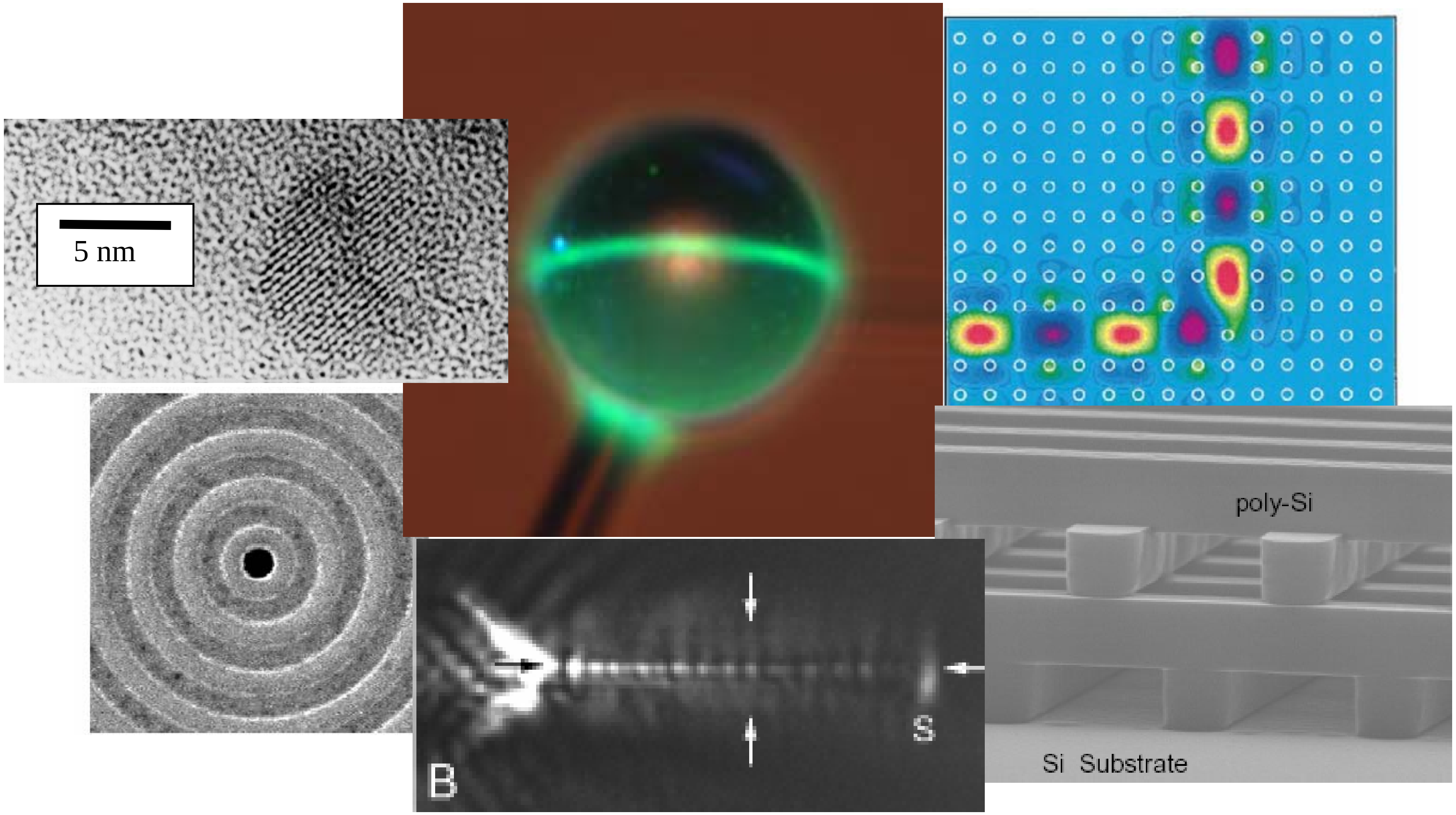
Nanophotonics and Metamaterials*

Professor Vladimir M. Shalaev



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Overview of the Course

Part I: a) Light interaction with matter and b) Photonic Crystals

a) Maxwell's equations
Dielectric properties of insulators, semiconductors and metals (bulk)
Light interaction with nanostructures and microstructures (compared with λ)

b) Electromagnetic effects in periodic media
Media with periodicity in 1, 2, and 3-dimensions
Applications: Omni-directional reflection, sharp waveguide bends,
Light localization, Superprism effects, Photonic crystal fibers

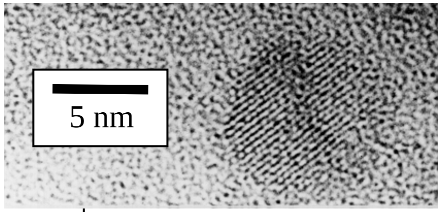
Part II: Metal optics (plasmonics) and nanophotonics

Light interaction with 0, 1, and 2 dimensional metallic nanostructures
Guiding and focusing light to nanoscale (below the diffraction limit)
Near-field optical microscopy
Transmission through subwavelength apertures

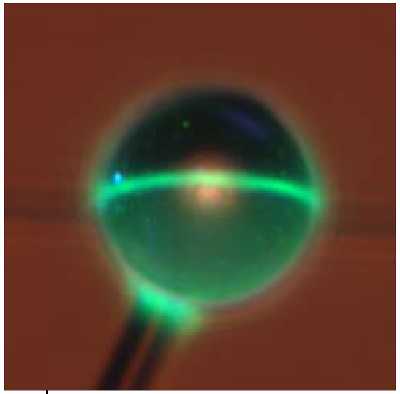
Part III: Metamaterials

Metamaterials, optical magnetism, and negative refractive index
Perfect lens
How to make objects invisible: *Cloaking*

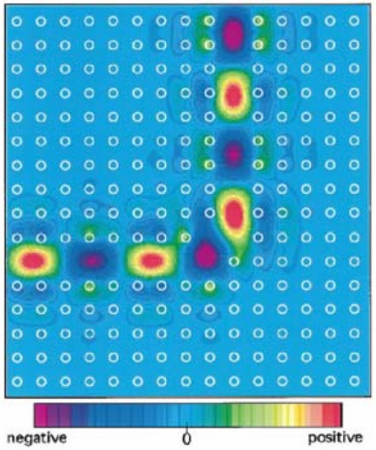
Overview in Images



K.S. Min et al. PhD Thesis



K.V. Vahala et al, Phys. Rev. Lett, 85, p.74 (2000)



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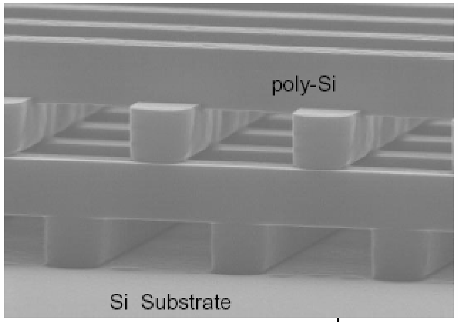
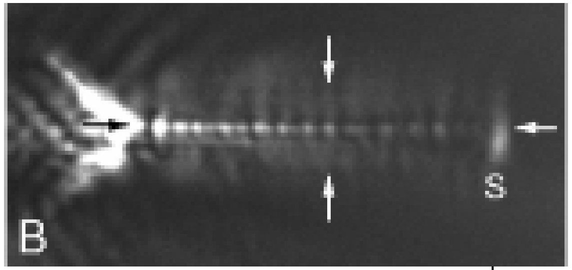


S. Lin et al, Nature, vol. 394, p. 251-3, (1998)



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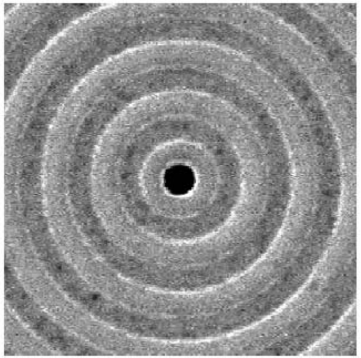


T.Thio et al., Optics Letters **26,** 1972-1974 (2001).

Motivation

Major breakthroughs are often materials related

- Stone Age, Iron Age, Si Age,….metamaterials
- People realized the utility of naturally occurring materials
- Scientists are now able to engineer new functional nanostructured materials

Is it possible to engineer new materials with useful optical properties

- Yes ! ☺

- Wonderful things happen when structural dimensions are $\approx \lambda_{light}$ and much less

This course talks about what these "things" are…and why they happen

What are the smallest possible devices with optical functionality ?

- Scientists have gone from big lenses, to optical fibers, to photonic crystals, to…
- Does the diffraction set a fundamental limit ?
- Possible solution: metal optics/plasmonics

A. Light Interaction with Matter: Maxwell Equations & Constitutive Relations

Maxwell's Equations

Divergence equations

$$\nabla \cdot \boldsymbol{D} = \rho_f$$

$$\nabla \cdot \boldsymbol{B} = 0$$

Curl equations

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial \boldsymbol{t}} + \boldsymbol{J}$$

$\boldsymbol{D}$ = Electric flux density

$\boldsymbol{E}$ = Electric field vector

ρ = charge density

$\boldsymbol{B}$ = Magnetic flux density

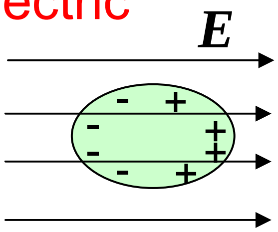
$\boldsymbol{H}$ = Magnetic field vector

$\boldsymbol{J}$ = current density

Constitutive Relations

Constitutive relations relate flux density to polarization of a medium

Electric



When $\boldsymbol{P}$ is proportional to $\boldsymbol{E}$

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}\left(\boldsymbol{E}\right) = \varepsilon \boldsymbol{E}$$

Electric polarization vector…… Material dependent!!

ε_0 = Dielectric constant of vacuum = 8.85 · 10^{-12} $C^2N^{-1}m^{-2}$ [F/m]

ε = Material dependent dielectric constant

Total electric flux density = Flux from external E-field + flux due to material polarization

Magnetic

$$\boldsymbol{B} = \mu_0 \boldsymbol{H} + \mu_0 \boldsymbol{M}\left(\boldsymbol{H}\right)$$

Magnetic flux density Magnetic field vector Magnetic polarization vector

μ_0 = permeability of free space = 4πx10^{-7} H/m

Note: For now, we will focus on materials for which

$$\boldsymbol{M} = 0 \Rightarrow \boldsymbol{B} = \mu_0 \boldsymbol{H}$$

Speed of an EM Wave in Matter

Speed of the EM wave:

Compare $\nabla^2 \boldsymbol{E} = \mu_0 \varepsilon_0 \varepsilon_r \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$ and $\nabla^2 \boldsymbol{E} = \frac{1}{v^2} \frac{\partial^2 \boldsymbol{E}}{\partial \boldsymbol{t}^2}$

$$\Rightarrow \quad v^2 = \frac{1}{\mu_0 \varepsilon_0} \frac{1}{\varepsilon_r} = \frac{c_0^2}{\varepsilon_r}$$

Where $c_0^2 = 1/(\varepsilon_0\, \mu_0) = 1/((8.85\text{x}10^{-12}\ \text{C}^2/\text{m}^3\text{kg})\ (4\pi \text{ x } 10^{-7}\ \text{m kg/C}^2)) = (\, 3.0 \text{ x } 10^8\ \text{m/s})^2$

Optical refractive index

Refractive index is defined by: $n = \frac{c}{v} = \sqrt{\varepsilon_r} = \sqrt{1 + \chi}$

Note: Including polarization results in same wave equation with a different ε_r ⇒ c becomes v

Dispersion Relation

Dispersion relation: ω = ω(k)

Derived from wave equation $$\nabla^2 \boldsymbol{E}\left(\boldsymbol{r},t\right) = \frac{n^2}{c^2}\frac{\partial^2 \boldsymbol{E}\left(\boldsymbol{r},t\right)}{\partial \boldsymbol{t}^2}$$

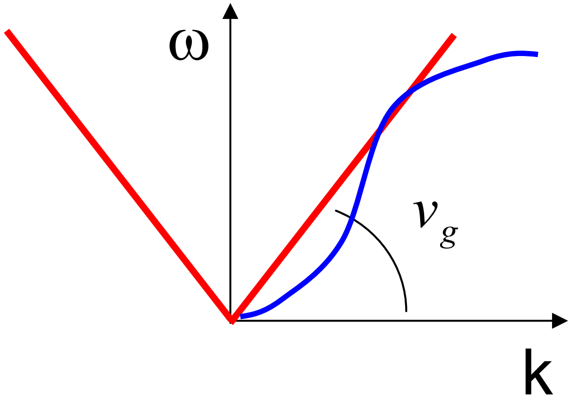
Substitute: $$\boldsymbol{E}\left(z,t\right) = \mathrm{Re}\left\{\boldsymbol{E}\left(z,\omega\right)\exp\left(-ikr + i\omega t\right)\right\}$$

Result: $$k^2 = \frac{n^2}{c^2}\omega^2$$

Check this!

$$\Downarrow$$

$$\omega^2 = \frac{c^2}{n^2}k^2$$



Group velocity: $$v_g \equiv \frac{d\omega}{dk}$$

Phase velocity: $$v_{ph} = \frac{\omega}{k} = \frac{c}{n} = \frac{c}{\sqrt{\varepsilon_r}} = \frac{c}{\sqrt{1+\chi}}$$

Electromagnetic Waves

Solution to: $$\nabla^2 \boldsymbol{E}(\boldsymbol{r},t) = \frac{n^2}{c^2}\frac{\partial^2 \boldsymbol{E}(\boldsymbol{r},t)}{\partial \boldsymbol{t}^2}$$

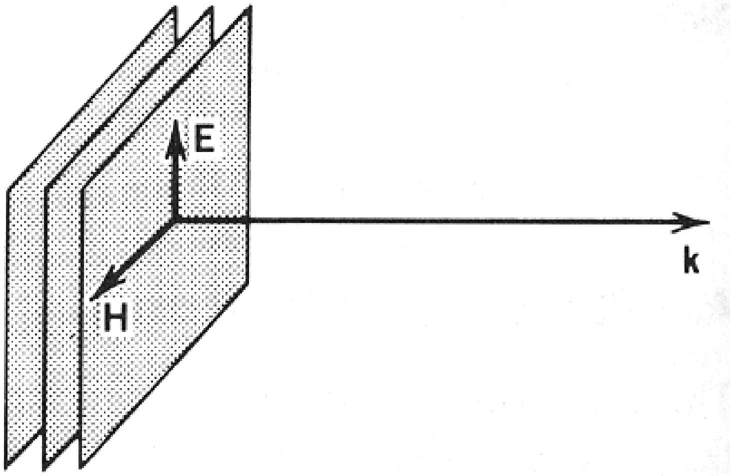
Monochromatic waves:

$$\boldsymbol{E}(\boldsymbol{r},t) = \mathrm{Re}\left\{\boldsymbol{E}(\boldsymbol{k},\omega)\exp\left(-i\boldsymbol{k}\cdot\boldsymbol{r}+i\omega t\right)\right\}$$

$$\boldsymbol{H}(\boldsymbol{r},t) = \mathrm{Re}\left\{\boldsymbol{H}(\boldsymbol{k},\omega)\exp\left(-i\boldsymbol{k}\cdot\boldsymbol{r}+i\omega t\right)\right\}$$

Check these are solutions!

TEM wave



Symmetry Maxwell's Equations result in $\boldsymbol{E} \perp \boldsymbol{H} \perp$ propagation direction

Optical intensity

Time average of Poynting vector: $\boldsymbol{S}(\boldsymbol{r},t) = \boldsymbol{E}(\boldsymbol{r},t) \times \boldsymbol{H}(\boldsymbol{r},t)$

EM waves in Dispersive Media

Relation between *P* and *E* is dynamic

$$\boldsymbol{P}(\boldsymbol{r},t)=\varepsilon_0\int_{-\infty}^{+\infty}dt'x(t-t')\boldsymbol{E}(\boldsymbol{r},t')$$

EM wave:

$$\boldsymbol{E}(\boldsymbol{r},t)=\mathrm{Re}\{\boldsymbol{E}(\boldsymbol{k},\omega)\exp(-i\boldsymbol{k}\cdot\boldsymbol{r}+i\omega t)\}$$

$$\boldsymbol{P}(\boldsymbol{r},t)=\mathrm{Re}\{\boldsymbol{P}(\boldsymbol{k},\omega)\exp(-i\boldsymbol{k}\cdot\boldsymbol{r}+i\omega t)\}$$

⇨

Relation between complex amplitudes

$$\boxed{\boldsymbol{P}(\boldsymbol{k},\omega)=\varepsilon_0\chi(\omega)\boldsymbol{E}(\boldsymbol{k},\omega)}$$

(Slow response of matter ⇨ ω-dependent behavior)

This follows by equation of the coefficients of exp(iωt) ..check this!

It also follows that: $\varepsilon(\omega)=\varepsilon_0\left[1+\chi(\omega)\right]$

Absorption and Dispersion of EM Waves

Transparent materials can be described by a purely real refractive index n

EM wave: $\boldsymbol{E}(z,t)=\mathrm{Re}\left\{\boldsymbol{E}(k,\omega)\exp(-ikz+i\omega t)\right\}$

Dispersion relation $\omega^2=\dfrac{c^2}{n^2}k^2 \Rightarrow k=\pm\dfrac{\omega}{c}n$

Absorbing materials can be described by a complex n: $\boxed{n=n'+in''}$

It follows that: $k=\pm\dfrac{\omega}{c}(n'+in'')=\pm\left(\dfrac{\omega}{c}n'+i\dfrac{\omega}{c}n''\right)\equiv\pm\left(\beta-i\dfrac{\alpha}{2}\right)$

Investigate + sign: $\boldsymbol{E}(z,t)=\mathrm{Re}\left\{\boldsymbol{E}(k,\omega)\exp\left(\underset{\substack{\uparrow\\ \text{Traveling wave}}}{-i\beta z}\underset{\substack{\uparrow\\ \text{Decay}}}{-\dfrac{\alpha}{2}z}+i\omega t\right)\right\}$

Note: $\beta=\dfrac{\omega}{c}n'=k_0n'$ ⇨ n’ act as a regular refractive index

$\alpha=-2\dfrac{\omega}{c}n''=-2k_0n''$ ⇨ α is the absorption coefficient

Absorption and Dispersion of EM Waves

n is derived quantity from χ (next lecture we determine χ for different materials)

Complex n results from a complex χ:

$$\left.\begin{aligned} \chi &= \chi' + i\chi'' \\ n &= \sqrt{1+\chi} \end{aligned}\right\} ➡$$

$$➡ \left.\begin{aligned} n = n' + in'' = \sqrt{1+\chi} &= \sqrt{1+\chi'+i\chi''} \\ \alpha &= -2k_0 n'' \end{aligned}\right\} ➡ \; n = n' - i\frac{\alpha}{2k_0} = \sqrt{1+\chi'+i\chi''}$$

Weakly absorbing media

When χ'<<1 and χ'' << 1: $\sqrt{1+\chi'+i\chi''} \approx 1 + \frac{1}{2}\left(\chi' + i\chi''\right)$

Refractive index: $n' = 1 + \frac{1}{2}\chi'$

Absorption coefficient: $\alpha = -2k_0 n'' = -k_0\chi''$

Intermediate Summary (A)

Maxwell's Equations

Bold face letters are vectors!

$$\nabla\cdot\boldsymbol{D}=\rho_f \qquad \nabla\cdot\boldsymbol{B}=0 \qquad \nabla\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial\boldsymbol{t}} \qquad \nabla\times\boldsymbol{H}=\frac{\partial\boldsymbol{D}}{\partial\boldsymbol{t}}+\boldsymbol{J}$$

Curl Equations lead to

$$\nabla^2\boldsymbol{E}=\mu_0\varepsilon_0\frac{\partial^2\boldsymbol{E}}{\partial t^2}+\mu_0\frac{\partial^2\boldsymbol{P}}{\partial t^2}$$

(under certain conditions)

Linear, Homogeneous, and Isotropic Media

$$\boldsymbol{P}=\varepsilon_0\chi\boldsymbol{E}$$

Wave Equation

$$\nabla^2\boldsymbol{E}(\boldsymbol{r},t)=\frac{n^2}{c^2}\frac{\partial^2\boldsymbol{E}(\boldsymbol{r},t)}{\partial\boldsymbol{t}^2}$$

Solutions: EM waves

$$\boldsymbol{E}(z,t)=\mathrm{Re}\left\{\boldsymbol{E}(z,\omega)\exp\left(-i\beta z-\frac{\alpha}{2}z+i\omega t\right)\right\}$$

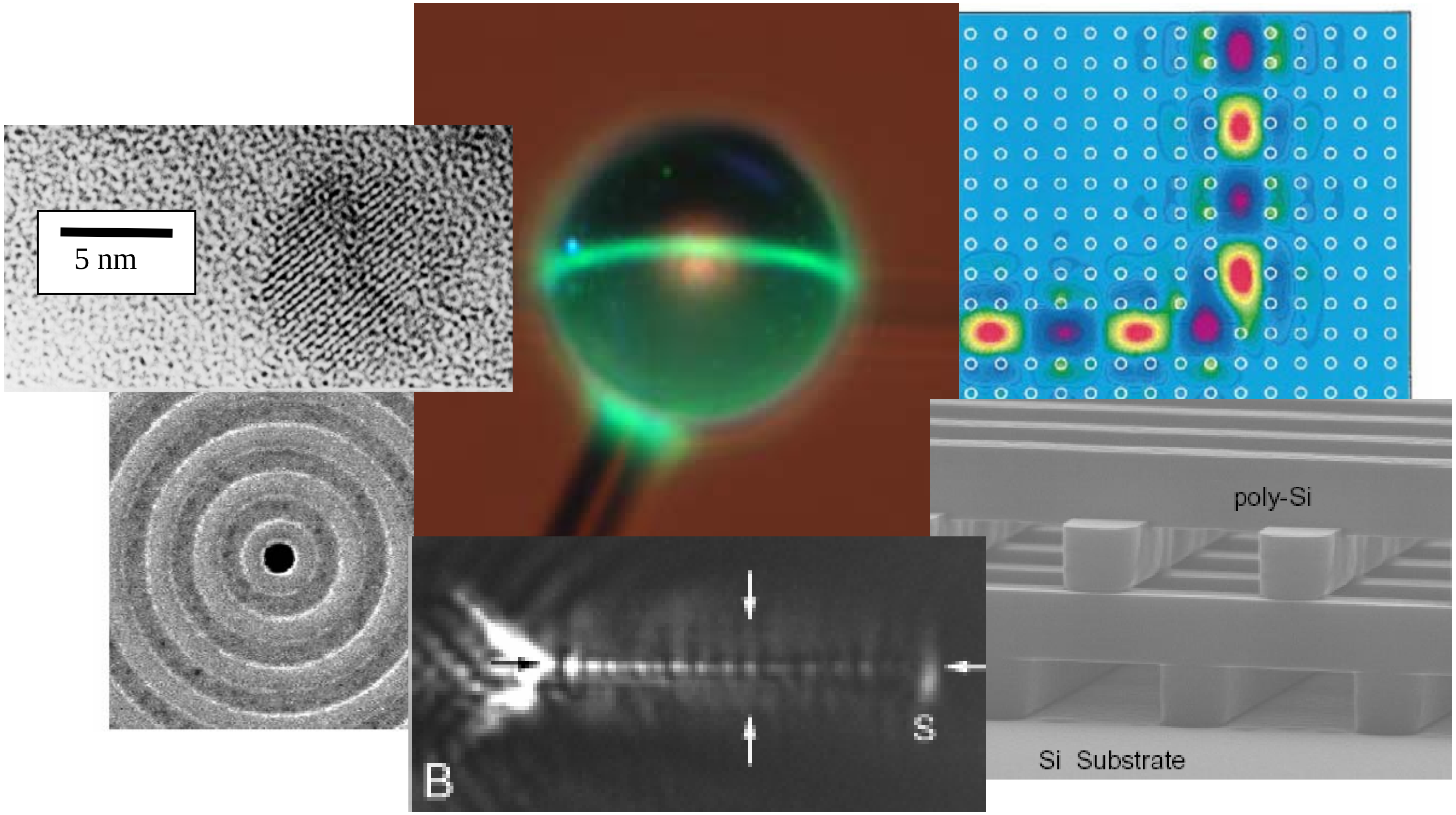
where $\underbrace{\beta=k_0n'}_{\text{Phase propagation}}$ and $\underbrace{\alpha=-2k_0n''}_{\text{absorption}}$

In real life: Response of matter (*P*) is not instantaneous

$$\boldsymbol{P}(\boldsymbol{r},t)=\varepsilon_0\int_{-\infty}^{+\infty}dt'x(t-t')\boldsymbol{E}(\boldsymbol{r},t') \Rightarrow \left.\begin{matrix}\chi'=\chi'(\omega)\\ \chi''=\chi''(\omega)\end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix}n'=n'(\omega)\\ n''=n''(\omega)\end{matrix}\right.$$

⤴

B. Optical Properties of Bulk and Nano



Microscopic Origin ω-Response of Matter

Origin frequency dependence of χ in real materials

- Lorentz model (harmonic oscillator model)
- Insulators (Lattice absorption, color centers…)
- Semiconductors (Energy bands, Urbach tail, excitons …)
- Metals (AC conductivity, Plasma oscillations, interband transitions…)

Real and imaginary part of χ are linked

- Kramers-Kronig

But first…..

- When should I work with χ, ε, or n ?

