latter outside the integral sign at the point of the maximum of $A(\mathbf{v})$. As a result $\langle Y\rangle_v$, $\langle \mathbf{v}Y\rangle_v$, and consequently also the drift velocity u, are determined only by the integrated intensity of the radiation:

$$\frac{u}{\bar{v}}=\frac{\nu_n-\nu_m}{\nu_n+\nu_m}\frac{\Omega\varkappa(\Omega)/k\bar{v}}{1+(\tau_\beta/\tau_\alpha)\varkappa(\Omega)}, \qquad (4.1)$$

$$\varkappa(\Omega)=[2\pi^{1/2}\tau_\alpha|G|^2/k\bar{v}]\exp[-(\Omega/k\bar{v})^2]\ll 1.$$

Thus, in the considered case the drift velocity does not depend on the form of the spectrum.

In another limiting case, when the spectrum width greatly exceeds the Doppler width of the transition, and the envelope of this spectrum is smooth, the quantity u is likewise independent of the form of the spectrum. Indeed, if $\Gamma\ll k\bar{v}$, we obtain

$$A\approx 2\pi|G|^2 g(\Omega').$$

However, calculating $\langle Y\rangle_v$ and $\langle \mathbf{v}Y\rangle_v$, we take into account the change of the spectral density of the intensity over an interval of the order of $k\bar{v}$:

$$Y(\Omega-\mathbf{kv})\approx Y(\Omega)-\tau_\alpha A'(\Omega)\mathbf{kv}/[1+\tau_\alpha A(\Omega)]^2,$$
$$A'(\Omega)=dA/d\Omega; \quad A(\Omega)=2\pi|G|^2 g(\Omega), \qquad (4.2)$$

where $A(\Omega)$ is dependent by the spectral density of the radiation at the frequency of the atomic transition. From (2.8) we obtain with the aid of (4.2)

$$\frac{u}{\bar{v}}=\frac{\nu_m-\nu_n}{\nu_m+\nu_n}\frac{\varkappa(\Omega)}{1+\varkappa(\Omega)}\frac{\tau_\alpha k\bar{v}}{\tau_\alpha+2\Gamma_m^{-1}\varkappa(\Omega)}\frac{I'(\Omega)}{2I(\Omega)},$$
$$\varkappa(\Omega)=2\pi|G|^2\tau_\alpha g(\Omega); \quad -I'(\Omega)k\bar{v}=\Delta I, \qquad (4.3)$$

where ΔI is the change of the spectral density of the intensity over the interval $k\bar{v}$ in the vicinity of the atomic-transition frequency. In the particular case of a Lorentz spectrum, formula (4.3) coincides with (3.1) for values $\delta\gg k\bar{v}$.

From formula (4.3) follows an important conclusion that in the case of broad radiation spectra the velocity of the light-induced drift is governed not by the form of the entire spectrum, but only by the character of its variation near the frequency of the atomic transition. In addition, it is easily seen that the maximum value of $|u|$ is limited by the factor $\Delta I/2I$, which is small in the employed approximation and at optimal detunings $\Omega\sim\delta$ is of the order of the ratio of the Doppler width of the transition to the width of the emission spectrum.

We turn now to an analysis of cases when the width of the emission spectrum is comparable with the Doppler line width. The influence of the form of the spectrum then turns out to be substantial. This region is all the more interesting, because the LID effect itself is manifest in it to the greatest degree. For comparison with the already invested example of the Lorentz spectrum, we consider another example—a Gaussian spectrum:

$$g(\omega-\omega')=\pi^{-1/2}\beta^{-1}\exp[-(\omega-\omega')^2/\beta^2].$$

Putting $\beta\gg\Gamma$, we obtain

$$A=(2\pi^{1/2}|G|^2/\beta)\exp[-(\Omega'/\beta)^2].$$

In the approximation of low radiation intensities $\tau_\alpha A\ll 1$ and $Y\approx\tau_\alpha A$, we can obtain the following expression

$$\frac{u}{\bar{v}}=\frac{\nu_n-\nu_m}{\nu_n+\nu_m}\frac{\varkappa_G(\Omega)\Omega k\bar{v}}{[1+(\tau_\beta/\tau_\alpha)\varkappa_G(\Omega)][\beta^2+(k\bar{v})^2]}, \qquad (4.4)$$

$$\varkappa_G(\Omega)=\frac{2\pi^{1/2}\tau_\alpha|G|^2}{[\beta^2+(k\bar{v})^2]^{1/2}}\exp\left[-\frac{\Omega^2}{\beta^2+(k\bar{v})^2}\right].$$

In the same approximation in the field, for the Lorentz spectrum it is necessary to leave out the unity from the denominator of (3.1).

