Analysis of BJT's, Pseudo-HBT's, and HBT's by Including the Effect of Neutral Base Recombination

Saeed Mohammadi, *Student Member, IEEE*, and C. R. Selvakumar, *Member, IEEE*

***Abstract*—An analytical study of the effect of neutral base recombination on various transistor performance characteristics is presented. Using an approximate life time model which is suitable for BJT's, pseudo-HBT's, and HBT's with small variations in bandgap, closed form analytical relations for common emitter current gain, Early voltage and base and emitter delay times are derived including the effect of neutral base recombination. The relations show that the current gain β and Early voltage V_A and, as a result, the figure of merit current gain-Early voltage product (βV_A) drop rapidly as the recombination increases. They also show that emitter delay time increases with an increase in the neutral base recombination while base delay time decreases though these delay times are not strong function of neutral base recombination. Computer simulations (MEDICI) are also shown for comparing with analytical results.**

I. Introduction

HETEROJUNCTION bipolar transistors (HBT's) have been extensively studied during the past few years [1], [2]. Several significant advantages over conventional homojunction bipolar transistors (BJT's) are apparent, such as higher emitter injection efficiency, reduced charge storage in the emitter, lower base transit time and higher Early voltage for devices with a graded bandgap in the base. An important figure of merit, especially for analog applications, namely common emitter current gain-Early voltage product βV_A of over 100 000 V has been reported for SiGe HBT which is much higher than that is usually achieved for conventional Si devices [3], [4]. This figure of merit is enhanced due to two effects: an increase in the emitter injection efficiency and an increase in the Early voltage in a graded bandgap structure.

The other less often considered devices called pseudo-heterojunction bipolar transistors, exploit the bandgap narrowing phenomena in the heavily doped base region to increase emitter injection efficiency at low temperatures. Even though base is doped higher than the emitter, the injection efficiency is greater than one since bandgap narrowing in the base increases the effective intrinsic carrier concentration n_{ie} compared to the intrinsic carrier concentration n_i in the emitter. As temperature drops, n_{ie}/n_i becomes larger, resulting in a higher current gain in contrast to conventional BJT's which suffer from gain reduction at low temperatures [5].

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The authors are with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1.

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Recombination across neutral base region degrades the characteristics of HBT's and pseudo-HBT's as well as some conventional BJT devices. Since the emitter injection efficiency is high in HBT's, recombination can severely reduce the common emitter current gain in such devices because of a parasitic component added to the base current due to neutral base recombination [6]. High base doping density as well as defects in the base which behave as recombination centers are the main causes of neutral base recombination in HBT's. Although SiGe HBT's without significant neutral base recombination have been fabricated [7], [8], there are still technological problems in growing low defect LRP or MBE grown SiGe HBT's and AlGaAs and InP based HBT's [9]–[12]. Polysilicon emitter approach which improves the emitter injection efficiency might not offer a significant advantage since recombination process inside the base is independent of the emitter injection efficiency.

In this paper we consider the effect of neutral base recombination on various device parameters using a simple analytical recombination model for a device with nonuniform energy gap and arbitrary doping profile in the base. The relations derived can be applied to BJT's, pseudo-HBT's, and HBT's as well. However, application of this model for HBT's with large bandgap variation across the base might lead to unacceptable errors. The derived relations offer significant insight into the effect of neutral base recombination on transistor characteristics.

II. Theory

In order to construct our model, we shall consider a general transistor with a base with an arbitrary doping profile of $N_A(x)$ and an arbitrary bandgap profile of $E_g(x)$. We call this transistor an HBT, however it can represent a pseudo-HBT or a conventional BJT as well. In this model, we use the notation for an NPN transistor, however, it can also be extended for PNP transistors.

To evaluate the behavior of an HBT, we derive expressions for current gain β, Early voltage V_A, base delay time t_{bb} and emitter delay time $t_{\rm em}$. Since base has a nonuniform bandgap profile ($E_g(x)$) and an arbitrary doping profile ($N_A(x)$), the diffusion coefficient $D_n(x)$ and the minority carrier lifetime $\tau_n(x)$ may no longer be constant inside the base. To develop the model, the following assumptions are made:

- Low level injection regime inside the base (which is mostly the case for the HBT's, because of the moderately to heavily doped base region).

- Neglecting the current due to the majority carriers in the base.
- No hot electron effect.
- Nondegenerate statistics.
- Infinite transition velocity of carriers inside the base-collector depletion layer.
- High surface recombination velocity at the emitter contact.

