Vortical and thermal interfacial layers in wall-bounded turbulent flows under transcritical conditions

Matthew X. Yao,1 Zeping Sun,1,2 Carlo Scalo,3 and Jean-Pierre Hickey1
1Department of Mechanical and Mechatronics Engineering, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1
2Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, Ontario, Canada M5S 3G8
3School of Mechanical Engineering and School of Aeronautics and Astronautics, College of Engineering, Purdue University, West Lafayette, Indiana 47907, USA

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The outer region of fully developed turbulent boundary layers can be viewed as a collection of uniform momentum zones separated by thin (but finite-thickness) shear layers referred to as momentum internal interface layers (MIILs). We first show the existence of such interfacial layers under transcritical thermodynamic conditions and introduce their thermal counterpart, named uniform thermal zones (UTZs), based on temperature. The UTZs, and the corresponding thermal internal interfacial layers (TIILs), are studied for a database of turbulent channel flow at transcritical thermal conditions [K. Kim et al., J. Fluid Mech. 871, 52 (2019)]. The thermal and vortical interfaces are identified using a recently proposed clustering approach by Fan et al. [D. Fan et al., J. Fluid Mech. 872, 198 (2019)].

It is shown that the MIILs and TIILs are correlated, but not collocated; their location is related to the underlying turbulent structures in the flow. On average, the TIILs are located about halfway between the wall and the MIIL, and the relationship between these layers is studied from the perspective of the attached eddy model. Under high near-wall thermal gradients the pseudoboiling line and the outer MIIL are collocated, which is explained using a shear stress balance analysis. Ultimately, the study of the thermal layering permits a simplification of the wall scaling in the presence of complex transcritical thermodynamics.

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I. INTRODUCTION

Turbulence-driven convective heat transfer is central to many engineering and scientific applications. The correlation between turbulent velocity and temperature fluctuations, driven by the background velocity gradient of a turbulent boundary layer, yields a higher conductive-heat-transfer rate between the solid wall and the fluid than under laminar flow conditions. The theory behind this heat-transfer mechanism rests upon an established framework of analytical and empirical relations [1] derived for single-phase turbulent boundary layers. Forced convection in more complex settings, such as in transcritical or supercritical flows, continues to be a topic of active research [2,3]. At these near-critical thermodynamic regimes, the heat transfer from the wall modifies the thermophysical properties of the fluid (e.g., density, viscosity, and specific heat capacity) and the underlying near-wall turbulence, which in turn affects the flow’s ability to transfer heat from the wall [4]. This coupling means that the classical Dittus-Boelter relations used for thermally developed, constant property flows require additional terms to estimate the effective Nusselt number Nu [5]. Even with these ad hoc modifications to the Nu estimation, a general physics-based parametrization of forced-heat-transfer rates is currently still missing.
Turbulence at supercritical conditions presents new challenges to existing predictive mass, momentum, and heat-transfer models. In the vicinity of the critical point [6], the thermophysical properties of the fluid are characterized by a sharp yet discrete peak in the specific heat capacity as well as a dramatic change in density, thermal conductivity, and dynamic viscosity with temperature. This pseudophase change occurs at or near the pseudoboiling line, which can be interpreted as an extension of the coexistence curve above the critical point of the fluid [7]. A system in which the thermodynamic state of the fluid crosses the pseudoboiling line is known as a transcritical flow. At these transcritical conditions, the established universal near-wall velocity scaling laws, as well as their mapping functions for variable property flows, break down [8,9]. A semilocal scaling can improve the collapse towards a universal profile [4], but discrepancies remain [8]. The difficulty in the near-wall velocity scaling under these conditions is well documented [4,8–11] and sets the stage for the significant modeling challenges of predicting forced convective heat transfer under these same conditions.

