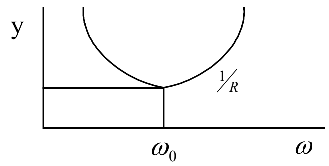
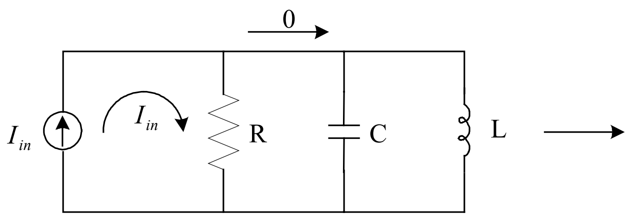
<u>RLC Networks</u>

Parallel RLC Tank





$$y = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

at resonance

Resonance → when capacitive and inductive admittances cancel

$$\omega_0 C - \frac{1}{\omega_0 L} = 0 \longrightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$V = I_{in} R$

at resonance:

$$I_{cap} = \frac{V}{Z_{cap}} = \frac{I_{in}R}{\frac{1}{j\omega_0 C}} = j\omega_0 C I_{in} R$$

$$I_{ind} = \frac{V}{Z_{ind}} = \frac{I_{in}R}{j\omega_0 L} = -j\omega_0 C I_{in} R$$

currents in inductor/capacitor branches can be very <u>high</u> but they cancel each other

how high ?

→ That depends on quality factor Q ?!

$$\left|I_{Cap}\right| = \left|I_{ind}\right| = \left|\frac{I_{in}R}{j\omega_0 L}\right| = \left|I_{in}R \cdot j\omega_0 C\right| = I_{in} \cdot \frac{R}{\sqrt{L/C}} = I_{in} Q$$

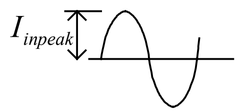
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = quality\ \ factor = \frac{power\ \ stored}{power\ \ dissipated} \qquad \boxed{Q = \frac{R}{\sqrt{L/C}}}$$

This is a general definition and applies to both <u>distributed</u> and <u>lumped</u> circuits

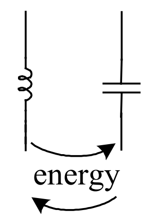
Power stored: for parallel RLC it is easy to find the stored energy in the capacitor at resonance!

$$at \quad \omega_0 \rightarrow V = RI_{inpeak}$$



$$E_{cap} = \frac{1}{2} C V_{peak}^2 = \frac{1}{2} C \left(RI_{inpeak} \right)^2$$

Resonance



$$P_{cap} = \omega_0 E_{cap} = \frac{R^2 C \omega_0 I_{inpeak}{}^2}{2}$$

Maximum energy
in capacitor

$$P_{diss} = \frac{1}{2} R I_{inpeak}^2$$

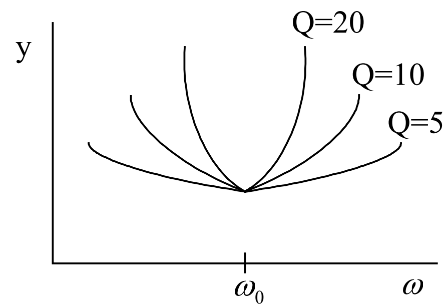
$$Q = \frac{P_{cap}}{P_{diss}} = \frac{\frac{1}{2} R^2 C \omega_0 I_{inpeak}^2}{\frac{1}{2} R I_{inpeak}^2} = RC\omega_0$$

$$Q = RC\omega_0 = \frac{R}{L\omega_0} = \frac{R}{\sqrt{L/C}}$$

remember → This definition of Q is only valid at resonance!

away from resonance
we still use this formula but it is not accurate!

<u>Relation between bandwidth and Q</u>



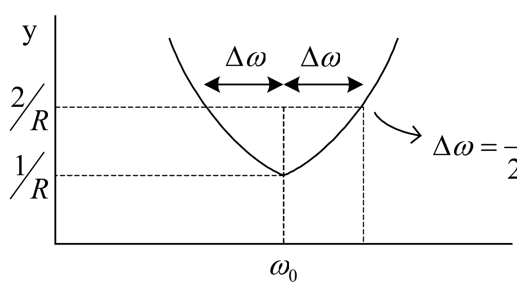
$$\text{assume} \quad \omega = \omega_0 + \Delta\omega$$

$$\left.\begin{aligned} \omega^2 &= \omega_0^{\ 2} + \Delta\omega^2 + 2\Delta\omega\omega_0 \\ LC &= \frac{1}{\omega_0^2} \end{aligned}\right\} \longrightarrow \omega^2 LC = 1 + \left(\frac{\Delta\omega}{\omega_0}\right)^2 + 2\frac{\Delta\omega}{\omega_0}$$