Ignoring the recombination inside the base region, one can write current gain and Early voltage as given by Prinz *et al.* [3]:

$$\beta = \frac{q}{J_{po} \int_0^w \frac{N_A}{D_n n_{ie}^2} dx} \tag{1}$$

$$V_A = \frac{q n_{ie}^2(w) D_n(w)}{C_{\mathrm{bc}}} \int_0^w \frac{N_A}{D_n n_{ie}^2} dx \tag{2}$$

where J_{po} is the thermal equilibrium hole current injected into the emitter; n_{ie}^2 is the effective intrinsic carrier density which reflects the variation of the bandgap inside the base due to the varying alloy composition as well as bandgap narrowing effect; C_{bc} is the base collector junction capacitance and w is the effective base width. From the two last equations, the figure of merit current gain-Early voltage product can be determined to be:

$$\beta V_A = \frac{q^2 n_{ie}^2(w) D_n(w)}{J_{po} C_{\mathrm{bc}}} \tag{3}$$

which is independent of bandgap variation and doping profile across the base and depends on the effective intrinsic carrier density at the edge of the collector base depletion region inside the base.

The base delay time t_{bb} may also be found, neglecting the recombination, as given by Kroemer [13]:

$$t_{bb} = \int_0^w \frac{n_{ie}^2}{N_A} \left(\int_x^w \frac{N_A}{D_n n_{ie}^2} dy \right) dx \cdot \tag{4}$$

If the assumption of infinite transition velocity of carriers inside the collector-base depletion region is not valid, there will be a nonzero minority carrier concentration at the boundary which gives rise to an increased base delay time. This effect is more pronounced in very thin base region [14]. The emitter delay time t_{em} may be calculated by dividing the net charge accumulated in the emitter by the collector current. To simplify the calculations, let us assume a thin emitter region with a width of W_e and a constant emitter doping of N_D. The emitter delay time will be

$$t_{\mathrm{em}} = \frac{W_e n_i^2}{2 N_D} \int_0^w \frac{N_A}{D_n n_{ie}^2} dx \tag{5}$$

where n_i is intrinsic carrier density inside the emitter. Again, if the assumption of high surface recombination velocity at the emitter contact is not valid, the net charge accumulated in the emitter region will become larger which gives rise to a larger emitter delay time.

The quantity $\int \frac{N_A dx}{D_n n_{ie}^2}$, called effective base Gummel number, is an important parameter in determining the transistor characteristics. If the effective base Gummel number is small, the current gain β will be large and the emitter delay time t_{em} will be small, however the Early voltage will suffer (see (2)). This reduction in V_A can be compensated by increasing $n_{ie}^2(w)$ and $D_n(w)$ which can be achieved by changing the alloy composition of the base material at the base-collector junction in order to have a smaller bandgap. The base delay time t_{bb} is significantly affected by the presence of an accelerating field in neutral base region produced by either bandgap variation or doping profile.

In the presence of neutral base recombination, the continuity equation should be solved to find the electron distribution in the base. It requires the knowledge of n_{ie}^2, N_A, D_n and the minority carrier lifetime τ_n as functions of position within the base. In some cases (like SiGe materials) n_{ie}^2 and N_A may be known but at present D_n and τ_n are less firmly known. In the absence of reliable data for τ_n and D_n for bandgap varying situations, we adopt a lifetime model proposed in [15] (see also [16]) to solve the continuity equation and hence an analytical solution can be found that includes the effect of neutral base recombination to compute transistor characteristics. The lifetime model is given by:

$$\tau_n = \frac{D_n}{C_s^2} \frac{n_{ie}^4}{N_A^2} \tag{6}$$

where C_s is a parameter which is nearly a constant. This model is shown to be in a good agreement with experimental results for Si BJT's [17] and for some III-V materials [18]. For pseudo-HBT's the model is still acceptable because according to bandgap narrowing models [19], [16], the variations in n_{ie}^2 and N_A almost cancel each other, resulting in a reasonable variation in lifetime across the base. For HBT's, however, the model may be unacceptable because in this case, (6) might not be an acceptable approximation across a thin base region. As a result, the parameter C_s might vary significantly. Nevertheless, in order to gain a reasonable understanding of recombination effects one may assume an average effect for C_s and treat this as a constant and apply this lifetime model for HBT's as well.

