8 Plasmonic Nanowire Metamaterials

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8.1 INTRODUCTION

The optical properties of nanostructured materials have been intensively studied during the last decade. Among particularly important problems in this field are the focusing and guiding light on nanometer scales beyond the diffraction limit of the conventional far-zone optics. In object imaging, the near-field part of the radiation contains all information about the scatterer. As the distance from the object increases, the evanescent portion of the scattered field exponentially decays, resulting in information loss on the "fine" (subwavelength) features of the scatterer. The usual way to solve this problem suggests either using shorter wavelengths or measuring in the near zone; both these methods have their limitations. A new way to solve this imaging problem has been proposed by Pendry, who further developed earlier studies on negative refraction [1, 2]. According to Pendry, when the scattered light passes through a material with a negative refractive index (specifically, it should be equal to -1), the evanescent components of the scattered field grow exponentially, allowing the restoration of the scatterer image with subwavelength resolution. Despite the obvious importance of such a superlens, it is worth noting here that possible applications for materials with negative refraction can go far beyond this idea. This is because the refractive index enters most of optical "laws" so that the possibility of its sign reversion can result in their serious revision and new exciting applications resulting from this.

Smith, Padilla, Vier, and Schultz [3] have demonstrated such negative-index materials (also referred to as double-negative or left-handed materials, LHMs, because the electric, magnetic vectors and the wavevector form a left-handed system, in this case) in the microwave range. (For recent references see the special issue of *Optics*

Express [4] and Refs. 5 and 6). In our earlier papers [4, 7], we proposed a first LHM (based on a nanowire composites [8]) that can have negative refraction in the near-IR and visible spectral ranges. A similar nanowire system was later considered by Panina et al. [9].

In this chapter we discuss the electrodynamics of nanowires materials and study the behavior of nanowire plasmon modes. We also describe how nanowire composites can be used for developing LHMs *in the near-IR and visible* parts of the spectrum.

The rest of the chapter is organized as follows. In the next section we discuss the interaction of a single metal nanowire with an electromagnetic wave (we refer to such a wire as a "conducting stick"). Section 8.3 describes the effective properties of composites comprising conducting sticks. In Section 8.4 we present computer simulations for the local electromagnetic field in stick composites. Section 8.5 discusses the magnetic response of two parallel conducting sticks and effective properties of composites comprising pairs of such sticks. In Section 8.6 we show that the forward and backward scattering by planar nanowire system can be characterized by their effective dipole and magnetic moments. Section 8.7 summarizes our results.

8.2 ELECTRODYNAMICS OF A SINGLE METAL NANOWIRE

Composite materials containing conducting sticks dispersed in a dielectric matrix have new and unusual properties at high frequencies. When frequency ω increases, the wavelength $\lambda = 2\pi c/\omega$ of an external electromagnetic field can became comparable in size with the stick length $2a$. In this case, one might think that the sticks act as an array of independent micro-antennas and an external wave should be scattered in all directions. Yet, we show that composite materials have well-defined dielectric and magnetic properties at high frequencies. Such "effective-medium" description is possible because a very thin conducting stick interacts with an external field like a dipole. Therefore, we can still use the effective dielectric constant ε_e or effective conductivity $\sigma_e = -i\omega\varepsilon_e/4\pi$ to describe the interaction of stick composites with an external electromagnetic wave. However, we note the formation of large stick clusters near the percolation threshold may result in scattering.

Since conducting stick composites are supposed to have effective parameters for all concentrations p outside the percolation threshold, we can use the percolation theory to calculate the effective conductivity σ_e. However, the theory has to be generalized to take into account the nonquasi-static effects. The problem of effective parameters of composites beyond the quasi-static limit has been considered in Refs. [8] and [10–15]. It was shown there that the mean-field approach can be extended to find the effective dielectric constant and magnetic permeability at high frequencies. Results of these considerations can be briefly summarize as follows. One first finds the polarizability for a particle in the composite illuminated by an electromagnetic wave (the particle is supposed to be embedded in the "effective medium" with dielectric constant ε_e). Then, the effective dielectric permittivity ε_e is determined by the self-consistent condition requiring that the averaged polarizability of all particles should vanish. Thus, for the nonquasi-static case, the problem is reduced to calculation of the polarizability of an elongated conducting inclusion. That is, we consider the retardation effects resulting from the interaction of a conducting stick with the electromagnetic wave scattered by the stick.

The diffraction of electromagnetic waves on a conducting stick is a classical problem of the electrodynamics. A rather tedious theory for this process is presented in several textbooks [16, 17]. We show below that the problem can be solved analytically in the case of very elongated sticks when the aspect ratio a/b is so large that $\ln(a/b) \gg 1$.

We consider a conducting stick of length $2a$ and radius b illuminated by an electromagnetic wave. We suppose that the electric field in the wave is directed along the stick and the stick is embedded in a medium with $\varepsilon = 1$. The external electric field excites in the stick and electric current $I(z)$, where z is the coordinate along the stick, measured from its midpoint. The dependence $I(z)$ is nontrivial when the wavelength λ is of the order of or smaller than the stick length. There is also a nontrivial charge distribution $q(z)$ along the stick in this case. The charge distribution $q(z)$ determines the polarizability of the stick. To find $I(z)$ and $q(z)$, we introduce the potential $U(z)$ of the charges $q(z)$ distributed over the stick surface. From the equation for the electric charge conservation we obtain the following formula:

$$\frac{dI(z)}{dz} = i\omega q(z) \tag{8.1}$$

which relates the charge per unite length $q(z)$ and the current $I(z)$. Note that the electric charges q are induced by the external field E_0 and they can be expressed as the divergence of the polarization vector, $q = -4\pi \operatorname{div} \mathbf{P}$. Then, the polarization $\mathbf{P}$ can be included in definition of the electric displacement $\mathbf{D}$ so that $\operatorname{div} \mathbf{D} = 0$. However, in calculating the high frequency field in a conducting stick it is convenient to explicitly consider charges generated by the external field.

To find an equation for the current $I(z)$ we treat a conducting stick as a prolate conducting spheroid with semiaxes a and b. The direction of the major axis is supposed to coincide with direction of the electric field $\mathbf{E}_0 \exp(-i\omega t)$ in the incident wave. The electric potential of the charge $q(z)$ is given by the following solution to Maxwell's equations (see, e.g., Ref. [17], p. 377):

$$U(z) = \oint \frac{[q(z')/2\pi\rho(z')] \exp(ik\,|\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|} ds' \cong \int_{-a}^{a} \frac{q(z') \exp(ik\,|z - z'|)}{\sqrt{(z - z')^2 + \rho(z)^2}} dz' \tag{8.2}$$

where the integration in the first integral is performed over the surface of the stick, $\mathbf{r}$ and $\mathbf{r}'$ are two points on the surface of the stick with the coordinates z and z', respectively, $\rho(z) = b\sqrt{1 - z^2/a^2}$ is the radius of the cross section at the coordinate z, and $k = \omega/c$ is the wavevector of the external field. In transition to the second expression in equation (8.2), we neglect terms of the order of $\rho(z)/a < b/a \ll 1$. We divide the last integral in equation (8.2) into two parts, setting $q(z') \exp(ik\,|z - z'|) =$

$q(z) + [q(z')\exp(ik\,|z-z'|) - q(z)]$, that is,

$$U(z) \cong q(z)\int_{-a}^{a} \frac{dz'}{\sqrt{(z-z')^2+\rho(z)^2}} + \int_{-a}^{a} \frac{q(z')\exp(ik\,|z-z'|) - q(z)}{|z-z'|}\,dz' \tag{8.3}$$

The first integral in equation (8.3) is given by

$$\int_{-a}^{a} \frac{dz'}{\sqrt{(z-z')^2+\rho(z)^2}} = 2\ln\left(2\frac{a}{b}\right) \tag{8.4}$$