They all seem to describe the optical properties of materials!

n’ and n’’ vs χ’ and χ’’ vs ε’ and ε’’

All pairs (n’ and n’’, χ’ and χ’’, ε’ and ε’’) describe the same physics

For some problems one set is preferable for others another

n’ and n’’ used when discussing wave propagation

$$\boldsymbol{E}(z,t)=\mathrm{Re}\left\{\boldsymbol{E}(z,\omega)\exp\left(-i\beta z-\frac{\alpha}{2}z+i\omega t\right)\right\}$$

where $\underline{\beta = k_0 n'}$ and $\underline{\alpha = -2k_0 n''}$

Phase propagation absorption

χ’ and χ’’
ε’ and ε’’ } used when discussing microscopic origin of optical effects

As we will see today…

Inter relationships

Example: n and ε

From $n=\sqrt{\varepsilon_r}$ $\Rightarrow$

$\downarrow$

$$n'+in''=\sqrt{\varepsilon_r'+i\varepsilon_r''}$$

$$\boxed{\begin{aligned}\varepsilon_r' &= (n')^2-(n'')^2\\ \varepsilon_r'' &= 2n'n''\end{aligned}}$$

and

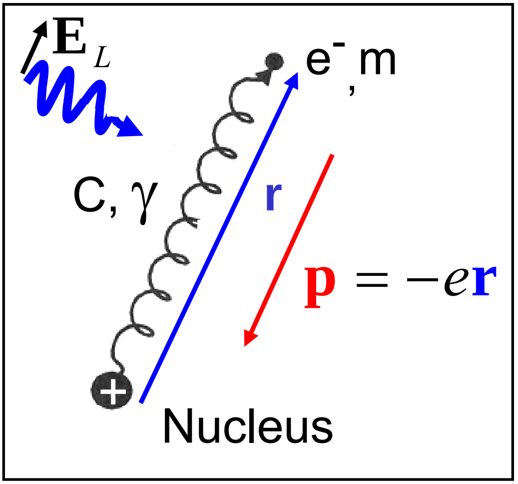
$$\boxed{\begin{aligned}n' &= \sqrt{\frac{\sqrt{(\varepsilon_r')^2+(\varepsilon_r'')^2}+\varepsilon_r'}{2}}\\ n'' &= \sqrt{\frac{\sqrt{(\varepsilon_r')^2+(\varepsilon_r'')^2}-\varepsilon_r'}{2}}\end{aligned}}$$

Linear Dielectric Response of Matter

Behavior of bound electrons in an electromagnetic field

- Optical properties of insulators are determined by bound electrons

Lorentz model



- Charges in a material are treated as harmonic oscillators

$$m\mathbf{a}_{el} = \mathbf{F}_{E,Local} + \mathbf{F}_{Damping} + \mathbf{F}_{Spring}$$ (one oscillator)

$$m\frac{d^2\mathbf{r}}{dt^2} + m\gamma\frac{d\mathbf{r}}{dt} + C\mathbf{r} = -e\mathbf{E}_L\exp(-i\omega t)$$

- The electric dipole moment of this system is: $\mathbf{p} = -e\mathbf{r}$

$$m\frac{d^2\mathbf{p}}{dt^2} + m\gamma\frac{d\mathbf{p}}{dt} + C\mathbf{p} = e^2\mathbf{E}_L\exp(-i\omega t)$$

- Guess a solution of the form:

$$\mathbf{p} = \mathbf{p}_0\exp(-i\omega t)\ ;\frac{d\mathbf{p}}{dt} = -i\omega\mathbf{p}_0\exp(-i\omega t)\ ;\frac{d^2\mathbf{p}}{dt^2} = -\omega^2\mathbf{p}_0\exp(-i\omega t)$$

$$\Rightarrow -m\omega^2\mathbf{p}_0 - im\gamma\omega\mathbf{p}_0 + C\mathbf{p}_0 = e^2\mathbf{E}_L \Rightarrow$$ Solve for $\mathbf{p}_0(\mathbf{E}_L)$

Atomic Polarizability

Determination of atomic polarizability

- Last slide: $-m\omega^2\mathbf{p}_0 - im\gamma\omega\mathbf{p}_0 + C\mathbf{p}_0 = e^2\mathbf{E}_L$

⇨ $$-\omega^2\mathbf{p}_0 - i\gamma\omega\mathbf{p}_0 + \frac{C}{m}\mathbf{p}_0 = \frac{e^2}{m}\mathbf{E}_L$$ (Divide by m)

Define as ω_0^2 (turns out to be the resonance ω)

⇨ $$\mathbf{p}_0 = \frac{e^2}{m}\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}\mathbf{E}_L$$

Atomic polarizability (in SI units)

- Define atomic polarizability: $\alpha(\omega) \equiv \dfrac{p_0}{\varepsilon_o \mathrm{E}_L} = \dfrac{e^2}{\varepsilon_0 m}\dfrac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}$

Resonance frequency

Damping term

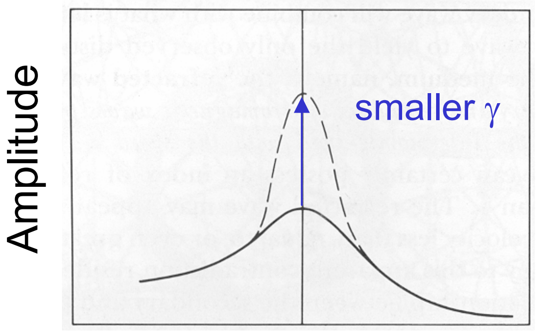
Characteristics of the Atomic Polarizability

Response of matter (***P***) is not instantaneous ⇨ ω-dependent response

- Atomic polarizability: $\alpha(\omega) = \dfrac{p_0}{\varepsilon_o \mathrm{E}_L} = \dfrac{e^2}{\varepsilon_0 m} \dfrac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} = A \exp\left[i\theta(\omega)\right]$

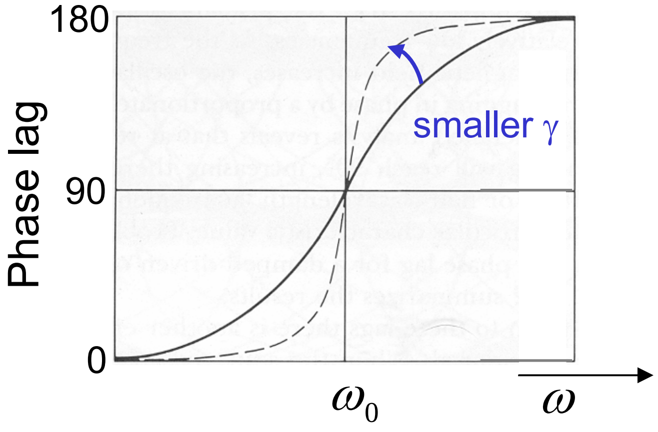
- Amplitude

$$A = \frac{e^2}{\varepsilon_0 m} \frac{1}{\left[\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2 \omega^2\right]^{1/2}}$$



- Phase lag of α with ***E*:**

$$\theta = \tan^{-1} \frac{\gamma\omega}{\omega_0^2 - \omega^2}$$



Relation Atomic Polarizability (α) and χ: 2 cases

Case 1: Rarified media (.. gasses)

- Dipole moment of one atom, j :

E-field photon

$$\mathbf{p}_j = \varepsilon_0 \alpha_j(\omega) \mathbf{E}_L$$

- Polarization vector:

Occurs in Maxwell's equation..

Density

$$\mathbf{P} = \underbrace{\frac{1}{V}\sum_j \mathbf{p}_j}_{\text{sum over all atoms}} = \frac{\varepsilon_0}{V}\sum_j \alpha_j \mathbf{E}_L = \varepsilon_0 N \alpha_j \mathbf{E}_L$$

$$\alpha_j(\omega) = \frac{e^2}{\varepsilon_0 m}\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

⇨ $$\mathbf{P} = \frac{Ne^2}{m}\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}\mathbf{E}_L \quad \left(= \varepsilon_0 \chi \mathbf{E}_L\right)$$

⇨ Microscopic origin susceptibility: $$\boxed{\chi(\omega) = \frac{Ne^2}{\varepsilon_0 m}\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}}$$

- Plasma frequency defined as: $\omega_p^2 = \frac{Ne^2}{\varepsilon_0 m}$ ⇨ $\chi(\omega) = \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\gamma\omega}$

Remember: ε and n follow directly from χ

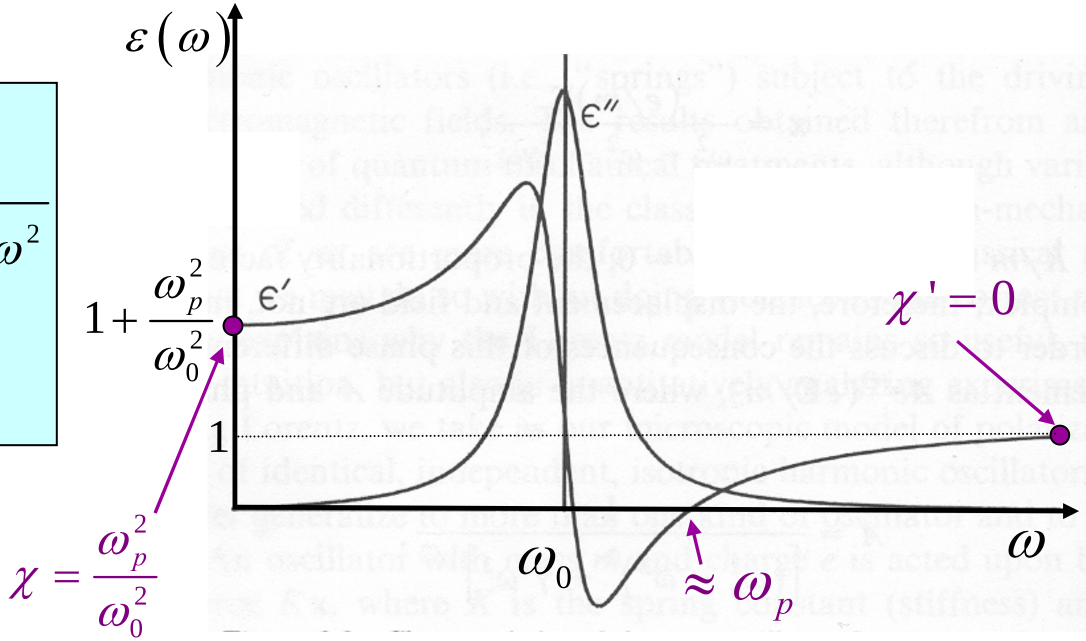
Frequency dependence ε

- Relation of ε to χ : $\varepsilon = 1 + \chi = 1 + \dfrac{\omega_p^2}{\omega_0^2 - \omega^2 - i\gamma\omega}$

➡ $$\varepsilon' + i\varepsilon'' = 1 + \chi' + i\chi'' = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

$$\varepsilon' = 1 + \chi'(\omega) = 1 + \frac{\omega_p^2\left(\omega_0^2 - \omega^2\right)}{\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2}$$

$$\varepsilon'' = \chi''(\omega) = \frac{\omega_p^2\gamma\omega}{\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2}$$



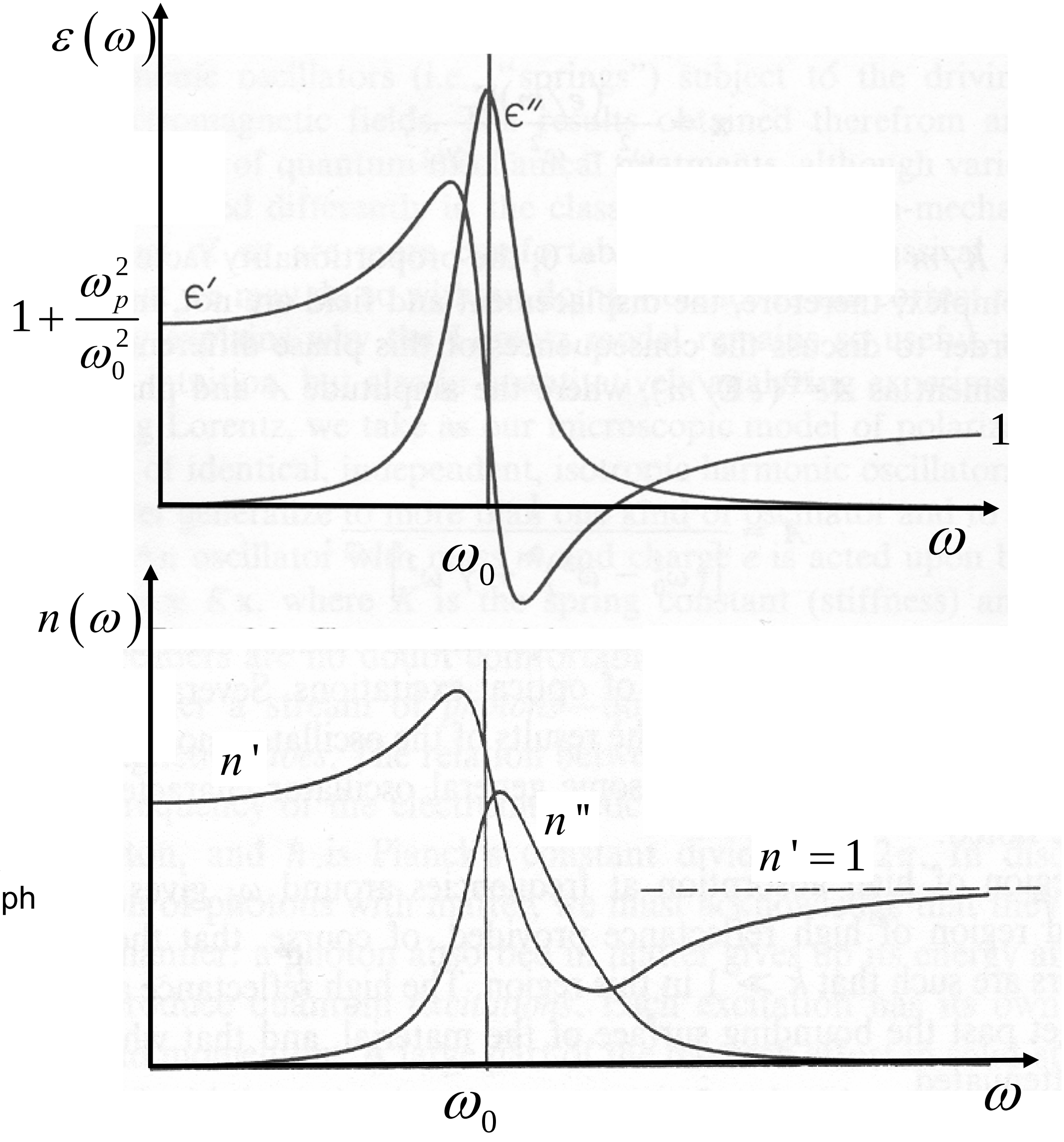
Propagation of EM-waves: Need n' and n''

Relation between n and ε

$$n = \sqrt{\varepsilon}$$

$$\varepsilon_r' = (n')^2 - (n'')^2$$

$$\varepsilon_r'' = 2n'n''$$



- $\omega \ll \omega_0$: High n' ⇨ low $v_{ph} = c/n'$
- $\omega \approx \omega_0$: Strong ω dependence v_{ph}

  Large absorption (~ n'')
- $\omega \gg \omega_0$: n' = 1 ⇨ $v_{ph} = c$

Realistic Rarefied Media

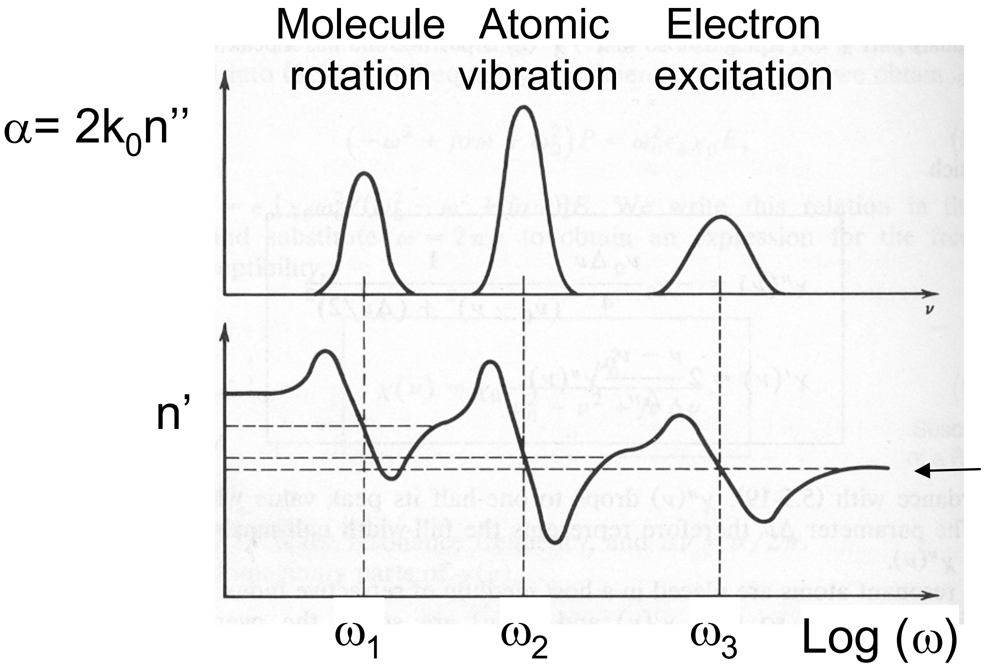
Realistic atoms have many resonances

- Resonances occur due to motion of the atoms (low ω) and electrons (high ω)

$$\Rightarrow \quad \chi = \sum_k \frac{N_k e^2}{\varepsilon_0 m} \frac{1}{\omega_k^2 - \omega^2 - i\gamma\omega}$$

Where N_k is the density of the electrons/atoms with a resonance at ω_k

Example of a realistic dependence of n' and n''

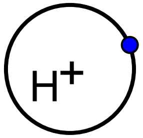


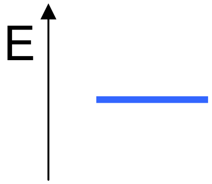
n' > 1 indicates presence high ω oscillators

Classification Matter: Insulators, Semiconductors, Metals

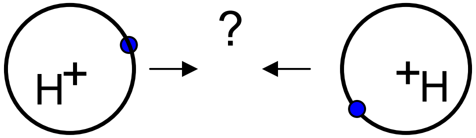
Bonds and bands

- One atom, e.g. H. Schrödinger equation:

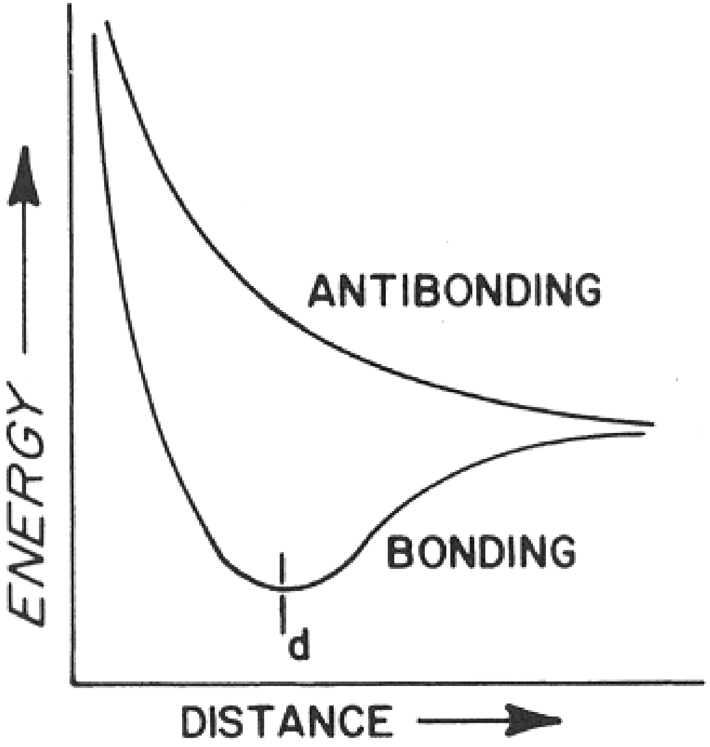




- Two atoms: bond formation

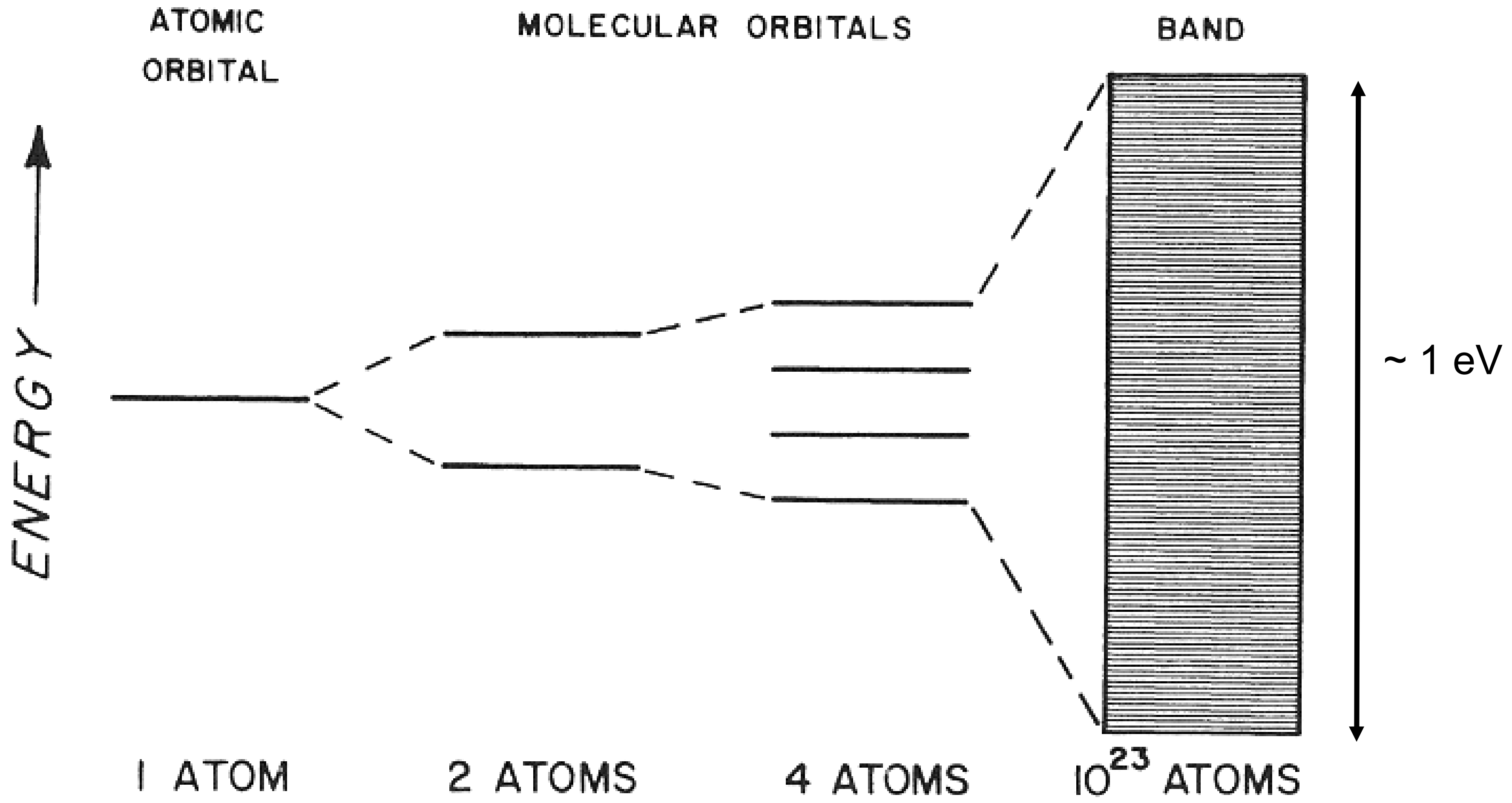


Every electron contributes one state ⟶



- Equilibrium distance d (after reaction)

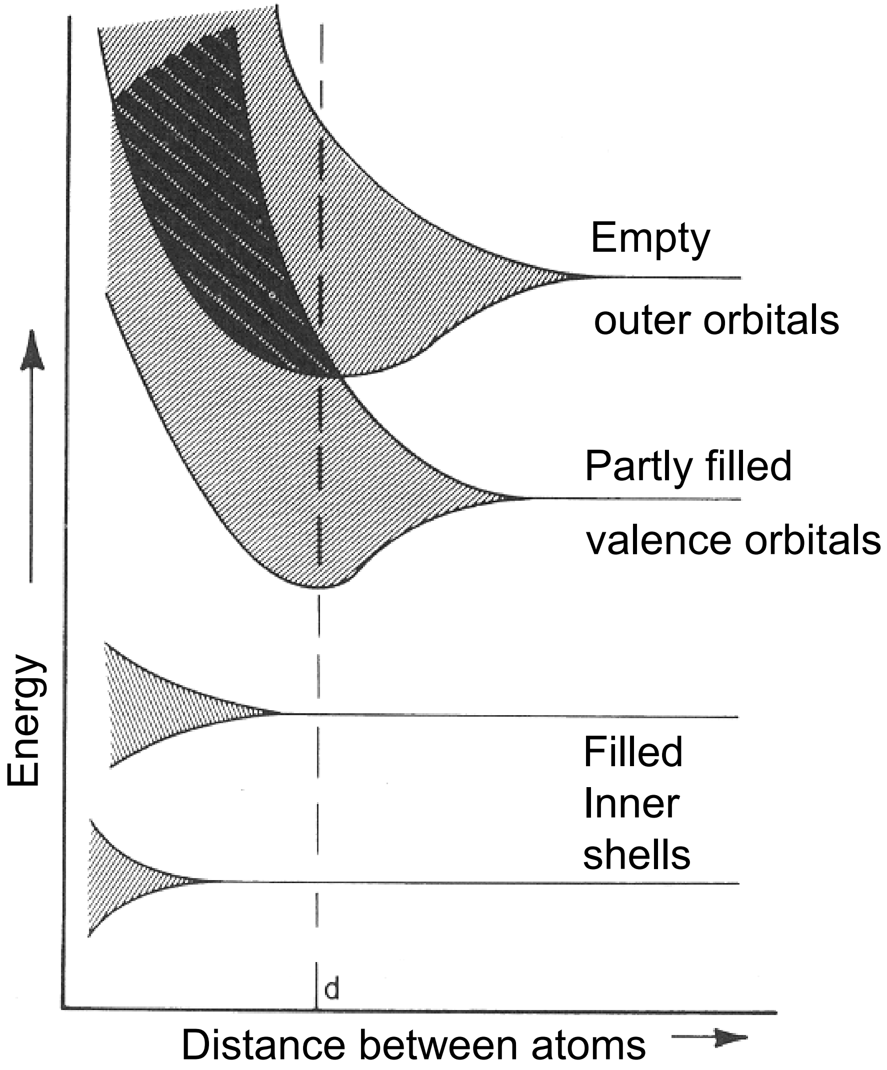
Classification Matter



- Pauli principle: Only 2 electrons in the same electronic state (one spin↑ & one spin↓)