Considering a regional model and solving the minority carrier continuity equation in neutral base using (6), one obtains minority carrier concentration $n(x)$ inside the base:

$$n(x) = \frac{n_{ie}^2}{N_A} e^{V_{BE}/V_t} \left[\frac{\sinh[Ki(w) - Ki(x)]}{\sinh[Ki(w)]} \right] \tag{7}$$

where $Ki(z)$ is defined as

$$Ki(z) = C_s \int_0^z \frac{N_A}{D_n n_{ie}^2} dx \cdot \tag{8}$$

Electron current density inside the base can also be found to be

$$J_n(x) = q C_s e^{V_{BE}/V_t} \left[\frac{\cosh[Ki(w) - Ki(x)]}{\sinh[Ki(w)]} \right]. \tag{9}$$

Having $n(x)$ and $J_n(x)$, we are now able to find various transistor parameters. We can write base delay time t_{bb} as

$$t_{bb} = q\int_0^w \frac{n(x)dx}{J_n(x)} = \frac{1}{C_s}\int_0^w \frac{n_{ie}^2}{N_A}\tanh[Ki(w) - Ki(x)]dx. \quad (10)$$

Early voltage calculated assuming a constant base current is (see Appendix for the derivation):

$$V_A = \frac{qD_n(w)n_{ie}^2(w)}{C_s C_{bc}} \times \left[\frac{qC_s(\cosh[Ki(w)] - 1) + J_{po}\sinh[Ki(w)]}{qC_s\sinh[Ki(w)] + J_{po}\cosh[Ki(w)]}\right] \quad (11)$$

and the current gain β is (see Appendix for the derivation):

$$\beta = \frac{qC_s}{qC_s(\cosh[Ki(w)] - 1) + J_{po}\sinh[Ki(w)]}. \quad (12)$$

The emitter delay time t_{em}, assuming a narrow emitter with a constant doping profile, will be:

$$t_{em} = \frac{W_e n_i^2 \sinh[Ki(w)]}{2N_D C_s}. \quad (13)$$

It is important to notice that (10)–(13) will readily reduce to (1), (2), (4), and (5) when Ki number is small. That is for small C_s which means that the recombination is negligible.

The figure of merit current gain-Early voltage product in the presence of recombination can be derived using (11) and (12):

$$\beta V_A = \frac{q^2 n_{ie}^2(w)D_n(w)}{C_{bc}(qC_s\sinh[Ki(w)] + J_{po}\cosh[Ki(w)])}. \quad (14)$$

The Ki number, which is a design parameter and appears in (10)–(14), should be minimized in order to improve β and t_{em}, however it results in a reduction of the Early voltage V_A. At the same time $n_{ie}^2(w)$ and $D_n(w)$ may be selected to be large so that V_A becomes acceptable. The delay time t_{bb} is again minimized by an accelerating field either produced by bandgap variation or by impurity profile. The figure of merit βV_A appears to diminish dramatically as the recombination increases, since both β and V_A fall off rapidly with recombination.

III. Application I (Pseudo-HBT)

The results obtained in the previous section can be applied to pseudo-HBT devices. Base doping concentration in pseudo-HBT's is usually higher than emitter doping concentration and hence these devices show a small common emitter current gain at room temperature. However, bandgap narrowing in the heavily doped base has pronounced effect at lower temperatures resulting in a considerably higher gain [5].

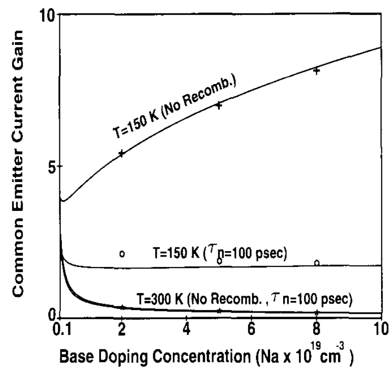

Fig. 1. Variation of the common emitter current gain β versus the acceptor doping density across the base in a pseudo-HBT at room temperature ($T = 300$ K) and $T = 150$ K. Two different lifetimes ($\tau_n = \infty$ and $\tau_n = 100$ ps) have been used for neutral base recombination. Results of MEDICI simulation are also shown in the figure: (*) corresponds to $T = 300$ K (No recombination and $\tau_n = 100$ ps).(+) corresponds to $T = 150$ K (No recombination).(o) corresponds to $T = 150$ K ($\tau_n = 100$ ps). The transistor has following specifications: Emitter doping concentration $N_D = 5 \times 10^{17}$ cm^{-3}, emitter width $W_e = 0.2$ μm and neutral base width $w = 0.08$ μm. Diffusion coefficients are calculated from analytical mobility model given in [22].

Let us consider a pseudo-HBT with a uniform emitter doping concentration of $N_D = 5 \times 10^{17}$ cm^{-3} and an emitter width of $W_e = 0.2$ μm. The base doping is uniform and its concentration varies from $N_A = 1 \times 10^{18}$ cm^{-3} to $N_A = 1 \times 10^{20}$ cm^{-3}. Effective base width is $w = 0.08$ μm.