$$\begin{aligned} y &= \frac{1}{R} + j\omega C + \frac{1}{j\omega L} \\ &= \frac{1}{R} + \left(\frac{j}{\omega L}\right)\left(\omega^2 LC - 1\right) \end{aligned}$$

$$\begin{aligned} y &= \frac{1}{R} + \frac{j}{\omega L}\left(\left(\frac{\Delta\omega}{\omega_0}\right)^2 + 2\frac{\Delta\omega}{\omega_0}\right) \\ &\approx \frac{1}{R} + \frac{j}{\omega L}\cdot\frac{2\Delta\omega}{\omega_0} = \frac{1}{R} + 2jC\Delta\omega \end{aligned}$$

$$\omega\omega_0 = \frac{1}{LC}$$



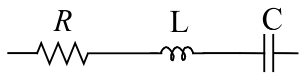
$$\Delta\omega = \frac{1}{2RC}$$

$$BW = \frac{1}{RC} \longrightarrow \text{or -3 dB bandwidth}$$

$$\frac{BW}{\omega_0} = \frac{1}{RC\omega_0} = \frac{1}{Q}$$

→ So high Q → sharper peak

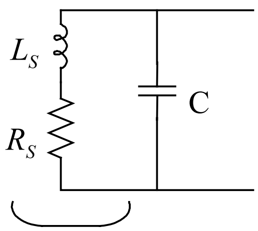
Series RLC



$$Z_{\min} = Z_{resonance} = R \longrightarrow \quad \text{L \& C cancel}$$

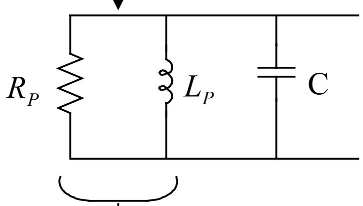
$$Q_{resonance} = \frac{L\omega_0}{R} = \frac{1}{RC\omega_0} = \frac{\sqrt{L/C}}{R}$$

to analyze



$$\longleftarrow \quad Q = \frac{\omega_0 L_S}{R_S} = \frac{R_P}{\omega_0 L_P}$$

transform series to parallel

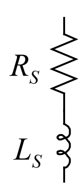


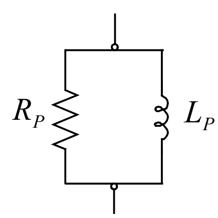
$$j\omega_0 L_S + R_S = j\omega_0 L_P \| R_P = \frac{j\omega_0 L_P R_P}{j\omega_0 L_P + R_P} = \frac{(\omega_0 L_P)^2 R_P + j\omega_0 L_P R_P{}^2}{R_P{}^2 + \omega_0{}^2 L_P{}^2}$$

valid at ~ resonance

$$R_P = R_S\left(Q^2 + 1\right)$$

$$L_P = L_S\left(\frac{Q^2 + 1}{Q^2}\right)$$

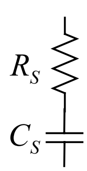


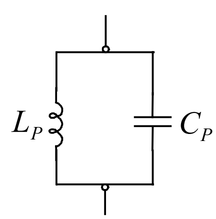


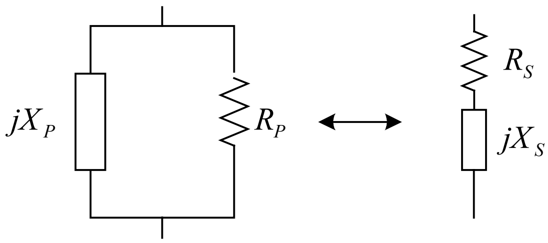
similarly

$$R_P = R_S\left(Q^2 + 1\right)$$

$$C_P = C_S\left(\frac{Q^2}{Q^2 + 1}\right)$$





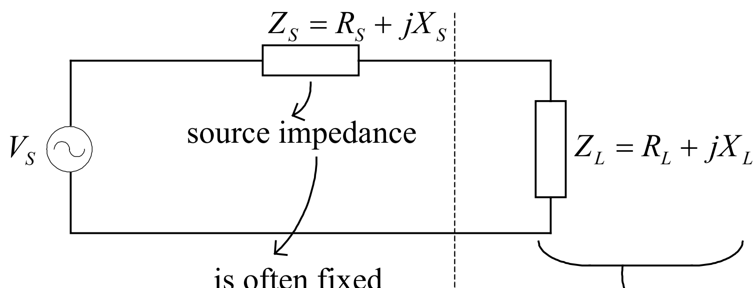


$$R_P = R_S\left(Q^2+1\right)$$

$$X_P = X_S\left(\frac{Q^2+1}{Q^2}\right)$$

$Q \gg 1 \quad \rightarrow \quad \left.\begin{array}{l} R_P = Q^2 R_S \\ X_P = X_S \end{array}\right\}$ only valid for frequencies around resonance