Classification Matter

Atoms with many electrons



Outermost electrons interact

➡ Form bands

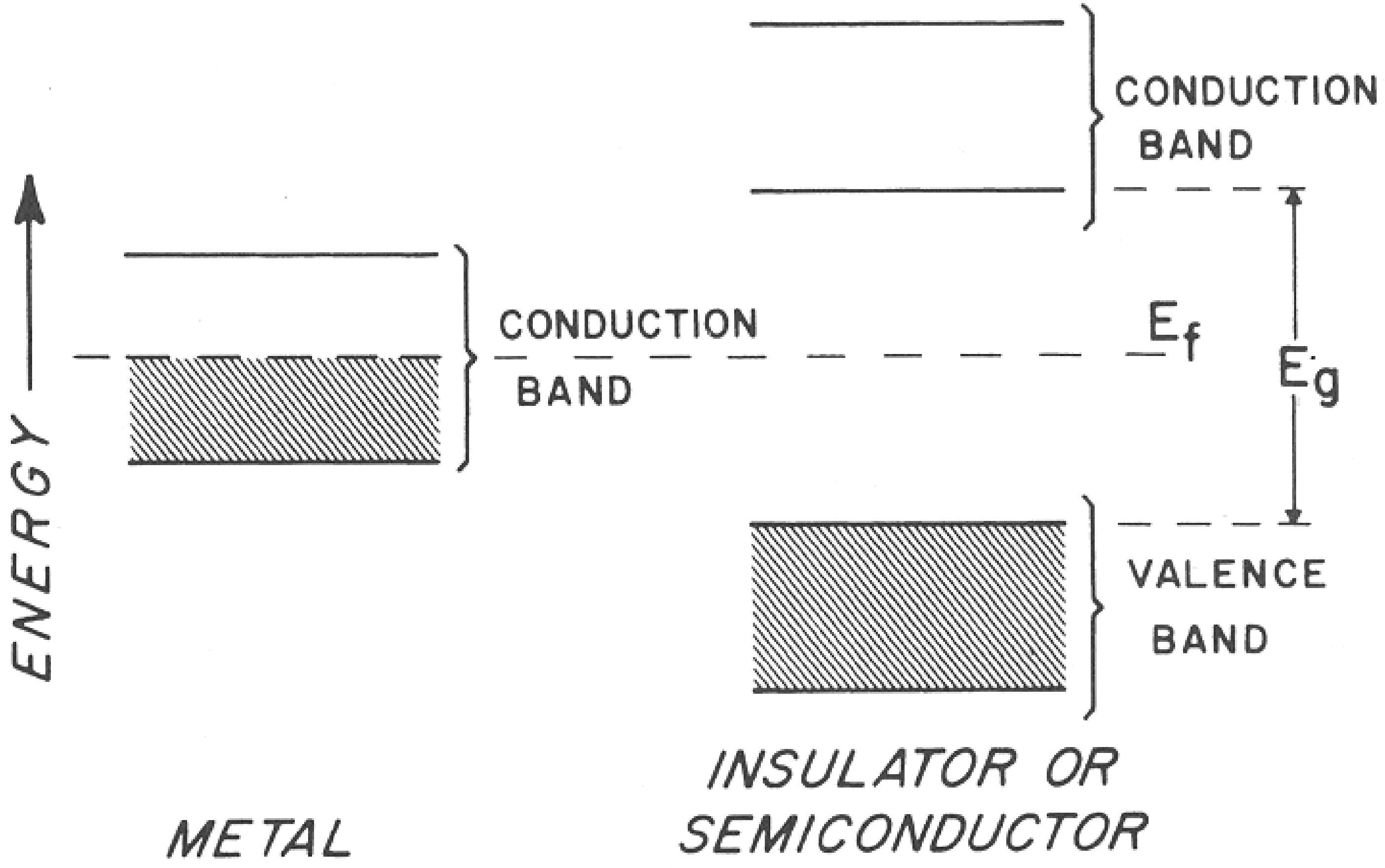
Electrons in inner shells do not interact

➡ Do not form bands

Classification Matter

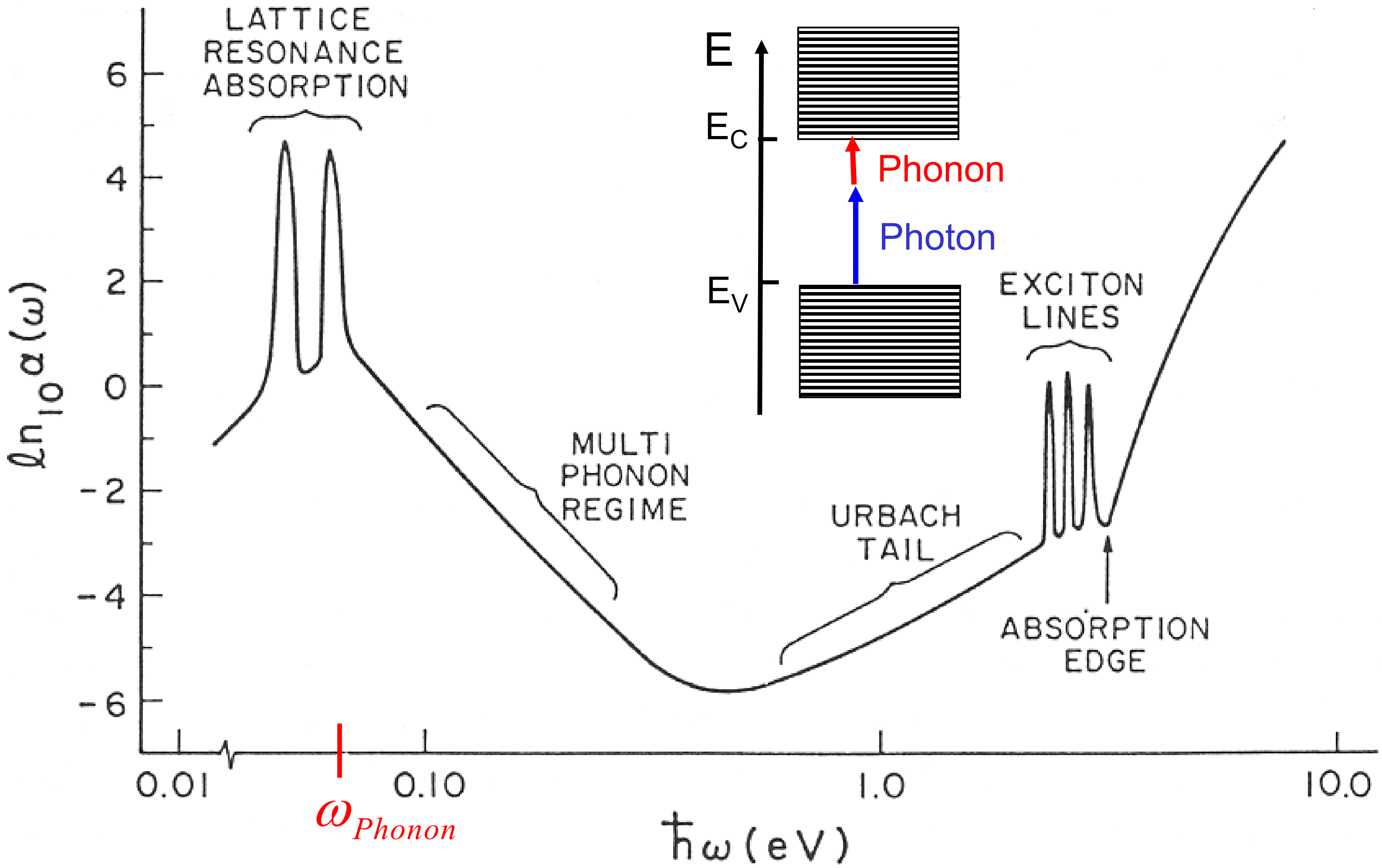
Insulators, semiconductors, and metals

- Classification based on bandstructure



Absorption Processes in Semiconductors

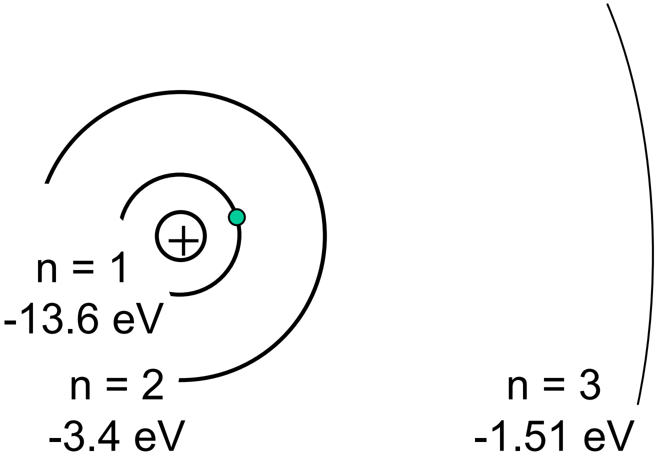
Absorption spectrum of a typical semiconductor



Excitons: Electron and Hole Bound by Coulomb

Analogy with H-atom

- Electron orbit around a hole is similar to the electron orbit around a H-core
- 1913 Niels Bohr: Electron restricted to well-defined orbits



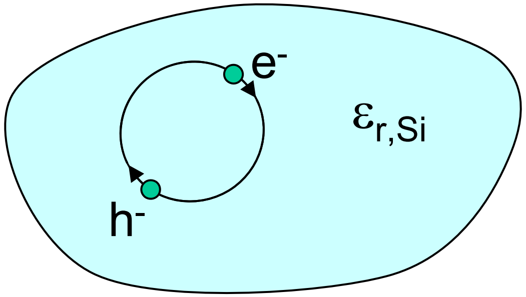
- Binding energy electron: $E_B = -\dfrac{m_e e^4}{2\left(4\pi\varepsilon_0\hbar n\right)^2} = -\dfrac{13.6}{n^2} eV, n = 1, 2, 3, \ldots$

Where: m_e = Electron mass, ε_0 = permittivity of vacuum, $\hbar$ = Planck's constant
n = energy quantum number/orbit identifier

Binding Energy of an Electron to Hole

Electron orbit "around" a hole

- Electron orbit is expected to be qualitatively similar to a H-atom.
- Use reduced effective mass instead of m_e: ⇨ $1/m^* = 1/m_e + 1/m_h$
- Correct for the relative dielectric constant of Si, $\varepsilon_{r,Si}$ (screening).

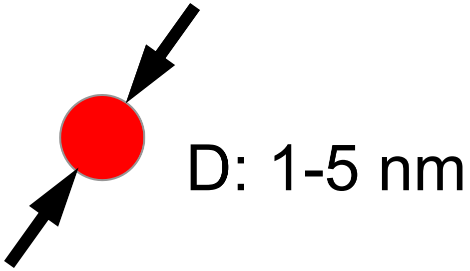


⇨ Binding energy electron: $E_B = \frac{m^*}{m_e}\frac{1}{\varepsilon^2}13.6eV, n = 1, 2, 3, \ldots$

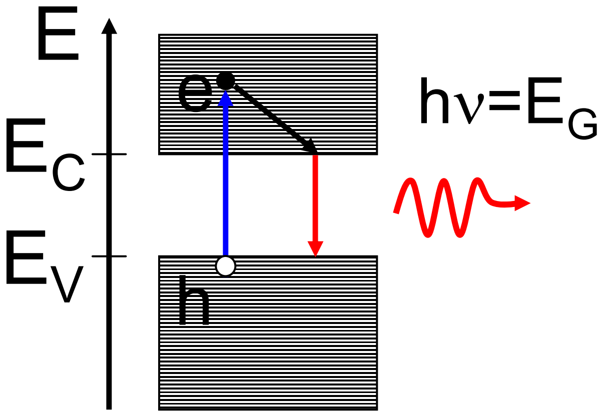
- Typical value for semiconductors: $E_B = 10meV - 100meV$
- Note: Exciton Bohr radius ~ 5 nm (many lattice constants)

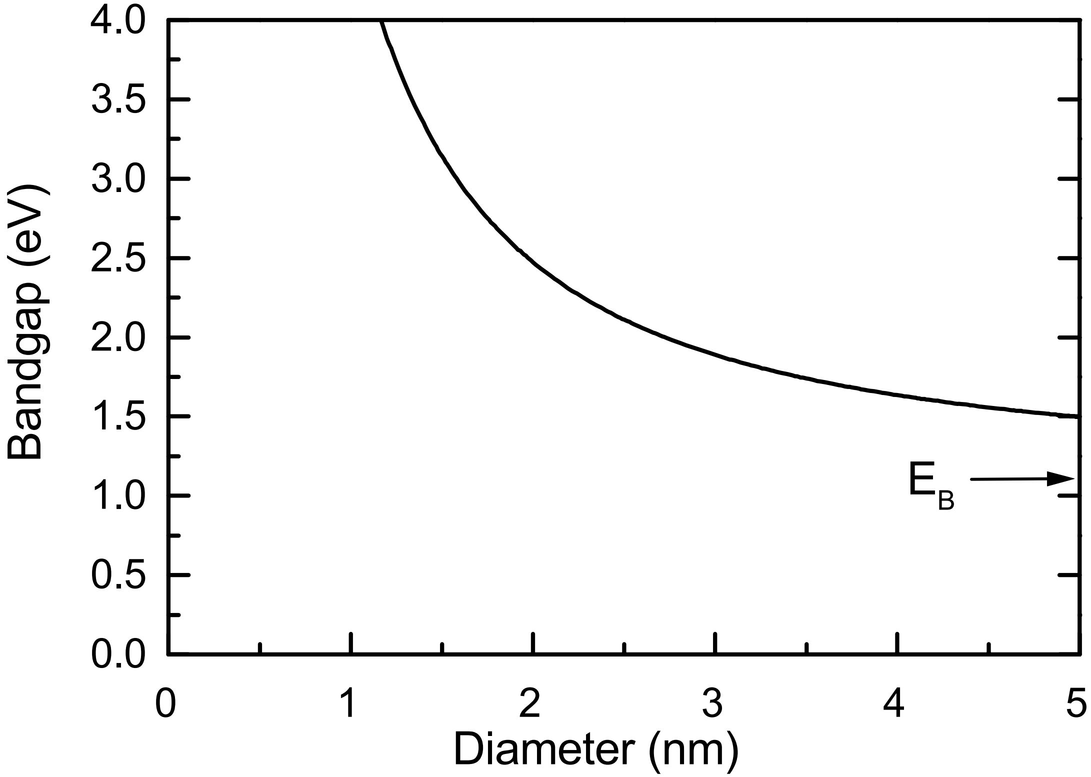
Semiconductor Nanoparticles

Example: Si nanocrystals



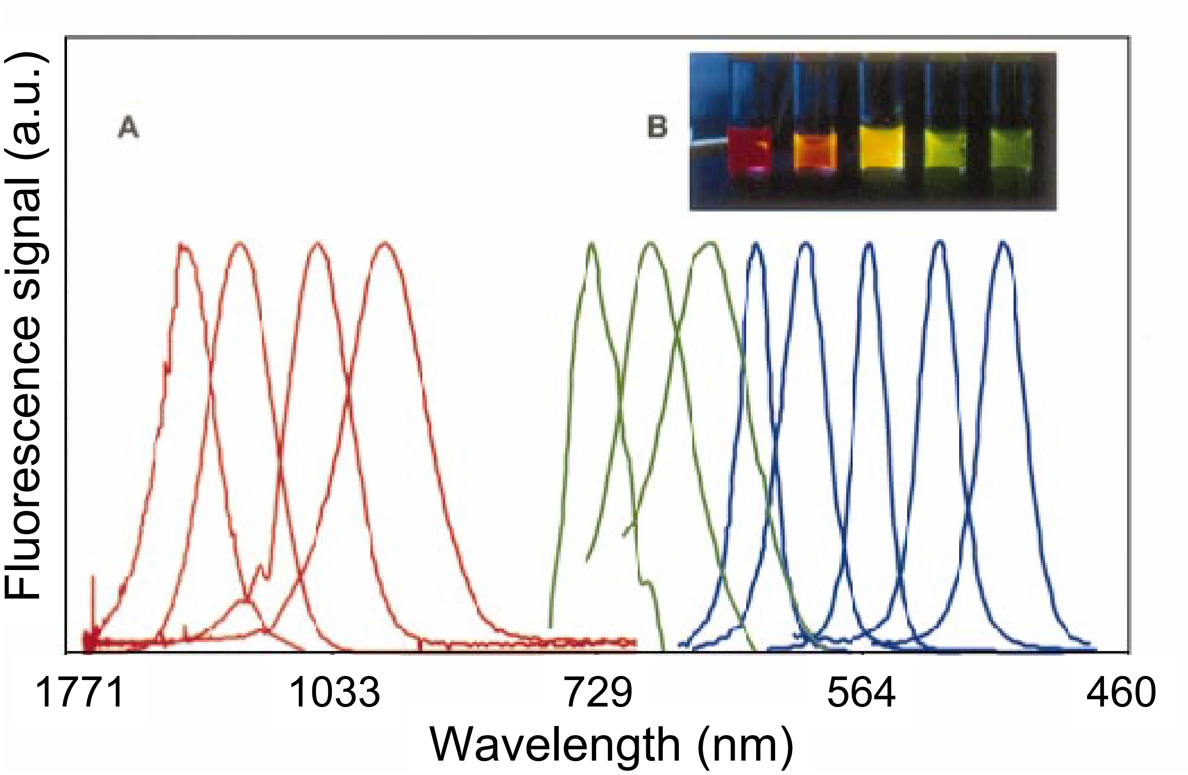
Photoluminescence





C.Delerue et al. Phys Rev. B 48, 11024 (1993)

Size and Material Dependent Optical Properties



- Red series: InAs nanocrystals with diameters of 2.8, 3.6, 4.6, and 6.0 nm
- Green series: InP nanocrystals with diameters of 3.0, 3.5,and 4.6 nm.
- Blue series: CdSe nanocrystals with diameters of 2.1, 2.4,3.1, 3.6, and 4.6 nm

M.Bruchez et al. (Alivisatos group), Science, 2013, ***281 (****2014)*

Optical Properties of Metals (determine ε)

Current induced by a time varying field

- Consider a time varying field:

$$\mathbf{E}(t) = \mathrm{Re}\left\{\mathbf{E}(\omega)\exp(-i\omega t)\right\}$$

- Equation of motion electron in a metal:

$$m\frac{d^2\mathbf{x}}{dt^2} = \frac{d\mathbf{v}}{dt} = -m\frac{\mathbf{v}}{\tau} - e\mathbf{E}$$

relaxation time ~ 10^{-14} s

- Look for a steady state solution:

$$\mathbf{v}(t) = \mathrm{Re}\left\{\mathbf{v}(\omega)\exp(-i\omega t)\right\}$$

- Substitution **v** into Eq. of motion:

$$-i\omega m\mathbf{v}(\omega) = -\frac{m\mathbf{v}(\omega)}{\tau} - e\mathbf{E}(\omega)$$

- This can be manipulated into:

$$\mathbf{v}(\omega) = \frac{-e}{m\left(1/\tau - i\omega\right)}\mathbf{E}(\omega)$$

- The current density is defined as:

$$\mathbf{J}(\omega) = -ne\mathbf{v}$$

Electron density

- It thus follows:

$$\mathbf{J}(\omega) = \frac{\left(ne^2/m\right)}{\left(1/\tau - i\omega\right)}\mathbf{E}(\omega)$$

Optical Properties of Metals

Determination conductivity

- From the last page: $\mathbf{J}(\omega)=\dfrac{\left(ne^2/m\right)}{\left(1/\tau-i\omega\right)}\mathbf{E}(\omega)$
- Definition conductivity: $\mathbf{J}(\omega)=\sigma(\omega)\mathbf{E}(\omega)$

$$\Rightarrow \boxed{\sigma(\omega)=\frac{\left(ne^2/m\right)}{\left(1/\tau-i\omega\right)}=\frac{\sigma_0}{\left(1-i\omega\tau\right)}}$$

where: $\sigma_0=\dfrac{ne^2\tau}{m}$

Both bound electrons and conduction electrons contribute to ε

- From the curl Eq.: $\nabla\times\boldsymbol{H}=\dfrac{\partial\boldsymbol{D}}{\partial\boldsymbol{t}}+\boldsymbol{J}=\dfrac{\partial\varepsilon_B\boldsymbol{E}(t)}{\partial\boldsymbol{t}}+\boldsymbol{J}$
- For a time varying field: $\mathbf{E}(t)=\mathrm{Re}\left\{\mathbf{E}(\omega)\exp(-i\omega t)\right\}$

$$\Rightarrow \nabla\times\boldsymbol{H}=\frac{\partial\varepsilon_B\boldsymbol{E}(t)}{\partial\boldsymbol{t}}+\boldsymbol{J}=-i\omega\varepsilon_B(\omega)\boldsymbol{E}(\omega)+\sigma(\omega)\boldsymbol{E}(\omega)=-i\omega\varepsilon_0\underbrace{\left(\varepsilon_B(\omega)-\frac{\sigma(\omega)}{i\varepsilon_0\omega}\right)}_{\varepsilon_{EFF}(\omega)}\boldsymbol{E}(\omega)$$

Currents induced by ac-fields modeled by $\varepsilon_{\mathrm{EFF}}$

- For a time varying field: $\varepsilon_{EFF}=\varepsilon_B-\dfrac{\sigma}{i\varepsilon_0\omega}=\varepsilon_B+i\dfrac{\sigma}{\varepsilon_0\omega}$

Bound electrons ↗ ε_B

Conduction electrons ↖ $\dfrac{\sigma}{i\varepsilon_0\omega}$

Optical Properties of Metals

Dielectric constant at $\omega \approx \omega_{visible}$

- Since $\omega_{vis}\tau >> 1$: $\sigma(\omega) = \frac{\sigma_0}{(1-i\omega\tau)} = \frac{\sigma_0(1+i\omega\tau)}{(1+\omega^2\tau^2)} \approx \frac{\sigma_0}{\omega^2\tau^2} + i\frac{\sigma_0}{\omega\tau}$

$$\left.\begin{array}{l} \text{• It follows that:}\ \varepsilon_{EFF} = \varepsilon_B + i\frac{\sigma}{\varepsilon_0\omega} = \varepsilon_B + i\frac{\sigma_0}{\varepsilon_0\omega^3\tau^2} - \frac{\sigma_0}{\varepsilon_0\omega^2\tau} \\ \text{• Define:}\ \omega_p^2 = \frac{\sigma_0}{\varepsilon_0\tau} = \frac{ne^2}{\varepsilon_0 m} \quad (\approx 10eV\ for\ metals) \end{array}\right\} \Rightarrow$$

$$\boxed{\varepsilon_{EFF} = \varepsilon_B \underbrace{- \frac{\omega_p^2}{\omega^2} + i\frac{\omega_p^2}{\omega^3\tau}}}$$

Bound electrons

Free electrons

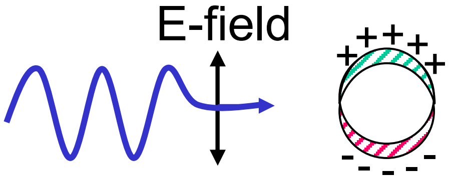
- What does this look like for a real metal?