The bandgap varies with temperature according to

$$E_g(T) = E_g(0) - \frac{\alpha T^2}{T + \beta} \quad (15)$$

where $E_g(0) = 1.125$ eV, $\alpha = 4.73 \times 10^{-4}$/K^2 and $\beta = 636$ K for Silicon [20]. Variation of the effective intrinsic carrier density versus temperature, exploiting the Slotboom–deGraaff bandgap narrowing model [19], can be written as (see bottom of page) where Co $= 1.686 \times 10^{43}$ cm^{-6}eV^{-3} for Silicon [21]. Since we assume a uniform doping concentration across the base, the minority carrier lifetime is constant and instead of using C_s as the recombination parameter, we can use minority carrier lifetime τ_n.

Diffusion coefficients D_n and D_p depend on actual doping concentration and operating temperature. Assuming Boltzmann statistics and the assumption that Einstein relation holds, we can represent diffusion coefficients in terms of electron and hole mobilities. We use analytical mobility model given in MEDICI simulation package [22] with all the default values for silicon to define the relation among diffusion coefficients and impurity concentrations and temperature.

The current gain of the pseudo-HBT device versus the base doping concentration is depicted in Fig. 1 for two different

$$n_{ie}^2 = \text{Co}\cdot(kT)^3 \times \exp\left(\frac{-E_g(T) + 0.009\ln(N_A/10^{17}) + 0.009\sqrt{\ln^2(N_A/10^{17}) + 0.5}}{kT}\right) \quad (16)$$

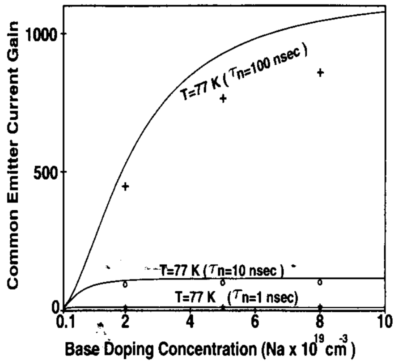

Fig. 2. Variation of the common emitter current gain β versus the acceptor doping density across the base in a pseudo-HBT at $T = 77$ K. Three different lifetimes ($\tau_n = 100$ ns, $\tau_n = 10$ ns and $\tau_n = 1$ ns) have been used for neutral base recombination. Results of MEDICI simulation are also shown in the figure: (+) corresponds to $\tau_n = 100$ ns. (o) corresponds to $\tau_n = 10$ ns. (*) corresponds to $\tau_n = 1$ ns. The transistor has following specifications: Emitter doping concentration $N_D = 5\times10^{17}$ cm^{-3}, emitter width $W_e = 0.2$ μm and neutral base width $w = 0.08$ μm. Diffusion coefficients are calculated from analytical mobility model given in [22].

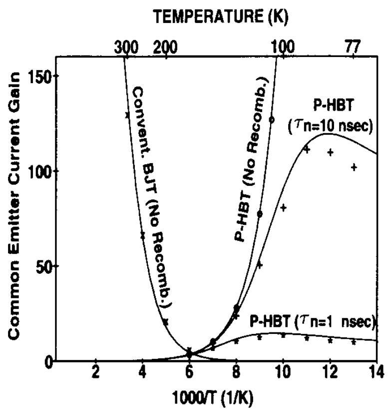

Fig. 3. Variation of the common emitter current gain β of a conventional BJT and a pseudo-HBT versus temperature. Three different lifetimes ($\tau_n = \infty$, $\tau_n = 10$ ns and $\tau_n = 1$ ns) have been used for neutral base recombination. Results of MEDICI simulation are also shown in the figure: (x) corresponds to conventional BJT with no recombination. (o) corresponds to pseudo-HBT with no recombination. (+) corresponds to pseudo-HBT with $\tau_n = 10$ ns.(*) corresponds to pseudo-HBT with $\tau_n = 1$ ns. The pseudo-HBT has following specifications: Emitter doping concentration $N_D = 5 \times 10^{17}$ cm^{-3}, Base doping concentration $N_A = 5 \times 10^{19}$ cm^{-3}, emitter width $W_e = 0.2$ μm and neutral base width $w = 0.08$ μm. The conventional device has the same specification except that the base doping concentration $N_A = 10^{17}$ cm^{-3} and the emitter doping concentration $N_D = 5 \times 10^{19}$ cm^{-3}. Diffusion coefficients are calculated from analytical mobility model given in [22].

operating temperature using (12). For the room temperature, as the doping density increases the current gain decreases, however, due to the bandgap narrowing effect in the base, the drop in the current gain is not so sensitive to the base doping. Neutral base recombination causes an additional drop in the current gain. When we reduce the operating temperature to 150 K, the effect of bandgap narrowing in the base becomes more pronounced and in this case, current gain even increases as the base doping increases (Fig. 1), since $\beta \propto \exp(\frac{\Delta_G}{kT})$ where Δ_G is the effective bandgap difference between the base and the emitter. As the base doping increases, Δ_G becomes larger, resulting in an enhancement in the current gain. This enhancement, however, cannot be seen at room temperature since Δ_G is small comparing to kT. Recombination in the neutral base region still causes a reduction in the gain.