Faced with the physical complexity of these flows, the attention has turned towards the modification of the established Reynolds-averaged Navier-Stokes turbulence models to predict near-wall turbulent-heat-transfer rates. The de facto established approach can be summarized as follows: Using existing turbulence models [12], modify their coefficients [13], or adopt other parameters such as damping functions [14], to heuristically capture supercritical effects. These changes to the turbulence models are made to match experimental [15] or direct numerical simulation [16,17] databases. These modeling approaches are based on integral-scale characteristics (mean quantities) and not on the underlying near-wall turbulence physics. Although these modeling approaches can capture some of the physical differences in high-pressure flows, they overlook the inherent changes to the underlying turbulence dynamics caused by the thermodynamic conditions. Despite this trend, some recent works have sought to gain an understanding of the thermodynamic effects on the near-wall turbulence structures in the transcritical flows [8,9,18]. Recent work by Ma et al. [9] showed evidence of a hierarchical attached-eddy-type behavior in the near wall at these transcritical conditions, supporting the idea that the size of the turbulent structures is related to the distance from the wall. We extend the implications of the analysis to argue that the presence of attached eddies at these thermodynamically complex conditions is associated with uniform momentum zones (UMZs), as observed in incompressible wall-bounded turbulence [19–21].

This work focuses on the identification and characterization of vortical interfacial layers (which separate the UMZs), as well as introducing the concept of thermal interfacial layers (which separate the uniform thermal zones, UTZs), in wall-bounded flows with heat transfer at transcritical conditions. The interface between UMZs can be interpreted as a jump between the different hierarchies of the attached-wall-bounded turbulent structures; the thermal interfaces are similarly tied to the heat transfer resulting from the dynamics of the turbulent structures. The identification of these vortical layers relies on a new method developed by Fan et al. [22] to detect uniform momentum zones, which we extend for the identification of thermal layers. In Sec. II we provide the theoretical framework for the concepts of uniform momentum and thermal zones and summarize the internal interfacial layer identification approach. The numerical database of transcritical channel flow is presented in Sec. III, followed by the results and characterization of the layers in Sec. IV. The findings are summarized in Sec. V.

II. THERMAL AND VORTICAL INTERFACES

The concept of uniform momentum zones in wall-bounded turbulence and its conceptual connection to the attached-eddy model (AEM) is central to the current study and is presented here for completeness. In the following sections, the UMZs are defined and we introduce an analogous concept of UTZ. Uniform thermal zones share some of the same characteristics with their momentum counterparts, most notably, a large gradient across the separating interfaces. Finally we summarize the concepts of the zonal identification method used in the present study.
A. Uniform momentum zones

Uniform momentum zones are characterized by regions of uniform momentum within the turbulent boundary layer and are identified through an analysis of the streamwise velocity. The existence of UMZs was first reported in incompressible flows by Meinhart and Adrian [20]; the UMZs have subsequently been studied in depth by other researchers [22–26]. Because UMZs have different bulk velocities resulting from the organization of coherent structures in the flow, continuity and kinematics require that two adjacent UMZs must have a thin but finite shear layer between them, herein referred to as a momentum internal interfacial layer (MIIL). These thin shear layers delineate the UMZs. Given the high vorticity at the interface, we will also refer to these thin shear layers as vortical interfaces. The MIILs can also provide important insight into the transfer processes within the boundary layer. The transfer of momentum between any of the UMZs must occur across these interfacial layers and as such the MIILs can govern many processes of engineering interest [27].

One of the overarching goals of studying UMZs is to gain insight into the structure of wall-bounded turbulent flows. The classical detection technique introduced by Adrian et al. [21] is based on an analysis of the histogram of the streamwise velocity, where each peak in the histogram correlates to the presence of a local UMZ. In their study, they found that the existence of uniform momentum zones can be explained by the spacing of different packets of hairpin vortices as they grow further from the wall. Kwon et al. [25] successfully applied this approach to study the structure of channel flow turbulence and found that although channel flows are commonly described as fully turbulent, very low turbulence intensities are present in the core region. A new method of identifying the UMZs and MIILs, based on a clustering algorithm applied to the streamwise velocity of fluid parcels, was proposed by Fan et al. [22]. This new method allows for the identification of three-dimensional (3D) internal interfaces. Having demonstrated the successful extraction of UMZs in two and three dimensions, the cluster-based method is used in our present study to identify internal zones, both relating to velocity and temperature fields.