Excitation of a Metal Nanoparticle

Particle

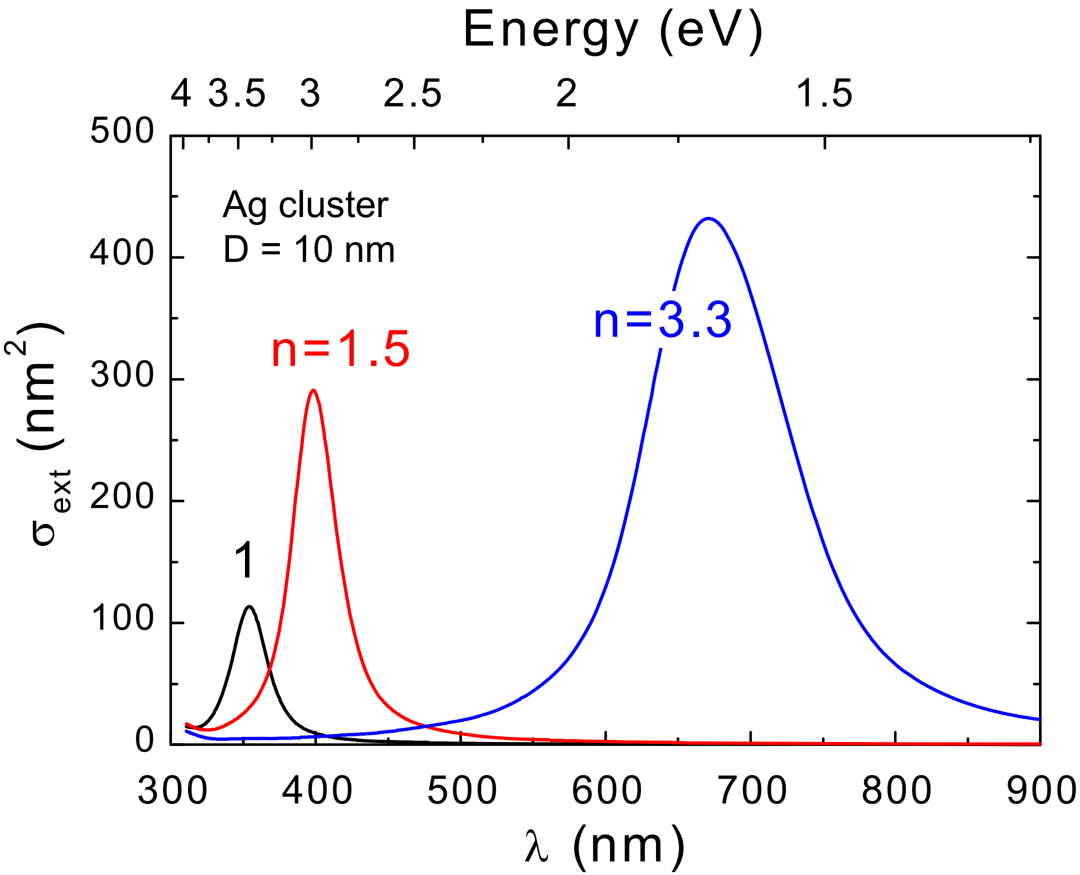
Volume = V_0

$\varepsilon_M = \varepsilon'_M + i\varepsilon''_M$



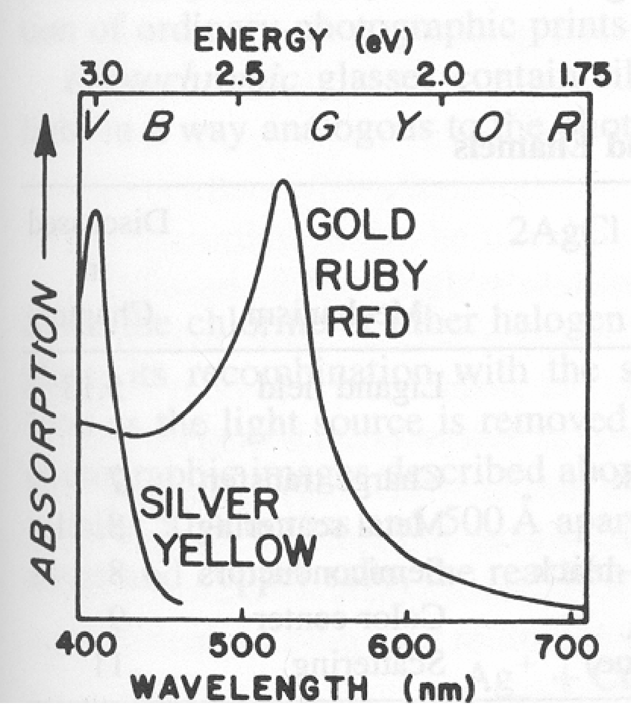
Host matrix

$\varepsilon_H = \varepsilon'_H = n_H^2$



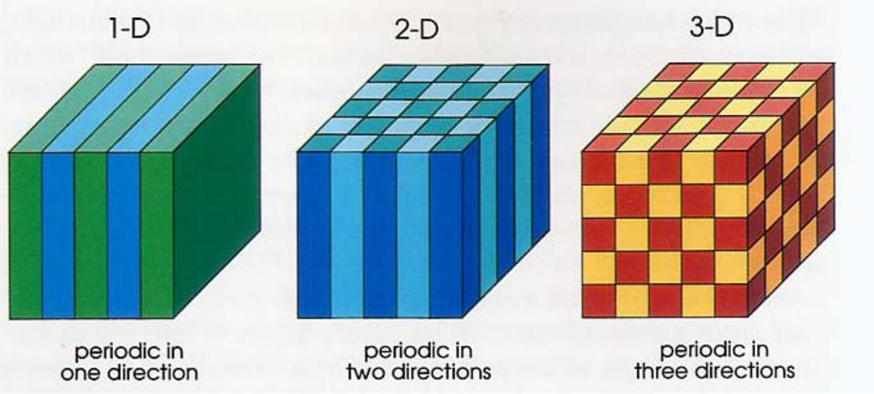
$$\sigma_{ext}(\omega) = 9\frac{\omega}{c}\varepsilon_H'^{\,3/2} V_0 \frac{\varepsilon_M'(\omega)}{\left[\varepsilon_M'(\omega) + 2\varepsilon_H'\right]^2 + \varepsilon_M''(\omega)^2}$$

Applications Metallic Nanoparticles



- Engraved Czechoslovakian glass vase
- Ag nanoparticles cause yellow coloration
- Au nanoparticles cause red coloration
- Molten glass readily dissolves 0.1 % Au
- Slow cooling results in nucleation and growth of nanoparticles

C. Photonic crystal: An Introduction



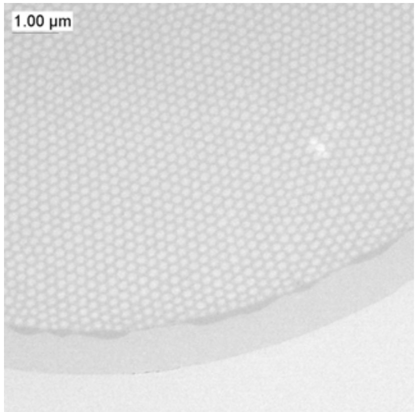
Photonic crystal:

Periodic arrangement of dielectric (metallic, polaritonic…) objects.
Lattice constants comparable to the wavelength of light in the material.

“ A worm ahead of its time”

Sea Mouse

and its hair




Normal incident light

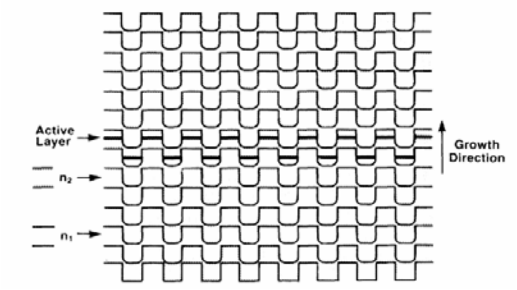

Off-Normal incident light

http://www.physics.usyd.edu.au/~nicolae/seamouse.html

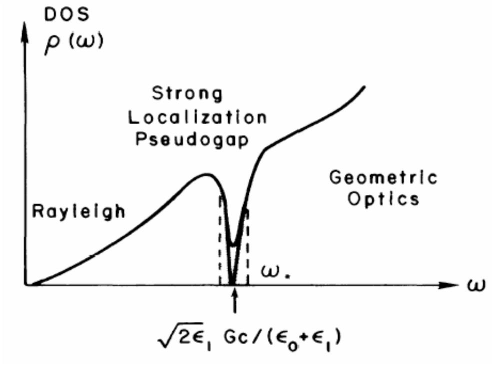
Fast forward to 1987……

E. Yablonovitch
"Inhibited spontaneous emission in solid state physics and electronics"
Physical Review Letters, vol. 58, pp. 2059, 1987

S. John
"Strong localization of photons in certain disordered dielectric superlattices"
Physical Review Letters, vol. 58, pp. 2486, 1987



Face-centered cubic lattice



Complete photonic band gap

The emphasis of recent breakthroughs

- The use of strong index contrast, and the developments of nano-fabrication technologies, which leads to entirely new sets of phenomena.

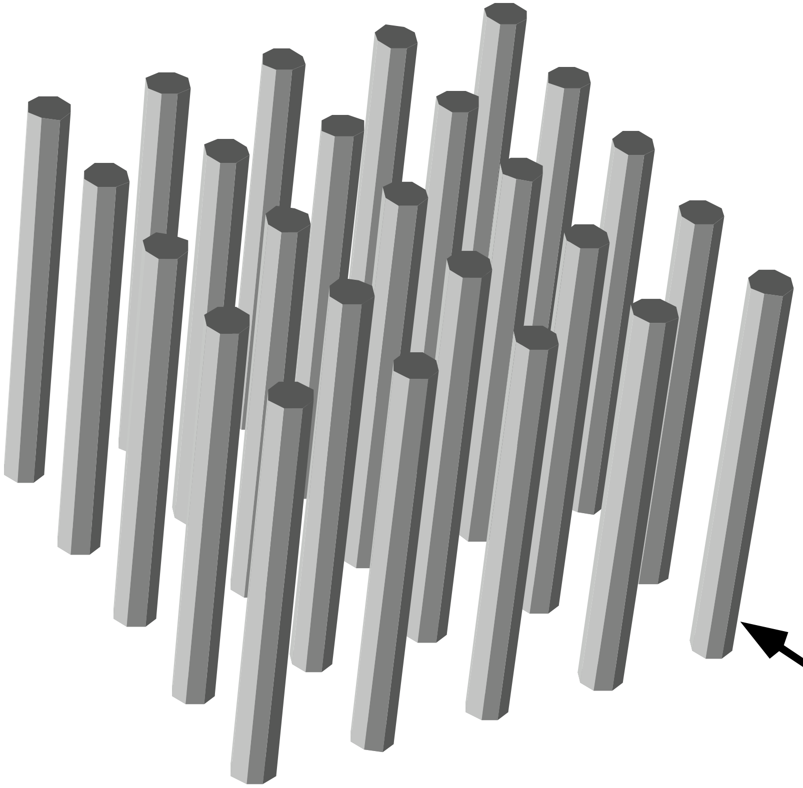
  Conventional silica fiber, $\delta n \sim 0.01$, photonic crystal structure, $\delta n \sim 1$

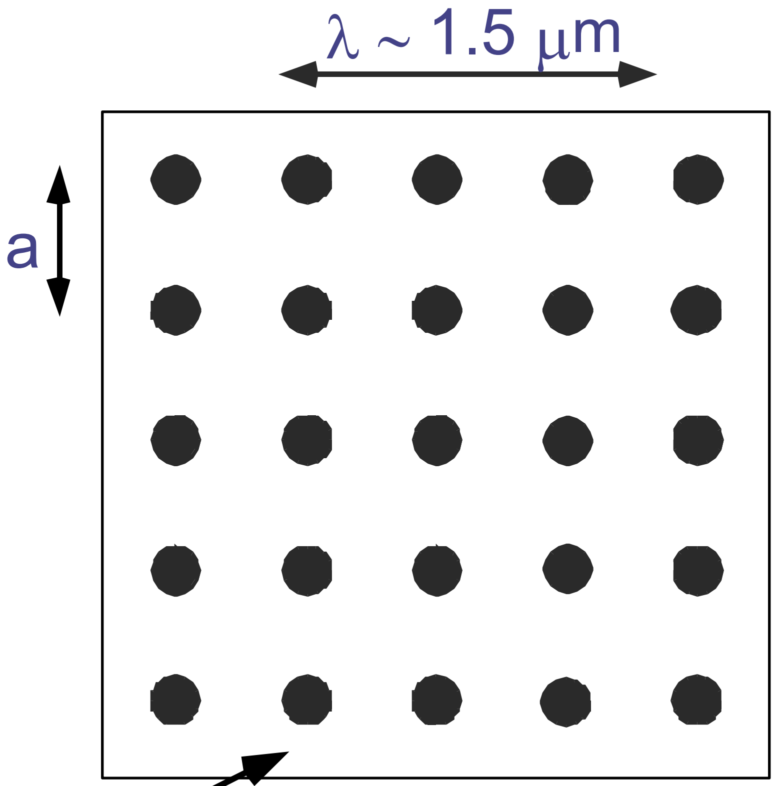
- New conceptual framework in optics

  Band structure concepts.
  Coupled mode theory approach for photon transport.

- Photonic crystal: semiconductors for light.

Two-dimensional photonic crystal

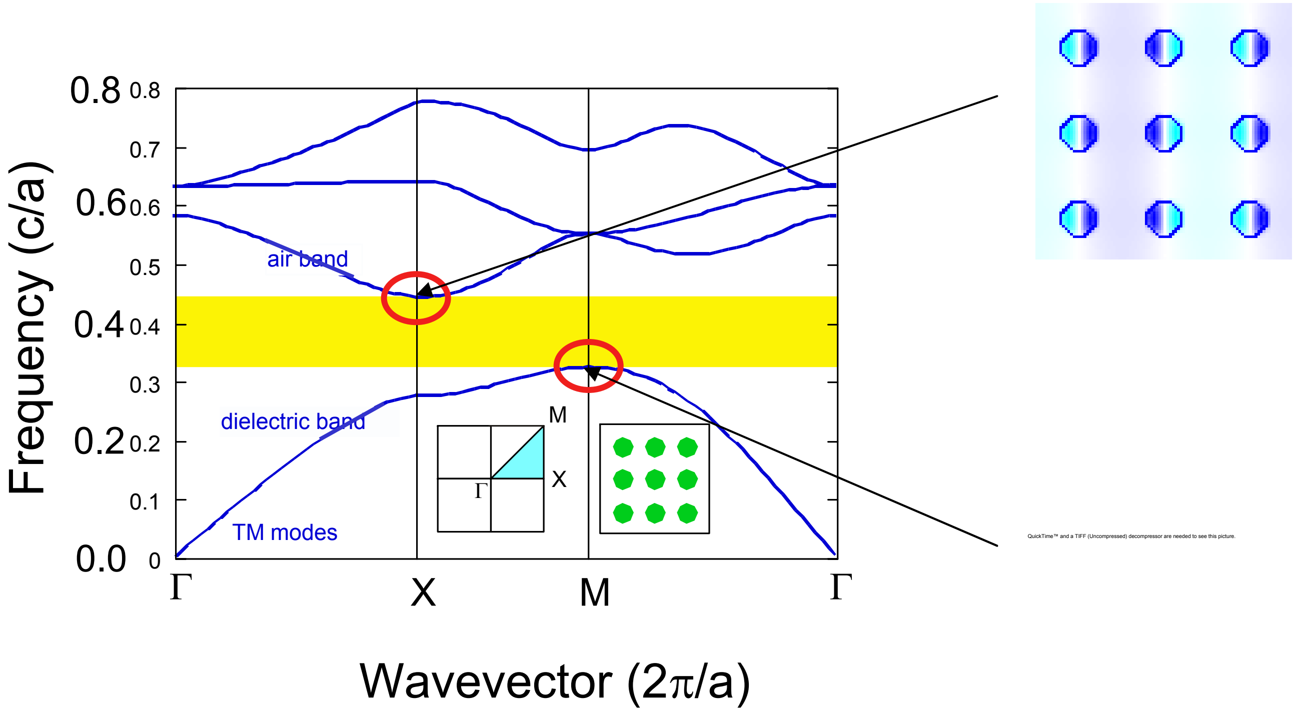




High-index
dielectric material,
e.g. Si or GaAs

Band structure of a two-dimensional crystal

Displacement field parallel to the cylinder

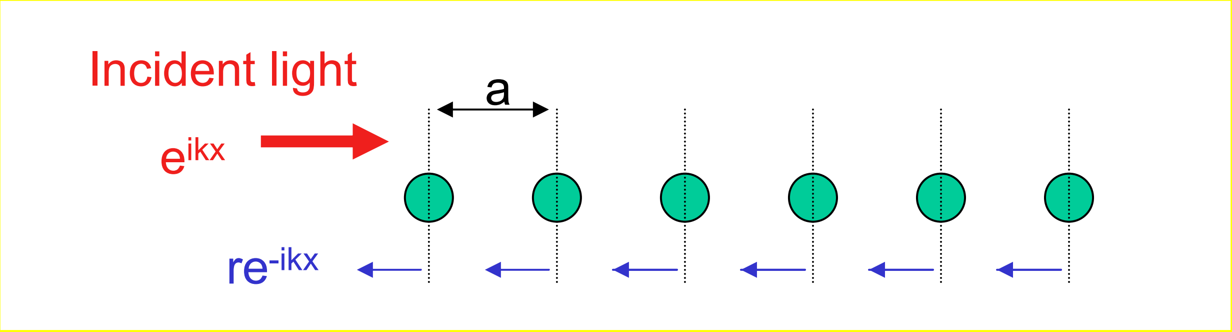


Wavevector determines the phase between nearest neighbor unit cells.

X: (0.5*2π/a, 0): Thus, nearest neighbor unit cell along the x-direction is 180 degree out-of-phase

M: (0.5*2π/a, 0.5*2π/a): nearest neighbor unit cell along the diagonal direction is 180 degree out-of-phase

Bragg scattering



Regardless of how small the reflectivity r is from an individual scatter, the total reflection R from a semi infinite structure:

$$R = re^{-ikx} + re^{-2ika}e^{-ikx} + re^{-4ika}e^{-ikx} + \ldots = re^{-ikx}\frac{1}{1-e^{-2ika}}$$

Diverges if

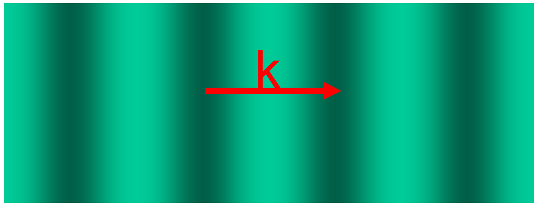
$$\boxed{e^{2ika} = 1 \qquad k = \frac{\pi}{a}}$$

← Bragg condition

Light can not propagate in a crystal, when the frequency of the incident light is such that the Bragg condition is satisfied → Origin of the photonic band gap

A simple example of the band-structure: vacuum (1d)

Vacuum: ε=1, μ=1, plane-wave solution to the Maxwell's equation:



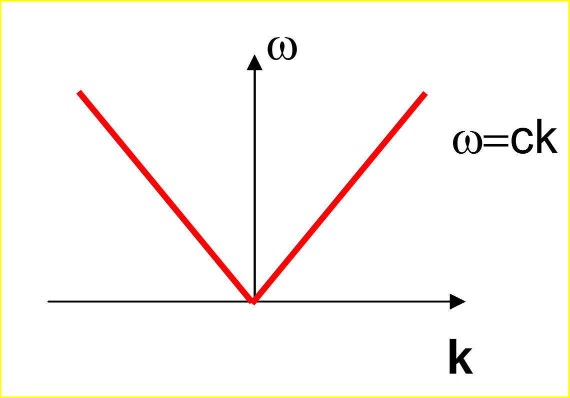
$$\boxed{\mathbf{H}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}}$$

with a transversality constraints: $\mathbf{k} \bullet \mathbf{H} = \mathbf{0}$

A band structure, or dispersion relation defines the relation between the frequency ω, and the wavevector k.

$$\boxed{\omega = c\left|\mathbf{k}\right|}$$

For a one-dimensional system, the band structure can be simply depicted as:

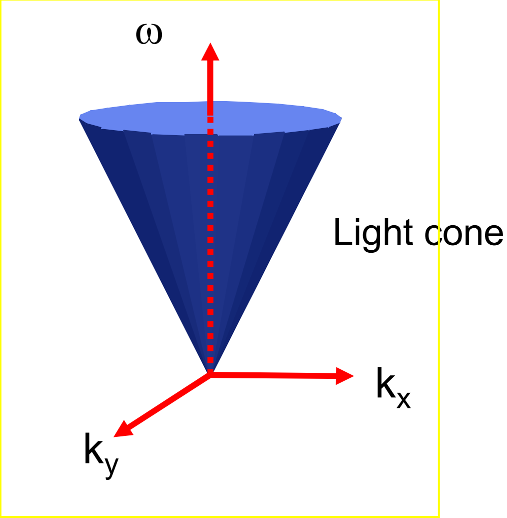


Visualization of the vacuum band structure (2d)

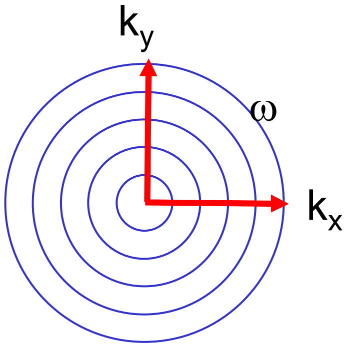
For a two-dimensional system:

$$\boxed{\omega = c\sqrt{k_x^2 + k_y^2}}$$

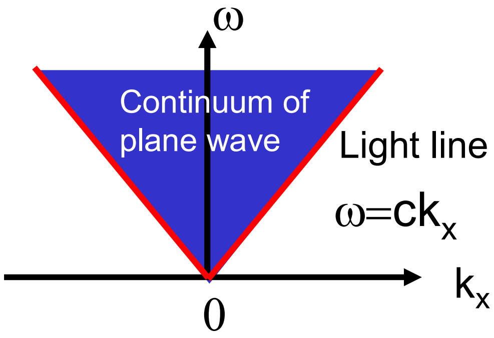
This function depicts a cone: light cone.



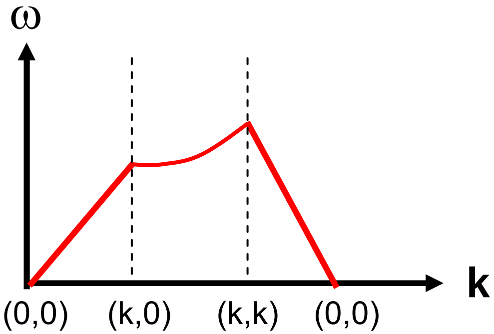
A few ways to visualize this band structure :



Constant frequency contour



Projected band diagram



Band diagram along several “special” directions

Maxwell’s equation in the steady state

Time-dependent Maxwell’s equation in dielectric media:

$$\boxed{\begin{array}{ll} \nabla \bullet \mathbf{H}(\mathbf{r},t)=0 & \nabla \times \mathbf{H}(\mathbf{r},t)-\varepsilon(\mathbf{r})\dfrac{\partial(\varepsilon_0\mathbf{E}(\mathbf{r},t))}{\partial \mathbf{t}}=0 \\ \nabla \bullet \varepsilon\mathbf{E}(\mathbf{r},t)=0 & \nabla \times \mathbf{E}(\mathbf{r},t)+\dfrac{\partial(\mu_0\mathbf{H}(\mathbf{r},t))}{\partial \mathbf{t}}=0 \end{array}}$$

Time harmonic mode (i.e. steady state):

$$\mathbf{H}(\mathbf{r},t)=\mathbf{H}(\mathbf{r})e^{-i\omega t}$$

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}(\mathbf{r})e^{-i\omega t}$$

Maxwell equation for the steady state:

$$\nabla \times \mathbf{H}(\mathbf{r})+i\omega(\varepsilon(\mathbf{r})\varepsilon_0\mathbf{E}(\mathbf{r}))=0$$

$$\nabla \times \mathbf{E}(\mathbf{r})-i\omega(\mu_0\mathbf{H}(\mathbf{r}))=0$$

Master's equation for steady state in dielectric

Expressing the equation in magnetic field only:

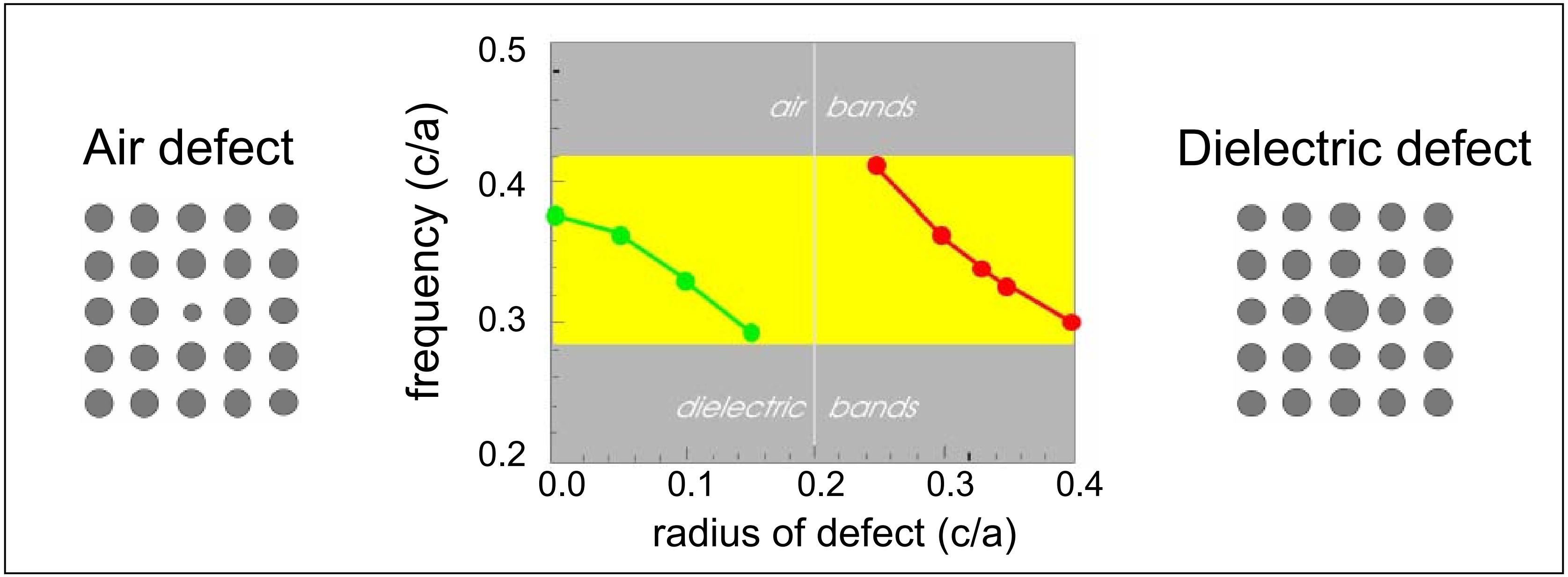
$$\boxed{\nabla\times\frac{\mathbf{1}}{\varepsilon(\mathbf{r})}\nabla\times\mathbf{H}(\mathbf{r})=\left(\frac{\omega}{c}\right)^2\mathbf{H}(\mathbf{r})}$$

$$c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

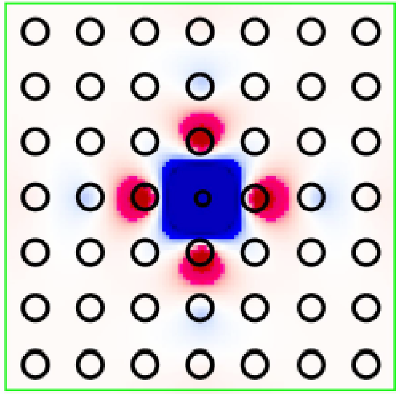
Thus, the Maxwell's equation for the steady state can be expressed in terms of an eigenvalue problem, in direct analogy to quantum mechanics that governs the properties of electrons.