* at high frequency <u>Power Gain</u> is important because active devices have limited gain so maximum power transform is an important issue



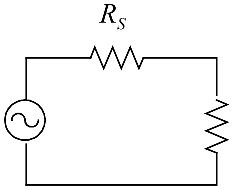
load impedance?
To maximize power transfer to R_L

Power delivered to load is entirely due to R_L

$$P_{R_L} = \frac{\left|V_{R_L}\right|^2}{R_L} = \frac{R_L\left|V_S\right|^2}{\left(R_L+R_S\right)^2+\left(X_L+X_S\right)^2}$$

P_{R_L} Maximize when $\left\{\begin{array}{l} X_L + X_S = 0 \\ R_L = R_S \end{array}\right. \rightarrow \left\{\begin{array}{l} X_L = -X_S \\ R_L = R_S \end{array}\right. \rightarrow$ maximum power transfer complex conjugate

complex conjugate match



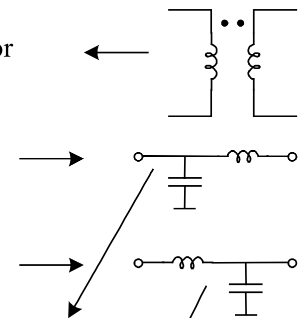
$R_L = R_S \longrightarrow$ only 1/2 power is delivered to the load <u>at best</u>!

Impedance transformation

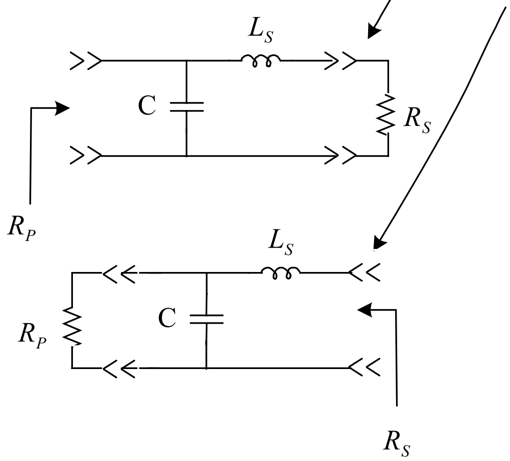
RF transformers are difficult to realize for turned circuits we use

↓

* L-Match



in IC



we do matching
* for transistors +
* to maximize
power transfer ratio
* to minimize noise

$Q \rangle\rangle 1 \quad (Q \geq 3 \sim 4) \quad \rightarrow \quad R_P \approx R_S Q^2$ around resonance

10% error

$$Q = \frac{1}{\omega_0 R_S C} = \frac{1}{R}\sqrt{\frac{L_S}{C}}$$

↓

$$R_P = R_S \times \frac{L_S}{C}$$

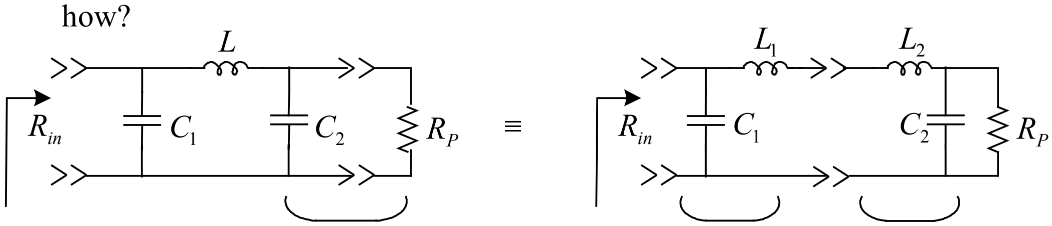
↓

impedance transformation

Problem with L-match:
impedance transformation is a function of Q
$\rightarrow Q = 10 \rightarrow R_P = 100 R_S$

$*\pi$-Match $\longrightarrow$ decouples transformation ratio from Q

how?

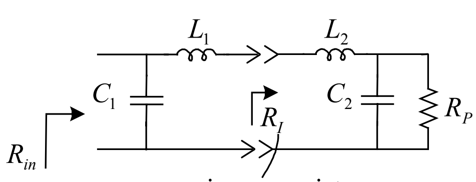


often the input impedance of transistor

L-match — brings up the resistance

L-match — brings down the resistance

in this process decouple impedance transformation from Q



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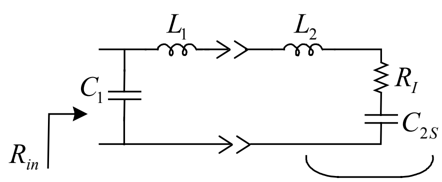
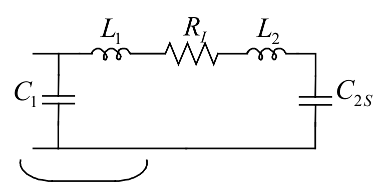