B. Attached-eddy hypothesis

One of the commonly invoked structural models of turbulent boundary layers is the attached-eddy model, first proposed by Townsend [1] and further refined by Perry and Chong [19]. The AEM hypothesizes that wall turbulence can be characterized by a hierarchy of wall-attached eddies, which are self-similar and grow in size the farther they move from the wall. Based on this hypothesis, the turbulent fluctuations of the flow can be calculated by considering the contributions from each eddy along with the probability that an eddy of that size exists [28]. The model has undergone many improvements and a comprehensive review was recently written by Marusic and Monty [28]. In turbulent wall-bounded flows, researchers have used spectral signatures based on the streamwise wave number to identify attached eddies in wall-bounded turbulent flows [29,30]. De Silva et al. [23] showed that a synthetic database created based on the AEM also exhibits the zonal-like organization of UMZs found in numerical and experimental data sets. Ma et al. [9] provide evidence for the validity of the AEM in the case of transcritical channel flow.

C. Uniform thermal zones

Analogous to the UMZs, which are identified by regions of uniform streamwise momentum, UTZs are conceptually defined as instantaneous layers of uniform thermodynamic properties, that is, regions of relatively uniform temperature distribution. Similar to UMZs, two UTZs are separated by a thermal internal interfacial layer (TIIL). Since adjacent UTZs have different bulk temperatures, the largest temperature gradients lie at the TIILs. It follows that we expect a local peak in heat transfer across these internal boundaries. By understanding the properties of the UTZs and UMZs, we can gain insight into the relationship between momentum and heat transfer within turbulent boundary layers and wall-bounded flows.
Supercritical flows require special consideration in the transcritical regime, where slight changes in the temperature can create large variations in the thermophysical properties. Studying the UTZs can inform our understanding of the location and nature of these sharp transitions, shedding light on phenomena such as heat-transfer deterioration. The complications arising from drastic thermophysical variations in transcritical flows have consistently impeded the development of near-wall heat-transfer scaling in this complex regime. This work is a step towards developing physics-based near-wall models for transcritical flows, relating the momentum- and heat-transfer scaling by drawing a connection between the momentum and heat transfer.

D. Zonal identification method

The thermal and momentum interfacial layers are identified via a fuzzy cluster algorithm proposed by Fan et al. [22]. The fuzzy cluster method is based on the seminal work by Dunn [31], which was further developed by Bezdek et al. [32]. In the fuzzy clustering algorithm, each data or grid point is a member of all clusters; we attribute each data point to a single cluster by identifying the cluster which has the largest membership function (defined below). A summary of the algorithm is as follows; readers can refer to Ref. [22] for more details. First, the number of clusters is selected (based on the Reynolds number) and the center of each cluster is randomly selected. Then, a membership coefficient is assigned to each data point representing the likelihood of the point belonging to each of the clusters. The cluster centers are recalculated and the membership coefficients are reassigned until the objective function is minimized.

Mathematically, consider a case where we wish to cluster the streamwise velocity of a given DNS data set. We denote all data points by \( \{U_m\}_{m=1}^{N} \), where \( N \) represents the total number of grid points of the instantaneous snapshot. These data are then organized into \( K \) clusters with the centroid of each cluster being denoted by \( c_k \). The distance measurement between any two observations is defined by calculating the norm in Euclidean space, \( D_{mn} = |U_m - U_n| \). The key to the fuzzy clustering method is that each observation \( U_m \) is assigned a membership coefficient \( u_{mk} \), which represents the probability that \( U_m \) belongs to cluster \( k \). The summation of the membership coefficient in all clusters equals one. The centroid of the cluster represents the average velocity of the cluster and is defined based on the membership coefficient

\[
c^k = \frac{\sum_{m=1}^{K} (u_{mk})^p U_m}{\sum_{m=1}^{N} (u_{mk})^p},
\]

where \( p \) is a fuzziness parameter, which is typically set to 2 [33]. The resulting objective function is

\[
J_p(\mathbf{U}, \mathbf{c}) = \sum_{m=1}^{N} \sum_{k=1}^{K} (u_{mk})^p (D_{mk})^2,
\]

where \( \mathbf{c} \) is the vector of all the cluster centroids. The objective function is then minimized to determine the optimal \( \mathbf{c} \). After the function is minimized, the observations are assigned to the clusters in which they have the highest membership coefficient. Fan et al. [22] show that the clustering results are robust against initial conditions and added noise. We should keep in mind that although the physical interfacial layers are finite thickness (albeit thin) layers of intense velocity and temperature gradients, the clustering method provides a precise delineation between UMZs and UTZs.