	Quantum mechanics	Electromagnetism
Field	$\Psi(\mathbf{r},t)=\Psi(\mathbf{r})e^{j\omega t}$	$\mathbf{H}(\mathbf{r},t)=\mathbf{H}(\mathbf{r})e^{i\omega t}$
Eigen-value problem	$\hat{H}\Psi(\mathbf{r})=E\Psi(\mathbf{r})$	$\Theta\mathbf{H}(\mathbf{r})=\left(\frac{\omega^2}{c^2}\right)\mathbf{H}(\mathbf{r})$
Operator	$\hat{H}=\frac{-\hbar^2\nabla^2}{2m}+V(\mathbf{r})$	$\Theta=\nabla\times\frac{\mathbf{1}}{\varepsilon(\mathbf{r})}\nabla\times$

Donor and Acceptor States

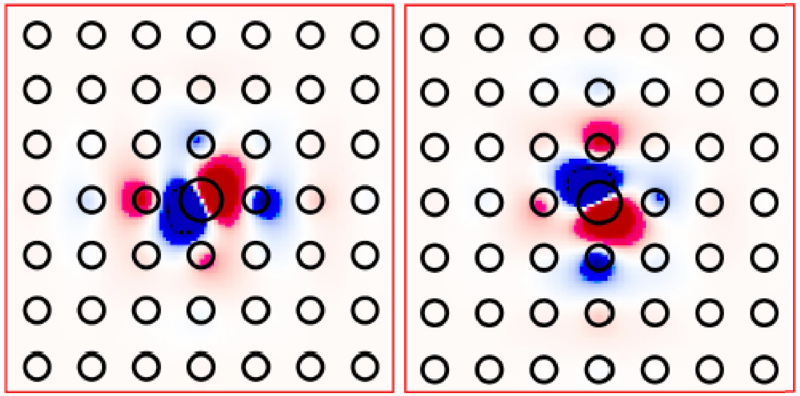


Air defect



s-state

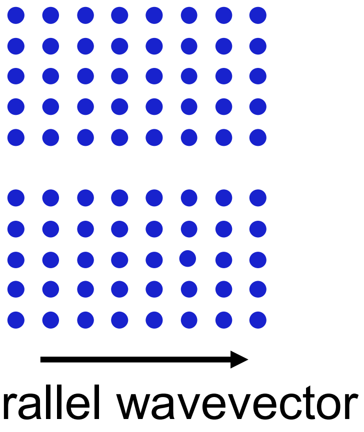
Dielectric defect

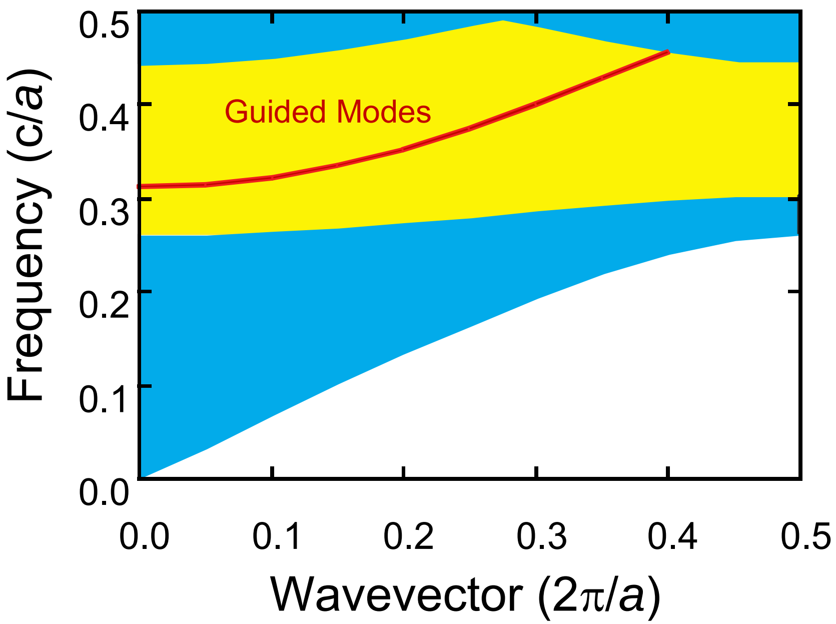


p-state

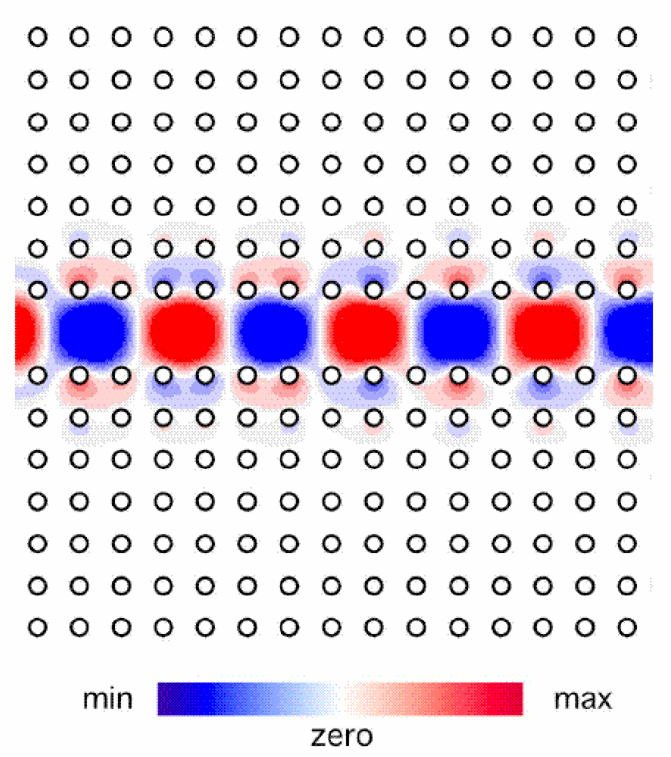
P. R. Villeneuve, S. Fan, and J. D. Joannopoulos, Phys. Rev. B 54, 7837 (1996)

Line defect states: projected band diagram

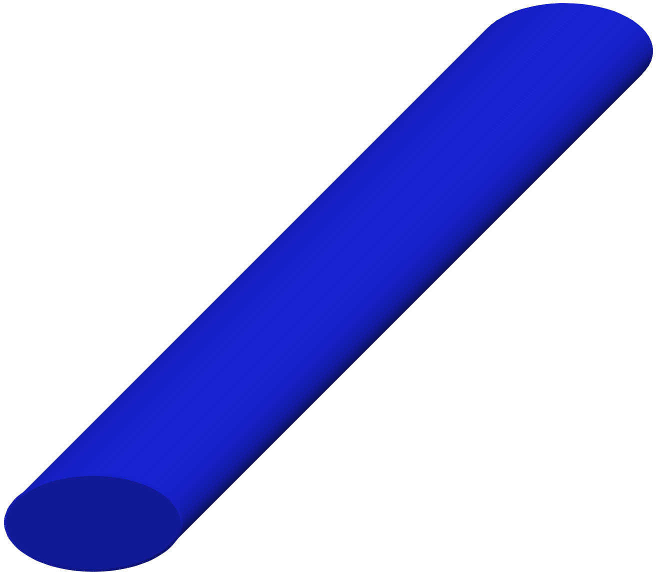




Electric field

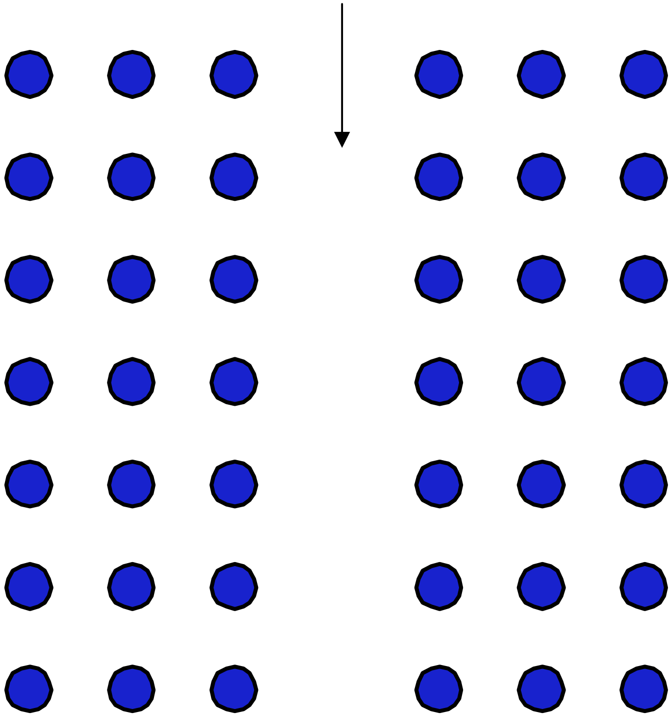


Photonic crystal vs. conventional waveguide



High-index region

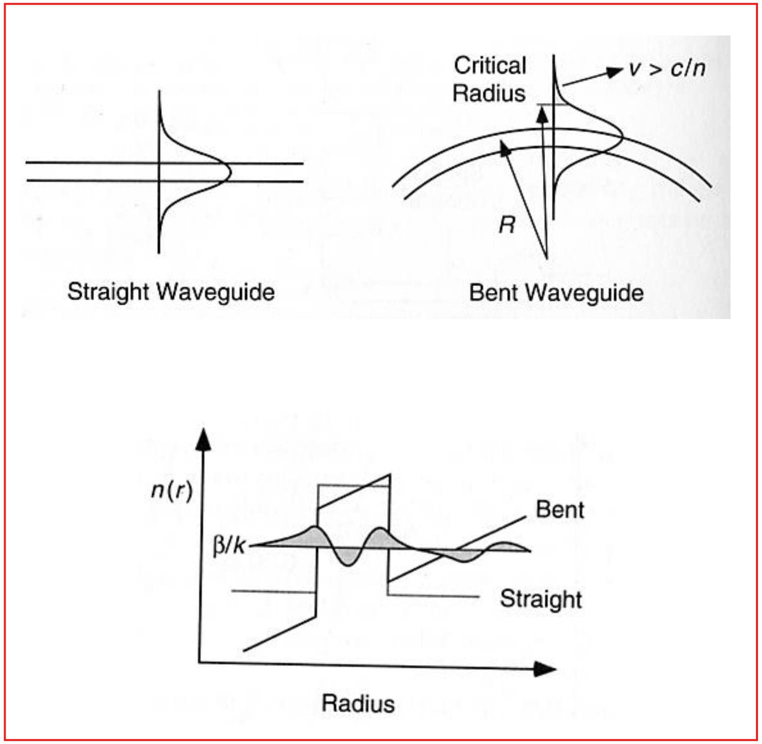
Conventional waveguide



Low index region

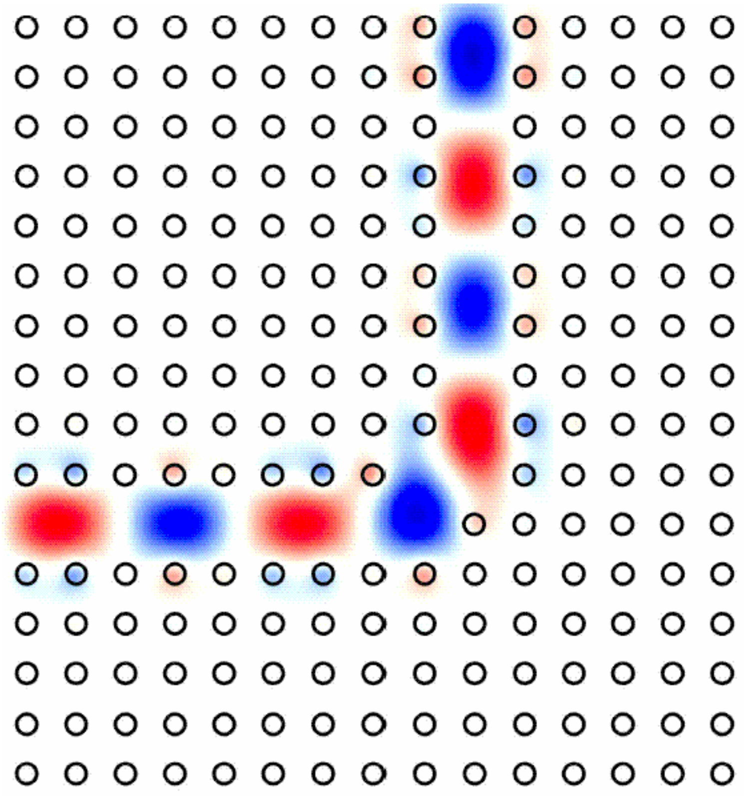
Photonic crystal waveguide

High transmission through sharp bends



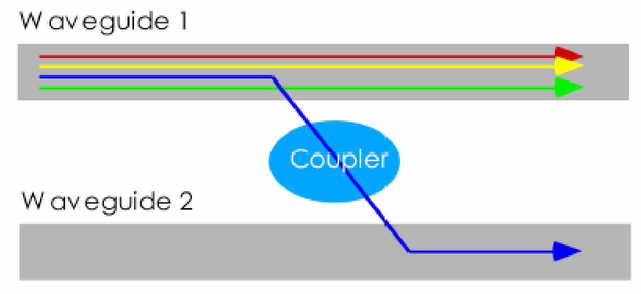
$$\alpha = \frac{1}{2}\left(\frac{\pi}{aV^3}\right)^{1/2}\left[\frac{\kappa a}{\gamma a K_1(\gamma a)}\right]^2 R^{-1/2} e^{-UR}$$

Polluck, Fundamentals of Optoelectronics, 1995

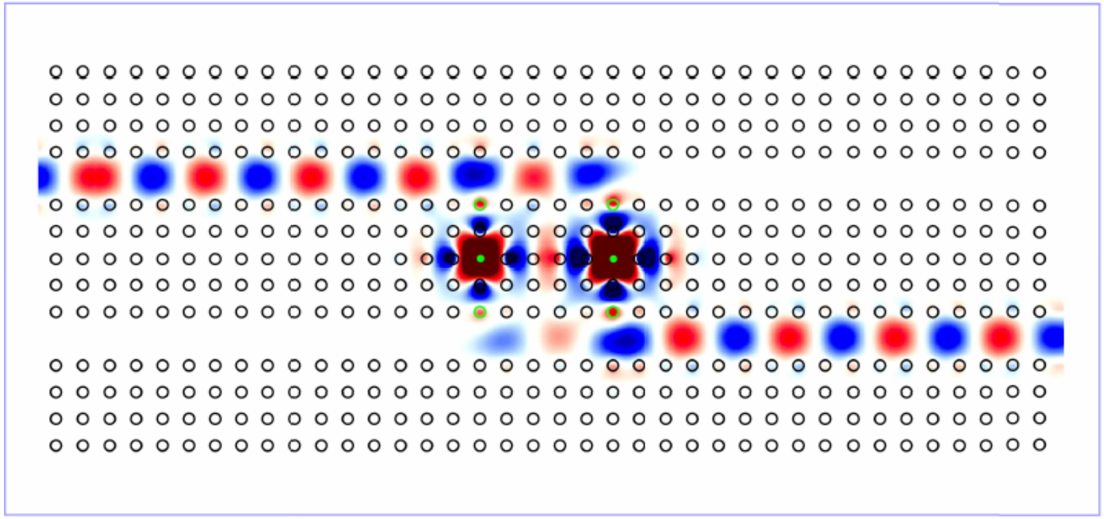


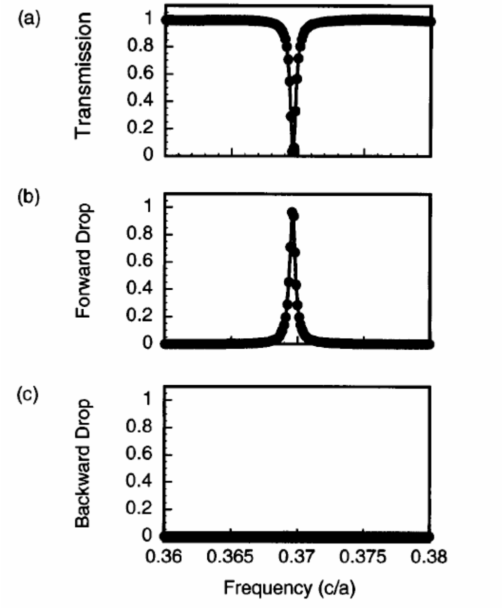
A. Mekis et al, PRL, 77, 3786 (1996)

Micro add/drop filter in photonic crystals



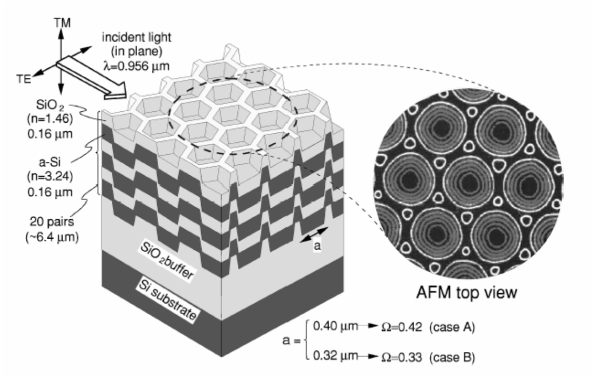
- *Two resonant modes with even and odd symmetry.*
- *The modes must be degenerate.*
- *The modes must have the same decay rate.*

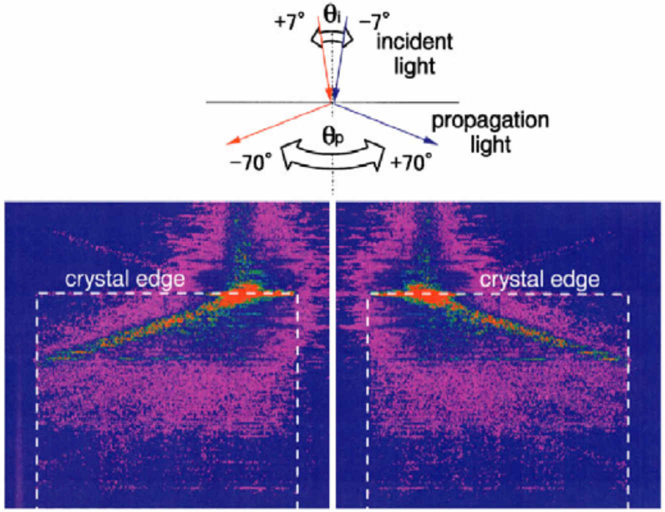




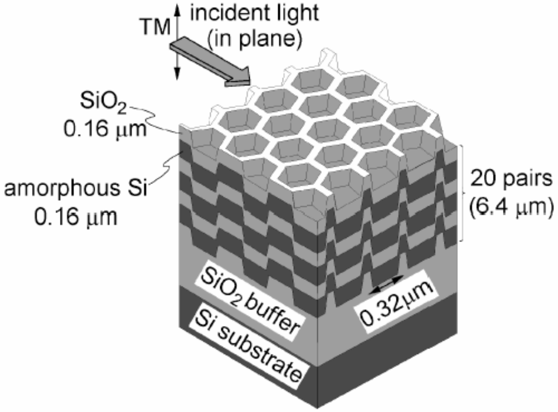
S. Fan, P. R. Villeneuve, J. D. Joannopoulos, and H. A. Haus, Physical Review Letters 80, 960 (1998).

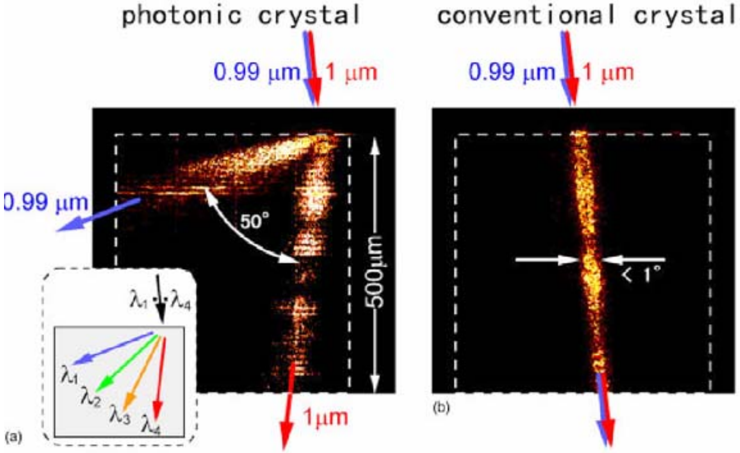
Super-lens and Super-prism effects





H. Kosaka et al, Phys. Rev. B. 58, 10096, 1998



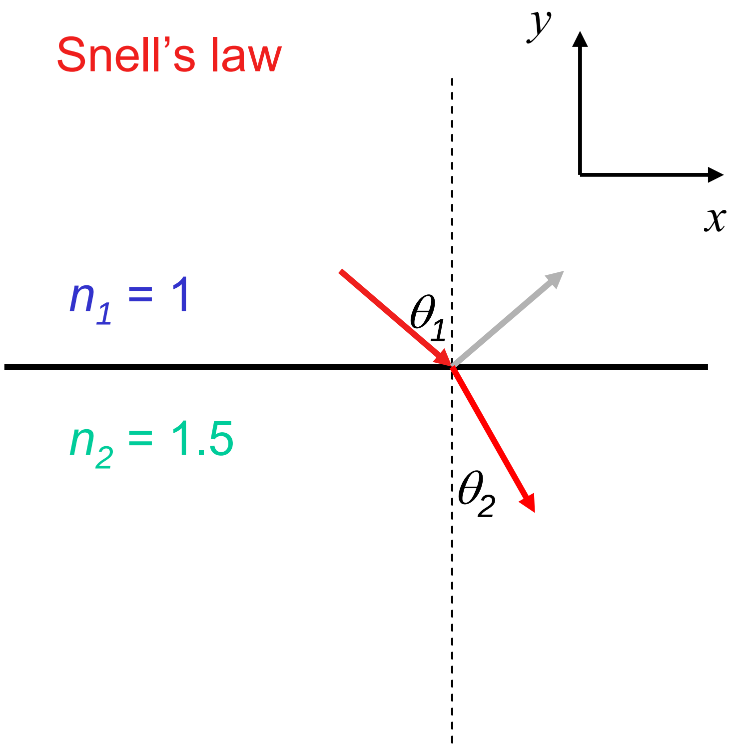


H. Kosaka et al, Appl. Phys. Lett. 74, 1370, 1999

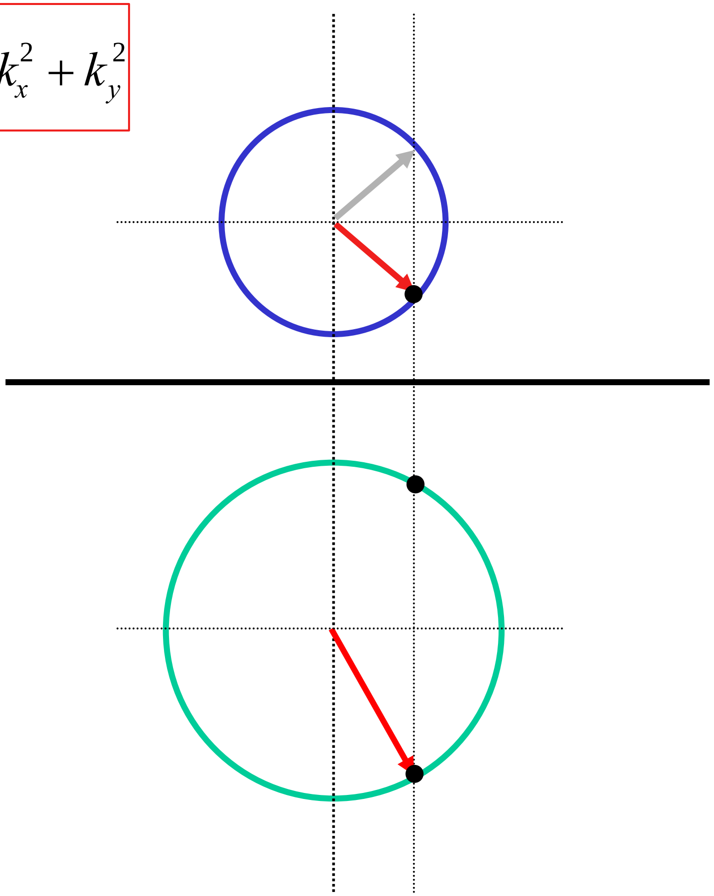
Snell's law in terms of a constant frequency circle

Example: using constant frequency diagram to derive Snell's law and the condition for total internal reflection.