$$Q_{right} = \frac{\omega_0 L_2}{R_I} = \sqrt{\frac{R_P}{R_I} - 1}$$



$$Q_{left} = \frac{\omega_0 L_1}{R_I} = \sqrt{\frac{R_{in}}{R_I} - 1}$$

overal Q $\rightarrow$ $Q = \dfrac{\omega_0 (L_1 + L_2)}{R_I}$

$$Q = Q_{left} + Q_{right} = \sqrt{\frac{R_{in}}{R_I} - 1} + \sqrt{\frac{R_P}{R_I} - 1} \longrightarrow \text{requires iteration}$$

$1st$ *order approximation* $\quad R_I = \dfrac{\left(\sqrt{R_{in}} + \sqrt{R_P}\right)^2}{Q^2}$

from Q and transformation ratio $\left(\dfrac{R_{in}}{R_P}\right)$ *calculate* R_I

then

$$\boxed{\begin{aligned} L &= L_1 + L_2 = \frac{QR_I}{\omega_0} \\ C_1 &= \frac{Q_{left}}{\omega_0 R_{in}} = \frac{\sqrt{\frac{R_{in}}{R_I} - 1}}{\omega_0 R_{in}} \\ C_2 &= \frac{Q_{right}}{\omega_0 R_P} = \frac{\sqrt{\frac{R_P}{R_I} - 1}}{\omega_0 R_P} \end{aligned}}$$

(*example*) *need transformation from* $200\,\Omega \rightarrow 50\,\Omega$ (*ratio* $= 4$)

with $Q \sim 10$ *at* $1.6\,GHz$

$$\left(Q = \frac{\omega_0}{\omega_{-3dB}}\right)$$

$$R_I \sim \frac{9 \times 50}{100} = 4.5\Omega$$

$$\omega_0 = 10G\, rad/\sec$$

$$L = \frac{QR_I}{\omega_0} = \frac{10 \times 4.5}{10 \times 10^9} = 4.5\,nH$$

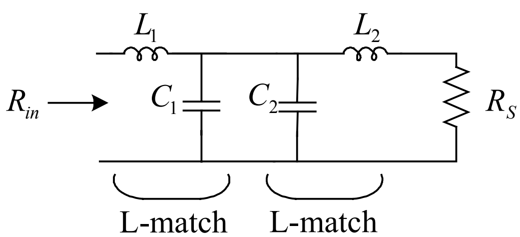
$$C_1 = \frac{\sqrt{\frac{R_{in}}{R_I} - 1}}{\omega_0 R_{in}} = \frac{\sqrt{50/4.5 - 1}}{10 \times 50 \times 10^9} = \frac{3.2}{5} \times 10^{-11} = 6.4\,pF$$

$$C_2 = \frac{\sqrt{200/4.5 - 1}}{500 \times 10^9} = \frac{6.5}{5} \times 10^{-11} = 13\,pF$$

if transistor gate cap $\approx 3\,pF$

$\rightarrow 4.5\,nH$

* T-Match



$$Q = \omega_0 R_I (C_1 + C_2) = \underbrace{\sqrt{\frac{R_I}{R_{in}} - 1}}_{Q_{left}} + \underbrace{\sqrt{\frac{R_I}{R_S} - 1}}_{Q_{right}}$$

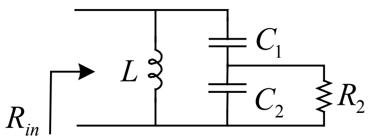
$$C_1 + C_2 = \frac{Q}{\omega_0 R_I}$$

$$\left.\begin{aligned} L_1 &= Q_{left} \frac{R_{in}}{\omega_0} \\ L_2 &= Q_{right} \frac{R_S}{\omega_0} \end{aligned}\right\} \longrightarrow$$

useful when the source and termination are inductive

Tapped Capacitor Resonance

⟶ You can set center frequency, Q and transformation ratio independently



$R_2 < R_{in} \quad \leftrightarrow \quad$ *we have a voltage divider*

often used in oscillators
tapped point does not degrade
the Q of the resonator

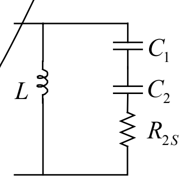
$$\frac{R_2}{R_{in}} \approx \left(\frac{C_1}{C_1 + C_2}\right)^2 = \frac{1}{n^2}$$

