Nonlinear Dynamics of Second Mode Waves on a Hypersonic Flared Cone

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A numerical analysis of spectral broadening of second mode waves over a hypersonic flared cone, designed to maintain a constant thickness boundary layer, was conducted through a comparison between direct numerical simulations (DNS) and nonlinear parabolized stability equations (NPSE). The current simulations show that the constant thickness focuses the spectrum of the second mode on one frequency band centered around 300 kHz; its amplitude grows by approximately 8 orders of magnitude over 0.45 m of cone length. This growth is shown to trigger nonlinear mechanisms such as mean flow distortion and spectral broadening, as expected. The DNS and NPSE exhibit very good agreement for the growth rates estimated for the principal frequency and the first two harmonics although the spatial location of the beginning of the growth differs for the higher frequency harmonics. It is also shown that the “rope waves” observed in Schlieren images in previous experiments of hypersonic transition are connected to the region of high nonlinear interaction of the second mode, where the pressure fluctuation amplitude is in the order of 50 Pa. This work is an intermediate step towards studying the relative importance of the different nonlinearities present in this flow and the energy source of the second mode waves at very high amplitudes.

I. Introduction

One major obstacle in the design of hypersonic vehicles is laminar-turbulent transition uncertainty. Transition significantly affects the values of heat transfer and shear stress in a hypersonic boundary layer. Reed et al.¹ assumed a fully turbulent and a fully laminar flow for a low-Earth orbit hypersonic flight vehicle and reported that the vehicle would experience 5 times more heating in the turbulent condition and would also need at least 2 times the weight in thermal protection systems. Another problem that arises is that transition leads to a local heat transfer flux that exceeds fully turbulent flow calculations because the newly transitioned boundary layer is thinner and with higher gradients than the fully turbulent model. On the other hand, there are some scenarios in which turbulent flow and mixing is desired, such as the flow over control surfaces and the inlet of a scramjet engine. This context leads to the conclusion that the optimum design of such vehicles is intrinsically associated with a fundamental understanding of the laminar-turbulent transition process.

Mack² ³ described the instability mechanisms in compressible boundary layers as a function of free stream velocity. As the speed increases from subsonic to supersonic and beyond, Tollmien-Schlichting waves starts to become three-dimensional, traveling at approximately 45 degrees to the free stream, and to decrease in relative importance to transition. This mode was named “first mode” and it is the principal disturbance

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for Mach numbers up to 4. At higher speeds and for canonical geometries, two dimensional acoustic modes take over, namely “second mode” waves. Such modes are expected to be predominant for transition in flight conditions with small angles of attack, on vehicles with a high lift to drag ratio, which tend to have predominantly 2D shapes and sharp tips, and in the presence of cooled walls. Second mode disturbances are acoustic waves trapped between the solid wall and the relative sonic line. Their resonance-like behavior is in accordance with experimental results that show that second-mode frequencies are strictly linked to the boundary layer thickness.

The path to transition, especially under low disturbance conditions, is described by the modal growth of the primary modes, such as the second mode. As their amplitude grow, the nonlinear interactions start to become important and effects such as spectral broadening of resonant modes, harmonic generation and saturation becomes important before the final breakdown to turbulence occurs. Experiments conducted by Berridge et al. of hypersonic flow over a cone were able to capture all these transition stages in different wind tunnels under noisy flow conditions. Demetriades performed transition experiments over a sharp cone at Mach 8 and reported the appearance of “rope waves” (Figure 1) in the region preceding transition and this was attributed to the effects of nonlinearities in the flow.

Figure 1: Schlieren images of a sharp cone at zero angle of attack in a Mach 8 free stream flow that show rope-like features at the edge of the boundary layer.

Although the linear regime of transition is well understood, the investigation of the influence of nonlinearities in the flow deserves more attention. Achieving high amplitude second mode waves is in fact challenging in experiments under quiet conditions on straight cones due to size restrictions. This problem, in the study of the second mode, can be overcome by the design of a compression cone specifically to maintain the boundary layer thickness constant along the length of the cone.