Snell's law

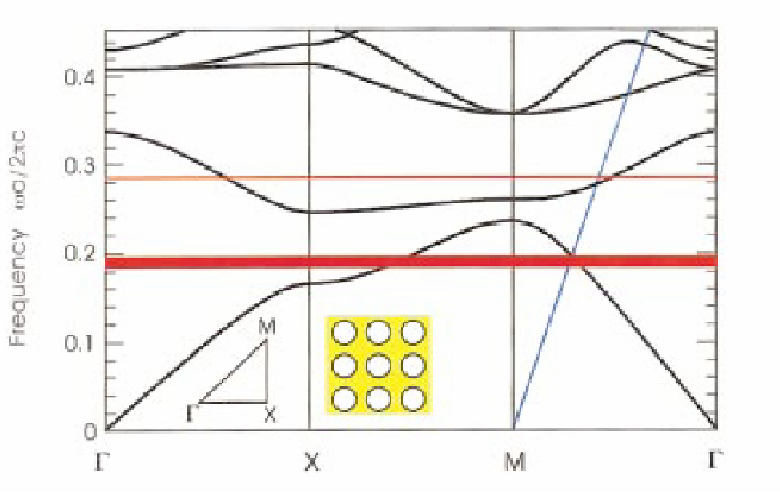


$$\boxed{n^2 \frac{\omega^2}{c^2} = k_x^2 + k_y^2}$$

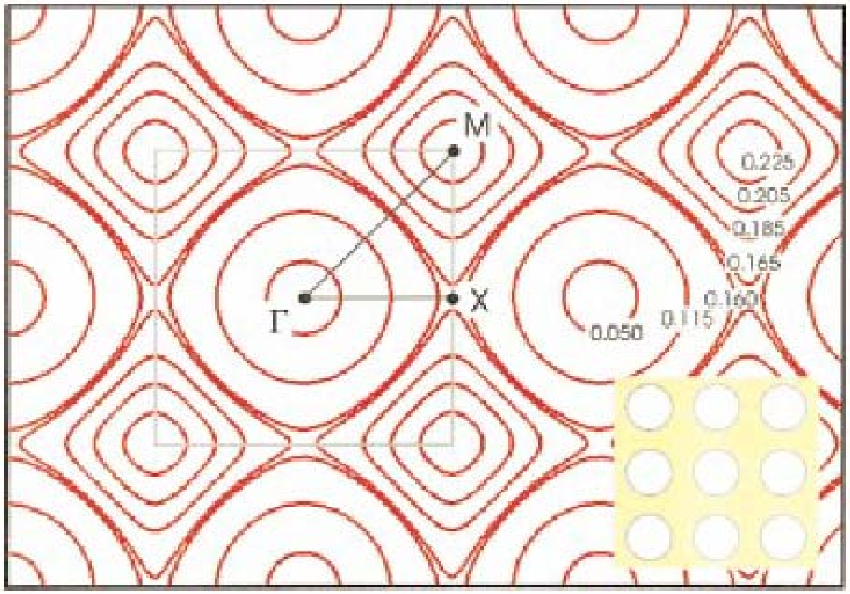


$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Constant frequency contour in a 2D crystal



Constant frequency diagram for the first band

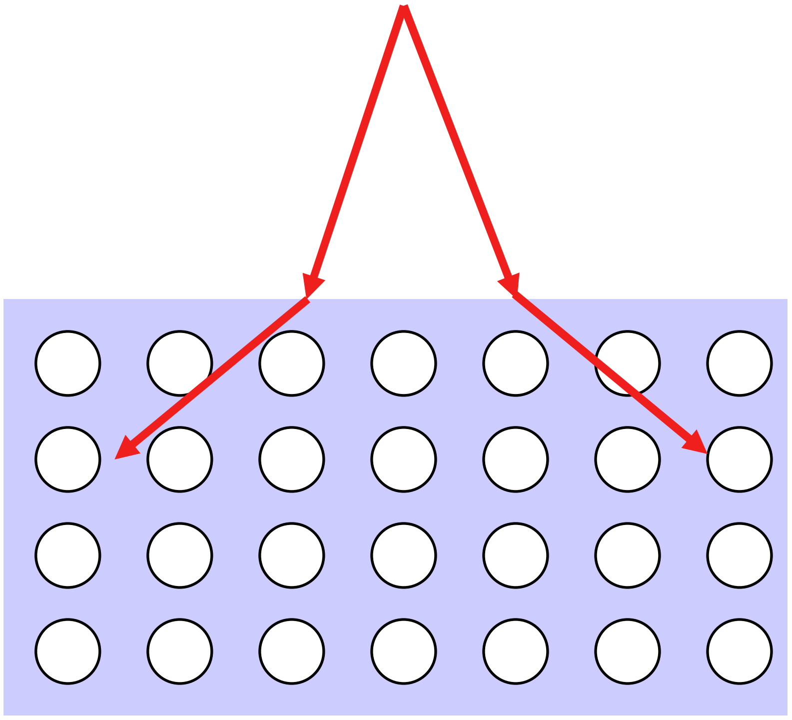


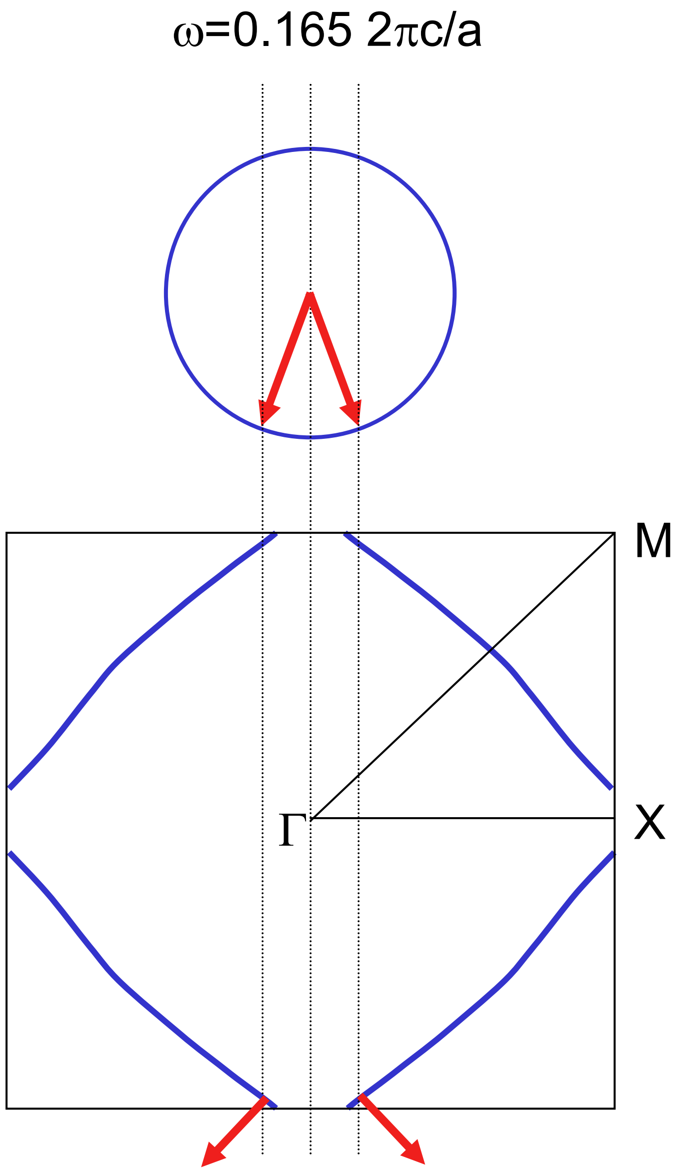
- *At low frequencies, the constant frequency diagram approaches a circle, the photonic crystal behaves as a uniform dielectric as far as diffraction is concerned*
- *With increasing frequencies, the constant frequency contour becomes more complicated, leading to effects including superprism, superlens, negative refraction, and self-collimation.*

Luo et al, Phys. Rev. B 65, 201104, 2002; M. Notomi, Phys. Rev. B 62, 10692, 2000

Super-lens and constant frequency

□□□





$V_g = \partial_{\mathbf{k}}\, \omega(\mathbf{k})$ group velocity

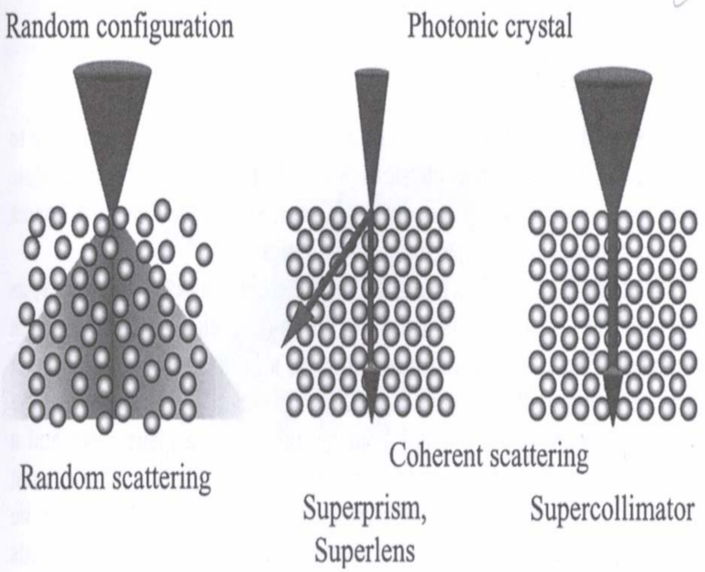


Figure 2.4.4 The comparison of ordinary scattering and scattering in photonic crystals. Scattering in photonic crystals occurs coherently. Moreover, propagation directions are divided into sensitive cases to incidental directions (these instances correspond to superprisms and superlenses) and insensitive cases (these instances correspond to supercollimators).

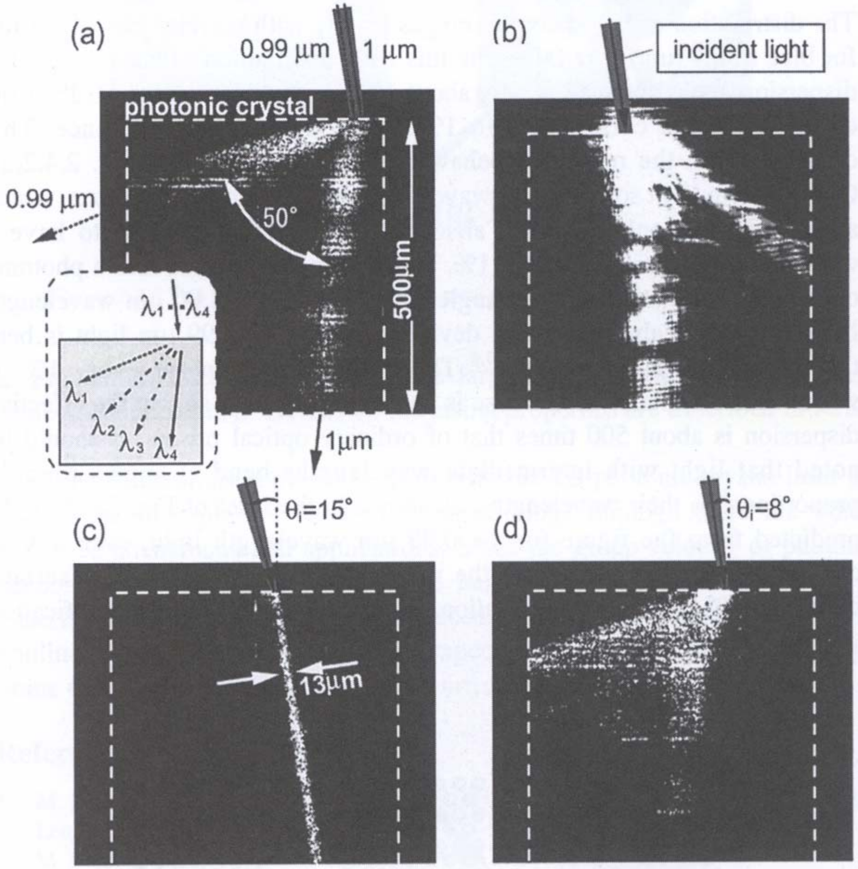


Figure 2.4.2 Phenomena observed in photonic crystals. (a) Superprism phenomenon,[2-4] (b) multi-refrigence phenomenon,[3] (c) supercollomator phenomenon,[5] and (d) superlens phenomenon.[5] The in-plane lattice constants in (a), (c) and (d) were all 0.32 µm, and 0.4 µm in (b). It was 0.32 µm in the layer normal direction for all examples. Micrographs were taken using a microscope equipped with a charge coupled detector (CCD) camera to show propagation of the laser light injected from the side of the photonic crystal.

Photonic Band Engineering

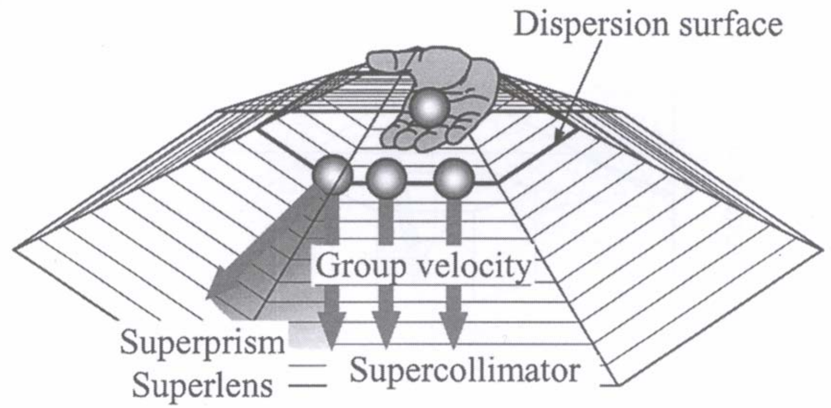
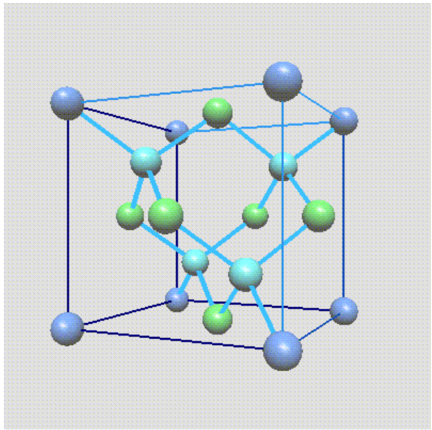
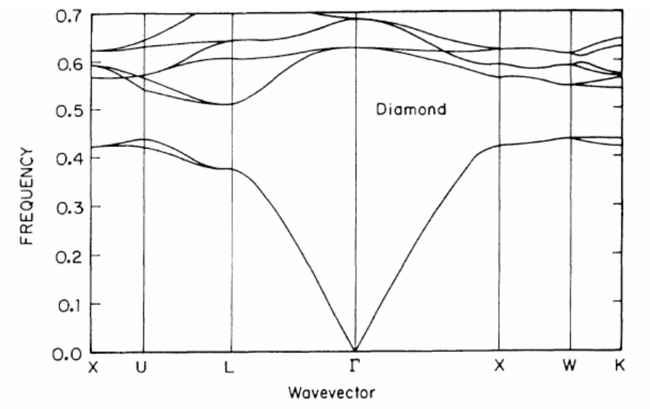


Figure 2.4.5 Conceptual figure to indicate guidelines for photonic band engineering for manipulation of light rays at will. Rays move along the potential gradient direction of an equal energy surface (dispersion surface) in photonic bands. Balls fall down in the same direction in a flat plane case, while they reflect initial conditions sensitively at angles. These behaviors correspond to the phenomenon describing supercollimators, superprisms, and superlenses.

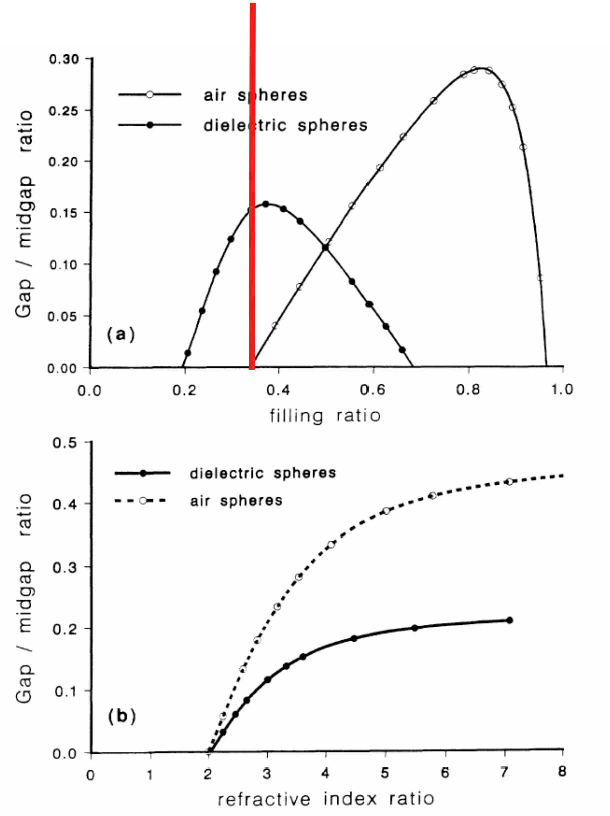
$V_g = \partial_{\mathbf{k}}\, \omega(\mathbf{k})$ group velocity

3D photonic crystal with complete band gap





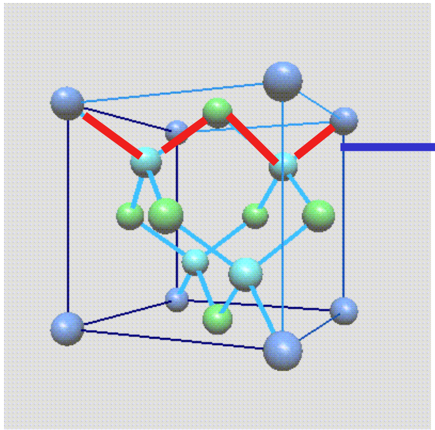
34 % when spheres are touching each other

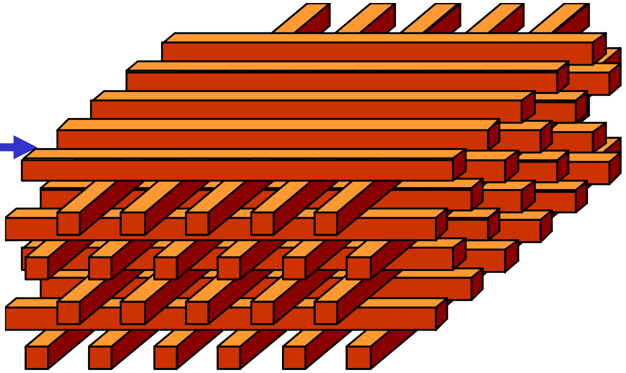


Complete band gap observed in both air spheres and dielectric spheres
Refractive index ratio needs to exceed 2 in order for band gap to open
Optimal structure consists of connected dielectric and air networks.

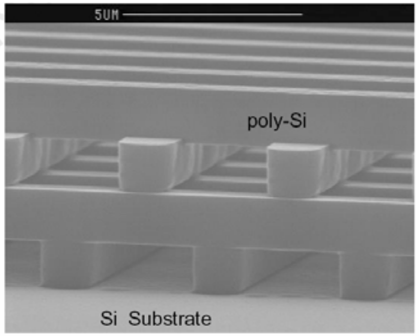
Ho, Chan, Soukoulis, PRL, 65, 3152 (1990)

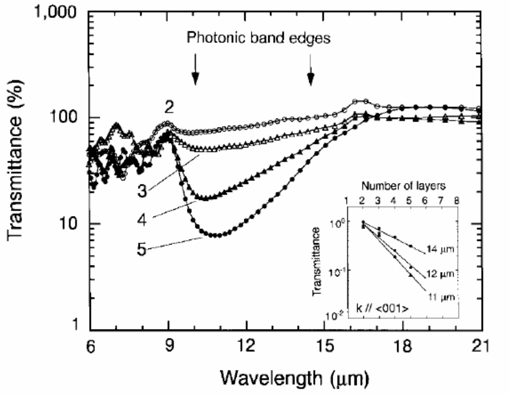
Variants of diamond structure, practical 3d structures

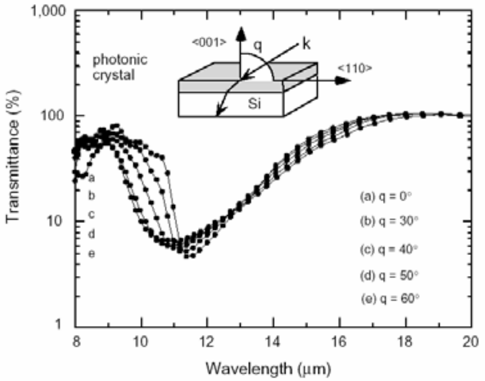




Chan et al, Solid State Communication, 89, 413-6 (1994)

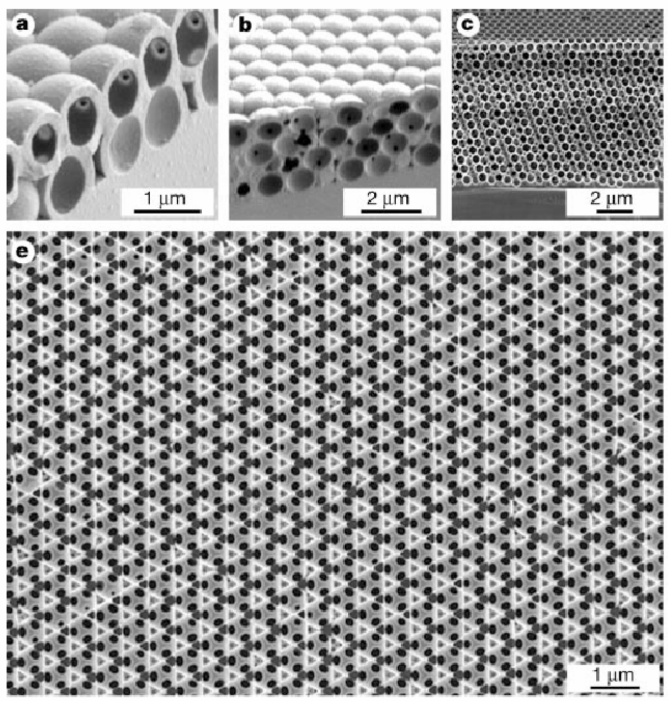


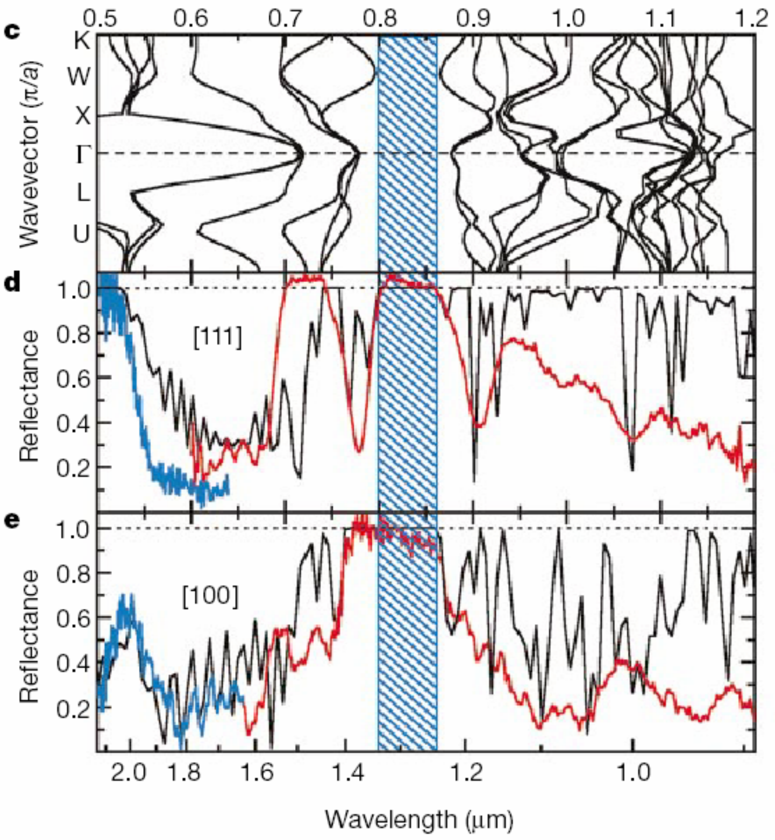




S. Lin et al, Nature, vol. 394, p. 251-3, (1998)

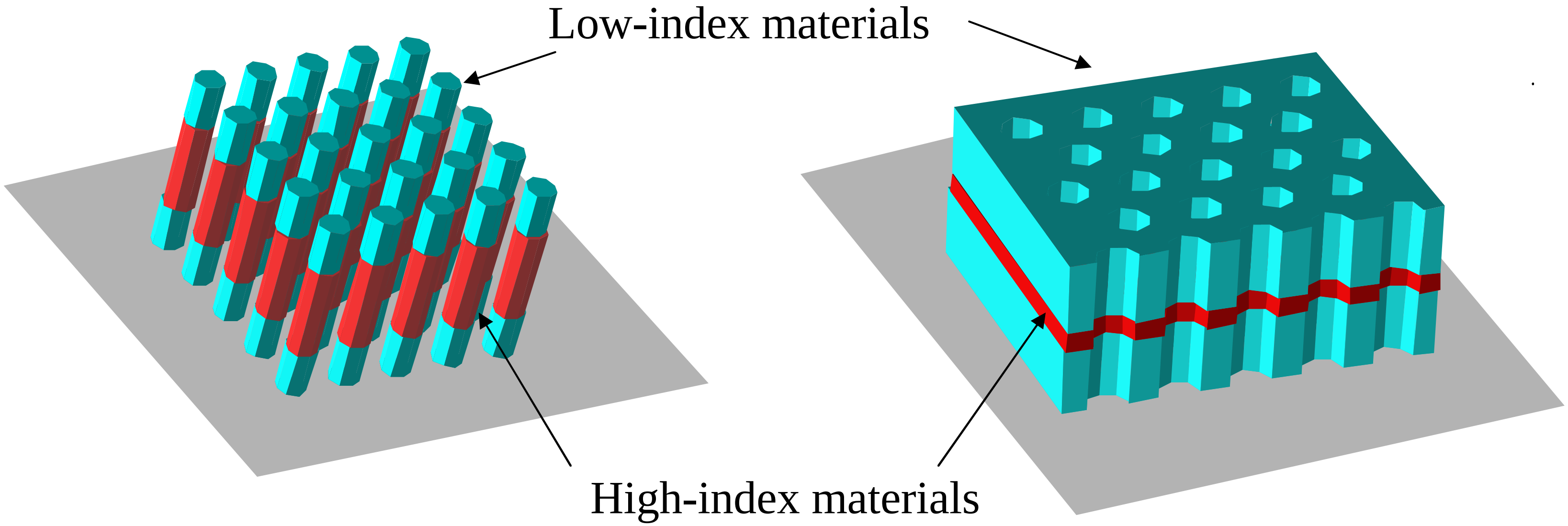
Self-assembled 3D photonic crystal structures