design methodology: ← more accurate analysis

$$Q = \frac{\omega_0}{\omega_{-3dB}}$$

span of frequency over which the impedance transformation ratio remains the same

$$Q = \frac{R_{in}}{\omega_0 L}$$



transfomaing R2
transfomaing Rin

$$R_{2S} = \frac{R_2}{Q_2^2 + 1} = \frac{R_{in}}{Q^2 + 1}$$

$$C_{2S} = C^2 \frac{Q_2^2 + 1}{Q_2^2}$$

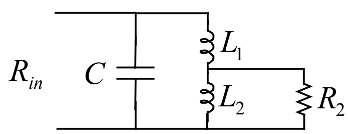
⟶ Q_2 as a function of Q

$$Q_2 = \sqrt{\frac{R_2}{R_{in}}\left(Q^2 + 1\right) - 1}$$

$$Q_2 = \omega_0 R_2 C_2$$

$$L = \frac{R_{in}}{\omega_0 Q} \quad , \quad C_2 = \frac{Q_2}{\omega_0 R_2} = \frac{\sqrt{\frac{R_2}{R_{in}}\left(Q^2 + 1\right) - 1}}{\omega_0 R_2} \quad , \quad C_1 = \frac{C_2\left(Q_2^2 + 1\right)}{Q \cdot Q_2 - Q_2^2}$$

Similarly tapped inductor resonator



$$Q = \frac{\omega_0}{\omega_{-3dB}} \quad , \quad Q_2 = \frac{R_2}{\omega_0 L_2}$$

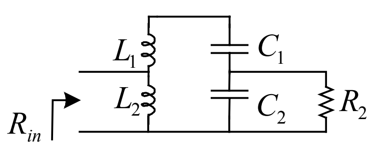
$$C = \frac{Q}{\omega_0 R_{in}}$$

$$L_2 = \frac{R_2}{\omega_0 \sqrt{\frac{R_2}{R_{in}}\left(Q^2 + 1\right) - 1}}$$

$$L_1 = L_2 \frac{[QQ_2 - Q_2{}^2]}{Q_2{}^2 + 1}$$

Double tapped resonator

→ gives additional degrees of freedom



you can set center frequency,
transformation ratio, Q, and
total inductance (or capacitance)

R_2 transforms to a higher resistance than usual then transferred to a lower value R_{in}

as a results $\Rightarrow \left.\begin{matrix} L \uparrow \\ C \downarrow \end{matrix}\right\} \longrightarrow$ both values now closer to realizable values

go through example 4.6 to understand the impedance transformation better
We'll have some homework

Distributed Systems

EM signals (like light) travel at a speed of

$$c = \frac{1}{\sqrt{\mu\varepsilon}} \qquad\qquad c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \leftarrow \text{speed of light in vaccum}$$

permittivity of the media

permeability of the media

$$c = \lambda \cdot f \qquad\qquad if \;\; \ell \;\; \langle\langle \;\; \lambda$$

$$\downarrow$$

lumped element

$$\downarrow$$

wave length of the signal

KVL / KCL hold

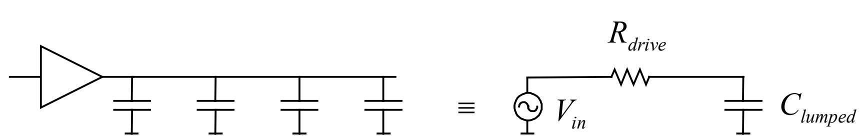
$\rightarrow$ *when* $\ell \sim \lambda$ *we need to consider distributed effect*