Assuming the width of the spectra to be the same and equal to the Doppler width of the transition $\beta\ln^{1/2}2=\delta=k\bar{v}$; $\tau_\beta/\tau_\alpha\leq 1$, we find with the aid of the tables of Ref. 10 that the maximum value of the drift velocity is reached at $|\Omega|\approx 1.1k\bar{v}$, for a Gaussian spectrum and at $|\Omega|\approx 1.2k\bar{v}$ for a Lorentz spectrum. In the case of different integrated intensities of the radiation, the drift velocity for a Gaussian spectrum is 1.8 times larger than the drift velocity for a Lorentz spectrum. This agrees with the concept developed above, since a Gaussian spectrum drops off more rapidly in the wings than a Lorentz spectrum, and consequently ensures higher selectivity of the excitation.

Under the conditions $\beta,\delta\gg k\bar{v},\Gamma$; $\nu_j\lesssim\Gamma_m$; $\varkappa_G,\varkappa_L\ll 1$, the expressions for the drift velocities u_g and u_L for Gaussian and Lorentz emission spectra take respectively the forms

$$\frac{u_G}{\bar{v}}=\frac{\nu_n-\nu_m}{\nu_n+\nu_m}\frac{\Omega k\bar{v}}{\beta^2}\varkappa_G(\Omega); \quad \varkappa_G(\Omega)=\frac{2\tau_\alpha\pi^{1/2}|G|^2}{\beta}\exp\left[-\left(\frac{\Omega}{\beta}\right)^2\right], \qquad (4.5)$$

$$\frac{u_L}{\bar{v}}=\frac{\nu_n-\nu_m}{\nu_n+\nu_m}\frac{\Omega k\bar{v}}{\Omega^2+\delta^2}\varkappa_L(\Omega); \quad \varkappa_L(\Omega)=\frac{2\tau_\alpha|G|^2\delta}{\delta^2+\Omega^2}. \qquad (4.6)$$

The maximum of $|u_G|$ is reached at $|\Omega|=2^{-1/2}\beta$, while that of $|u_L|$ at $|\Omega|=3^{-1/2}\delta$. Assuming that $\beta=\delta\ln^{-1/2}2$, we find that $u_G/u_L\approx 1.6$ at equal integrated radiation intensities. A similar estimate can be obtained also with the aid of the general formula (4.3). Thus, in the latter case the difference is less pronounced, this being due to the overall decrease of the selectivity of the excitation in the case of spectra that are excessively broad.

With the aid of (4.3) we can obtain for the drift velocity an expression that is valid at arbitrary radiation intensities with a broad Gaussian spectrum:

$$\frac{u_G}{\bar{v}}=\frac{\nu_n-\nu_m}{\nu_n+\nu_m}\frac{\Omega k\bar{v}}{\beta^2}\frac{\varkappa_G(\Omega)[1+\varkappa_G(\Omega)]^{-1}}{1+(1+\tau_\beta/\tau_\alpha)\varkappa_G(\Omega)}. \qquad (4.7)$$

According to (4.7), the maximum value of $u_G/\bar{v}$ is described by the expression

$$\frac{u_G}{\bar{v}}=\frac{\nu_n-\nu_m}{\nu_n+\nu_m}\frac{\Omega k\bar{v}}{\beta^2}\left[1+\left(2\frac{\Gamma_m+\nu_m}{\Gamma_m}\frac{\nu_n}{\nu_n+\nu_m}\right)^{1/2}\right]^{-2} \qquad (4.8)$$

and is reached at radiation intensities satisfying the condition

$$\frac{2|G|^2}{\beta(\Gamma_m+\nu_m)}=\left(\frac{1}{2\pi}\frac{\Gamma_m}{\Gamma_m+\nu_m}\frac{\nu_n}{\nu_n+\nu_m}\right)^{1/2}\exp\left(\frac{\Omega}{\beta}\right)^2. \qquad (4.9)$$