The second integral in equation (8.3) has no singularity at $z = z'$, and therefore its value is $\sim q(z)$, which is an odd function of the coordinate z. We assume for simplicity that $q(z)$ is proportional to z and in this approximation

$$\int_{-a}^{a} \frac{q(z')\exp(ik\,|z-z'|) - q(z)}{|z-z'|}\,dz' = 2\,q(z)\,\left[-e^{i\,a\,k} + \mathrm{Ei}(a\,k)\right] \tag{8.5}$$

where the $\mathrm{Ei}(x)$ function is defined as

$$\mathrm{Ei}(x) = \int_0^x \left[\exp(it) - 1\right]/t\,dt \tag{8.6}$$

By substituting equations (8.4) and (8.5) in equation (8.3), we obtain

$$U(z) = q(z)/C \tag{8.7}$$

where the capacitance C is given by

$$C = \frac{1}{2\left[\ln(2a/b) - e^{i\,a\,k} + \mathrm{Ei}(a\,k)\right]} \tag{8.8}$$

The capacitance C takes the usual value $C = 1/\left[2\ln(2a/b) - 2\right]$, in the quasi-static limit $ka \to 0$. The retardation effects result in additional terms in equation (8.8) that have small magnitudes in comparison with the leading logarithmic term. This result is obtained within the logarithmic accuracy: its relative error is on the order of $1/\ln(a/b)$, and the ratio a/b is assumed so large that its logarithm is also large.

By substituting equation (8.7) into equation (8.1), we obtain the following equation:

$$\frac{dI(z)}{dz} = i\omega C U(z) \tag{8.9}$$

which relates the current $I(z)$ and the surface potential $U(z)$. The electric current $I(z)$ and electric field $E(z)$ on the stick surface are related by the usual Ohm's Law

$$E\,(z) = RI\,(z) \tag{8.10}$$

where R is the impedance per unit length. Since the stick is excited by the external field $E_0\exp(-i\omega t)$ which is parallel to its axis, the electric field $E(z)$ is equal to

$$E(z) = E_0 - \frac{dU(z)}{dz} + i\frac{\omega}{c}A_z(z) \tag{8.11}$$

We consider now the vector potential $A_z(z)$ induced by the current $I(z)$ flowing in the stick and obtain $A_z(z)$ by the same procedure as was used to estimate the potential U. Thus, with the same logarithmic accuracy we find

$$A_z(z) = \frac{1}{c}\int_{-a}^{a} \frac{I(z')\exp\left(ik\left|z-z'\right|\right)}{\sqrt{\left(z-z'\right)^2+\rho(z)^2}}\,dz'$$
$$\simeq \frac{2}{c}I(z)\ln\left(\frac{2a}{b}\right) + \frac{1}{c}\int_{-a}^{a} \frac{I(z')\exp(ik\left|z-z'\right|) - I(z)}{\left|z-z'\right|}\,dz' \qquad (8.12)$$

where c is the speed of light. To estimate the second integral in equations (8.12), we approximate the current $I(z)$, which is an even function of z, as $I(z) = I(0)\left[1-(z/a)^2\right]$; thus we obtain for $z \ll a$ that

$$\frac{1}{c}\int_{-a}^{a} \frac{I(z')\exp\left(ik\left|z-z'\right|\right) - I(z)}{\left|z-z'\right|} \simeq \frac{1}{c}I(z)\left[-1+l\,(ka)\right] \qquad (8.13)$$

where the function $l(x)$ is given by

$$l(x) = \left[2+2e^{i\,x}\,(i\,x-1)+x^2\right]x^{-2} + 2\,\mathrm{Ei}(x) \qquad (8.14)$$

By substituting equation (8.13) in equation (8.12), we obtain the following relation between the vector potential and current:

$$A_z(z) = \frac{L}{c}I(z) \qquad (8.15)$$

where L is the inductance per unit length,

$$L \simeq 2\ln\left(\frac{2a}{b}\right) - 1 + l(ka) \qquad (8.16)$$

The last term in equation (8.16) is much smaller than the first one when $2\ln(2a/b) \gg 1$. Nevertheless, we keep this term since, as we show below, it plays an important role in the electromagnetic response. Equation (8.16) is invalid near the ends of the stick; however, in calculating the polarizability, this region is unimportant.

By substituting equations (8.11) and (8.15) in equation (8.10), we obtain the following form of Ohm's Law:

$$-\frac{dU(z)}{dz} = \left(R - i\frac{\omega L}{c^2}\right)I(z) - E_0 \qquad (8.17)$$

To obtain a closed equation for the current $I(z)$, we differentiate equation (8.9) with respect to z and substitute the result into equation (8.17) for $dU(z)/dz$. Thus we obtain

$$\frac{d^2I(z)}{dz^2} + i\omega C\left[\left(R - i\frac{\omega L}{c^2}\right)I(z) - E_0\right] = 0 \qquad (8.18)$$

with the boundary conditions requiring the vanishing current at the ends of the stick,

$$I(-a) = 0, \qquad I(a) = 0 \tag{8.19}$$

A solution for equation (8.18) gives the current distribution $I(z)$ in a conducting stick irradiated by an electromagnetic wave. Then we can calculate the charge distribution and the polarizability of the stick.

As mentioned, we consider the conducting stick as a prolate spheroid with semi-axes such that $a \gg b$. To determine the impedance R in equation (8.18) we recall that the cross-section area of a spheroid at coordinate z is equal to $\pi b^2[1-(z/a)^2]$; thus we have the following expression for the impedance:

$$R = \frac{1}{\pi b^2[1-(z/a)^2]\sigma_m^*} \tag{8.20}$$

where σ_m^* is the renormalized stick conductivity taking into account the skin effect. We assume that the conductivity σ_m changes due to the skin effect in the same way as the conductivity of a long wire of radius b (see, e.g., Ref. [18], Section 61),

$$\sigma_m^* = \sigma_m f(\Delta), \qquad f(\Delta) = \frac{1-i}{\Delta}\frac{J_1[(1+i)\Delta]}{J_0[(1+i)\Delta]} \tag{8.21}$$

where J_0 and J_1 are the Bessel functions of the zeroth and first order, respectively, and the parameter Δ is equal to the ratio of the stick radius b and the skin depth,

$$\Delta = b\sqrt{2\pi\sigma_m\omega}/c \tag{8.22}$$

When the skin effect is weak (i.e., $\Delta \ll 1$) the function $f(\Delta) = 1$ and the renormalized conductivity σ_m^* is equal to the stick conductivity $\sigma_m^* = \sigma_m$. In the opposite case of a strong skin effect ($\Delta \gg 1$), the current I flows within a thin skin layer at the surface of the stick. Then equation (8.22) gives $\sigma_m^* = (1-i)\sigma_m/\Delta \ll \sigma_m$.