Reducing the temperature to 77 Kelvin results in a significant enhancement in the current gain as shown in Fig. 2. It is worth noting that the effect of recombination at low temperatures (e.g., 77 K) is more pronounced since the injection efficiency of the device is very high at these temperatures and the base current is dominated by the neutral base recombination. As can be seen, lifetimes of the order of nanoseconds can degrade the device performance at 77 K while lifetimes even as low as 100 ps do not affect the current gain of the device at room temperature much (Fig. 1).

Shown in Fig. 3 is a comparison between the common emitter current gain of a pseudo-HBT and a conventional BJT. As the temperature decreases, the gain of the conventional device falls off while the gain of the pseudo-HBT device increases. Neutral base recombination causes a drop in the current gain and becomes more important in lower temperatures as shown in the figure.

Current gain of pseudo-HBT's is influenced by base doping concentration in two different aspects: in one aspect, it is inversely proportional to the base doping while in another aspect it is affected by the bandgap narrowing effect by a rather complicated function (see (12) and (16)). At high temperatures, the term inversely proportional to base doping is dominant while at lower temperatures, the term involving the bandgap narrowing effect is dominant, resulting in an increase in current gain as base doping concentration increases. The effect of bandgap narrowing becomes more apparent at higher temperatures if Δ_G, the effective bandgap difference between the base and the emitter is higher.

IV. Application II (HBT)

Next, we apply our model to gain an understanding about the HBT's performance. We will consider a general case of linearly varying bandgap. The slope of the bandgap is considered to be a parameter while the average bandgap reduction may be considered as a constant [23]. To simplify the case even more, we assume that the impurity profile N_A is also a constant across the base. The diffusion coefficient D_n is also assumed to be a constant, however it may vary a little for the practical case [24].

The transistor used in this example has the following specifications:

- An NPN Si-Ge HBT with an average base bandgap discontinuity of $\Delta E_{\text{ave}} = 0.16$ eV and with a linearly varying bandgap with the slope η defined by

$$\eta = \frac{\Delta E(w) - \Delta E(0)}{kT} \qquad (17)$$

where $\Delta E(w)$ and $\Delta E(0)$ are the bandgap discontinuities at the collector base and emitter base junctions, respec-

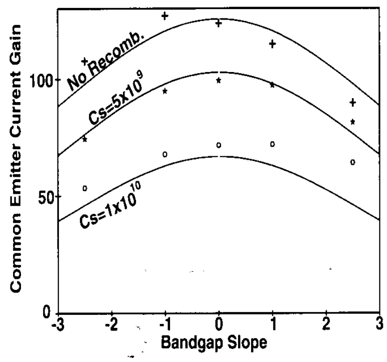

Fig. 4. Variation of the current gain β versus the slope of the bandgap η when C_s is a parameter. Results of MEDICI simulation are also shown in the figure: (+) corresponds to no recombination case. (*) corresponds to $\tau_n = 4$ ns. (o) corresponds to $\tau_n = 1$ ns. The HBT is assumed to be an NPN SiGe-base transistor with a linear bandgap variation across the base having a base doping N_A of 10^{18} cm^{-3}, emitter doping N_D of 2×10^{17} cm^{-3}, effective base width of $w = 0.1$ μm and emitter width of $W_e = 0.1$ μm. Diffusion coefficients D_n and D_p are assumed to be constant, $D_n = 8$ cm^{2}s^{-1} and $D_p = 6$ cm^{2}s^{-1}. The Ki number calculated using (8) varies from 0.059 for $\eta = 0$ to 0.084 for $\eta = \pm 3$ for the case $C_s = 5 \times 10^9$.

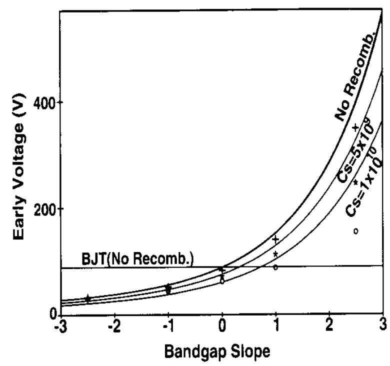

Fig. 5. Variation of the Early voltage V_A versus the slope of the bandgap η when C_s is a parameter. The Early voltage of the equivalent BJT is also shown. Results of MEDICI simulation are also shown in the figure: (+) corresponds to no recombination case. (*) corresponds to $\tau_n = 4$ ns. (o) corresponds to $\tau_n = 1$ ns. The HBT is assumed to be an NPN SiGe-base transistor with a linear bandgap variation across the base having $N_A = 10^{18}$ cm^{-3}, $N_D = 2 \times 10^{17}$ cm^{-3}, $w = W_e = 0.1$ μm and $D_n = 8$ cm^{2}s^{-1}. The base collector junction capacitance C_{bc} is assumed to be 18 nFcm^{-2}. The Ki number calculated using (8) varies from 0.059 for $\eta = 0$ to 0.084 for $\eta = \pm 3$ for the case $C_s = 5 \times 10^9$.

tively. The parameter η varies between -3 to 3 to ensure that the variation in n_{ie}^2 is not unacceptably large ((6) holds). Negative bandgap slopes are generally undesirable for device operation because of increased minority carrier concentration in the base.