This strategy relies on the observations that the frequency of the most amplified waves in a hypersonic boundary layer is closely related to the boundary layer thickness. Therefore, a geometry that is capable of maintaining a constant boundary layer allows the tuning of a specific band of frequencies over a larger spatial span and, in the end, allows for higher disturbance amplitudes and the study of nonlinear interactions. Since this configuration highlights the second mode characteristics it is the optimum configuration for a fundamental study of this instability mechanisms for both experiments and simulations.

Lachowitz et al. performed experiments of transition on a flared cone in the NASA Langley Nozzle Test Chamber Facility quiet tunnel and reported the existence of higher harmonics. It was proven that the the existence of the higher harmonics in the flow is attributed to an energy cascade instead of being a consequence of noisy flow conditions existent in previous experiments. He also reported “rope-like” waves with a wavelength of approximately two times the boundary layer thickness in the Schlieren images in the region right before transition. Pruett and Chang performed a DNS of the transition on a flared cone replicating the experiments performed by Lachowitz and were able to replicate the most unstable frequency in the experiments and also show the presence of rope-like waves in their numerical simulations. More recently Sivasubramanian and Fasel performed a DNS of hypersonic transition of over a flared cone replicating experiments performed in the BAM6QT at Purdue University and were able to identify that both fundamental (K-type) and oblique breakdown are viable paths for transition on this geometry.

This work intends to replicate the nonlinear mechanisms of the flow over the Purdue compression cone with NPSE and DNS to provide a basis for a future theoretical studies of energy transfer mechanisms from...
the mean flow to the disturbance in the linear and nonlinear regime from a nonlinear acoustics perspective.

II. Computational Tools

A. Parabolized Stability Equations Solver: JoKHeR

The JoKHeR code was developed by Kuehl\textsuperscript{15–17} and will be utilized in this work. Herbert and Bertolotti, during a critical review of Gaster’s early nonparallel work, originally identified the parabolized stability equations (PSE) method as an efficient and powerful tool for studying the stability of advection-dominated laminar flows. An excellent introduction to the PSE method and summary of its early development are provided by Herbert.\textsuperscript{18}

The stability analysis begins by considering a generic disturbance and decomposing the flow variables into a steady basic state plus a disturbance (1) and assuming the basic state is a solution to the governing equations of motion, which represents the flow that exists in the absence of any environmental disturbances or forcing. Assuming, without loss of generality, a 2-D basic state and 2-D disturbances, substitution into the governing set of equations yields the equations describing the evolution of the disturbances.

\[
\phi(x, y, t) = \hat{\phi}(x, y) \phi(x, y, t)
\]

Here \(x, y\) are the wall-parallel and -normal coordinates, \(t\) is time, \(u, v\) are the wall-parallel and -normal velocity components, \(\rho\) is the density and \(T\) is the temperature. Different stability methods solve the disturbance equations under different sets of assumptions. Formally, Linear Stability Theory (LST) imposes the assumptions of infinitesimal (linear) disturbances and parallel flow upon which separable solutions may be found. That is, the basic state is assumed to be only a function of \(y\) and \(\phi' = \hat{\phi}(y)e^{i(\alpha x-\omega t)}\). Here \(\alpha\) and \(\omega\) are the streamwise wavenumber and frequency, respectively, of the disturbance. Substitution of this form into the disturbance equations yields a generalized eigenvalue problem.

PSE analysis is similar to the Fourier/Laplace transform in that it considers an initial-value problem. However, an assumption is made for PSE based on the observation that the basic flow state is slowly varying in the streamwise direction, an assumption the Laplace transform is not capable of accommodating because it is needed to allow the wave number to be variable.

Instead a method-of-multiple-scales approach is taken in which a disturbance is assumed of the form \(\phi' = \hat{\phi}(\bar{x}, y) \Phi(x, t)\) where the wave part satisfies \(\frac{\partial \hat{\phi}}{\partial x} = i\alpha(\bar{x})\Phi\) and \(\frac{\partial \hat{\phi}}{\partial t} = -i\omega\Phi\). Notice the multiple-scales introduction of a slow variable \(\bar{x} = \frac{xt}{\nu_e}\) into the shape function and streamwise \((x)\) complex exponential \(\alpha(\bar{x})\), where \(Re_e = \frac{\nu_e}{\delta_e}\) is a Reynolds number based on characteristic values of edge velocity \((U_e)\), edge kinematic viscosity \((\nu_e)\), and reference boundary-layer length scale \((\delta_e = \sqrt{\frac{\nu_e}{U_e}})\). Formally, this is the result of a Fourier transform. Thus, PSE considers disturbances of the form