One limitation of this fuzzy clustering approach is that it relies on a user-defined number of clusters (for classical incompressible flows, Ref. [22] provides a quantitative approach to estimate the number of zones). Although researchers have argued that the number of uniform momentum zones and internal layers may vary based on Reynolds number [22,23,34], in our study we select three zones per half channel based on physical assumptions due to the lower Reynolds number of the flows. The approach has been successfully applied by Fan et al. [22] to identity internal interfacial layers within the turbulent boundary layer by clustering based on the streamwise velocity. It is
also shown that the solution is not sensitive to the preselected number of zones, as the addition of a zone will simply identify a layer in addition to the ones defined previously. In this work, the momentum and thermal internal interfacial layers are identified by clustering based, respectively, on the streamwise velocity and temperature. As the clustering method identifies interfaces located in high-gradient regions, the clustering based on \( u \) and \( T \) is more effective than, for example, a clustering based on \( \rho u \) or \( \rho e \), especially in transcritical flows. Moreover, we know that a vortical layer delineates the boundary between two UMZs, and thus the identification using the velocity remains more appropriate. As discussed in the following section, for the data set the clustering based on \( u \) or \( T \) identifies the exact same interface as \( \rho u \) or \( \rho e \) except for the upper wall at \( \Delta T = 20 \) in which the interfacial layers and pseudoboiling point are in close proximity.

### III. DESCRIPTION OF THE NUMERICAL DATABASE

For the present investigation, we analyze a DNS database of compressible turbulent channel flow at transcritical thermodynamic conditions (Fig. 1) by Kim et al. \[8\]. The working fluid is R-134a and the real fluid effects are accounted for by closing the Navier-Stokes equations with the cubic Peng-Robinson equation of state. All thermophysical quantities are thermodynamically consistent with the selected state equation and derived using the appropriate departure functions. The setup consists of differentially heated walls with the top- and bottom-wall temperature, respectively, at a fixed \( \Delta T/2 \) above and below the pseudoboiling temperature at a slightly supercritical base pressure \( p = 1.1 p_c \). The pseudoboiling point is determined by the local peak in specific heat capacity at the given pressure. This setup isolates the effects of pseudophase change within the turbulent channel, and, by varying the differential temperature of the walls, the pseudoboiling point is shifted from the center of the domain to within the boundary layer near the channel walls. An illustration of the numerical setup and the isobaric fluid properties of R-134a can be seen in Fig. 1. The database contains three different \( \Delta T \): 5, 10, and 20 K. The friction Reynolds number of the turbulent channel flow is around \( Re_\tau = 360 \). A very careful grid convergence study was undertaken by Kim et al. \[8\] to ensure adequate resolution of all scales in the flow. Despite the modest friction Reynolds number and the adoption of a high-order scheme, a grid of \( 512 \times 256 \times 512 \) over an appropriately sized domain was needed to reach DNS resolution. The average velocity and thermodynamic profiles of the three cases are shown in Fig. 2, where the symbols correspond to the average pseudophase locations. The reader is referred to \[8\] for more details on the simulation database. We do note that given the asymmetry in the thermodynamic mean quantities relative to the center plane, we consider the top and bottom walls separately for the analysis.

![Figure 1](image.png)

**FIG. 1.** Shown on the left is an illustration of the numerical setup of a turbulent channel flow with differential wall heating. Shown on the right are the density (solid line) and Prandtl number (dashed line) of R-134a versus temperature at \( p = 1.1 p_c \). Closed circles represent pseudoboiling conditions.
FIG. 2. Reynolds-averaged velocity and thermodynamic profiles of the differentially heated simulations for \( \Delta T = 5 \) K (solid line), 10 K (dashed line), and 20 K (dotted line). The symbol corresponds to the pseudoboiling point based on the averaged density and temperature.

IV. RESULTS

A. Existence of thermal internal interfacial layers

The identification of UMZs and UTZs is conducted according to the method proposed by Fan et al. [22], summarized in Sec. IV A. An instantaneous snapshot of the \( \Delta T = 20 \) K channel flow with three-momentum and thermal zones is shown in Figs. 3 and 4. The UMZs and the UTZs are separated by the MIILs (solid lines) and TIILs (dotted lines), respectively. Note that some regions of the zones fold over on themselves, thereby enveloping a small portion of another zone. For the purposes of our analysis, we consider the interfacial layers to follow the outer-enveloped regions.