- Constant doping profile in the base, emitter and collector. $N_D = 2 \times 10^{17}$ cm^{-3}, $N_A = 10^{18}$ cm^{-3} and $N_{\rm epi} = 10^{16}$ cm^{-3}.
- Effective base width $w = 0.1$ μm and effective emitter width $W_e = 0.1$ μm.
- Minority carrier diffusion coefficient inside the base $D_n = 8$ cm^{2}s^{-1} and inside the emitter $D_p = 6$ cm^{2}s^{-1}.
- The junction capacitances are calculated to be: $C_{\rm bc} = 18$ nFcm^{-2} and $C_{\rm be} = 460$ nFcm^{-2} under bias condition of $V_{\rm be} = 0.65$ V and $V_{ce} = 2$ V.

Fig. 4 shows the variation of current gain β as a function of slope of the bandgap η (see (17)) when C_s is a parameter (see (12)). As can be seen from the figure, the maximum gain is achieved when η is zero, that is when bandgap profile is uniform across the base. The reason is that $Ki(w)$ is dominated by the regions in the base where n_{ie}^2 is small, i.e., E_g is large [3]. So, for the case where the slope is not zero, $Ki(w)$ will be larger than that of $\eta = 0$ and the current gain becomes smaller. Note that when C_s increases, i.e., recombination increases, the current gain β drops. Also shown in the figure are the results of MEDICI simulation for the same device operating at three different conditions: no recombination, $\tau_n = 4$ ns and $\tau_n = 1$ ns.

Fig. 5 depicts the variation of the Early voltage with respect to the bandgap slope η, when C_s is a parameter (see (11)). Also shown in the figure are the results of MEDICI simulation for the same device operating at no recombination, $\tau_n = 4$ ns and $\tau_n = 1$ ns. Simulation results at positive bandgap slopes are less than what are expected from the model since in the model, we use a constant C_s while in the simulation, we define a constant τ_n. For positive slope where the bandgap discontinuity at the collector base junction is larger than the bandgap discontinuity at the emitter base junction, $n_{ie}^2(w)$ becomes large, resulting in an increase in the Early voltage compared to the equivalent BJT. As C_s increases (i.e., recombination increases), the Early voltage drops.

The reason for the drop in Early voltage is stated as follows: As V_{ce} increases, the width of the collector base depletion region extending into the base region increases resulting in an increase in the slope of the minority carrier concentration profile inside the base and consequently an increase in collector current which is generally known as Early effect. In the presence of neutral base recombination, with an increase in V_{ce}, the neutral base width becomes thinner and therefore the recombination component of the base current becomes smaller. To keep the base current constant, the injection component of the base current has to increase resulting in a further increase in the slope of the minority carrier concentration profile inside the base when compared to no recombination case. Therefore in the presence of neutral base recombination, the Early effect is more pronounced and the Early voltage decreases with an increase in the neutral base recombination.

The variation of base delay time t_{bb} with respect to the bandgap slope η, where C_s is a parameter, is depicted in Fig. 6 (see (10)). As can be seen from the figure, for positive bandgap slope, the delay time becomes less than that of equivalent BJT due to the accelerating field produced by the bandgap slope. As C_s increases, the base delay time reduces since the net charge stored in the base is reduced by the recombination process.

Shown in Fig. 7 is the emitter delay time which may be minimized when the slope of the bandgap η is zero and recombination is negligible (see (13)). As recombination increases, the collector current decreases for the same base current (drop in β), resulting in an increase in the emitter delay time. As may be noticed, in this example, emitter delay

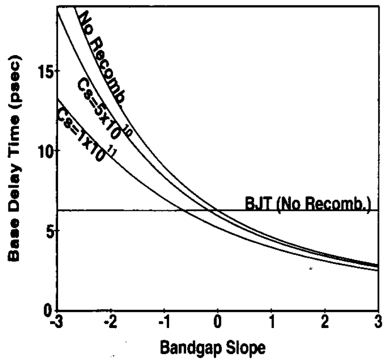

Fig. 6. Variation of the base delay time t_{bb} versus the slope of the bandgap η when C_s is a parameter. The base delay time of the equivalent BJT is also shown. The HBT is assumed to be an NPN SiGe-base transistor with a linear bandgap variation across the base having $N_A = 10^{18}$ cm^{-3}, $N_D = 2\times10^{17}$ cm^{-3}, $w = W_e = 0.1$ μm and $D_n = 8$ cm^{2}s^{-1}.