\[
\phi' = \int_{-\infty}^{\infty} \hat{\phi}(\bar{x}, y, \omega) A(\bar{x}, \omega) e^{-i\omega t} d\omega
\]

where \(A(\bar{x}, \omega) = e^{i\int \alpha(\bar{x}, \omega) dx}\) and the dependence of the shape function \((\hat{\phi})\) and amplitude function \((A)\) on \(\omega\) has been made explicit. The shape and amplitude functions are essentially the Fourier transform of the disturbance. Upon expansion of the streamwise derivatives it is found that the second spatial derivative \(\frac{\partial^2 \phi}{\partial x^2}\) is of highest order and a perturbation expansion may be consistently truncated resulting in the neglect of this term. This leaves the disturbance equation nearly parabolized and an efficient marching solution may be sought. Recently, Kuehl\textsuperscript{16} has extended to NPSE methodology to consider gaussian wave packets, which allows for spectral broadening, low frequency generation\textsuperscript{19} and more physically consistent representation of the instability physics than the traditional NPSE formulation.
B. High-order structured compact finite-difference solver in curvilinear coordinates: CFDSU

CFDSU solves fully-compressible Navier-Stokes equations on a structured curvilinear grid using sixth-order compact finite difference scheme.\textsuperscript{20} The staggered variable arrangement used in this code in particular was shown to retain near spectral accuracy at very high wavenumbers,\textsuperscript{21} which is an indispensable characteristic to solve the ultrasonic waves present at hypersonic flight conditions.

The discretized Navier stokes equations are solved in a uniform and orthogonal spatial domain and, to do that, a coordinate system transformation of a general curvilinear coordinate system \((x_1, x_2, x_3)\) based on the contravariant velocity terms \(v^i = dx^i/dt\) but we also need to define the cartesian coordinate system \((x_1, x_2, x_3)\) where the Navier stokes equation is originally derived. The equations shown here were presented by Nagarajan et al.\textsuperscript{22}

\[
\frac{\partial J\rho}{\partial t} + \frac{\partial}{\partial x^j}(J\rho v^j) = 0 \tag{3}
\]

\[
\frac{\partial J\rho v^i}{\partial t} + \frac{\partial}{\partial x^j}(J\rho v^i v^j + J\rho g^{ij} - J\sigma^{ij}) = -J\Gamma_{qj}^i (\rho v^q v^j + pg^{qj} - \sigma^{qj}) \tag{4}
\]

\[
\frac{\partial J\rho v^i}{\partial t} + \frac{\partial}{\partial x^j}(J(E + p)v^j) = \frac{\partial}{\partial x^k}(J\sigma^{ij} g_{ik} v^k) \tag{5}
\]

In this frame of reference the total energy, the viscous stress tensor and the heat flux vector are described by slightly modified relations described below.

\[
E = \frac{p}{\gamma - 1} + \frac{1}{2} \rho g_{ij} v^i v^j \tag{6}
\]

\[
\sigma^{ij} = \mu (g^{ik} \frac{\partial v^i}{\partial x^k} + g^{ik} \frac{\partial v^j}{\partial x^k} - \frac{2}{3} g^{ij} \frac{\partial v^k}{\partial x^k}) \tag{7}
\]

\[
q_i = -k g^{ij} \frac{\partial T}{\partial x^j} \tag{8}
\]

In the notation used the superscripts indicate the contravariant tensors and the subscripts the covariant ones. Following the convention, \(g^{ij}\) is the contravariant metric tensor and the covariant is \(g_{ij}\). Following, \(\Gamma^{ij}_{qj}\) is the Christoffel symbol of the second kind, which is the representation of a second order derivative of a vector or a first order derivative of a second order tensor and \(J\) is the Jacobian of this transformation, which is the determinant of the Jacobi matrix \((J_{ij} = \partial x_i / \partial x^1)\).