Y. Vaslov et al, Nature, vol. 414, p. 289, (2001)

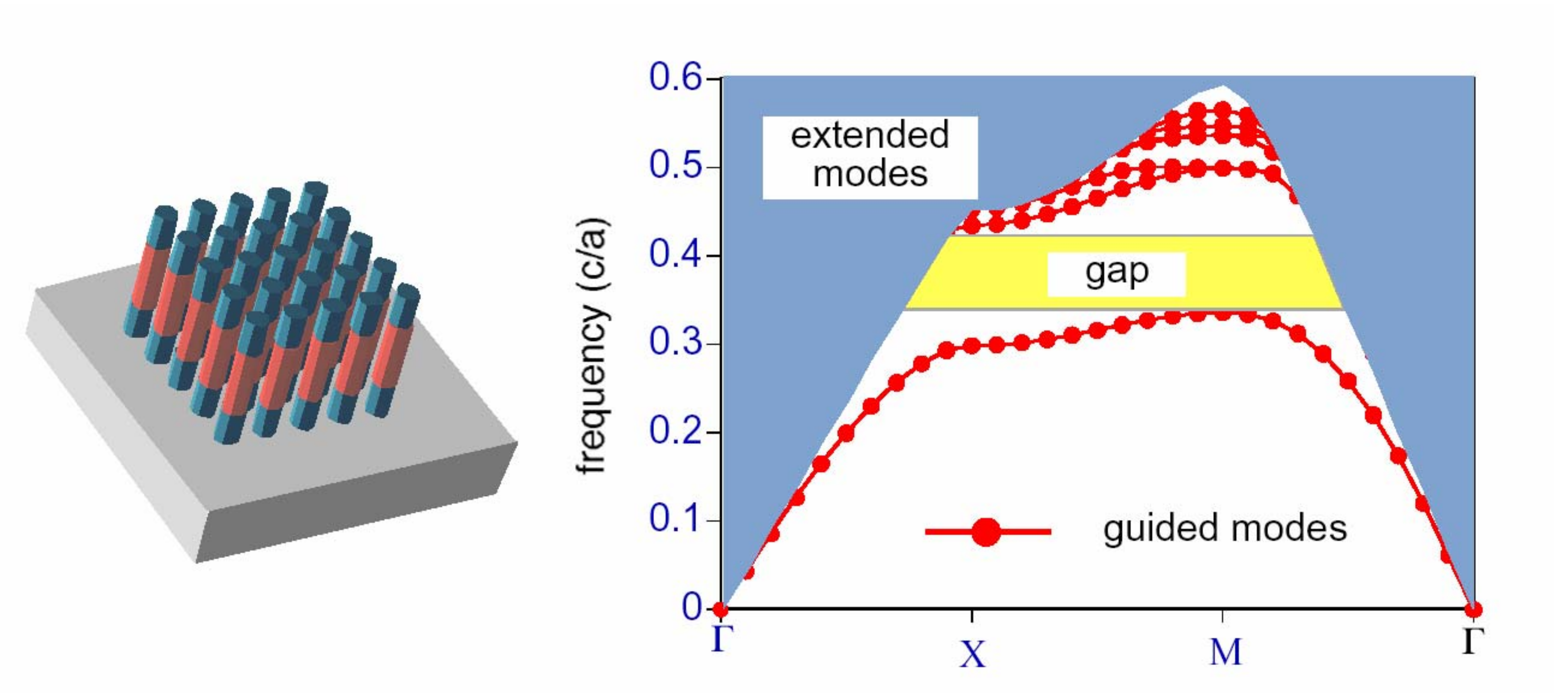
Photonic crystal slab structures



In plane 2D photonic band gap provides complete in plane confinement.
Out of plane confinement provided by high index guiding

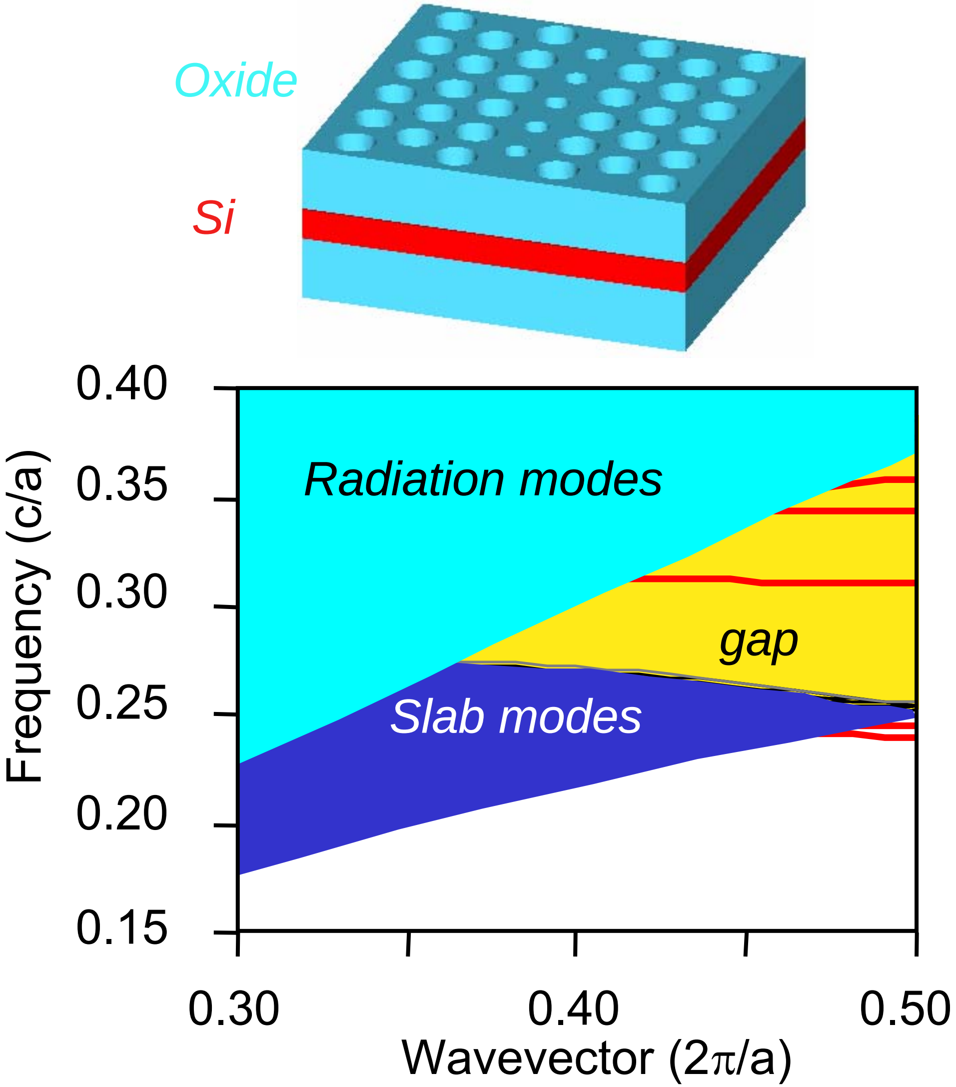
Ease of fabrication
In complete confinement in the third dimension

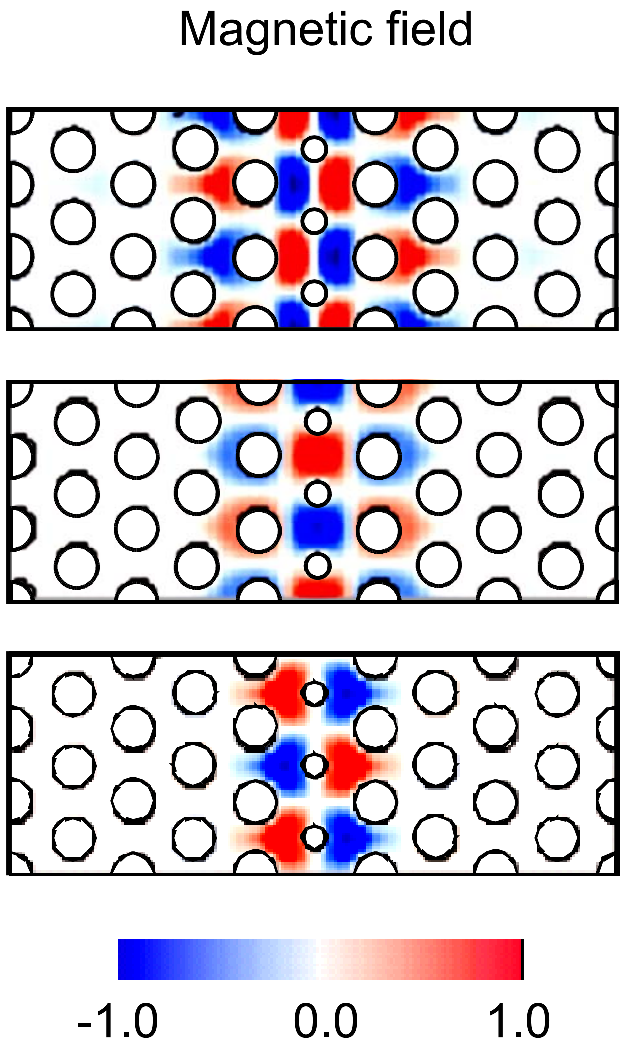
Photonic band diagram for photonic crystal slabs



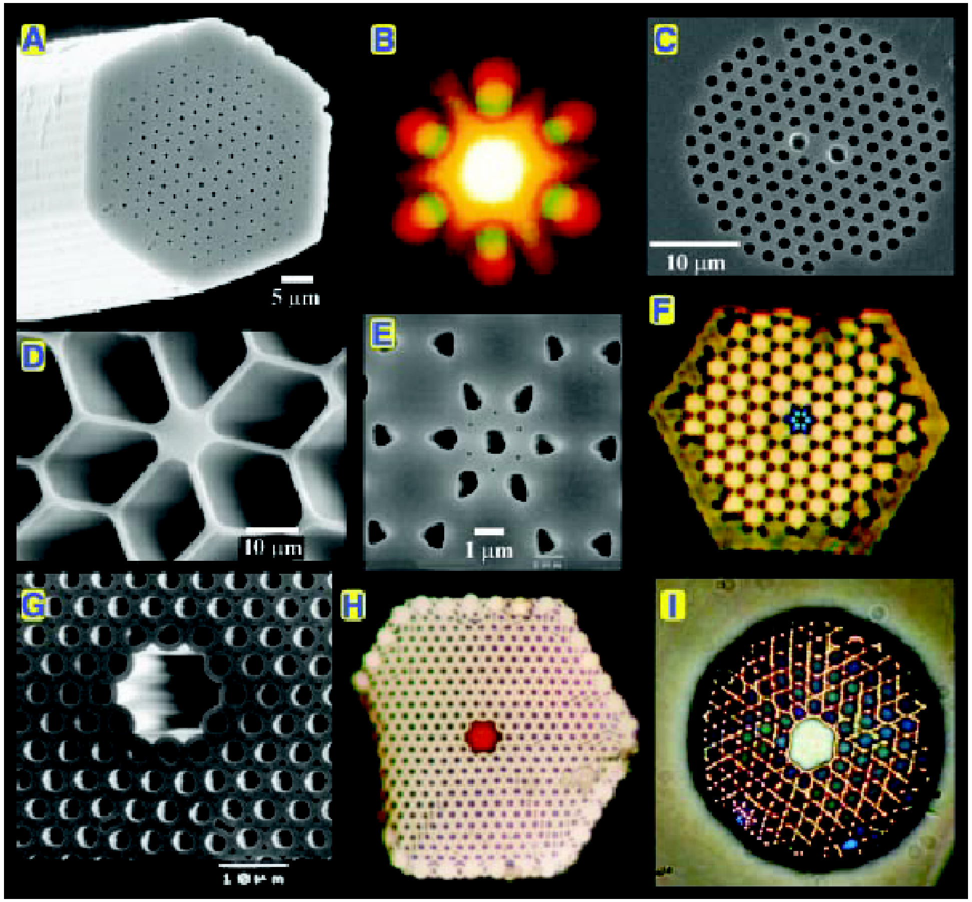
Radiation modes above the light line.
Losslessly guided modes below the light line.
Incomplete band gap in the guided mode spectrum

Waveguides in dielectric slabs





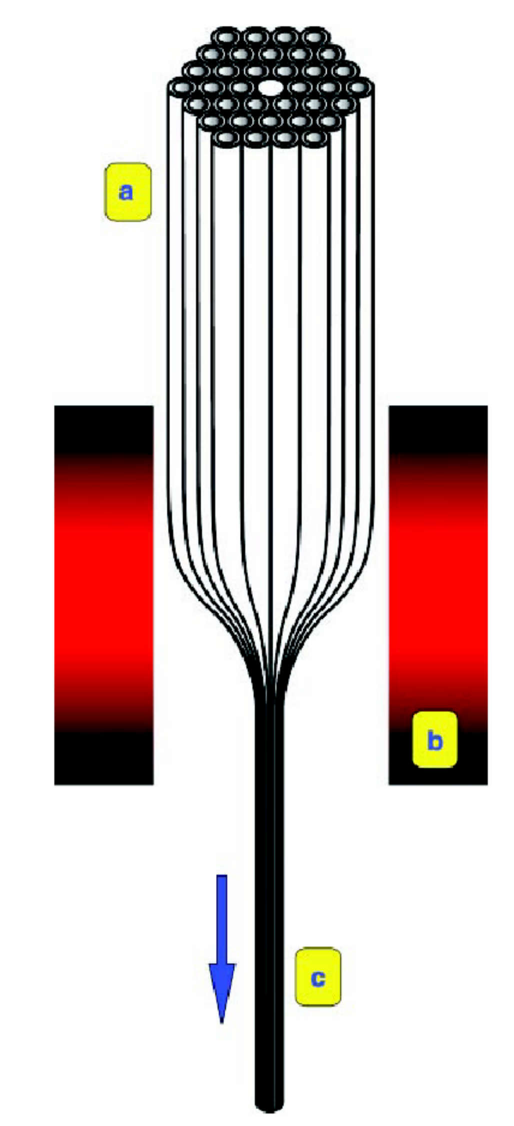
Photonic crystal fibers



“Photonic crystal fibers guide light by corralling it within a periodic array of microscopic air holes that run along the entire fiber length….”

P. Russell, Science, 299, 358, 2003

Stack and Draw Technique

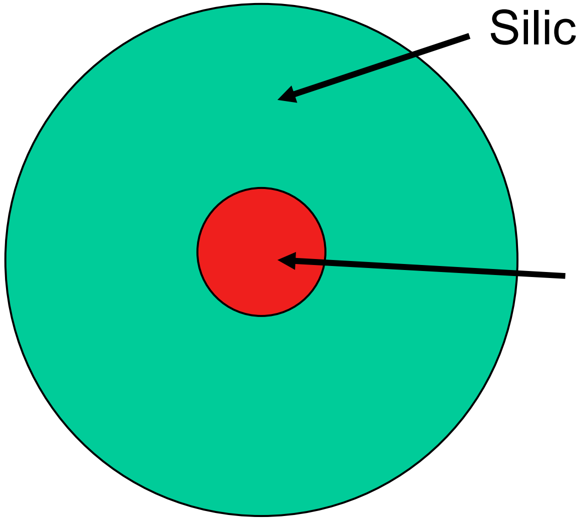


Macroscopic “preform” with the required periodicity

Furnace to soften the silica gas

Photonic crystal fiber

A brief overview of conventional fiber structure



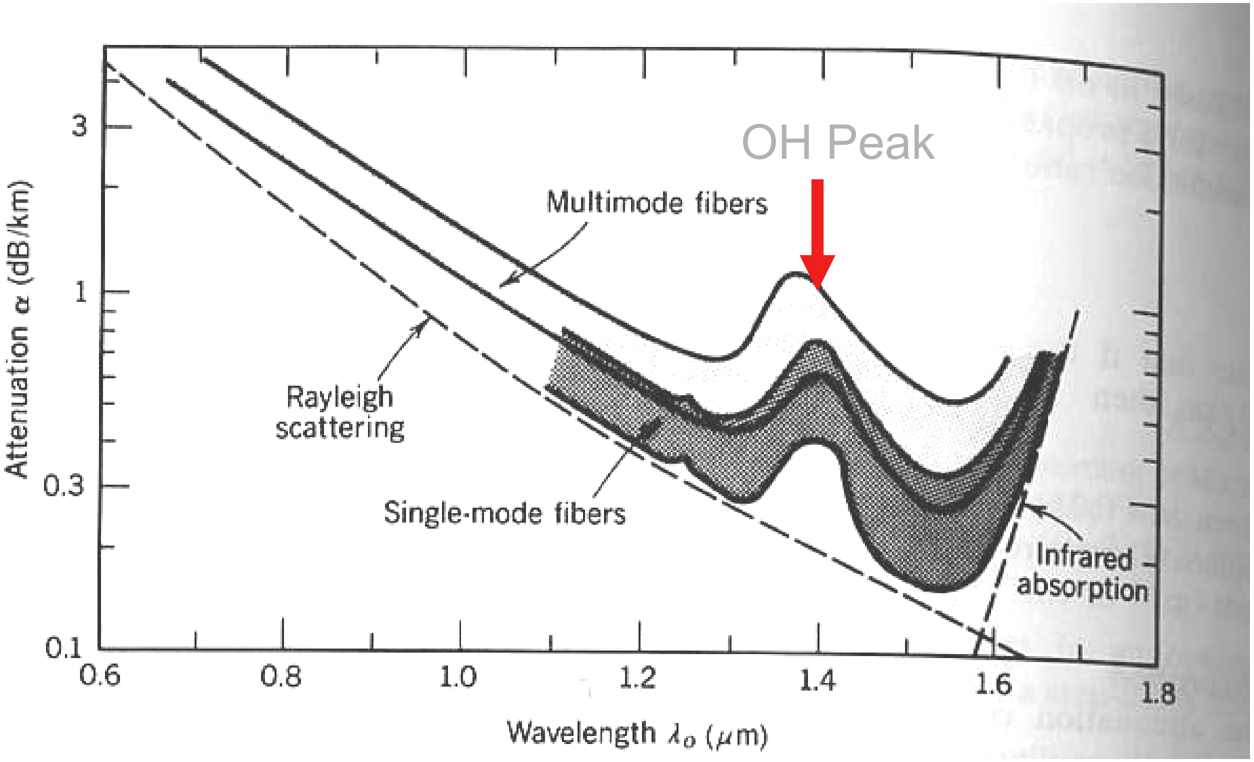
Silica cladding: n_1 = 1.44 to 1.46

Silica core doped with Germanium, Boron or Titantium

$$\frac{n_2 - n_1}{n_2} = 0.001 \sim 0.02$$

Core diameter for single mode fiber about 8 μm.

Propagation loss in conventional optical fiber



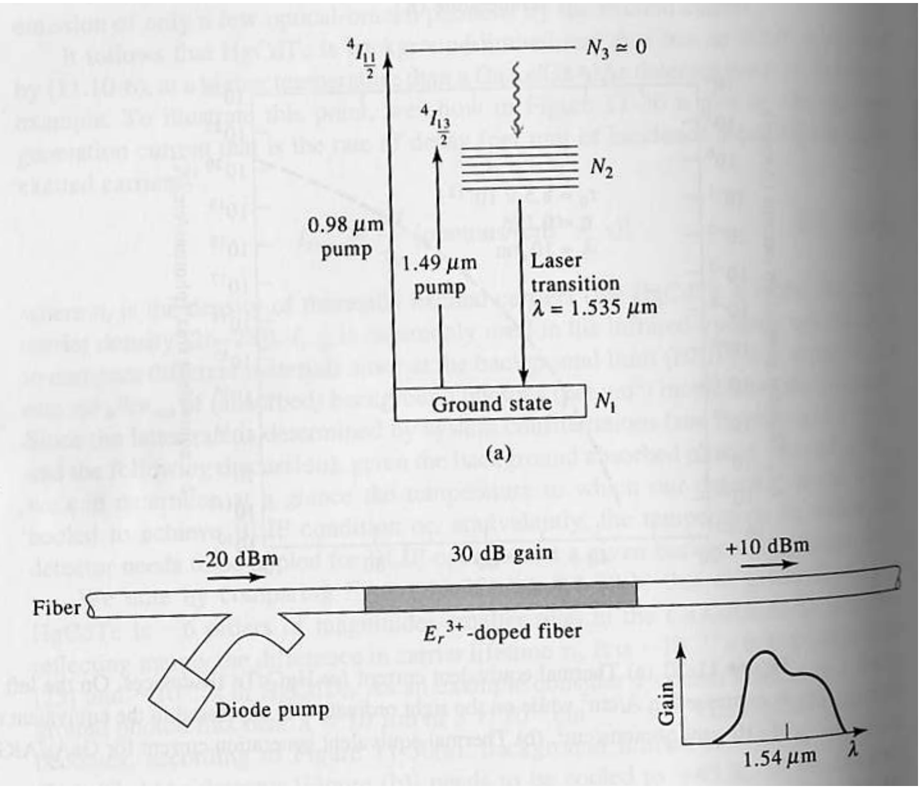
Rayleigh scattering: from random localized variation of the molecular positions in glass which creates random inhomogeneities in index.

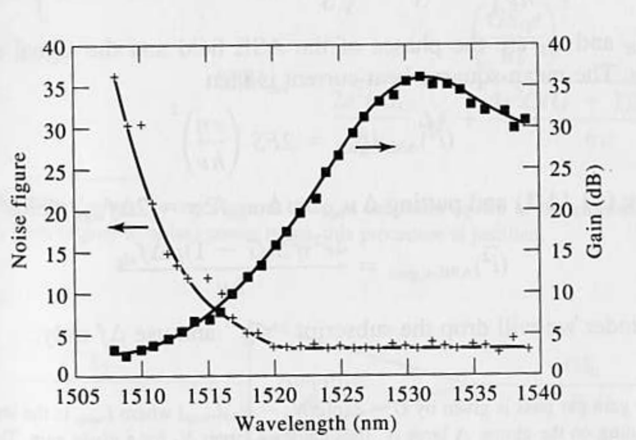
Infrared absorption: from vibrational transitions.

Absolute minimum at 1.55 micron, at 0.16dB/km, about 3.6% per km.

Saleh and Teich, Fundamentals of Photonics, 1991

fiber optical amplifier for long distance communication



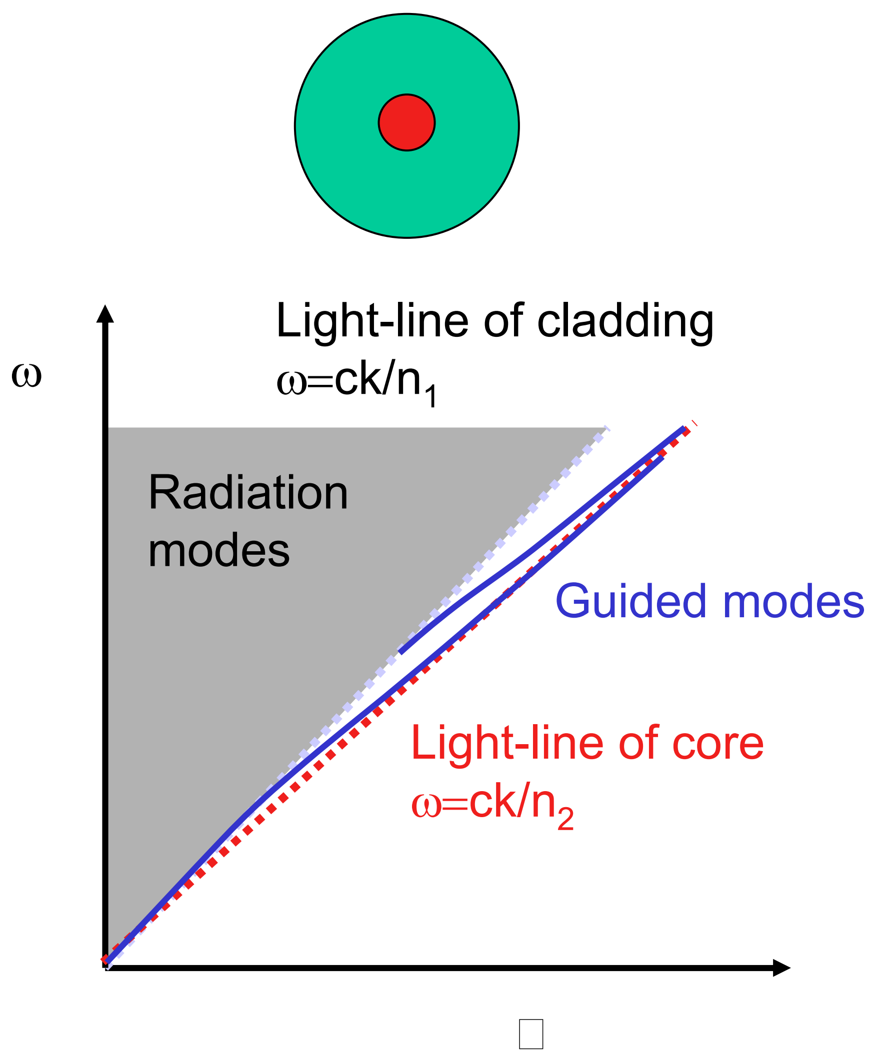


Er, gain maximum close to 1.55 micron

Usable bandwidth limited by the amplifier bandwidth to be approximately 30nm

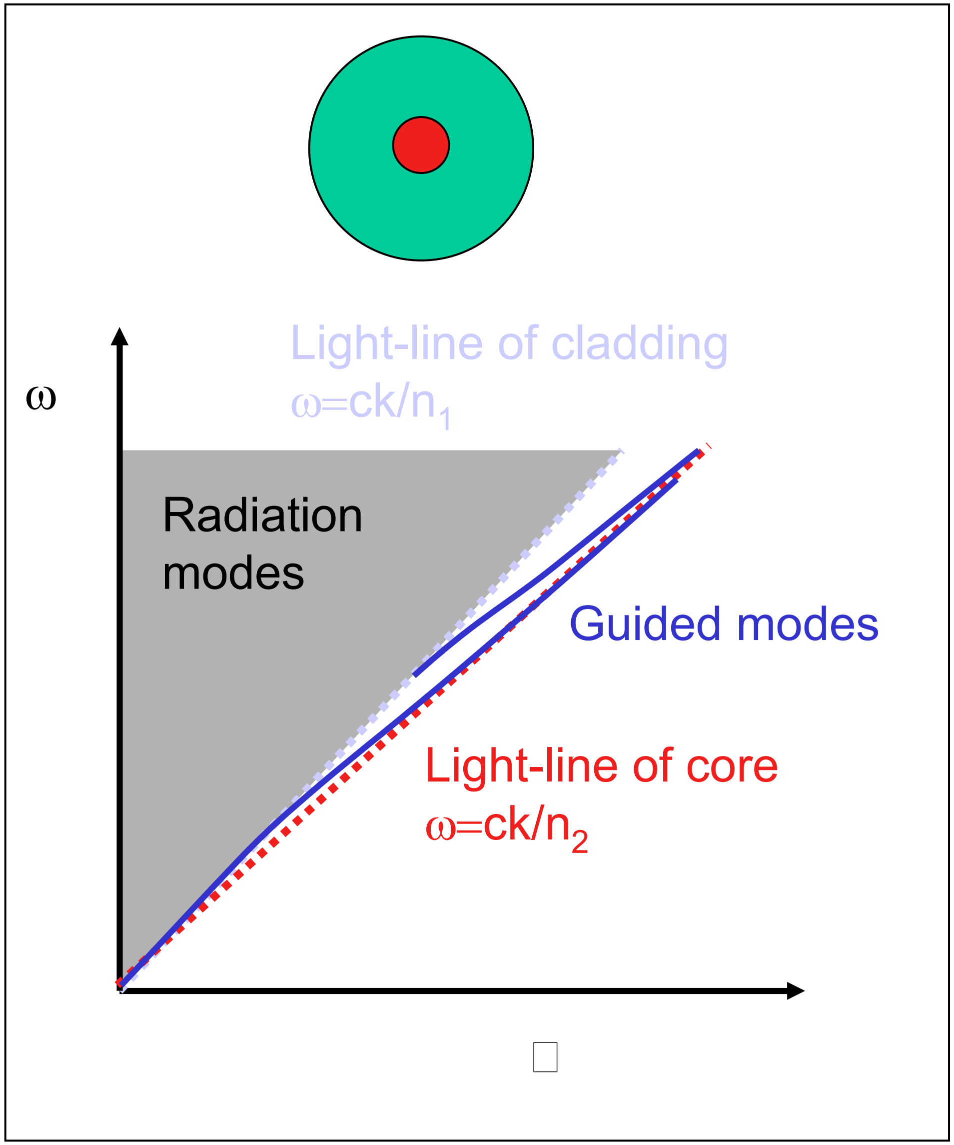
Improving bandwidth by removing amplifiers, guiding in air?