FIG. 3. Instantaneous slice of the \( \Delta T = 20 \) K channel flow showing the MIILs (solid lines) separating the UMZs at the top and bottom walls.
FIG. 4. Instantaneous slice of the $\Delta T = 20$ K channel flow showing the TIILs (dotted lines) separating the UTZs at top and bottom walls.

(we neglect the fold-over regions), as suggested by Kwon et al. [25]. The MIILs and TIILs are not fully coincident, although they share similar topological features, extending into the core of the channel from the near-wall region at approximately the same streamwise locations. The quantitative relationship between the MIILs and TIILs is established in Sec. IV C.

Since the UMZs are characterized by different bulk velocities, it is expected that the MIILs identify a thin (albeit finite-thickness) layer of concentrated vorticity. Similarly, the TIILs separating the UTZs are expected to manifest themselves as layers of concentrated thermal gradients. To illustrate this, portions of the instantaneous MIILs and TIILs identified in Figs. 3 and 4 are shown in Fig. 5. On the left, the upper image shows the MIILs and TIILs overlaid on the magnitude of the temperature gradient; the lower image shows the MIILs and TIILs overlaid on the magnitude of the normalized vorticity. The TIILs closely follow the regions of the flow with the largest temperature gradients, and similarly, the MIILs follow the regions of largest velocity gradients (or more precisely, spanwise vorticity magnitude). The temperature gradient is especially high at the near-wall TIIL, being almost twice as large in magnitude as the temperature gradient at the outer TIIL. Figure 5 (left) shows that the TIIL lies along a ridge of higher local vorticity. Although the

FIG. 5. Shown on the left are instantaneous MIILs (solid lines) and TIILs (dashed lines) overlaid on vorticity magnitude (top) and temperature gradient magnitude (bottom) contours. Shown on the right are instantaneous (dotted lines) and mean (solid lines) profiles overlaid with local location of MIILs (solid lines) and TIILs (dashed lines). Data are taken from the $\Delta T = 20$ K transcritical channel flow.

MIIL lies along regions of higher vorticity as well, the distance is much farther from the wall, and as such, the temperature is much closer to the mean fluid temperature. The TIILs necessarily espouse the regions of higher mixing closer to the wall where the temperature differences are greater.

The nature of the UMZs and UTZs can be seen by examining the instantaneous velocity and temperature profiles at a given streamwise location, shown in Fig. 5 (right). The instantaneous profiles exhibit sharp jumps delineating between zones of either uniform momentum or temperature at a particular streamwise location. The jumps in the instantaneous profiles outline the UMZs and UTZs, and thus the location of the jumps correspond to the MIILs and TIILs, respectively.

FIG. 6. Conditional sampling of the magnitude of the temperature gradient across the outer TIIL and the conditional sampling of the vorticity across the outer MIIL of the bottom wall for the $\Delta T = 20$ K channel flow. The conditionally sampled vorticity is computed from the curl of the instantaneous velocity field (not the fluctuating component).

FIG. 7. Histogram of (a) the inner MIIL bottom wall, (b) the inner MIIL top wall, (c) the outer MIIL bottom wall, and (d) the outer MIIL top wall.
FIG. 8. Histogram of (a) the inner TIIL bottom wall, (b) the inner TIIL top wall, (c) the outer TIIL bottom wall, and (d) the outer TIIL top wall.

To further confirm the existence of these separating layers, a conditional averaging is conducted across the internal layers. Here, conditional averaging is the average relative to the interfacial layer. The conditional sampling of the outer MIIL and TIIL is shown in Fig. 6. The image on the left shows the conditionally sampled magnitude of the temperature gradient across the TIIL and the image on the right shows the conditionally sampled vorticity across the MIIL. Since the TIIL is a region of relatively high temperature gradient and the MIIL is a region of relatively high vorticity, the peaks in the conditionally sampled profiles at the internal interface location support the existence of the TIILs and MIILs.