For further consideration, it is convenient to rewrite equations (8.18) and (8.19) in terms of the dimensionless coordinate $z_1 = z/a$ and dimensionless current $I_1 = I/\left(\sigma_m^* \pi b^2 E_0\right)$. We introduce the dimensionless relaxation parameter

$$\gamma = 2\,i\frac{b^2\,\pi\,\sigma_m^*}{a^2\,C\,\omega} = \varepsilon_m^*\left[g + \frac{b^2}{a^2}\left(1 - e^{i\,a\,k} + \mathrm{Ei}(a\,k)\right)\right] \tag{8.23}$$

where $\varepsilon_m^* = i4\pi\sigma_m^*/\omega$ is the renormalized dielectric constant for metal, and

$$g = (b/a)^2\left[\ln(2a/b) - 1\right] \tag{8.24}$$

is the depolarization factor for a very prolate ellipsoid (see, e.g., Ref. [18], Section 4). We also introduce the dimensionless frequency

$$\Omega\,(ak) = ak\sqrt{LC} \simeq ak\left[1 + \frac{1 + e^{i\,a\,k}\left(-1 + i\,a\,k + a^2\,k^2\right)}{2\,(ak)^2\log(2\,a/b)}\right] \tag{8.25}$$

Above, we assumed that the stick is aligned with the electric field of the incident electromagnetic wave. Stick composites can be formed by randomly oriented rods. In this case, we have to modify equation (8.31) for the stick polarizability. We consider a conducting stick directed along the unit vector $\mathbf{n}$ and suppose that the stick is irradiated by an electromagnetic wave with the electric field

$$\mathbf{E} = \mathbf{E}_0 \exp\left[i\left(\mathbf{k}\cdot\mathbf{r}\right)\right] \tag{8.32}$$

where $\mathbf{k}$ is the wavevector inside the composite. The current I in a strongly elongated stick is excited by the component of the electric field, which is parallel to the stick

$$\mathbf{E}_{\parallel}\left(z\right) = \mathbf{n}\left(\mathbf{n}\cdot\mathbf{E}_0\right)\exp\left[i\left(\mathbf{k}\cdot\mathbf{n}\right)z\right] \tag{8.33}$$

where z is the coordinate along the stick.

The field $\mathbf{E}_{\parallel}$ averaged over the stick orientations is aligned with the external field $\mathbf{E}_0$ and has the following magnitude:

$$E_0^*\left(z\right) = \frac{E_0}{\left(kz\right)^2}\left[\frac{\sin\left(kz\right)}{kz} - \cos\left(kz\right)\right] \tag{8.34}$$

The current in the stick is a linear function of the filed $\mathbf{E}_{\parallel}$. Since the average field $\mathbf{E}_{\parallel}$ is aligned with $\mathbf{E}_0$, the current averaged over the stick orientations is also parallel to the external field $\mathbf{E}_0$.

To obtain the current $\langle\langle I(z)\rangle\rangle$ averaged over the stick orientations and the average stick polarizability $\langle\langle P_m\rangle\rangle$, we substitute the field $E_0^*\left(z\right)$ given by equation (8.34) into equation (8.18) for the field E_0. Hereafter, the sign $\langle\langle\ldots\rangle\rangle$ stands for the average over stick orientations. Then, the current $\langle\langle I\rangle\rangle$, the polarizability $\langle\langle P_m\rangle\rangle$, and the effective dielectric permittivity depend on frequency ω and, in addition, on the wavevector k. This means that a conducting stick composite is a medium with spatial dispersion. This result is easy to understand, if we recall that a characteristic scale of inhomogeneity is the stick length $2a$, which can be of the order of or larger than the wavelength. Therefore, it is not surprising that the interaction of an electromagnetic wave with such composite has a nonlocal character and, therefore, the spatial dispersion is important. One can expect that additional waves can be be excited in the composite in the presence of strong spatial dispersion.

Below we consider wavelengths such that $\lambda > \lambda_2$; therefore, we can expand $E_0^*(z)$ in a series as

$$E_0^*\left(z\right) = \frac{E_0}{3}\left(1 - \frac{\left(kz\right)^2}{10}\right) \tag{8.35}$$

and restrict ourselves to the first term, when considering the dielectric properties. Since the average field is given by $E_0^*\left(z\right) = E_0/3$, then the average current is equal to $\langle\langle I\rangle\rangle = I/3$, where the current I is defined by equation (8.30). As a result, the stick polarizability averaged over the orientations is equal to $\langle\langle P_m\rangle\rangle = P_m/3$, where P_m is given by equation (8.31).

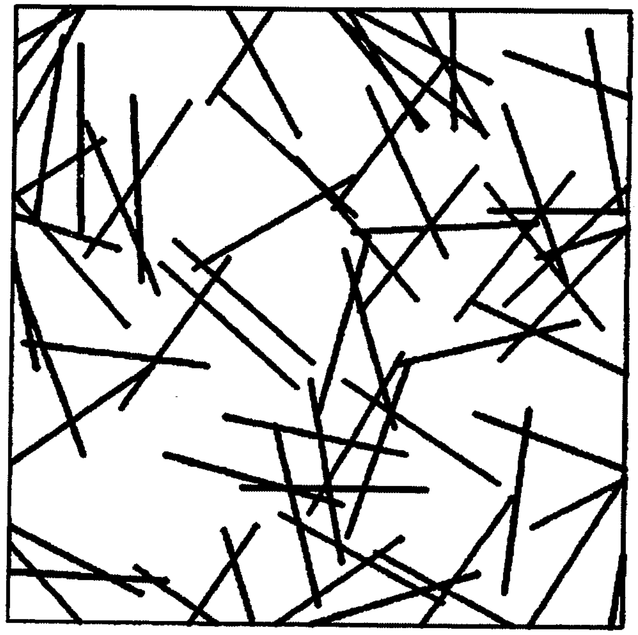

Fig. 8.1 Conducting stick composite.

8.3 CONDUCTING STICK COMPOSITES: EFFECTIVE MEDIUM APPROACH

Here we consider composites that contain very elongated conducting inclusions, "sticks," embedded in a dielectric host with dielectric constant ε_d as shown in Fig. 8.1. The sticks are randomly distributed in the host. The problem here is to calculate the macroscopic dielectric response of such a composite. Metal–dielectric composites, where conducting inclusions are very elongated, can have various important applications (see, e.g., Refs. [8, 13, 15, 20–22] and references therein). Here we show that conducting stick composites can be employed as metamaterials with tunable effective dielectric and magnetics properties.

To calculate the effective properties of a composite, we use a self-consistent approach known as the effective medium theory (EMT) [23–25]. The EMT has the virtue of mathematical and conceptual simplicity, and it is a method that provides a quick insight into the effective properties of metal–dielectric composites. Usually, the EMT is based on the assumption that metal and dielectric grains are embedded in the same homogeneous effective medium whose properties should be determined self-consistently. This assumption should be modified to take into account intrinsic structures of conducting stick composites.

Let us consider a small domain of the composite with the size $l \sim b \ll a$. The probability that the domain contains a conducting stick is estimated as $p(l) \sim$

$l^3 N(p) \sim b^3 N(p)$, where $N(p)$ is the stick concentration. The probability $p(l)$ is small even for the concentrations p corresponding to the percolation threshold $p_c \sim b/a$ [8], where it is estimated as $p(l) \sim b^3 N(p_c) \sim (b/a)^2 \ll 1$. Therefore, the dielectric constant of such a domain is equal to ε_d, with the probability close to unity. On the other hand, we can prescribe the bulk effective dielectric constant ε_e to the domain with the size l much larger than the stick length $2a$. Thus we obtain that the local dielectric constant $\varepsilon(l)$ depends on the scale under consideration: for $l < a$, the dielectric constant $\varepsilon(l)$ is equal to $\varepsilon(0) = \varepsilon_d$ and, for $l > a$, $\varepsilon(l) = \varepsilon_e$. We use a simple assumption that a conducting stick is surrounded by a medium with the dielectric constant given by

$$\begin{aligned} \varepsilon(l) &= \varepsilon_d + (\varepsilon_e - \varepsilon_d) l/a, \quad l < a \\ \varepsilon(l) &= \varepsilon_e, \quad l > a \end{aligned} \tag{8.36}$$

We can summarize the main assumptions for our effective-medium theory suggested first in Ref. [8] as follows:

1. Each conducting stick is embedded in the effective medium with the dielectric constant $\varepsilon(l)$ that depends on the scale l as described by equation (8.36). The value of ε_e is to be determined self-consistently.
2. The dielectric regions are assumed to be spherical and they are embedded in the effective medium with the dielectric constant ε_e.
3. The effective permittivity ε_e is determined by the condition that the polarizability averaged over all inclusions should vanish [10–12].

