EPSILON-NEAR-ZERO (ENZ) AND MU-NEAR-ZERO (MNZ) MATERIALS

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How can this be achieved?

Some materials show near zero index of refraction at certain frequency bands.

By matching the resonances in series-parallel circuit of a DNG metamaterial, the propagation constant as a function of frequency continuously passes through zero index with a non-zero slope with non-zero group speed in its transition from DNG to DPG.

At matched cut-off frequencies, when the series-parallel lumped circuit unit cell is small ($\Delta z < \lambda$) and

\[
\beta_{\text{Case1 matched}} = \omega \sqrt{L_1 \varepsilon} C_2 (1 - \frac{\varepsilon_0^2}{\varepsilon^2})
\]

Dielectric with arrays of metal inclusions

- L-C configuration for a DPG medium, $\beta > 0$ ($\beta = \omega \sqrt{\varepsilon \mu}$)
- C-L configuration of a DNG medium, $\beta < 0$

Near-zero refractive index materials

Near-zero metamaterials

- A structure composed of a periodic mesh of metallic thin wires, when its characteristic dimensions (period, section of the wires) are small in comparison to the wavelength, behaves as a homogenous material with low plasma frequency.

- At $\omega = \omega_0$, wave number, $k=0$
- Wavelength, $\lambda = \infty$, $\vartheta = \infty$
- Group velocity is finite
- This behavior corresponds to plasmon resonance, where $\varepsilon \sim 0$ or/and $\mu \sim 0$
- Varying effective impedance, we can achieve particular group speed at $\omega = \omega_0$

• **DNG**
• **DPS**

\[ \omega / \omega_0 \]

\[ \text{Amplitude} \]

\[ -1000 \quad -500 \quad 0 \quad 500 \quad 1000 \]
Near-zero refractive index materials
1. Light manipulation occurs with artificially designed metamaterial structure
2. ENZ and MNZ materials have near zero parameters ($\varepsilon \approx 0$ or/and $\mu \approx 0$)
3. Refractive index, $n = \sqrt{\varepsilon \mu} \approx 0$
4. Low wave number (due to $n \approx 0$) and hence no phase variation inside the medium
5. Static like character even when dynamically oscillating

Decoupling of electric and magnetic fields

\[\nabla \times E = i\omega \mu H \Rightarrow \nabla \times E = 0\]
\[\nabla \times H = -i\omega \varepsilon E \Rightarrow \nabla \times H = 0\]
Let us consider a point source inside a low refractive index metamaterial.
Incidence on top slab coming from a source inside the slab of metamaterial.
For low n, the refracted ray is nearly normal to the slab.
The second figure assumes that light is dispersing in all directions.
Experimental work has been done with Cu wires.
Can be used as directional antennas.

Tunneling through distorted channels

- Since refractive index is low, phase variation is slow
- Wave fronts are radiated in parallel to the ENZ material interface

Motivation: Since the wavelength of radiation inside the ENZ material is extremely large, the wave must be able to propagate inside the ENZ material with no relevant reflection losses at abrupt bends or junctions

Equations

\[ E = \frac{1}{\sqrt{-\varepsilon}} \nabla H_z \times \hat{u}_z \]

Inside ENZ, \( \nabla H_z \) vanishes

Outside,

\[ \nabla \cdot \left( \frac{1}{\varepsilon} \nabla H_z \right) + \omega^2 \mu H_z = 0 \]

Using Boundary conditions of matched magnetic fields

\[ \rho = \frac{(a_1 - a_2) + ik_0 \mu_{r,p} A_p}{(a_1 + a_2) - ik_0 \mu_{r,p} A_p} \]

Tunneling through distorted channels

\[
\rho = \frac{(a_1 - a_2) + ik_0 \mu_{r,p} A_p}{(a_1 + a_2) - ik_0 \mu_{r,p} A_p}
\]

\[
a_1 \approx a_2 \quad \text{Relative permeability} \approx 0
\]

Cross-sectional area or width small

Outcome:
As the ENZ channel becomes increasingly narrower, the reflectance decreases more

**Supercoupling and squeezing of wave energy**

- Interaction with electromagnetic waves is independent of the granularity or lattice arrangement of the specific material and the channel size or shape.

- Confining EM field in very small subwavelength air cavity with enormous field enhancement.