The probability density functions of the MIIL and TIIL heights are shown in Figs. 7 and 8, respectively, for all $\Delta T$ cases. The height distribution of the MIILs and TIILs is qualitatively symmetric with respect to the channel center plane. However, we note that the outer MIIL of the bottom wall is typically slightly farther from the wall than the outer MIIL of the upper wall; this same feature was qualitatively observed in Fig. 3.

To investigate the mean properties of the layers, the average distances between the wall and interfacial layers are summarized in Table I. On average, the inner TIIL is around 36–55% of the height of the MIIL. For the outer layers, the bottom wall shows a similar trend, as the TIIL is approximately 53–67% of the height of the MIIL. However, on average, the distances between the outer layers of the top wall are much smaller. Despite the fairly consistent correlation between the average heights of the MIIL and TIIL, a clear explanation is rendered difficult due to the strong variation in Prandtl number in these thermodynamically complex flows (see Fig. 1).

Another notable trend is the difference between the thicknesses of the top-wall and bottom-wall outer MIILs. As the temperature difference is increased, the thickness of the outer MIIL at the bottom wall remains relatively stable, whereas the thickness of the outer MIIL at the top wall decreases by about 30%. This phenomenon is a result of the pseudoboiling within the channel flow. In the original paper, Kim et al. [8] showed that the turbulence intensity near the wall decreases monotonically with increasing $\Delta T$. This suppression of the turbulence results in a much thinner UMZ along the top wall. The effect can also be seen in Fig. 5 (right), where the jumps in the
TABLE I. Average distance of all TIILs and MIILs from the wall.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Wall</th>
<th>$\Delta T = 5$ K</th>
<th>$\Delta T = 10$ K</th>
<th>$\Delta T = 20$ K</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIIL</td>
<td>top</td>
<td>0.017</td>
<td>0.016</td>
<td>0.018</td>
</tr>
<tr>
<td>MIIL</td>
<td>top</td>
<td>0.037</td>
<td>0.035</td>
<td>0.033</td>
</tr>
<tr>
<td>TIIL</td>
<td>bottom</td>
<td>0.014</td>
<td>0.016</td>
<td>0.020</td>
</tr>
<tr>
<td>MIIL</td>
<td>bottom</td>
<td>0.039</td>
<td>0.038</td>
<td>0.042</td>
</tr>
</tbody>
</table>

inner layer

<table>
<thead>
<tr>
<th>Layer</th>
<th>Wall</th>
<th>$\Delta T = 5$ K</th>
<th>$\Delta T = 10$ K</th>
<th>$\Delta T = 20$ K</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIIL</td>
<td>top</td>
<td>0.216</td>
<td>0.130</td>
<td>0.155</td>
</tr>
<tr>
<td>MIIL</td>
<td>top</td>
<td>0.241</td>
<td>0.196</td>
<td>0.166</td>
</tr>
<tr>
<td>TIIL</td>
<td>bottom</td>
<td>0.150</td>
<td>0.160</td>
<td>0.132</td>
</tr>
<tr>
<td>MIIL</td>
<td>bottom</td>
<td>0.253</td>
<td>0.240</td>
<td>0.251</td>
</tr>
</tbody>
</table>

outer layer

instantaneous velocity and temperature profiles at the bottom wall are much more pronounced than at the top wall. The thinner MIILs at the top wall are also consistent with the results from Ma et al. [9], who showed a thinner boundary layer on the heated wall. The damped turbulence and thinner boundary layer explain the similarity between the heights of the outer MIIL and TIIL at the top wall.

One benefit of the proposed interfacial layer identification method is that it allows for the extraction of UMZs and UTZs in three dimensions. The structures of the 3D MIILs and TIILs

FIG. 9. Structure of the thermal and vortical interface 2D contour plot for the bottom wall of the $\Delta T = 20$ K channel flow. (a) The inner layer is much smoother than (b) the outer layer. $Y_{\text{TIIL}}$ is the height of the TIIL and $Y_{\text{MIIL}}$ is the height of the MIIL.
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FIG. 10. Structure of the thermal and vortical interface 2D contour plot for the top wall of the $\Delta T = 20$ K channel flow. (a) The outer layer is much rougher than (b) the inner layer. $Y_{\text{TIIL}}$ is the height of the TIIL and $Y_{\text{MIIL}}$ is the height of the MIIL.

for the $\Delta T = 20$ K case are shown in Figs. 9 and 10 by plotting the contour of their heights across the wall-normal direction of the channel. For both the top (heated) and bottom (cooled) walls, the near-wall MIIL and TIIL are relatively smooth and are characterized by thin streaky patterns, whereas the outer layers have much larger fluctuations in the layer height. Qualitatively, it can be seen that the structures of the MIILs and TIILs are very similar for both the inner and outer layers.