Since sticks are randomly oriented, the dielectric regions of the composite are supposed to have spherical shapes, as assumed above. The specific polarizability of a dielectric region is given then by the known quasi-static equation (see, e.g., Ref. [23])

$$\alpha_d = \frac{3(\varepsilon_d - \varepsilon_e)}{2\varepsilon_e + \varepsilon_d} \tag{8.37}$$

The polarizability of a conducting stick embedded in the effective medium (8.36) is obtained from equation (8.31), by replacing in the numerator ε_m^* with $\varepsilon_m^*/\varepsilon_e$. The scale dependence of the local dielectric constant in equation (8.36) results in a modification of the the parameter γ [see equation (8.23)] to

$$\tilde{\gamma} = \frac{\varepsilon_m^*}{\varepsilon_d}\left[\tilde{g} + \frac{b^2}{a^2}\left(1 - e^{ix} + \mathrm{Ei}(x)\right)\right]$$

where $\tilde{g} = (b/a)^2 \left[\ln(1 + 2a\varepsilon_d/b\varepsilon_e) - 1\right]$ and $x = \sqrt{\varepsilon_d} ka$ [8]. Then the condition that the average polarizability should vanish gives the following equation:

$$\langle\langle 4\pi\alpha_m \rangle\rangle + (1-p)\, 4\pi\alpha_d = \frac{1}{3} p \frac{\varepsilon_m^*}{\varepsilon_e} \frac{1}{1 + \tilde{\gamma}\cos\Omega} + 3\frac{\varepsilon_d - \varepsilon_e}{2\varepsilon_e + \varepsilon_d} = 0 \tag{8.38}$$

where p is the volume concentration of the conducting sticks and the sign $\langle\langle\cdots\rangle\rangle$ denotes the average over the orientations.

To understand the composite properties at high frequencies, we consider a solution of equation (8.38) for the stick concentration p below the percolation threshold $(b/a)^2 \ll p \ll b/a$. Assuming that $\varepsilon_d \ll |\varepsilon_e| \ll a/b$, we obtain the explicit equation for the effective dielectric permittivity

$$\varepsilon_e \simeq \varepsilon_d \frac{2}{9} p \frac{a^2}{b^2} \frac{1}{[\ln(2a/b) - e^{ix} + \mathrm{Ei}(ka)] \cos[\Omega(ka)] + \varepsilon_d/\varepsilon_m^*} \tag{8.39}$$

where functions $\mathrm{Ei}(x)$ and $\Omega(x)$ are defined in equations (8.6) and(8.25), respectively.

We consider now the effective dielectric permittivity ε_e for the case of a perfect metal ($|\varepsilon_m| \to \infty$). Then the electromagnetic field does not penetrate in a metal and, as follows from equation (8.21), and the renormalized conductivity ε_m^* also tends to infinity. We neglect the second term in the denominator of equation (8.39) and obtain that the effective permittivity ε_e has maxima when $\mathrm{Re}\,\Omega = \pi/2 + n\pi$, $n = 0, 1, 2\ldots$, which approximately corresponds to the wavelengths $\lambda_n = \left(4a/\sqrt{\varepsilon_d}\right)/(1+2n)$.

Now we consider the behavior of the effective dielectric constant near the lowest resonance frequency $\omega_0 = \pi c/\left(2a\sqrt{\varepsilon_d}\right)$. By expanding the denominator of equation (8.39) in a power series of $\omega - \omega_0$ and taking into account that $\varepsilon_m^* \to \infty$ and $\ln(a/b) \gg 1$, we obtain

$$\varepsilon_e \simeq \varepsilon_d p \frac{4}{9\pi \log(2a/b)} \frac{a^2}{b^2} \frac{1}{(\omega_0^* - \omega)/\omega_0 - i\gamma} \tag{8.40}$$

where $\omega_0^* = \omega_0 \left[1 + 2(\pi - 2)/\left(\pi^2 \log(2a/b)\right)\right] \approx \omega_0$ and the loss factor $\gamma = \left(\pi^2 - 4\right)/\left[\pi^2 \log(2a/b)\right] \ll 1$.

It is interesting to point out that the effective dielectric constant is independent of the metal conductivity ε_m, as it should be for the limiting case when the electromagnetic field does not penetrate to the metal. At the resonance frequency $\omega = \omega_0^*$ the real part of ε_e changes its sign and it becomes negative when $\omega > \omega_0$. The imaginary part of ε_e has a maximum at the resonance and its magnitude

$$\varepsilon_e''(\omega_0) \simeq \varepsilon_d \frac{4\pi}{9(\pi^2 - 4)} p \frac{a^2}{b^2} \tag{8.41}$$

does not depend on the conductivity of the sticks. We obtain that the imaginary part of the effective dielectric constant does not vanish for composites with perfectly conducting sticks. The presence of the effective losses, in this case, is due to the excitation of the internal modes in the composite. When ε_m and the dielectric host have no losses, the amplitudes of these modes continuously increase with time. In real composites, there are always some losses in the conducting sticks as well as in the dielectric host. Therefore, the internal field should stabilize at some large values. Thus, one can anticipate the existence of giant local fields in conducting stick composites.

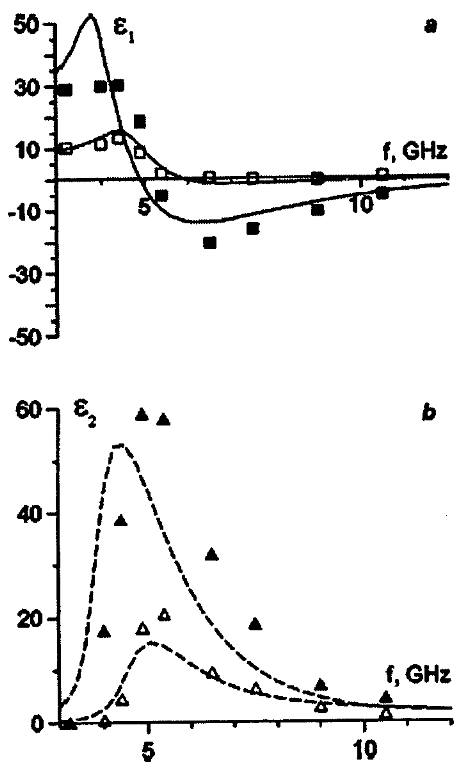

Fig. 8.2 Real (a) and imaginary (b) parts of the permittivity for a composite filled with aluminum-coated fibers 20 mm long (thickness $\sim 1\ \mu$m). The fiber volume concentration is 0.01% and 0.03%. Points indicate experimental data and line describe theoretical results.

Microwave metamaterials with negative dielectric permittivity were first obtained earlier [13,20]. In Fig. 8.2 we present experimental and theoretical results obtained in Ref. [13] for the microwave dielectric function of composites containing very thin aluminum microwires. In such metamaterials the real part of ε_e becomes negative for the frequency above the resonance as seen in Fig. 8.2.

8.4 CONDUCTING STICK COMPOSITES: GIANT ENHANCEMENT OF LOCAL FIELDS

We consider now the field distribution in thin (~ 10 nm) but relatively long ($\sim 1\ \mu$m) metal sticks. A problem of field distribution around such metal particles can hardly be solved analytically. We describe a numerical model based on the discrete dipole approximation (DDA) following our papers [4,7]. This approach was first introduced by Purcell and Pennypacker [26].

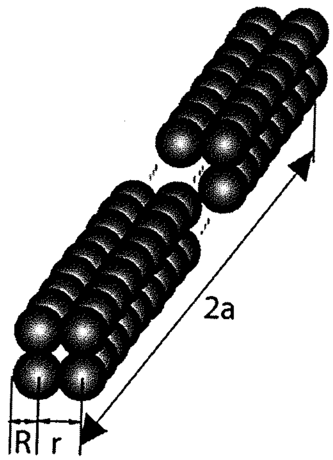

Fig. 8.3 Long stick modeled by chains of spheres. After Ref. [4]. Copyright © 2003 Optical Society of America, Inc.