$$\downarrow$$

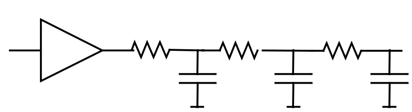
Transmission line effect

* Interconnect models

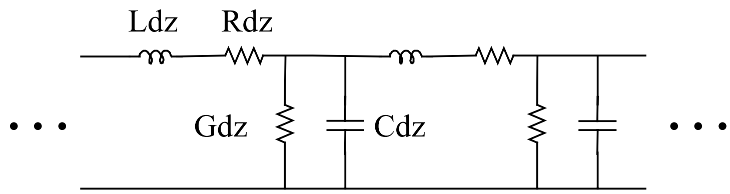
* lumped capacitive



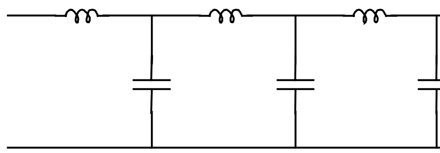
* lumped RC model



* Transmission Line (TRL) model

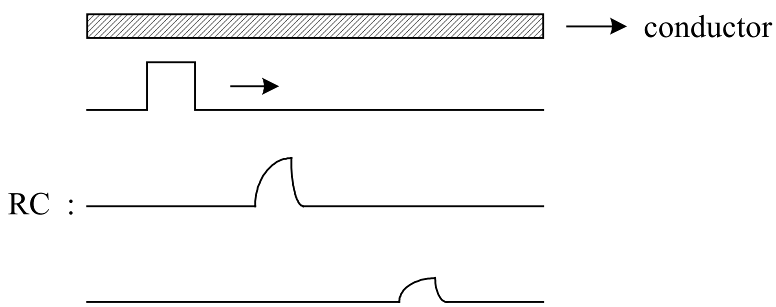


lossy line

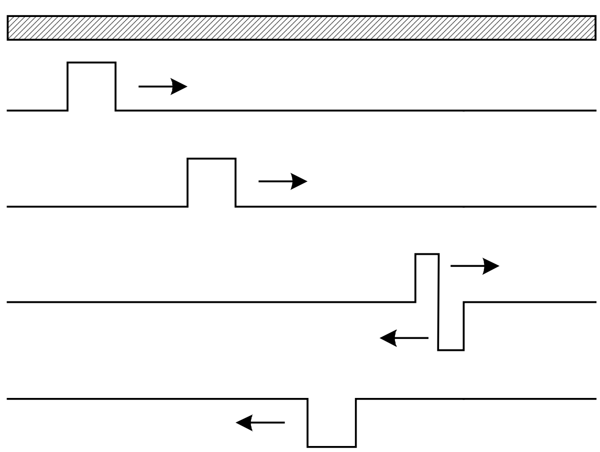


loss-less line

in RC model signal diffuses as it moves



in TRL models signal propagates like a wave



in loss less transmission line model

wave equation → $\dfrac{\partial^2 v}{\partial z^2} = \ell c \dfrac{\partial^2 v}{\partial t^2}$

unit inductance (H/m) unit capacitance (F/m)

$\longrightarrow \quad \ell c = \dfrac{1}{v^2} \quad \longrightarrow \quad v = \dfrac{1}{\sqrt{\ell c}} = \dfrac{1}{\sqrt{\mu \varepsilon}} = \dfrac{c_0}{\sqrt{\mu_0 \varepsilon_0}}$

speed of light

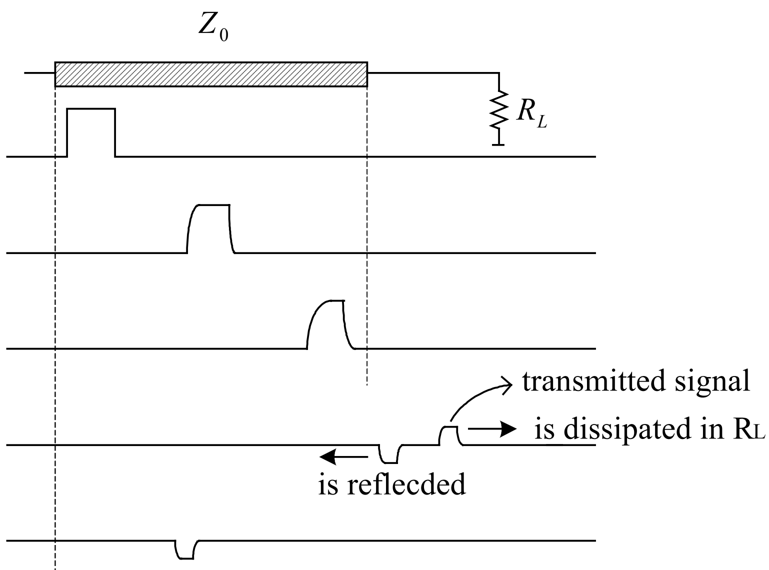
≈ 1 average dielectric constant

propagation delay : $T_d = \sqrt{\ell c}$

Impedance of the line $Z_0 = \dfrac{V}{I} = \sqrt{\dfrac{\ell}{c}}$

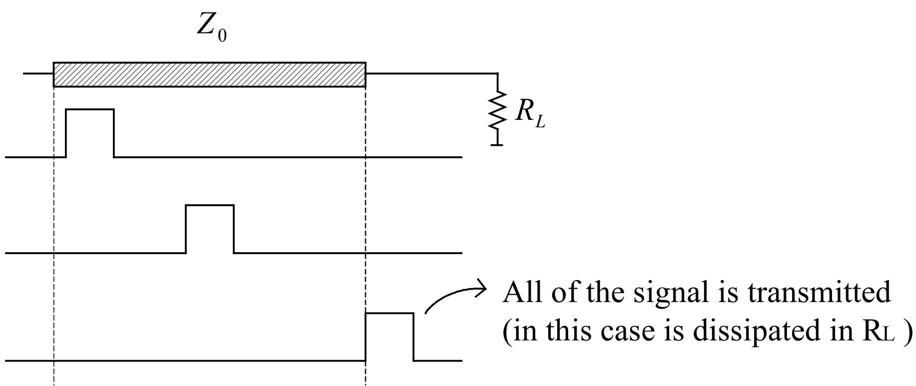
only true for loss-less or close to loss-less lines

Signal propagates like a wave → what does it mean?