B. Pseudoboiling and interfacial layers

The correlation between the instantaneous pseudophase change and the interfacial layers is considered. At the pseudoboiling point, the density is most sensitive to temperature variations; the slightest thermal fluctuation results in a large density fluctuation. Furthermore, the pseudoboiling is characterized by a local minimum in the viscosity and maxima in the specific heat capacity and Prandtl number. Figure 11 overlays the instantaneous pseudophase change location on the MIIL and TIIL in the present cases. The pseudoboiling point remains uncorrelated for the cases at $\Delta T = 5$ and 10 K; at $\Delta T = 20$ K, the pseudoboiling point is collocated with the MIIL. This collocation is a rather surprising observation, but the presence of a vortical layer at the pseudophase change location can be understood by applying a simple stress balance analysis at the pseudoboiling interface. At the interface between the pseudogas (PG) and pseudoliquid (PL) phase, the viscous nature of the fluid ensures that the interfacial velocities of the PG and PL are the same. Furthermore, as there is no surface tension in supercritical flows, the hydrodynamic stresses must also be in equilibrium. If we neglect the effect of interfacial turbulence, we can show that the velocity gradients on both sides
of the interface are inversely proportional to the viscosity ratio

$$\left. \frac{du}{dy} \right|_{PL} = \frac{\mu_{PG} du}{\mu_{PL} dy} \bigg|_{PG}. \quad (3)$$

Given the large change in molecular viscosity at the pseudoboiling point, we expect a similarly large change of velocity gradient as well. This local change in the velocity gradient is captured by the present interface identification method, and thus the surprising collocation of the pseudoboiling and MIIL can be understood.

C. Relationship between vortical and thermal interfaces

From Figs. 3 and 4 it can be seen that although the identified MIILs and TIILs are not coincident, they appear to be qualitatively similar. That is, the MIILs and TIILs rise and fall in similar locations. However, at other snapshots, the qualitative relationship between the MIILs and TIILs is not as clear. For example, consider the slice from the $\Delta T = 20$ K channel flow shown in Fig. 12. While the height of the inner TIIL is consistent with the inner MIIL, the outer TIIL fluctuates quite drastically between the inner and outer MIILs.

We aim to explain the relationship between the MIILs and TIILs by considering the existence of UMZs and UTZs under the paradigm of the AEM. The attached-eddy model proposes that the turbulent fluxes in a wall-bounded flow can be obtained by considering the sum contribution from a hierarchy of wall-attached eddies. Adrian et al. [21] conducted a detailed study of the vortex organization within the boundary layer. In their work, they proposed the existence of groups or packets of hairpin vortices which propagate at similar velocities. These packets of hairpin vortices induce $Q_2$ (ejection) and $Q_4$ (sweep) events, creating shear layers in their wake as they propagate. The importance of this result is twofold. First, they show that these packets of hairpin vortices are consistent with the attached-eddy hypothesis, according to their growth with distance from the wall and the existence of hierarchies of hairpin vortex packets. Second, they show that the induction by the vorticity of each hairpin in a packet creates a region of lower momentum flow in the packet to create uniform momentum zones. Accordingly, they identify the MIIL as a line following the shear

FIG. 12. Slice of the bottom wall for the $\Delta T = 20$ K channel flow showing the MIILs (solid lines) and TIILs (dashed lines).
layer and connecting the hairpin vortex heads. More recently, Hwang and Sung [35] showed more evidence for relating wall-attached structures and uniform momentum zones.