In the DDA approximation, a conducting stick is represented by a large amount of small spherical particles of some radius R as shown in Fig. 8.3. Each particle is placed in a node of a cubic lattice with period a. The position of individual particles is denoted by $\mathbf{r}_i$. It is supposed that the particle radius R is much smaller than the wavelength λ so that interactions of particles are well described by their dipole moments $\mathbf{d}_i$. Each particle is subjected to an incident field $\mathbf{E}_0$ and to the field scattered by all other particles. Therefore, the dipole moments of particles are coupled to the incident field and to each other and can be found solving the following coupled-dipole equations (CDEs):

$$\mathbf{d}_i = \alpha \left[\mathbf{E}_0 \left(\mathbf{r}_i \right) + \sum_{j \neq i} \hat{G}(\mathbf{r}_i - \mathbf{r}_j) \mathbf{d}_j \right] \tag{8.42}$$

where α is the polarizability of a particle, $\mathbf{E}_0(\mathbf{r}_i)$ is the incident field at point $\mathbf{r}_i$, and $\hat{G}(\mathbf{r}_i - \mathbf{r}_j)\mathbf{d}_j$ gives the field produced by dipole $\mathbf{d}_j$ at the point $\mathbf{r}_i$ and $\hat{G}(\mathbf{r}_i - \mathbf{r}_j)$ is the free-space dyadic Green's function:

$$\begin{aligned} G_{\alpha\beta} &= k^3[A(kr)\delta_{\alpha\beta} + B(kr)r_\alpha r_\beta] \\ A\,(x) &= [x^{-1} + ix^{-2} - x^{-3}] \exp(ix) \\ B(x) &= [-x^{-1} - 3ix^{-2} + 3x^{-3}] \exp(ix) \end{aligned} \tag{8.43}$$

with $\hat{G}\mathbf{d} \equiv G_{\alpha\beta} d_\beta$. The Greek indices represent Cartesian components and the summation over the repeated indices is implied. The polarizability α of an individual dipole is given by Lorentz–Lorenz formula with the radiative correction introduced

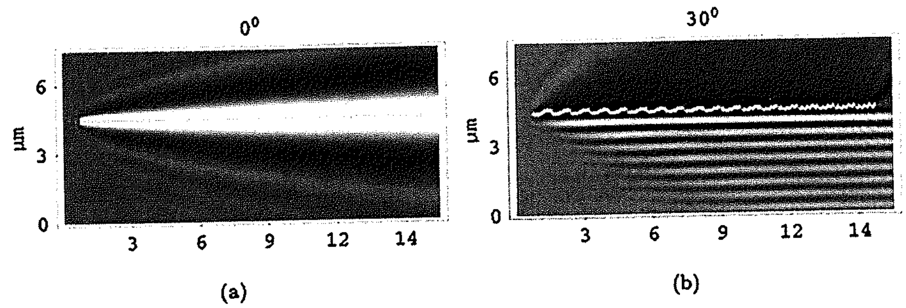

Fig. 8.4 EM field distribution for a long needle. The wavelength of incident light is 540 nm. The angle between the wavevector of incident light and the needle is (a) 0° and (b) 30°. After Ref. [4]. Copyright © 2003 Optical Society of America, Inc.

by Draine [27]:

$$\alpha_{LL} = R^3 \frac{\varepsilon_m - 1}{\varepsilon_m + 1}, \qquad \alpha = \frac{\alpha_{LL}}{1 - i\,(2/3)\,(kR)^3\,\alpha_{LL}} \tag{8.44}$$

Results of calculations for (8.42) depend on the intersection ratio r/R—that is, the ratio of the distance between neighboring particles and its radius (see Fig. 8.3). We choose the ratio as $r/R \approx 1.66$ to reproduce the quasi-static polarizability of an elongated metal ellipsoid.

In our numerical simulations [4, 7], a single nanostick was represented by four parallel chains of spherical particles to take into account the skin effect (see Fig. 8.3). Specifically, we consider the field distribution in the vicinity of a conducting stick with roughly $2b = 30$ nm thickness and $2a \approx 15\,\mu$m long, illuminated by a plane wave with the wavelength of 540 nm. Our results, shown in Fig. 8.4, clearly identify the interference pattern between irradiation and the plasmon polariton wave excited on the metal surface. Similar interference patterns were observed in experiments [28, 29]. Note that the electromagnetic field is concentrated around the wire surface, which suggests the possibility to use nanowires as nano waveguides.

Simulations for shorter sticks ($2a = 480$ nm) presented in Fig. 8.5 also show the existence of sharp plasmon resonances [4, 7] when the wavelength of the light is a multiple of surface plasmon (half) wavelengths. The enhancement of the local field intensity in the resonance can reach the magnitude of 10^3. The spatial area where the field concentrates is highly localized around the nanowire, and it can be as small as 100 nm. This plasmon resonance is narrowband, with the spectral width in a single nanowire about 50 nm, which corresponds to the discussed above equations (8.39) and (8.40).

In a composite with metal sticks randomly distributed in a dielectric substrate, the metal–dielectric transition occurs at a significantly smaller metal concentration than in the case of percolation films formed by spherical particles. In the 2-D case of a

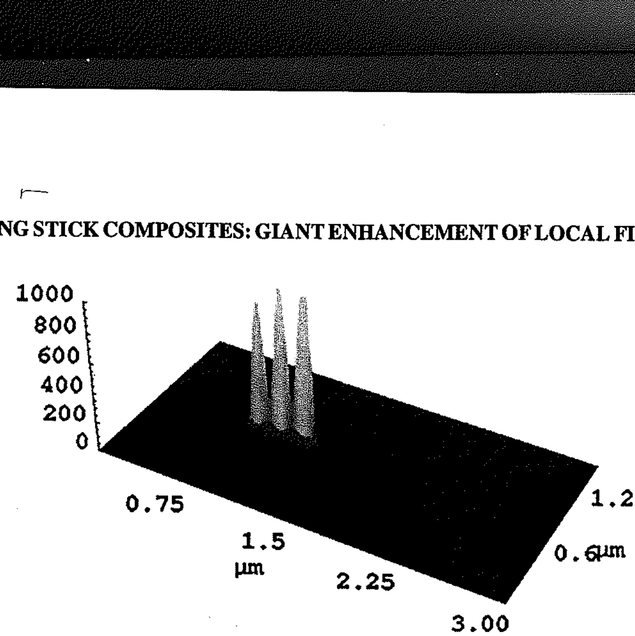

Fig. 8.5 The intensity distribution of the electric field at surface plasmon polariton resonance in a silver nanowire excited by a plane electromagnetic wave. The angle between the nanowire and the wavevector of the incident light is 30 degrees. The wavevector and **E** vector of the incident irradiation are in the plane of the figure; the needle length is 480 nm. After Ref. [4]. Copyright © 2003 Optical Society of America, Inc.

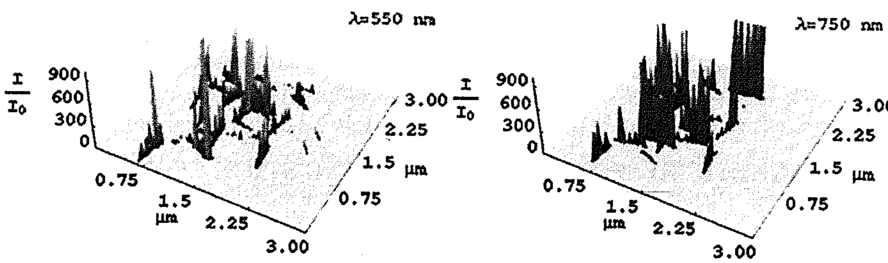

Fig. 8.6 Field distribution in nanowire percolation Ag composite for the incident wavelength of 550 nm (left) and 750 nm (right). In both figures the case of normal incidence with $(E||x)$ is considered. After Ref. [4]. Copyright © 2003 Optical Society of America, Inc.

wire composite, the percolation threshold is close to the inverse of the stick aspect ratio [8], and hence it can be arbitrary small for sufficiently long sticks.