Sometimes you don't get reflection

→ condition for no reflection : $Z_0 = R_L$

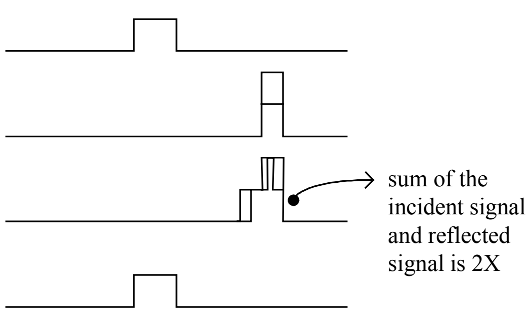


Amount of reflection is determined by reflection coefficient

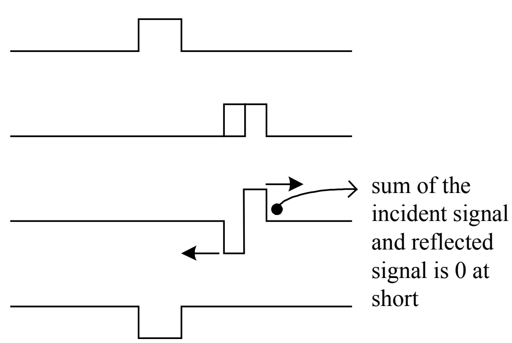
$$\rho = \frac{V_{ref}}{V_{inc}} = \frac{R - Z_0}{R + Z_0}$$

1. matched $R = Z_0 \rightarrow \rho = 0$ no reflection
2. open end $R = \infty \rightarrow \rho = 1$ all the signal is reflected without phase dhange
3. short end $R = 0 \rightarrow \rho = -1$ all the signal is reflected with 180 phase change

2. open end

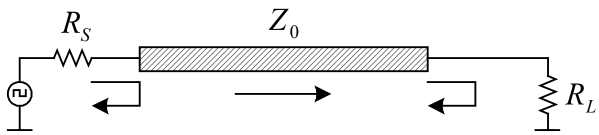


3. short end



$V_{@termination}=V_{inc}(1+P)$

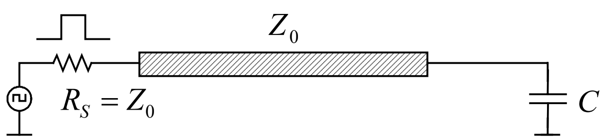
we also have reflection @ source



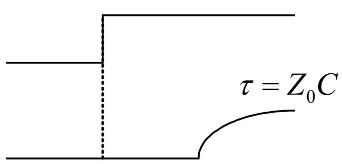
To have faster settling time → no reflection is desired ($R_S=R_L=Z_0$)

otherwise it takes a few propagation for the signal to reach steady-state

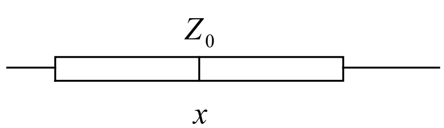
<u>* But in CMOS loads are usually capacitive</u>

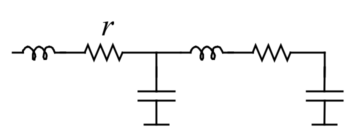


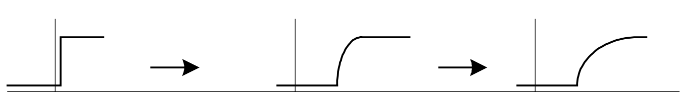
→ will be charged with
a time constant of $\tau = Z_0 C$



<u>Lossy transmission line</u>







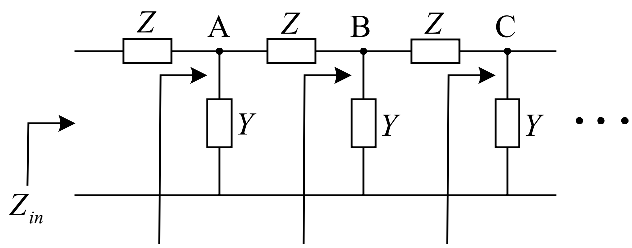
→ like distributed
RC line

$$\frac{V(x)}{V(0)} = e^{-\frac{r}{2Z_0}x}$$

Lossy line

when $R < 5Z_0$ R : resistance of interconnect
Z_0 : char. impedance

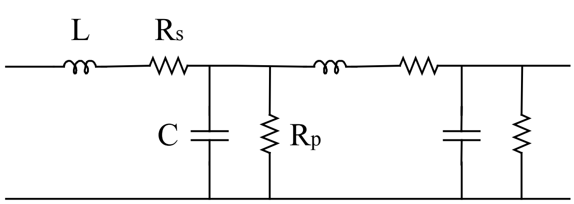
Then use loss-less TRL model
otherwise use RC model - lossy transmission model?