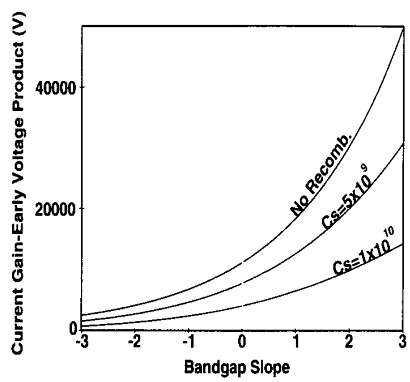

Fig. 8. Variation of figure of merit βV_A versus the slope of the bandgap η when C_s is a parameter. The HBT is assumed to be an NPN SiGe-base transistor with a linear bandgap variation across the base having $N_A = 10^{18}$ cm^{-3}, $N_D = 2 \times 10^{17}$ cm^{-3}, $w = W_e = 0.1$ μm, $D_n = 8$ cm^{2}s^{-1}, $D_p = 6$ cm^{2}s^{-1} and $C_{bc} = 18$ nFcm^{-2}.

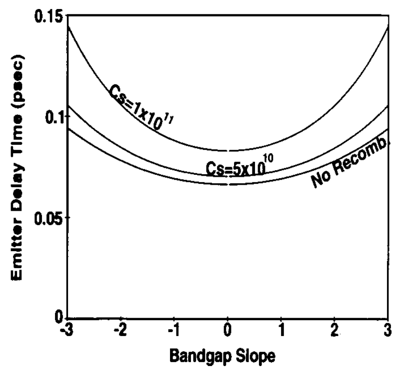

Fig. 7. Variation of the emitter delay time t_{em} versus the slope of the bandgap η when C_s is a parameter. The HBT is assumed to be an NPN SiGe-base transistor with a linear bandgap variation across the base having $N_A = 10^{18}$ cm^{-3}, $N_D = 2 \times 10^{17}$ cm^{-3}, $w = W_e = 0.1$ μm and $D_n = 8$ cm^{2}s^{-1}.

time is negligible comparing to the base delay time. However, it may not be the case always, especially when the average bandgap discontinuity is small. In such a situation emitter delay time is comparable or even larger than base delay time and in the presence of significant neutral base recombination, the difference becomes much more.

The current gain-Early voltage product can also be shown to increase as the bandgap slope η becomes more positive and reduces dramatically when neutral base recombination increases (Fig. 8).

V. Summary

In this paper, the effect of neutral base recombination on various transistor parameters using an approximate lifetime model was presented and discussed. The model developed in this paper is applicable for a transistor having a nonuniform bandgap and an arbitrary doping profile inside the base provided the bandgap variation is not too great. In order to achieve a better performance using a linearly varying bandgap HBT, it was found that one may design for a slightly positive slope in bandgap in order to have high Early voltage, low transition times and a reasonably large common emitter current gain. Aiming to achieve an extremely high current gain might prove to be counterproductive in some situations since neutral base recombination can become comparable to injection component of base current and this leads to a dramatic decrease in current gain. The insights gained from this study will be useful to consider the current gain trade off for higher frequency of operation by increasing the base doping and similar consideration when neutral base recombination might be a factor to be considered.

Appendix

In order to calculate Early voltage, (11), we change the base-collector voltage V_{cb}, which results in a change in base thickness. Assume that the new base thickness is w' where $w' = w + dw$. We may find the electron current density for w and w' as

$$J_n(x) = qC_s e^{V_{be}/V_t}\left[\frac{\cosh[Ki(w) - Ki(x)]}{\sinh[Ki(w)]}\right]$$
$$J_n'(x) = qC_s e^{V_{be}'/V_t}\left[\frac{\cosh[Ki(w') - Ki(x)]}{\sinh[Ki(w')]}\right]. \quad \text{(A.1)}$$

Since base current is constant while base-emitter voltage can

$$\frac{dJ_n(w)}{J_n(w)} = \frac{J_{po}[\sinh[Ki(w)] - \sinh[Ki(w')]] + qC_s[\cosh[Ki(w)] - \cosh[Ki(w')]]}{J_{po}\sinh[Ki(w')] + qC_s[\cosh[Ki(w')] - 1]} \quad \text{(A.4)}$$

vary, we can write:

$$e^{V'_{be}/V_t} = e^{V_{be}/V_t} \cdot \frac{J_{po} + qC_s \frac{\cosh[Ki(w)]-1}{\sinh[Ki(w)]}}{J_{po} + qC_s \frac{\cosh[Ki(w')]-1}{\sinh[Ki(w')]}}. \qquad (A.2)$$