We simulate composites by a random distribution of identical metal nanowires over a dielectric surface. In these simulations, the length of individual nanowires is given by $2a = 480$ nm, while their diameter is 30 nm. Figure 8.6 shows the intensity $|E|^2$ of the local electric field at wavelengths $\lambda = 540$ and 750 nm. Our simulations exhibit the existence of localized plasmon modes in such composites. Similar to localization of quasi-static plasmon modes [30], the localization of plasmon-polaritons bounded in the metal nanowires leads to large enhancement of local optical fields. Our simulations suggest that the local intensity enhancement factor reaches 10^3.

Our simulations also show that plasmon modes cover a broad spectral range. The incident field at a given wavelength excites small resonant parts of the percolation

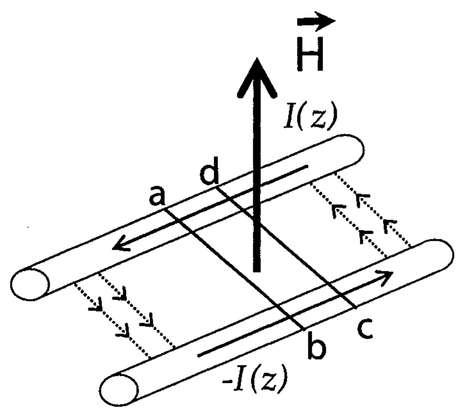

Fig. 8.7 Current in the two-stick circuit excited by external magnetic field **H**. The displacement currents, "closing" the circuit, are shown by dashed lines.

system, resulting in a large enhancement of the local fields in these elements. In our case, such resonating elements can be single nanowires or groups of nanowires. Different clusters of wires resonate at different frequencies and all together cover a broad spectral range where the stick composite has plasmon modes.

8.5 MAGNETIC RESPONSE OF CONDUCTING STICK COMPOSITES

We consider now a metal–dielectric composite consisting of pairs of parallel metal sticks embedded in a dielectric host. We assume that the volume concentration p of the sticks is less than the percolation threshold $p < p_c \sim b/a \ll 1$. We also suppose that neither the sticks nor the dielectric have magnetic properties. One might think that the composite has no magnetic properties under such conditions. Indeed, the magnetic response of a single conducting stick is small even at high frequencies (Ref. [18], Section 59). Since we have concentration $p \ll 1$, one could anticipate that the response of the entire composite is also small.

In reality, as we show below, the composite may have a giant magnetic response at some frequencies. The reason for such a behavior is the resonant response of the stick pairs to a high-frequency magnetic field. The external magnetic field excites electric currents in these stick pairs. The magnetic moments for the currents flowing in the stick pairs result in the magnetic response of the composite. Consider a pair of the sticks and suppose that an external magnetic field $H = H_0 \exp(-i\omega t)$ is applied perpendicular to their plane. This field excites a circular current I in the system of two parallel sticks. The circular current I flows in one stick in one direction and in the opposite direction in another stick as shown in Fig. 8.7. The displacement currents flowing between the two sticks close the circuit. The considered two-stick circuit acts as the well-known two-wire transmission line excited by an external magnetic field. The current I in the two-wire line can be calculated from the Telegrapher's equation (see, e.g., Refs. [17] and [31]). The electrodynamics processes in the line of two wires, separated by a distance d are determined by the impedance per unit length,

$$Z = \frac{2}{\sigma_m^* \pi b^2} - i\frac{\omega}{c^2} L_2 \tag{8.45}$$

where σ_m^* and b are the renormalized stick conductivity [see equation (8.21)] and radius, respectively. The parameter L_2 is the self-inductance per unit length for a system of two parallel straight wires having a cross section of radius b. We define the inductance L_2, following the procedure used to define the inductance of a single stick [see derivation of equation (8.16)]. Thus we obtain

$$L_2 = 4\ln(d/b) - (d/a)^2 + \frac{1}{6}(dk)^2\,[3 + 4iak + 6\,\log(2\,a/d)] \tag{8.46}$$

where d is the distance between the axes of the wires. Another important parameter is the mutual capacity per unit length C_2 of two wires. The approach that has been used to define a capacitance of a single stick [see equation (8.8)] results in the following equation for C_2

$$C_2 = \frac{\varepsilon_d}{4\,\log(d/b) - 3\,(d/a)^2 + (d\,k)^2\,[2\,\log(2\,a/d) - 1]\,/2} \tag{8.47}$$

where ε_d is the permittivity of the dielectric host. The capacitance C_2 determines the value of the displacement currents flowing between the two wires. Following the procedure described in Section 8.2, we introduce the current I as the current in a single stick. This current depends on the coordinate z along the stick. We also introduce the potential difference $U(z)$ between the two sticks. Using Faraday's Law

$$\oint_{(a,b,c,d)} \mathbf{E}\,d\mathbf{l} = i\frac{\omega}{c}\iint_S \mathbf{H}\,d\mathbf{s} \tag{8.48}$$

where $S = d \times dz$ is the area restricted by the contour (a, b, c, d) as shown in Fig. 8.7, we find

$$-\frac{dU(z)}{dz} = ZI(z) + idkH_0 \tag{8.49}$$

The current $I(z)$ depends on the coordinate z since it can go out from one stick and come into another stick. The second equation for $I(z)$ and $U(z)$ is obtained from the charge conservation law. Considering the currents in the sticks and the displacement current between them we find

$$\frac{dI(z)}{dz} = i\omega C_2 U(z) \tag{8.50}$$

The combination of equations (8.49) and equation (8.50) gives the second-order differential equation for the current,

$$\frac{d^2 I(z)}{dz^2} = -g^2 I(z) + \frac{C_2 d\omega^2}{c} H_0 \tag{8.51}$$

$$-a < z < a, \quad I(-a) = I(a) = 0$$

where the parameter g equals

$$g^2 = k^2 \left[1 + \frac{1}{\log(d/b)} \left(\frac{d^2}{2\,a^2} + \frac{(dk)^2}{4} + \frac{i}{6}\, ak(dk)^2 \right) - \frac{8C_2}{(kb)^2 \varepsilon_m^*} \right] \tag{8.52}$$

and $\varepsilon_m^* = i4\pi\sigma_m^*/\omega$ is the renormalized metal permittivity [see equation (8.21)]. Note that we still assume that $d/a \ll 1$ and $dk \ll 1$. For the perfect metal ($|\varepsilon_m^*| \to \infty$), the parameter g does not depend on the metal properties. We solve equation (8.51) for the current $I(z)$ and calculate the magnetic moment $\mathbf{m}$ for the circuit in the two sticks,

$$\mathbf{m} = \frac{1}{2c} \int \left[\mathbf{r} \times \mathbf{j}\left(\mathbf{r}\right) \right] \, d\mathbf{r} \tag{8.53}$$

where $\mathbf{j}(\mathbf{r})$ is the density of the current in the two conducting sticks and the density of the displacement currents. Integration in equation (8.53) goes over the two conducting sticks as well as over the space between them where the displacement currents are flowing. From equations (8.51)–(8.53) we obtain the magnetic moment for the system of two sticks:

$$m = 2H_0 a^3 C_2 (kd)^2 \frac{\tan(ga) - ga}{(ga)^3} \tag{8.54}$$

Let us now estimate quantitatively the effective magnetic permeability μ_e of the conducting stick composite. We suppose that the stick pairs are oriented in one direction. Taking into account the definition of the effective magnetic permeability $\mu_e \mathbf{H}_0 = \mathbf{H} + 4\pi\mathbf{M}$, where $\mathbf{M}$ is the magnetic moment per unite volume, we obtain from equation (8.54) the following equation for the component of μ_e perpendicular to the pairs:

$$\mu_e \approx 1 + 4\pi n \frac{m}{H_0} \approx 1 + 4\pi p \frac{a}{b} C_2 a d k^2 \frac{\tan(ga) - ga}{(ga)^3} \tag{8.55}$$

where n is the density of the stick pairs, $p = bdan$ is the volume concentration of the pairs, and parameters C_2 and g are given by equations (8.47) and (8.52), respectively. The effective magnetic permeability μ_e of the conducting stick composite is shown in Fig. 8.8 for the concentration $p = 0.2$. The permeability μ_e reaches its maximum at the resonance and becomes negative for the wavelength below the resonance. The length of a pair $2a = 400$ nm is much smaller than the resonance wavelength $\sim 2\,\mu$m. Therefore, the spatial dispersion effects, discussed at the end of Section 8.2, can be neglected and the composite has a well-defined magnetic permeability. Thus, composite materials formed by pairs of metal nanowires can act as left-handed material with negative refraction in the optical range.

8.6 PLANAR NANOWIRE COMPOSITES

In the sections above, we considered the response of conducting stick composites to the electric and magnetic fields. In this section, we consider a planar composite comprising regular array of pairs of parallel nanowires (see Fig. 8.9), which is

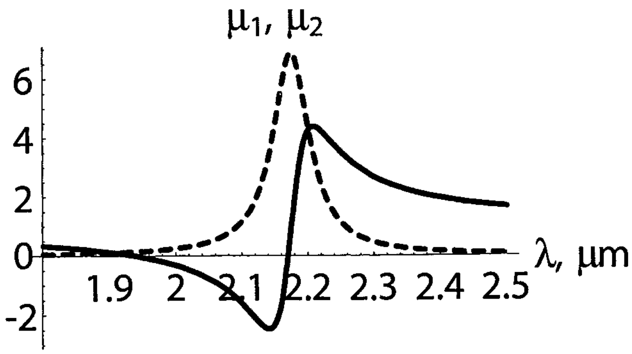

Fig. 8.8 Optical magnetic permeability $\mu = \mu_1 + \mu_2$ (μ_1, continuous line; μ_2, dashed line) of the composite containing pairs of silver sticks; $a = 200\,\text{nm}$, $d = 50\,\text{nm}$, $b = 10\,\text{nm}$.

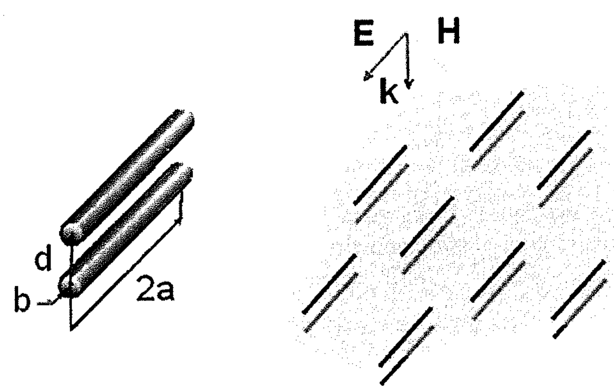

Fig. 8.9 A layer of pairs of parallel nanowires. After Ref. [4]. Copyright © 2003 Optical Society of America, Inc.

illuminated by a plane electromagnetic wave impinging perpendicular to the plane of the composite. First, we show that in the far zone the field scattered by pairs of nanowires can be approximated by the effective dipole and magnetic moments even when the size of the pair is comparable with the wavelength λ of the incident light. Then we consider the optical properties of a layer of such nanowire pairs.

Electric and magnetic fields at the distance R away from the nanowire pair with dimensions $2a \times d \times 2b$ (see Fig. 8.9) are derived from the vector potential $\mathbf{A}$ that for large distances, $R \gg \lambda, b_1, b_2, d$, takes the following standard form:

$$\mathbf{A} = \left(e^{ikr}/cR\right) \int e^{-ik(\mathbf{n}\cdot\mathbf{r})}\mathbf{j}(\mathbf{r})\, d\mathbf{r}$$

where $\mathbf{j}(\mathbf{r})$ is the current density inside the nanowires and vector $\mathbf{n}$ is the unit vector in the observation direction. We introduce the vector $\mathbf{d}$ directed from one nanowire to another, and assume that the coordinate system has its origin in the center of the system so that the centers of the wires have the coordinates $\mathbf{d}/2$ and $-\mathbf{d}/2$, respectively. The electromagnetic wave is incident in the plane of the wires perpendicular to them (see Fig. 8.9), that is, the wavevector $\mathbf{k} \parallel \mathbf{d}$. Then, the vector potential $\mathbf{A}$ can be written as

$$\mathbf{A} = \frac{e^{ikR}}{cR}\left[e^{-\frac{ik}{2}(\mathbf{n}\cdot\mathbf{d})}\int_{-b_1}^{b_1} e^{-\frac{ik}{2}(\mathbf{n}\cdot\mathbf{z})}\mathbf{j}_1(\rho)\,d\rho + e^{\frac{ik}{2}(\mathbf{n}\cdot\mathbf{d})}\int_{-b_1}^{b_1} e^{-ik\mathbf{n}\cdot\mathbf{z}}\mathbf{j}_2(\rho)\,d\rho\right] \tag{8.56}$$

where $\mathbf{j}_1$ and $\mathbf{j}_2$ are the currents in the wires, and $\mathbf{z}$ is the coordinate along the wires ($\mathbf{z} \perp \mathbf{d}$). As known, the dipole component is dominated in scattering by a thin antenna even for an antenna size comparable to the wavelength (see, e.g., Ref. [34]). Therefore we can approximate the term $e^{-ik\mathbf{n}\cdot\mathbf{z}}$ in equation (8.56) by unity. Note that for the forward and backward scattering, which are responsible for the effective properties of a medium, this term is exactly equal to one.

We consider the system where the distance d between the wires is much smaller than the wavelength and expand equation (8.56) in a series over d. This results in

$$\mathbf{A} = \frac{e^{ikR}}{cR}\left[\int_{-b_1}^{b_1}(\mathbf{j}_1 + \mathbf{j}_2)\,d\rho - \frac{ik}{2}(\mathbf{n}\cdot\mathbf{d})\int_{-b_1}^{b_1}(\mathbf{j}_1 - \mathbf{j}_2)\,d\rho\right] \tag{8.57}$$

The first term in the square brackets in equation (8.57) gives the effective dipole moment $\mathbf{P}$ for the system of two nanowires and its contribution to the scattering can be written as $\mathbf{A}_d = -ik\left(e^{ikR}/R\right)\mathbf{P}$, where

$$\mathbf{P} = \int \mathbf{p}(\mathbf{r})\,d\mathbf{r} \tag{8.58}$$

and $\mathbf{p}$ is the local polarizations. The integration in equation (8.58) is over the volume of both wires. The second term in equation (8.57) gives the magnetic dipole and quadrupole contributions to the vector potential:

$$\mathbf{A}_{mq} = \frac{ike^{ikR}}{R}\left[[\mathbf{n}\times\mathbf{M}] - \frac{\mathbf{d}}{2c}\int_{-b_1}^{b_1}(\mathbf{n}\cdot(\mathbf{j}_1 - \mathbf{j}_2))\,d\rho\right] \tag{8.59}$$

where $\mathbf{M}$ is the magnetic moment of two wires,

$$\mathbf{M} = \frac{1}{2c}\int[\mathbf{r}\times\mathbf{j}(\mathbf{r})]\,d\mathbf{r} \tag{8.60}$$

and the integration is over the volume of the wires as in equation (8.58).