These metrics components are responsible for translating the curvilinear coordinates into its cartesian component and accounting for the effects of using a non-inertial reference frame.

\[
g^{ij} = \frac{\partial x_i \partial x_j}{\partial x^k \partial x^k} \tag{9}
\]

\[
g_{ij} = \frac{\partial x^k \partial x^j}{\partial x_i \partial x_j} \tag{10}
\]

\[
\Gamma^{ij}_{qj} = \frac{\partial x_i}{\partial x^j} \frac{\partial^2 x^j}{\partial x_q \partial x_j} \tag{11}
\]

For this work, the three general cartesian coordinate system directions are named as \(x_1 = X\), \(x_2 = Y\), \(x_3 = Z\) and the three general curvilinear directions are related to a cylindrical grid transformation where \(x^1 = X\), \(x^2 = r\), \(x^3 = \Theta\). In addition, the simulations performed were limited to axisymmetric, therefore the third direction is discarded from this point on. Another grid transformation step is introduced in order to display results in the streamwise \((x)\) and wall normal \((y)\) directions.
Table 1: Flow parameters from Purdue’s compression cone experiments\textsuperscript{12}

<table>
<thead>
<tr>
<th>$Re_m$</th>
<th>$\rho_e$ [kg/m\textsuperscript{3}]</th>
<th>$U_e$ [m/s]</th>
<th>$T_e$ [K]</th>
<th>$T_w$ [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10.8 \times 10^6$</td>
<td>0.0417</td>
<td>864.31</td>
<td>51.65</td>
<td>300</td>
</tr>
</tbody>
</table>

III. Physical Model and Computational Approach

The BAM6QT tunnel conditions are used as the basis of this numerical study (Table 1). These flow conditions are applied over the flared cone geometry used in the Purdue University experiments.\textsuperscript{12} The cone was modeled as a smooth surface with a 3-meter-radius flare. The initial half angle is $\theta_i = 1.32$ degrees, the total axial length is $L = 516.89$ mm, the half base is $b = 57.15$ mm, and the nose-tip radius is $r_{tip} = 0.15$ mm, as shown in figure 2.

![Figure 2: Flared cone geometry and computational domain arrangement.](image)

The grid used for the boundary layer computations is a structured polar grid with the origin at the center of the circle that describes the surface of cone. The choice of polar coordinates guarantees orthogonality of gridlines, which leads to a well-conditioned grid transformation step, and is advantageous since it can be simply described analytically. However, using this grid generation strategy has the disadvantage of retaining an unnecessary high resolution in the streamwise direction away from the wall. This concern is mitigated by observing that the radius of curvature is large compared to the height of the computational domain, their ratio being of the order of 100. The polar grid is stretched in the wall-normal direction with a half-tangent hyperbolic law to resolve the flow gradients in the boundary layer.

DNS of hypersonic flows typically\textsuperscript{23,24} use low-order precursor simulations, capturing the laminar flow around the geometry, to inform inlet and initial conditions for high-order structured simulations. The latter are designed to solve all the scales involved in the transition dynamics of the boundary layer. The high-order simulations are first carried out with no perturbations, allowing the mean flow to be fully established and all residual perturbations that arise from flow initialization and retained by the low-dissipation numerics to be convected out of the domain.

The left and the top boundary of the high-order axisymmetric simulations are treated as inflow, i.e. with Dirichlet boundary conditions imposing the flow conditions established by a precursor calculation. At the wall, no-slip and no penetration conditions are imposed everywhere except when suction and blowing is applied to generate perturbations. The wall is also assumed to be isothermal at the temperature of 300 K justified by the small duration of the experimental runs, which do not allow sufficient time for the...
model to change its temperature. Neumann boundary conditions are imposed at right boundary. To prevent oscillations that could appear in the subsonic portion of the domain and to ensure the numerical stability of the simulations, the inlet and outlet boundaries are coupled with a buffer layer.

IV. Disturbance Evolution over a Hypersonic Flared Cone

This section aims to simulate and analyze the perturbation growth mechanism in a hypersonic flow over a flared cone through the excitation of the boundary layer with a broadband frequency pulse. In the future, we aim to use this canonical setup with an almost constant boundary layer thickness to study the principal energy source an dissipation mechanisms for second mode waves at very large amplitudes.