Now we can find the ratio of $J'_n(w')$ to $J_n(w)$ using (A.1) and (A.2):

$$\frac{J'_n(w')}{J_n(w)} = \frac{J_{po}\sinh[Ki(w)] + qC_s(\cosh[Ki(w)] - 1)}{J_{po}\sinh[Ki(w')] + qC_s(\cosh[Ki(w')] - 1)}. \qquad (A.3)$$

Having calculated this ratio, we can find $\frac{dJ_n(w)}{J_n(w)}$ as (see bottom of previous page). If dw is infinitesimal, then the following equations hold:

$$\sinh[Ki(w)] - \sinh[Ki(w')] = dKi \cdot \cosh[Ki(w')]$$
$$\cosh[Ki(w)] - \cosh[Ki(w')] = dKi \cdot \sinh[Ki(w')] \qquad (A.5)$$

where

$$dKi(w) = \frac{C_s N_A(w) dw}{D_n(w) n_{ie}^2(w)}. \qquad (A.6)$$

Now Early voltage can be calculated as follows:

$$V_A = dV_{cb} \cdot \frac{J_n}{dJ_n} \qquad (A.7)$$

where

$$dV_{cb} = dw \frac{qN_A(w)}{C_{bc}}. \qquad (A.8)$$

Combining (A.4) and (A.8), we obtain (11) for the Early voltage.

In order to prove (12), we express the current gain β as the ratio between the collector current and base current. In the presence of recombination, base current composed of injection and recombination components. Thus β can be written as

$$\beta = \frac{J_n(w)}{J_{po} e^{V_{be}/V_t} + J_n(0) - J_n(w)} = \frac{qC_s \frac{1}{\sinh[Ki(w)]}}{J_{po} + qC_s \frac{\cosh[Ki(w)]-1}{\sinh[Ki(w)]}} \qquad (A.9)$$

which leads to (12).

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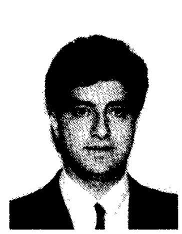

Saeed Mohammadi (M'90–S'92) received the B.S. degree in electrical engineering from Iran University of Science and Technology, Tehran, Iran, in 1989, and the M.Sc. degree in electrical and computer engineering from the University of Waterloo, Waterloo, Ontario, Canada, in 1994.

His research interests include the physics, modeling, and design of heterostructure devices.

C. R. Selvakumar (S'80–M'86) was born in Karur, Tamil Nadu, India, on March 17, 1950. He received the B.E. in electrical engineering from the College of Engineering, Guindy, Madras, India, in 1972, and the M.Tech. and Ph.D. degrees in electrical engineering from the Indian Institute of Technology (Bombay and Madras), in 1974 and 1985, respectively. His Ph.D. dissertation was based on a new analytical approach to solve semiconductor device equations in heavily and nonuniformly doped regions.

From 1975 to 1978, he was a Research Scholar at IIT Madras, where he was partly responsible for setting up the Microelectronics Research Laboratory. From 1978 to 1984 he was a Project Associate at IIT Madras, where he was responsible for the development of ultrasensitive photo p-i-n diodes for UV, visible, and IR radiation, as well as lifetime control using platinum for high-speed switching devices. Since 1985, he has been with the University of Waterloo, first as a Postdoctoral Fellow in the Electrical Engineering Department, and later as an Associate Professor in the Department of Electrical and Computer Engineering. In early 1992 he was a Visiting Faculty Member of Stanford University's Department of Electrical Engineering. In 1989 he contributed an invited review article of polysilicon bipolar transistors (published in *Polysilicon Emitter Bipolar Transistors*, A. Kapoor and D. J. Roulston, Eds., IEEE Press). His current research interests are in bipolar transistors, SiGe channel MOSFET's and HBT's, vertical polysilicon emitter bipolar transistors on SOI and novel group IV binary and ternary alloy devices, and quantum effect devices.

Dr. Selvakumar is serving as a Guest Editor for the upcoming *IEEE Transactions on Electron Devices* special issue on Submicron Bipolar /BiCMOS Devices and Technologies. He was a member of the Technical Program Committee of IEEE Bipolar Circuits and Technology Meeting and is the Technical Program Chairperson for 1994. He is a Chairman of the Kitchener–Waterloo IEEE Section Electron Devices, Circuits and Systems Chapter. He has been an associate Key Researcher of the Provincial Centre of Excellence (ITRC) since 1991, and an Associate Researcher for MICRONET. He is a member of MRS.