Figures 1 and 2 show a graphic analysis of expression (4.7) for the values of the parameters $\beta=10k\bar{v}$, and $\nu_m/\nu_n=10$. Figure 1 shows the dependence of the drift velocity on the radiation intensity I in W/cm^2 at a fixed value of the detuning $\Omega=\beta$. To estimate the radiation intensity we used also the values of the parameters ($\beta=10^{10}$ sec^{-1}, $\nu_m\ll\Gamma_m=10^7$ sec^{-1}, $|d|=1D$), at which $\varkappa_G=0.28I$ [W/cm^2]. It is seen from the figure that with increasing intensity the drift velocity increases

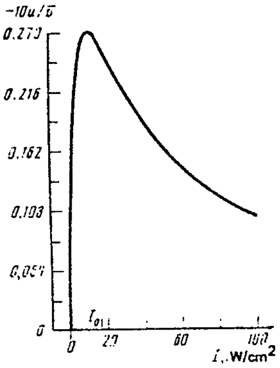

FIG. 1. Dependence of the drift velocity on the radiation intensity I with a broad Gaussian spectrum, $\Omega=\beta=10\,k\bar{v}$; $\nu_m/\nu_n=10$; I [W/cm²]$\approx 3.5\,\varkappa_G(\Omega)$.

and reaches at the chosen value of Ω a maximum equal to $u/\bar{v}\approx 1/37$ at $I\approx 12$ W/cm², which corresponds to $\varkappa_G(\Omega)\approx 3.4$. Next, with increasing radiation intensity the field broadening of the Bennett structure and the corresponding loss of selectivity lead to a decrease of the drift velocity.

Figure 2 shows the dependence of the drift velocity $-u/\bar{v}$ on the detuning Ω/β at a fixed value of the radiation intensity $I\sim 48$ W/cm² ($|G|^2/\beta\Gamma_m=1$). The remaining parameters are the same as in Fig. 1. Since the chosen value of the intensity is somewhat larger than that corresponding to the maximum of the plot of $u/\bar{v}$ against I in Fig. 1, it follows that, in accordance with (4.8) and (4.9), an increase took place both in the optimal value of the detuning ($|\Omega/\beta|\approx 1.87$) and in the ensuing maximum value of the drift velocity $u/\bar{v}\approx 1/20$. It is seen from the figure that the drift velocity reverses sign with change of the detuning.

The general regularities clarified in the present section allow us to proceed to an analysis of a situation wherein, using the dependence of the LID on the character of the spectrum, it is possible to obtain the most striking results, namely to obtain light-induced drift streams with velocity exceeding the sound velocity (SVLID).

5. SVLID OF GASES IN A NONMONOCHROMATIC FIELD WITH A "RECTANGULAR" SPECTRUM

We consider radiation with a rectangular spectrum $g(\omega'-\omega)$, having a width Δ:

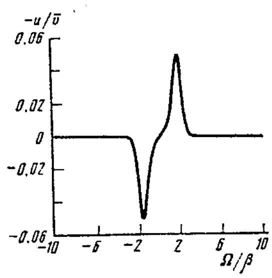

FIG. 2. Dependence of the drift velocity in a radiation field with fixed intensity and broad Gaussian spectrum on the detuning resonance from Ω/β. $\beta=10\,k\bar{v}$; $\nu_m/\nu_n=10$; $|G|^2/\beta\Gamma_m=1$.

$$g(\omega'-\omega)=\Delta^{-1}\quad \text{if}\quad \omega\leqslant\omega'\leqslant\omega+\Delta,$$
$$g(\omega'-\omega)=0\quad \text{if}\quad \omega'<\omega \text{ and } \omega'>\omega+\Delta.$$