Precursor simulations of the Purdue’s compression cone flow conditions (Table 1) were obtained using US3D\textsuperscript{25–27} and they were applied as initial condition to a high order simulation in CFDSU. The initialization step relies on the interpolation of the precursor solution onto a new grid. This triggered a transient adjustment phase of the numerical solution by CFDSU. Figure 3 shows the boundary layer profiles obtained after an unperturbed state is reached over the flared cone geometry. An excellent agreement for different grid refinement levels is shown. It is important to highlight that the adverse pressure gradient, induced by the flared geometry, effectively keeps the boundary layer thickness constant along the majority of the length of the cone.

![Figure 3: Boundary layer profiles for temperature and streamwise velocity as a function of distance from the wall at different locations at the cone surface for the precursor and high order calculations. Symbols are from CFDSU simulations and the solid line is US3D.](image)

A. Comparison with NPSE

Initially a pulse with a duration of approximately 1.67 $\mu$s and maximum amplitude of $1.0 \times 10^{-6}$ m/s was applied via suction and blowing at the wall near to the beginning of the computational domain, after the inlet buffer layer becomes negligible. The region of excitation was centered around X = 0.075 m and had a length of 2.0 mm, with the following expression for the wall normal velocity at the wall:

$$v = \cos(\pi x_1)^3 \sin(2\pi ft), \text{ for } 0 \leq t \leq 1/f$$

where $x_1$ is a variable with values between $[-1, 1]$, which is mapped to the actual spatial interval of application of the pulse, and $f = 600$kHz is the frequency around the which the pulse spectrum is centered.

A Fourier analysis of the wall pressure disturbance at different streamwise spatial locations as the pulse advects downstream is shown in figure 4. The initial pressure wave amplitude effectively received by the
boundary layer from the disturbance imposed at the wall is of the order of $1.0 \times 10^{-4}$ Pa and, therefore, only linear mechanisms of growth are expected to be excited initially and any subsequent excitation of higher harmonics can be attributed to nonlinear effects developed within the flow, and not to the external forcing.

Initially, only one narrow frequency band, centered around 296 kHz is observed. This result is consistent with previous experiments performed by Wheaton et al., who reported a maximum amplitude at 295 kHz. As the waves propagates downstream along the flared cone two trends can be highlighted. First, there is a small shift towards lower frequencies consistent with a small growth in the boundary layer thickness, no fully counteracted by the adverse pressure gradient effects, which establish themselves in full later. Second, the continuous energy injection focused on a narrow band of frequencies makes them increase rapidly in amplitude, i.e. by 8 orders of magnitude in less than half a meter of streamwise evolution and triggering nonlinear effects, such as the appearance of higher harmonics. A disturbance with the same initial amplitude evolving over a straight cone at similar flow conditions will have a much lower cumulative growth, since, as the waves propagate into regions of higher boundary layer thicknesses, the most amplified frequencies shift to lower values.

In order to depict better some characteristics of the perturbation advection and spectral broadening, the power spectrum of the pressure signal at the wall at specific locations is show in figure 5. At $X = 0.11 \ m$, one can observe the pressure perturbation near the excitation location and its broadband characteristic and low amplitude. At this location, it is possible to observe the beginning of the energy concentration in the
unstable frequency band of the primary perturbation. At \( X = 0.35 \) m the first evidence of nonlinear dynamics starts to appear as the first harmonic rises from the round-off noise floor. In the following two locations the excitation of the next two harmonics is observed. Another interesting fact that can be outline from this plot is the growth of a zero frequency perturbation. This component is associated with the mean flow distortion caused by the relatively high amplitude perturbations and it is another evidence of the presence of nonlinearities in the flow.

![Figure 6: Comparison of the current DNS simulations with results obtained from nonlinear PSE.](image)

We compared the results obtained in the current DNS simulations with NPSE calculations made in the JoKHeR (see section II.A) and these are gathered in figure 6. The results shows that, for all grids simulated, the growth rate of the primary frequency is in good agreement. As the grid is refined the estimated growth rates for the higher harmonics also start to exhibit agreement with NPSE but there is disagreement in the spatial location where the harmonics start to become important and in the region where saturation the wave is reached. This is consistent with the fact that disturbances of higher frequencies need more points to be accurately resolved with minimum numerical dissipation and therefore it is expected that increasing the grid resolution will lead to better matching of the results. It is noteworthy to say that the noise floor decreases monotonically as the grid is refined. To capture the growth rates of the 900 kHz mode, a grid resolution of at least \( n_x = 9216 \) and \( n_y = 640 \) was required.