Figure 13 shows a section of the MIIL from the $\Delta T = 20$ K channel flow overlaid on the velocity vector plot with a frame of reference convecting at $U_c = 0.9U_\infty \approx 36$ m/s. Along the MIIL lie a number of vortex heads, which is consistent with the observations made by Adrian et al. [21]. The vortex heads along the MIILs work to slow down the flow within the UMZ. Simultaneously, lower momentum fluid is kicked up into the UMZ through the legs of the hairpin vortices and the heads of smaller vortices. Along with the fluid packets of lower momentum being brought into the UMZ, fluid packets of different temperatures are also brought into the UMZ where they are mixed. Although the combined effect of the various hairpin vortices in the flow create UMZs, they do not necessarily dictate uniform mixing within the UMZ itself. Since the MIILs are identified between regions of relatively uniform velocity, they represent the strongest shear layers and as such follow the regions of largest vorticity. However, within the UMZ there is residual mixing from the presence of smaller hairpin vortices within the hierarchy. The strongest mixing within the UMZ occurs at these secondary shear layers and as such also represents the largest temperature gradients as the fluids of different temperature from above and below are mixed. This effect can be seen in Fig. 5, where the TIIL is seen following the regions of larger vorticity within the UMZ itself. As can be seen, in regions where the internal vorticity is very weak, the effective thermal mixing occurs at the MIIL itself. The uneven nature of the mixing within the UMZ results in the more sporadic nature of the TIIL shown in Fig. 12.

For a qualitative analysis, Fig. 14 presents a 3D visualization of the outer MIIL and TIIL for a section of the bottom wall of the $\Delta T = 20$ K channel flow. The MIIL typically lies higher than the TIIL and as such the MIIL envelops many of the near-wall vortical structures which are not covered by the TIIL. For more detail, Fig. 15 shows the 2D slices of the MIILs and TIILs for two spanwise locations. The TIIL lies along the contours of the $Q$-criterion structures which are enveloped by the MIIL, supporting the notion that the location of the internal structures is dependent on the hierarchy of wall-attached eddies.

V. CONCLUSION

The highly varying thermophysical properties characteristic of the transcritical regime introduce new challenges in the wall modeling of turbulent flows. In this work we introduced the concept of uniform thermal zones, which are analogous to the commonly studied uniform momentum zones. The UTZs and UMZs were extracted and the separating layers were identified using a recently proposed identification method based on a fuzzy clustering approach. Similar to how UMZs are separated by a thin layer of concentrated vorticity, we found that UTZs are separated by a thin layer of strong temperature gradient. The existence and characteristics of the UTZs and TIILs were interpreted under the existing framework of the attached-eddy hypothesis.
The characteristics of the MIILs and TIILs were analyzed qualitatively and quantitatively. On average, the TIIL is located at approximately half the distance between the wall and the MIIL. The TIIL is approximately 36–55% of the height of the MIIL for the inner layers on both walls and approximately 53–67% for the outer layers of the bottom wall. The outer layers of the top wall showed some different characteristics, which were found to be a result of the transcritical nature of the flow. For the top wall, the thickness of the outer MIIL decreased as the temperature difference increased. Concurrently, the difference between the height of the outer TIIL and the MIIL at the top wall also decreased. The results are consistent with previous studies, most notably that the pseudoboiling effect dampens the turbulent intensity at the top wall and that the boundary layer at the top wall is also reduced in thickness.

The identification of regions with uniform features presents opportunities for understanding and modeling these thermodynamically complex flows. The variability of these features (such as streamwise velocity and temperature), within each of these zones, is far less than within the

FIG. 14. Three-dimensional visualization of the outer MIIL (left) and TIIL (right) for a section of the bottom wall. Structures are identified using the $Q$ criterion.

FIG. 15. Two-dimensional slices of the MIILs (solid blue lines) and TIILs (dashed red lines) at two different spanwise locations overlaid on the contour plot of the $Q$ criterion.
entire boundary layer; this greatly facilitates the modeling assumptions for the thermodynamic and thermophysical variation within each zone. As a result, if one is able to characterize the interfacial heights as a function of Reynolds number and wall temperature (or temperature difference with the freestream), for example, a better estimate of the near-wall density and thermophysical is expected. Although outside the scope of the present work, the extension of these ideas to near-wall modeling appears to be the logical next step, similarly to the kinematic turbulence models which emerged from the conceptual understanding afforded by the attached-eddy model [19].

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