Putting $\Gamma\ll k\bar{v}$, Δ (it follows from the results of Sec. 3 that these assumptions correspond to the optimal conditions for obtaining maximum drift velocities u), we can obtain with the aid of (2.6) the expression

$$\frac{u}{\bar{v}}=\frac{1}{2\pi^{1/2}}\frac{\nu_n-\nu_m}{\nu_n+\nu_m}\left\{\exp\left[-\left(\frac{\Omega}{k\bar{v}}\right)^2\right]-\exp\left[-\left(\frac{\Omega+\Delta}{k\bar{v}}\right)^2\right]\right\}\left\{1+\varkappa_{\text{rect}}^{-1}+\frac{\tau_\phi}{2\tau_a}\left[\Phi\left(\frac{\Delta+\Omega}{k\bar{v}}\right)-\Phi\left(\frac{\Omega}{k\bar{v}}\right)\right]\right\}^{-1}, \quad (5.1)$$

$$\Phi(x)=2\pi^{-1/2}\int_0^x e^{-t^2}\,dt;\quad \varkappa_{\text{rect}}=\frac{2\pi\tau_a|G|^2}{\Delta};\quad \Omega=\omega-\omega_{0\alpha}.$$

An analysis of (5.1) shows that in the case of a rectangular spectrum at $\nu_m\sim\nu_n$ the optimal conditions for the value of the flux are $\varkappa_{\text{rect}}\gtrsim 1$, $\Omega\approx 0$, and $\Delta\gtrsim k\bar{v}$. A rectangular spectrum ensures the maximum velocity selectivity of atom excitation that is possible for nonmonochromatic radiation. Therefore, when radiation with this spectral composition is used, the entire capability of the LID of gas can be realized.

We consider now in greater detail the case of a "semi-infinite" spectrum as $\Delta\to\infty$. The corresponding expression for the drift velocity is

$$\frac{u}{\bar{v}}=\frac{1}{2\pi^{1/2}}\frac{\nu_n-\nu_m}{\nu_n+\nu_m}\exp\left[-\left(\frac{\Omega}{k\bar{v}}\right)^2\right]\left\{1+\varkappa_{\text{rect}}^{-1}+\frac{\tau_\phi}{2\tau_a}\left[1-\Phi\left(\frac{\Omega}{k\bar{v}}\right)\right]\right\}^{-1}. \quad (5.2)$$

The function $\Phi(\Omega/k\bar{v})$ is equal to zero at $\Omega=0$. With increasing value of $|\Omega/k\bar{v}|$, its modulus reaches quite rapidly values close to unity, and as $|\Omega/k\bar{v}|\to\infty$ it approaches unity asymptotically.

Thus, it is seen from (5.2) that at $\Omega=0$ the maximum drift velocity $|u|$ is reached when $\nu_n\to 0$ and $\varkappa_{\text{rect}}\to\infty$, and amounts to $|u|=\pi^{-1/2}\bar{v}$. However, when the detuning $|\Omega/kv|$ is increased, it becomes possible in principle to reach values $|u|>\bar{v}$, i.e., exceeding the speed of sound.

The SVLID phenomenon can be qualitatively explained in the following manner. Let $\nu_n=0$, $\nu_m\neq 0$, $-\Omega>k\bar{v}$, and $\Delta\to\infty$. The condition $\nu_n=0$ means that if the velocity of the unexcited atom is such that it does not interact with the field, then this velocity cannot change. In the case considered, the atoms that do not interact with the field are those whose velocity projection on the wave vector is $v_k<\Omega/k$. At $-\Omega>k\bar{v}$, this condition means $v_k<-\bar{v}$. Any unexcited atom whose velocity does not satisfy the described condition interacts with the field, and has a nonzero probability of going over in a unit time into an excited state and of acquiring as a result of a collision ($\nu_m\neq 0$) a velocity such that $v_k<-\bar{v}$, and go over via the spontaneous relaxation channel into an unexcited state. It is obvious that the indicated optical-collisional transfer processes establish a stationary state in which all the atoms are not excited, and their velocities are such that $v_k<-\bar{v}$. This means that a supersonic flux of absorbing particles is established in a direction opposite to the field. While the described tendency towards optical-collisional transfer is not so brightly pronounced at $\nu_n\neq 0$ and $\nu_m/\nu_n\gg 1$, it can nevertheless ensure the onset of SVLID.