### B. Spectral Broadening and Physical Implications

![Figure 7: Amplitude spectrum of the pressure disturbance at the wall caused by advection of a relatively high amplitude broadband pulse with amplitude of \( 1.0 \times 10^{-4} \) m/s over a flared cone.](image)
The results shown in the previous section focus on the development of the perturbation from linear to nonlinear and its comparison against another prediction methods. Because of that, the imposed disturbance only reaches a weakly nonlinear region in the limited spatial domain simulated. In order to show interesting phenomena occurring when highly nonlinear effects are present in the boundary layer, a pulse with the same period and spatial distribution but a higher amplitude of $1.0 \times 10^{-4}$ m/s was applied.

Figure 7 gathers all the information about power distribution for different frequencies and for different locations at the surface of the flared cone. This figure depicts all the trends of power concentration on specific frequency bands, higher harmonics and mean flow distortion discussed previously with the difference that it acts in a different amplitude range. This increase in amplitude enhances the importance of the nonlinear effects to the point that very high frequency harmonics are fully developed and have similar amplitudes, indicating the presence of saturation.

Figure 8: Amplitude distribution over frequency at different spatial locations of the pressure signal at the wall caused by advection of a relatively high amplitude broadband pulse with amplitude of $1.0 \times 10^{-4}$ m/s.

Towards the end of the computational setup it is also observed, in figure 7, that the frequency bands corresponding to the principal wave and its harmonics become wider, an evidence of spectral broadening. The amplitude distribution over frequency for the advected disturbance is shown for specific spatial locations in order to better depict the described phenomenon in figure 8. In this plot it is possible to observe that the excited frequency bands stay concentrated up to $X = 0.43$ m but, at the downstream probed location ($X = 0.48$ m), the disturbance is spread out and spans a larger portion of the frequency domain. This is an intermediate step before the final breakdown into turbulence, not captured in the current axysymmetric simulations, where the energy is distributed into all the scales until viscous dissipation allows.

Figure 9: Appearance of “rope waves” in the numerical Schlieren is connected to non-linear interactions between second mode waves of high amplitude.

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The increase in amplitude and the highly nonlinear interactions that happen in the aft region of the flared cone lead to the appearance of entangled structures at the edge of the boundary layer when the magnitude of the density gradient becomes significant (figure 9). Since the optical technique of measuring second mode waves based on Schlieren relies on density gradients, a numerical Schlieren rendering is used to create a parallel with experimental results. It is interesting to highlight that these structures of fluctuations is only present in highly nonlinear regions and this is consistent with the experimental and numerical observations that the “rope waves” appear in the region preceding transition to turbulence. The simulated “rope waves” are also in agreement with the observations made by Stetson that the second mode has a wavelength of approximately two times the boundary layer thickness.

Since the physical domain is transformed into wall normal and streamwise coordinates in figure 9, the numerical Schlieren is reploted in figure 10 in physical coordinates. In this figure, the entanglement of the spatial variation of amplitude is more visible and, in addition, regions of compression and rarefaction inside the boundary layer are observed.

V. Summary

The spectral broadening of the second mode over a hypersonic flared cone was addressed in this work. A high order compact finite difference scheme was initialized from the laminar mean flow solution of a precursor simulation and was used to numerically predict the perturbation’s evolution downstream. It was observed that the flared cone designed was able to create an adverse pressure gradient that maintained the boundary layer thickness approximately constant over the entire cone. This effect concentrated the growth mechanisms of the second mode wave in a single frequency band, up to triggering nonlinear effects such as mean flow distortion and higher harmonics. The results of linear growth and nonlinear interaction was compared with a nonlinear parabolized stability equations (NPSE) solver and a good agreement was achieved. A pulse of higher amplitude was then introduced to visualize the phenomena of spectral broadening and that the so called “rope waves”, which develop only in the region of highly nonlinear interactions.

In conclusion, this work represents an intermediate step towards the understanding the relevant contributions of the nonlinearities present in a hypersonic flow and the principal energy source mechanism for second mode waves when they reach very high amplitudes. In the end, we demonstrate that we have developed the necessary tools for analyzing this in a future contribution.

Acknowledgments

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