We show now results of our numerical simulations for the optical properties of gold nanowires (Fig. 8.10). According to our simulations, both the dielectric and magnetic moments excited in the nanowire system are opposite to the excited field when the wavelength of the incident field is below resonance. Thus, in this frequency range, a composite material based on parallel nanowire pairs have the dielectric permittivity

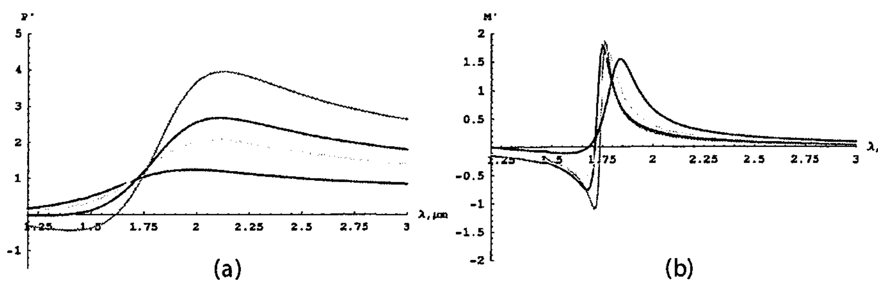

Fig. 8.10 Dielectric (a) and magnetic (b) moments in nanowire pairs as functions of wavelengths. The distance between the nanowires in the pairs is varied: $d = 0.15\,\mu$m (1), $d = 0.23\,\mu$m (2), $d = 0.3\,\mu$m (3), and $d = 0.45\,\mu$m (4); for all plots, $a = 0.35\,\mu$m and $b = 0.05\,\mu$m. The moments are normalized to the unit volume. After Ref. [4]. Copyright © 2003 Optical Society of America, Inc.

and magnetic permeability both negative and thus the composite acts as a left-handed material. These results are in good qualitative agreement with equations (8.31) and (8.54) which were derived for the case of needles with a high aspect ratio.

We consider now the transmittance and reflectance of a planar nanowire composite when an electromagnetic wave impinges normal to its plane. We take into account the dipole $\mathbf{P}$ and magnetic $\mathbf{M}$ moments given by equations (8.58) and (8.60), respectively, since they are responsible for the main contribution to forward and backward scattering. The second term in equation (8.59) describes a quadrupole contribution, which vanishes for the forward direction. (see discussion in Ref. [4]).

Maxwell's equations for the composite can be written in the following form

$$\operatorname{curl}\mathbf{E} = ik\mathbf{H}, \qquad \operatorname{curl}\mathbf{H} = \frac{4\pi}{c}\mathbf{j} - ik\mathbf{E} \tag{8.61}$$

where $\mathbf{j}$ is the current in the nanowires. We split the current $\mathbf{j}$ in two parts $\mathbf{j} = \mathbf{j}_P + \mathbf{j}_M$. Here $\mathbf{j}_M$ is the circular current in the nanowire pair. This current can be presented as $\mathbf{j}_M = c \operatorname{curl}\mathbf{m}$ with the vector $\mathbf{m}$ vanishing outside the composite. Then equations (8.61) can be rewritten as

$$\operatorname{curl}\mathbf{E} = ik\left(\mathbf{H}' + 4\pi\mathbf{m}\right), \qquad \operatorname{curl}\mathbf{H}' = \frac{4\pi}{c}\mathbf{j}_P - ik\mathbf{E} \tag{8.62}$$

where $\mathbf{H}' = \mathbf{H} - 4\pi\mathbf{m}$. We suppose that $z = 0$ is the principal plane of the composite and the electromagnetic wave is incident along the z-axis. We average equations (8.62) over the $\{x, y\}$ plane and integrate them over the space between the two reference planes placed in front ($z = -a$) and behind ($z = a$) the composite. The distance a is chosen so that $d \ll a \ll 1/k$. After the integration, equations (8.62) take the following form

$$\mathbf{E}_2 - \mathbf{E}_1 \cong ik4\pi dp\mathbf{M}_1, \quad \mathbf{H}_2 - \mathbf{H}_1 = -ik4\pi dp\mathbf{P}_1 \tag{8.63}$$

where $\mathbf{E}_1 = \mathbf{E}(-a)$, $\mathbf{E}_2 = \mathbf{E}(a)$, $\mathbf{H}_1 = \mathbf{H}(-a)$, $\mathbf{H}_2 = \mathbf{H}(a)$; $\mathbf{P}_1 = \mathbf{P}/(b_1 b_2 d)$ and $\mathbf{M}_1 = \mathbf{M}/(b_1 b_2 d)$ are the dipole and magnetic moments of the nanowire pairs. These moments are given by equations (8.58) and (8.60) which are normalized to the volume of the pairs, p is the filling factor, that is, the ratio of the area covered by the nanowires and the total area of the film.

The moments $\mathbf{P}_1$ and $\mathbf{M}_1$ are proportional to the effective electric and magnetic fields, respectively. For the dilute case ($p \ll 1$) considered here we can write $\mathbf{P}_1$ and $\mathbf{M}_1$ as $4\pi p \mathbf{P}_1 = \varepsilon(\mathbf{E}_2 + \mathbf{E}_1)/2$ and $4\pi p \mathbf{M}_1 = \mu(\mathbf{H}_2 + \mathbf{H}_1)/2$, where the coefficients ε and μ are the effective dielectric constant and magnetic permeability of the nanowire composite. Then equations (8.63) take the following form:

$$\mathbf{E}_2 - \mathbf{E}_1 \cong ikd\mu(\mathbf{H}_1 + \mathbf{H}_2), \quad \mathbf{H}_2 - \mathbf{H}_1 = -ikd\varepsilon(\mathbf{E}_1 + \mathbf{E}_2) \tag{8.64}$$

We match equation (8.64) at $z = -a$ with the plane wave solution

$$E = E_0[\exp(ikz) + r\exp(-ikz)]$$

that holds in front of the film ($z < -a$) and match equations (8.64) at $z = a$ with the solution $E = E_0 t \exp(ikz)$ that holds behind the film. E_0 is the amplitude of the impinging wave, r and t are reflection and transmission coefficients, respectively. This matching results in two equations for r and t. Solutions to these equations allow us to find the reflection R and transmittance TT coefficients of the nanowire composite in the following form

$$R = \left|\frac{2dk(\varepsilon - \mu)}{(-2 + idk\varepsilon)(-2 + idk\mu)}\right|^2, \quad T = \left|\frac{4 + d^2k^2\varepsilon\mu}{(-2 + idk\varepsilon)(-2 + idk\mu)}\right|^2 \tag{8.65}$$

When $\varepsilon = \mu$, the reflectance vanishes while the transmittance is given by $T = |(2 + idk\varepsilon)/(2 - idk\varepsilon)|^2$. If $\varepsilon = \mu$ and it is a real number, the reflectance $T = 1$. Still, the interaction of the electromagnetic wave with the composite results in the phase shift $2\arctan(dk\varepsilon/2)$ for the transmitted wave. The phase shift is positive if $\varepsilon = \mu > 0$ and the shift is negative when $\varepsilon = \mu < 0$. The last case corresponds to a left-handed material. Thus, a negative phase of the transmitted electromagnetic wave indicates the left-handedness of the composite.

8.7 CONCLUSIONS

We presented a detailed study of the electrodynamic properties of metal–dielectric composites consisting of elongated conducting inclusions—conducting sticks—embedded in a dielectric host. Conducting stick composites have new and unusual properties at high frequencies when surface plasmon-polaritons are excited in the sticks. The effective dielectric permittivity has strong resonances at some frequencies. The real part vanishes at the resonance and acquires negative values for frequencies above resonance. The dispersion behavior does not depend on the stick conductivity

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