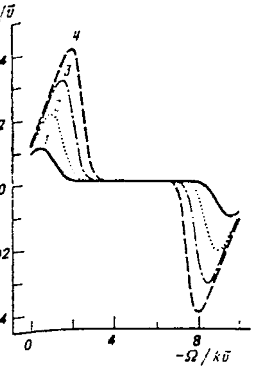

FIG. 3. Dependence of the drift velocity in a radiation field with rectangular spectrum of width $\Delta = 10\,k\bar{v}$ on the distance from the left edge of the spectrum to the center of the transition at different values of the parameter $\bar{\varkappa}_{rect} = 2|G|^2/\Delta\,(\Gamma_m + \nu_m)$ and on the ratio of the collision frequencies ν_m/ν_n. The maximum values of $|u/\bar{v}|$ are reached at $\bar{\varkappa}_{rect} \gg \varkappa_0$. *1)* $\nu_m/\nu_n = 10$, $\varkappa_0 = 0.08$; *2)* $\nu_m/\nu_n = 10^2$, $\varkappa_0 = 0.03$; *3)* $\nu_m/\nu_n = 10^3$, $\varkappa_0 = 0.03$; *4)* $\nu_m/\nu_n = 10^4$, $\varkappa_0 = 0.02$.

The qualitative difference between the case of the rectangular spectrum and the other spectra is that only for this spectrum there is no field broadening of the Bennett structures. Therefore [see (5.1) and (5.2)] for each set of values of Ω and ν_m/ν_n there exists a parameter value $\bar{\varkappa}_{rect} = \varkappa_0(\Omega)$ which, when substantially exceeded, makes u practically independent of the intensity of the radiation, and the limiting values of the drift velocity are then determined by the parameter ν_m/ν_n. For all the remaining spectrum types, to each value of the parameters ν_m/ν_n and Ω there corresponds a definite optimal intensity.

Figure 3 shows a graphic analysis of Eq. (5.1) for different values of the parameters ν_m/ν_n and $\bar{\varkappa}_{rect} \gg \varkappa_0(\Omega)$. The left and right sides of the plot correspond respectively to interaction between the rectangular spectrum and the right and left parts of a Maxwellian distribution. The curves are symmetrical, as they should be. The larger the parameter ν_m/ν_n, the larger the part of the Maxwellian distribution that can be converted into a wing by optical-collision transfer. Therefore the maxima of the curves shift towards larger values of $|\Omega/k\bar{v}|$ and increase with increasing parameter ν_m/ν_n.

From a comparison of the maximum of curve 1 with the corresponding maxima of the curves in Figs. 1 and 2 one can see the advantages of the rectangular spectrum over the Gaussian one. Curves 2–4 demonstrate the possibilities of reaching supersonic fluxes at quite moderate radiation intensities and demonstrate the decisive role of the factor ν_m/ν_n when it comes to obtaining large drift velocities.

Supersonic drift velocities can be obtained also with the aid of radiation with a Gaussian spectrum, but in this case the radiation intensity must satisfy rather stringent requirements. For a Gaussian spectrum with optimal value $\beta \approx k\bar{v}$, it is impossible to obtain for u an analytic formula that is valid at large values of $\varkappa_G$. From (4.8) and (4.9) it is seen, however, than at $\nu_n \ll \nu_m$ values $u_G > \bar{v}$ can be obtained at $|\Omega| \gtrsim \beta^2/k\bar{v}$. With increasing parameter $\beta/k\bar{v}$, the required intensities increase exponentially.

It is seen from (3.1) that for monochromatic radiation and for radiation with a Lorentz spectrum, no supersonic light-induced fluxes are possible. The reason is that the wings of the Lorentz contour decrease more slowly, and as a result the excitation of the atoms turns out to be insufficiently selective in velocity.

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