- For U-shaped tunneling, $A_p = (L_1 + L_2)a + L\alpha_{ch}$

\[ J_c = \hat{v} \times \mathbf{H} \]

For ENZ material, its reflectivity only depends on the volume, independent of the geometry and transverse cross section.

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Silveirinha, M. G. & Engheta, N. Theory of supercoupling, squeezing wave energy, and field confinement in narrow channels and tight bends using $\varepsilon$ near-zero metamaterials. Phys. Rev. B 76, 245109 (2007)
ENZ Field concentration and confinement

When EM wave is squeezed, field is highly enhanced
• Magnetic field is constant
• The electric field is such that at the boundary it depends on $a_{ch}/a_1$
• Electric field amplified by a factor of $a/a_{ch}$
• Magnetic field nearly constant when losses are moderate

$$E_y \approx E_{y}^{inc}|_{x=0} \frac{1}{\cos(k_0d) \frac{a_{ch}}{a} - \frac{i}{2} \sin(k_0d)}$$
MNZ Supercoupling, field concentration and confinement

- parallel plate waveguide can be modeled as a transmission line with $C$ and $L$
- $\eta_0 d_0 = \eta_1 d_1$

\[
\mu_{r1} = \left(\frac{d_0}{d_1}\right)^2, \quad \varepsilon_{r1} = \left(\frac{d_0}{d_1}\right), \quad \frac{d_1}{d_0} \to 0 \quad \varepsilon_{r1} \to 0
\]

\[
T = \frac{1}{1 - i \frac{\omega \mu_{r1}}{\varepsilon_{r1}} \frac{d_0}{d_1}}, \quad R = T - 1
\]
1. For ENZ, both electric field and power are compressed into the channel
2. Phase constant
3. They are independent of bending due to ‘supercoupling’
4. Supercoupling basically emerges from the combination of a constant transverse magnetic field, produced by the enlargement of the wavelength in ENZ media, and the associated reduction of the magnetic flux induced by narrowing the channel, which, in turn, imposes a zero circulation of the electric field and enforces full transmission
5. For MNZ medium, tunneling will be observed if the height of the channel is substantially increased
6. EMNZ acts like an electromagnetic ‘point’, thus seemingly ‘directly connecting’ the input and output ports and enabling full transmission
• Layered metamaterial board with effective epsilon-near-zero (ENZ) property, over which nanocircuit traces may be carved out
• A metamaterial substrate that may be formed by alternating thin layers of epsilon-positive Si₃N₄ and epsilon-negative Ag
• The carved loop, filled with air, is surrounded by an effective ENZ substrate
• This prevents leakage of displacement current.
• As a result, $J_d$ is confined within the air groove,
• This is analogous to optical nanowire
• An ENZ material surrounding a lumped nanocircuit element would indeed “insulate” the element from the unwanted coupling with other neighboring elements

Requisite?
- Silicon based circuit parameters
- Problem with proper coupling
- Leakage current

If we could confine the current?

Design a nanoinsulator using ENZ

M. G. Silveirinha, A. Alu’, J. Li, and N. Engheta, J. Appl. Phys. 103, 064305 (2008)
• Light, not charge, plays the role of information carrier, with small (subwavelength) components playing the roles of lumped circuit elements
• Positive permittivity represents a capacitor; negative permittivity an inductor, while loss represents a resistor.
• The photonic wires are fabricated on epitaxially grown highly doped semiconducting layers, which serve as the plasmonic material base for the hybrid waveguide geometry
• Can work in the optical domain

“Photonic wire” is a cylindrical waveguide with deeply subwavelength cross section that is formed by a core with ENZ surrounded to bounds the displacement current flow

Oxygen-Deprived AZO Thin Films

- Optical switching at ENZ regime
- Operating in this regime (i.e. n<1) produces larger absolute changes in the reflection for a fixed change in the refractive index
- Traditional materials cannot achieve this at telecom wavelengths
- AZO has large intrinsic carrier concentration to enable ENZ properties at the technologically important telecommunication wavelengths (which enhances the optical/electrical tunability)

- EMNZ to transform isotropic source into plane-wave like beam with low ohmic losses
- EMNZ gives an enhancement of directivity at 5.24 nm
EPSILON-NEAR-ZERO (ENZ) AND MU-NEAR-ZERO (MNZ) MATERIALS

Materials

Basic Structure

Features

Applications

Acknowledgement

TAKEAWAYS
THANK YOU FOR YOUR ATTENTION ANY QUESTIONS?