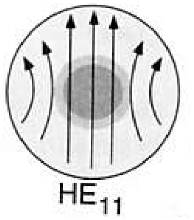
Band diagram for conventional fibers



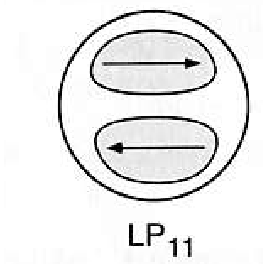
Guiding mode exists between the light line of the cladding and light line of the core.

Lower and higher order modes





Fundamental mode

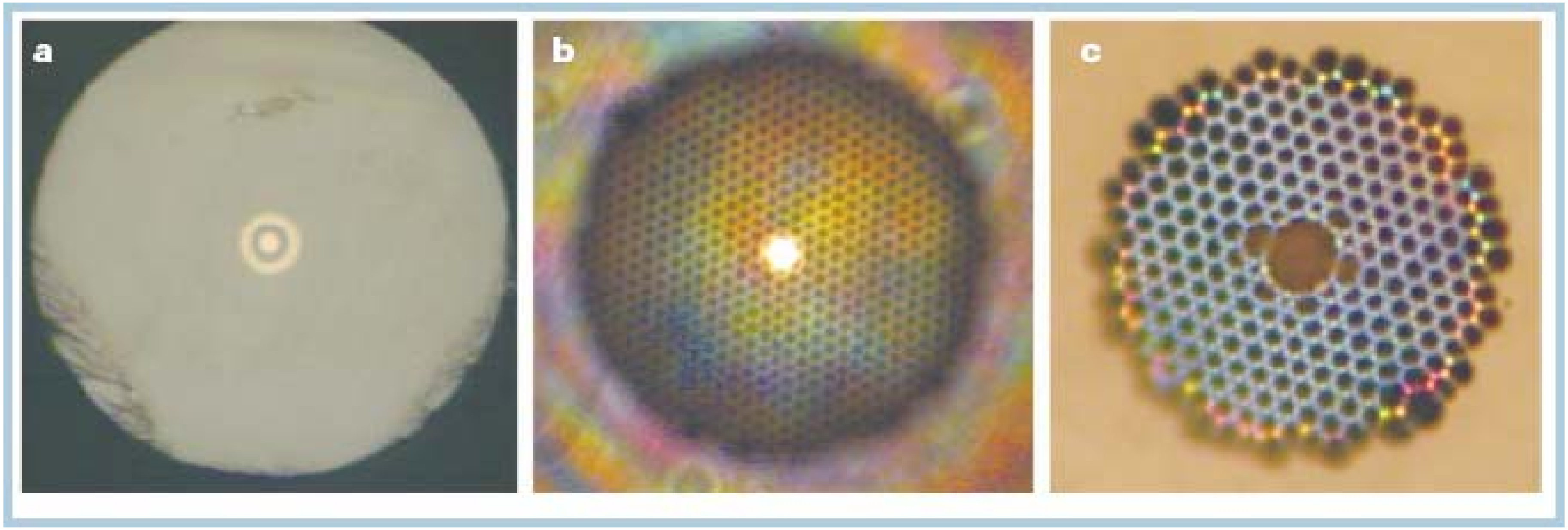


Higher order mode

The V-number determines the number of modes in the fiber

$$V = \frac{2\pi a}{\lambda}\left(\sqrt{n_{core}^{2} - n_{cladding}^{2}}\right)$$

Conventional vs Photonic Crystal Fibers



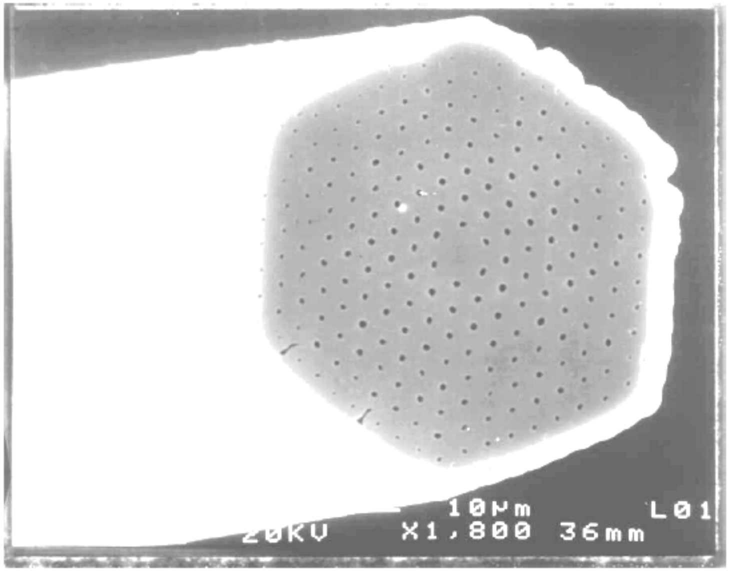
Conventional fiber
Core diameter 9 micron

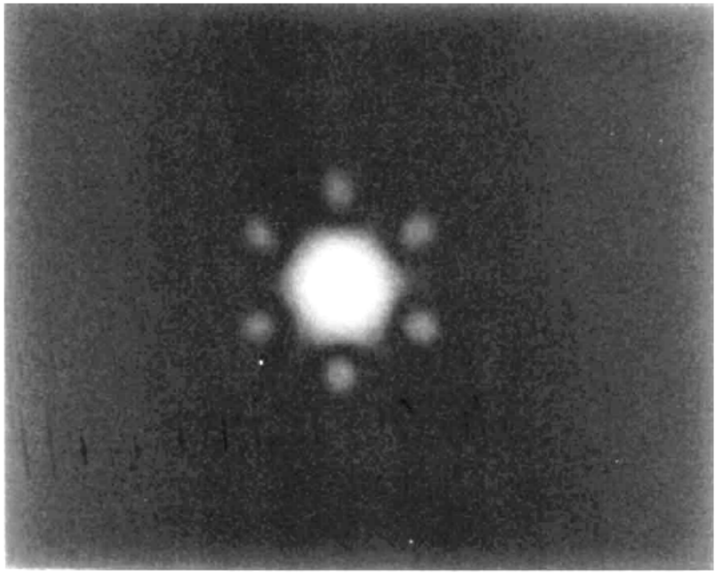
Dielectric-core PCF
Core diameter 5 micron

Air-core PCF
Core diameter 9 micron

Endless single mode photonic crystal fiber

Solid core photonic crystal fiber.



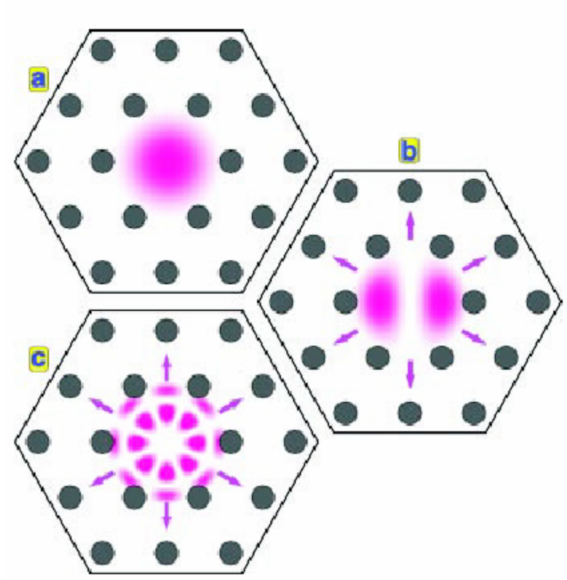


Solid core region nominally 4.6 μm wide

The fiber supports a single mode over the range of at least 458-1550nm

Knight et al, Opt. Lett. 21, 1547, 1996

The cladding as a mode sieve



The lower modes can not escape as the wire mesh are too narrow.

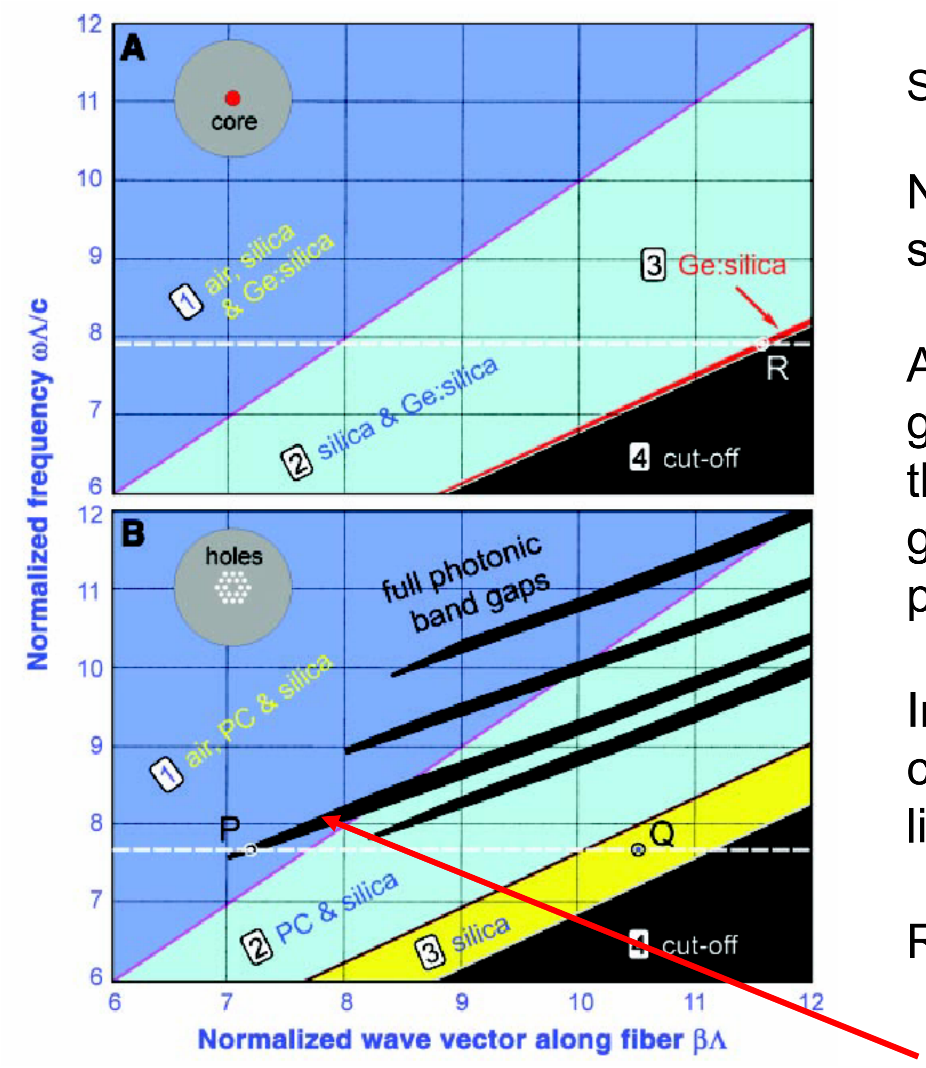
The higher order modes can leak through the narrow strip.

Increasing the relative size of the diameters of holes (d) with respect to the pitch (Λ) leads to the trapping of higher modes

Single mode behavior occurs when $d/\Lambda < 0.4$

Knight et al, Opt. Lett. 21, 1547, 1996

The band structure picture



Much larger room for dispersion management.

State-of-art loss figure at 0.58dB/km

No complete band gap at $\beta = 0$ for silica/air type of index contrast.

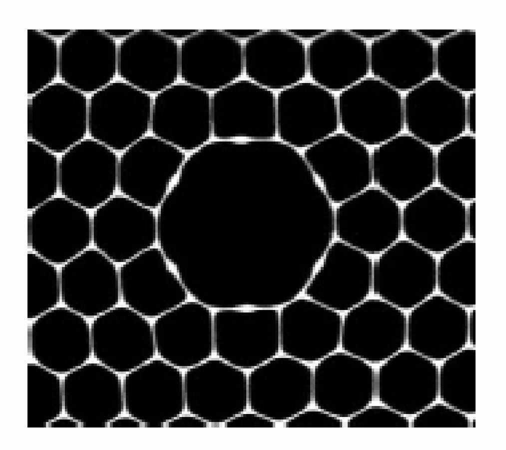
At finite β, band gap can appear. Band gaps arises from multiple reflection at the interfaces. At finite β, the reflectivity goes up, effectively increasing the in-plane index contrast.

In order to achieve guiding in air, the criteria is to find a band gap above the light line of air.

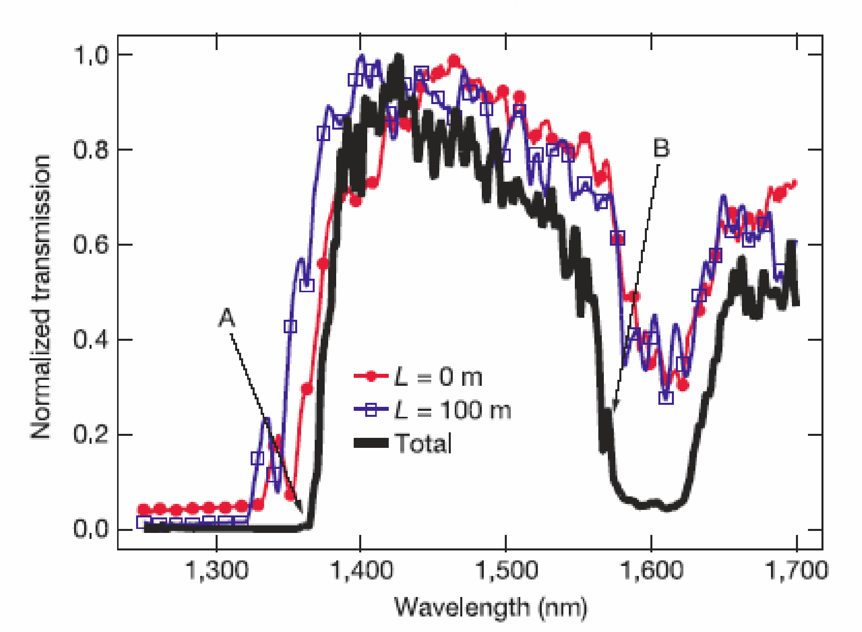
Requires fairly large air holes ($r \sim 0.47\Lambda$)

Possible region for air guiding

Air core photonic band gap fibers, experiments



Transmission through a 100m fiber

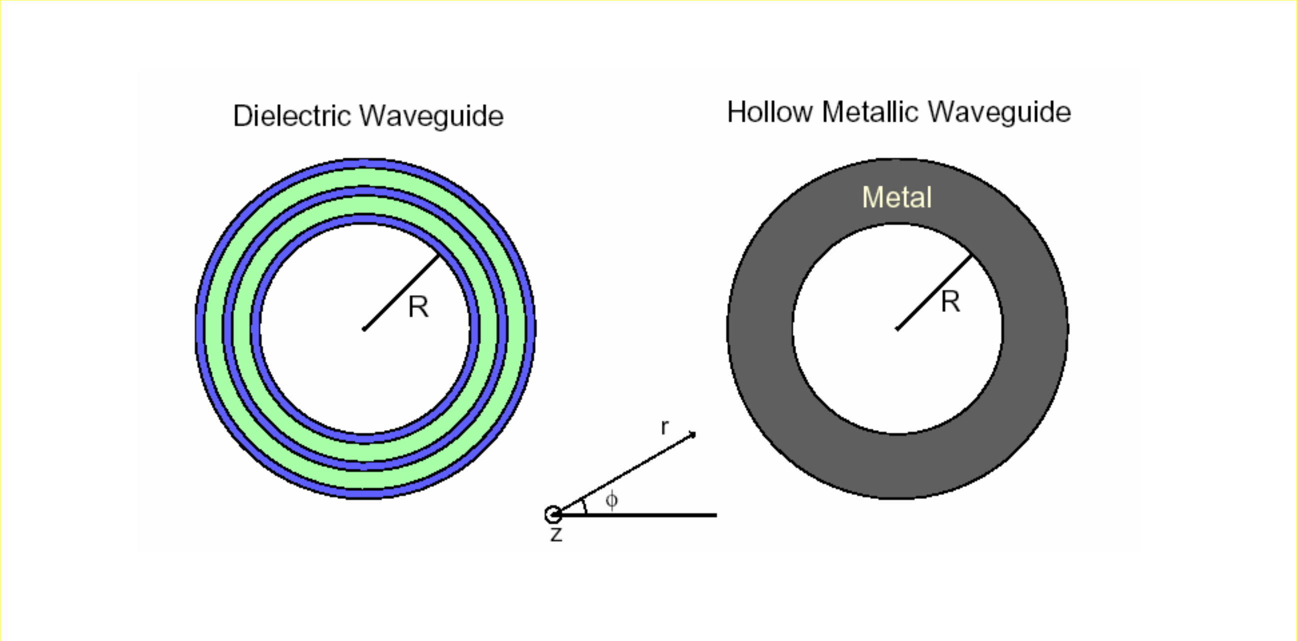


13 db/km in propagation loss, comparable to early days of conventional optical fiber.

Loss primarily due to the coupling of core modes to surface modes, and likely can be further reduecd significantly in newer design.

Smith et al, Nature, 424, 657 (2003)

The Bragg fiber

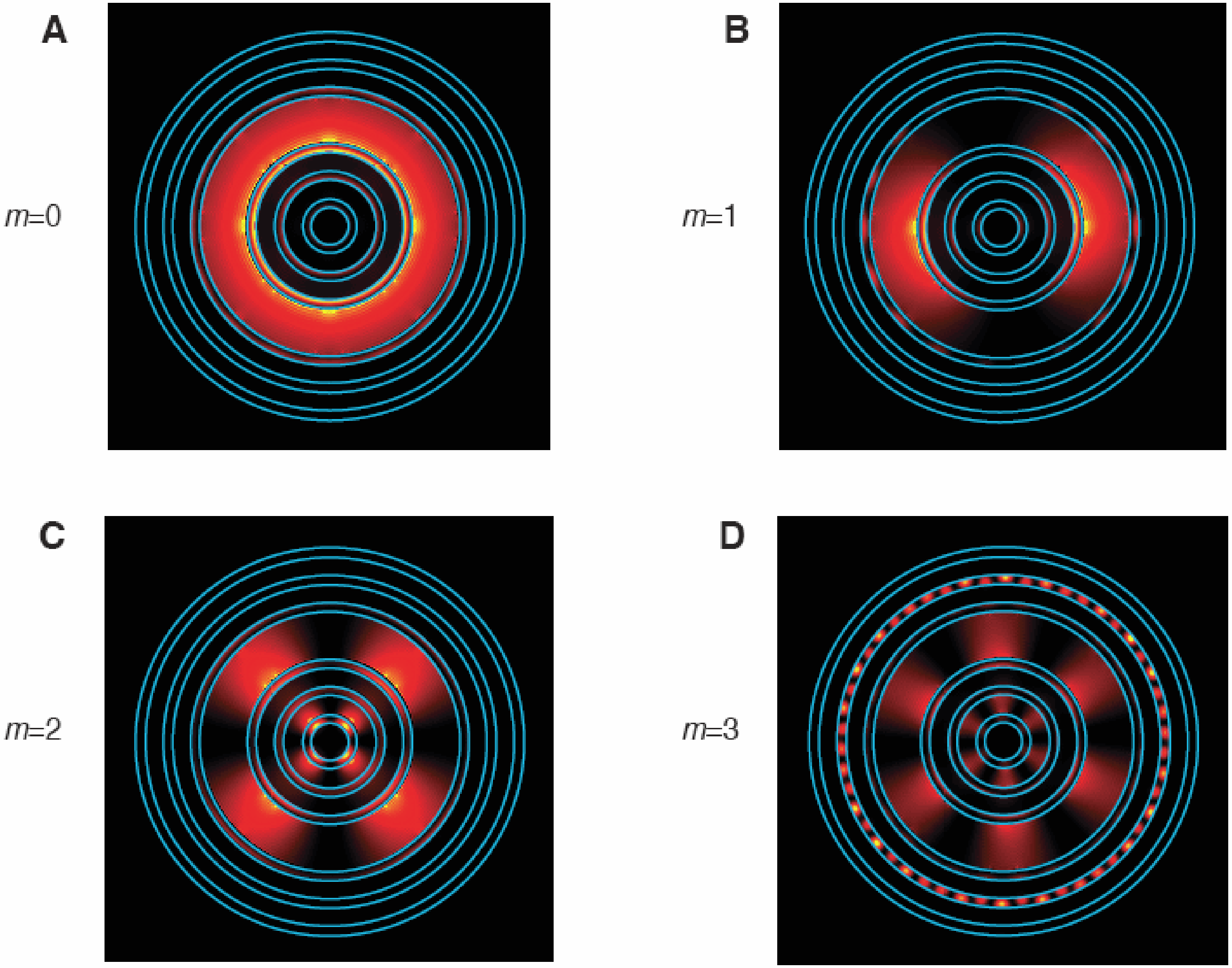


Using multilayer-film reflection to replace metal and create a light pipe.

The boundary condition for EM field at the boundary of core-film boundary can be designed to be rather similar to that at the metal boundary.

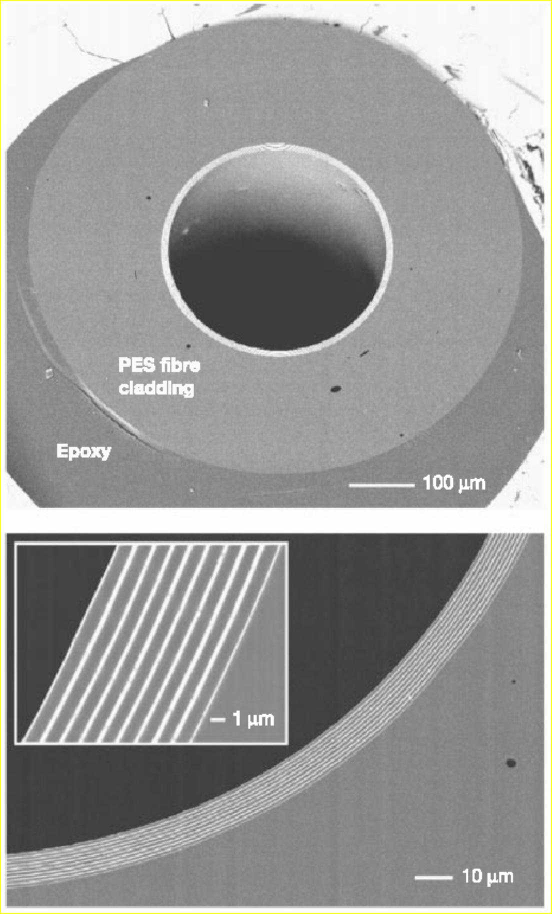
P. Yeh, A. Yariv and E. Marom, J. Opt. Soc. Am. 68, 1196 (1978).

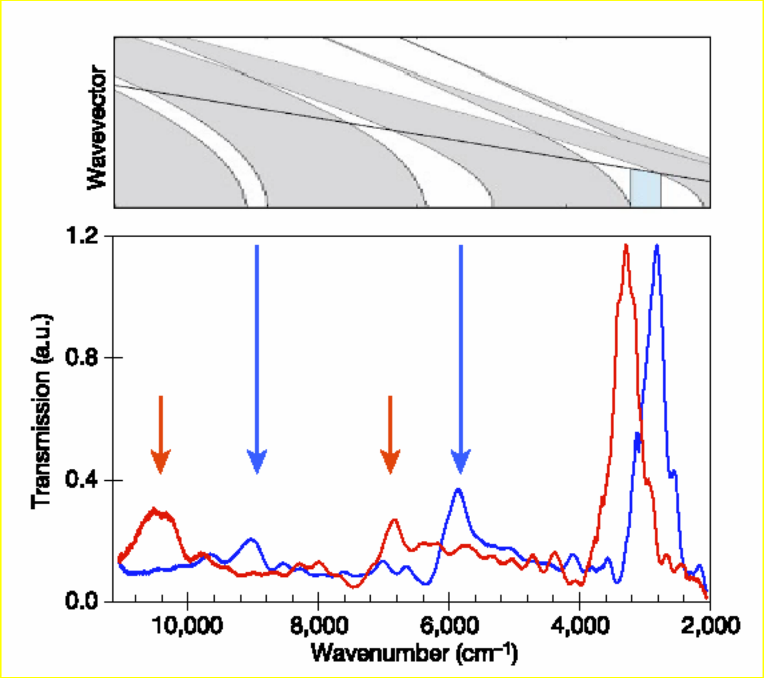
All dielectric co-axial waveguide



Single polarization mode in dielectric waveguide, similar to the TEM mode

Hollow optical fiber, experiments





Guiding of intense CO2 laser light at 10 micron wavelength range for high power applications

Temelkuran et al, Nature, 420, 650 (2002).