$$Z_{in} = Z_A = Z_B = Z_C \quad \rightarrow \quad Z_{in} = Z + \frac{Z_{in}/Y}{1/Y + Z_{in}}$$

$$solve\ for\ Z_{in} \quad \rightarrow \quad Z_{in} = \frac{Z}{2}\left[1 \pm \sqrt{1 + \frac{4}{ZY}}\right]$$

$$when\ |YZ| << 1 \quad \rightarrow \quad Z_{in} \approx \sqrt{\frac{Z}{Y}}$$



L : represents the magnetic energy stored around the wire

R_s : represents the electric energy stored in dielectric

R_p : represents the loss (ohmic) in the wire

C : represents the loss (leakage + dielectric) in the dielectric media for lossy transmission line

$$Z_{in} = Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad \rightarrow \textit{ not a real value}$$

$$\left.\begin{array}{ll} \text{when R, G <<} & \text{(negligible loss)} \\ \text{or} \quad \dfrac{R}{G} = \dfrac{L}{C} & \rightarrow \quad RC = GL \end{array}\right\} \longrightarrow Z_0 = real$$

$$in\ \ these\ case \ : \quad Z_0 = \sqrt{\frac{L}{C}}$$

Propagation constant:

write the wave equation ⟶

$$V(z) = V_0 e^{-\sqrt{YZ}z}$$

$\sqrt{YZ}$ → dimension; Impedance x admittance

$$\gamma = \sqrt{YZ} = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

$$V(z) = V_0 \underbrace{e^{-\alpha z}}_{\text{attenuation}} \underbrace{e^{-j\beta z}}_{\text{phase term}}$$

$$\alpha \approx \frac{R}{2}\sqrt{\frac{C}{2}} + \frac{6}{2}\sqrt{\frac{L}{C}} \qquad \text{attenuation factor}$$

$$\approx \frac{R}{2Z_0} + \frac{GZ_0}{2}$$

attenuation per length is small as long as resistance/conductance per length are small with respect to Z_0/Y_0

$\beta = \omega\sqrt{LC}$

$$T_{delay} = -\frac{\partial\phi}{\partial\omega} = -\frac{\partial(-\beta z)}{\partial\omega} = \sqrt{LC}\,z$$

delay of the loss less line is constant and independent of frequency and depends on the length of the line

$\alpha = 0 \quad in\ \ loss\ \ less\ \ line$

→ signal does not attenuate

→ loss-less line
- * no bandwidth limit as signal does not attenuate
- * delay not a function of f, only a function of z and geometry

phase shape will be preserved in loss less TRL

as
- * no attenuation
- * different frequency components of the pulse see exactly the same delay

↓

no dispersion of the signal

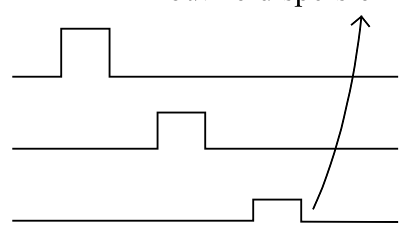
even lossy lines can be dispersion-less

by choosing $RC = GL \quad \rightarrow \quad Z_0 = real$

attenuation exist but no dispersion

$\alpha = \sqrt{RG}$

$\beta = \omega\sqrt{LC} \quad \rightarrow \quad same\ \ as\ \ loss\ \ less$



we talked about reflection

$$\rho = \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

at any arbitrary point along the line

$$\Gamma(z) = \Gamma_L e^{2\gamma z} \quad \rightarrow \quad \begin{array}{ll} z = 0 & \textit{at the load} \\ z = -l & \textit{at the source} \end{array}$$

$$\frac{Z(z)}{Z_0} = \frac{Z_L \cos\beta z - jZ_0 \sin\beta z}{Z_0 \cos\beta z - jZ_L \sin\beta z} \longrightarrow$$ transmission line also does impedance transformation

we use Smith chart to understand the impedance transformation of TRL

Spice model for lines

* Distributed RC	RPERL

  CPERL

  L

* loss less TRL

  Z0 → characteristic impedance

  TD → transmission delay

  NL → electrical length of the line $\left(L/\lambda\right)$

$$NL = \frac{L}{\lambda} = fTD \longleftarrow \lambda = \frac{v}{f} \; , \; TD = \frac{L}{v}$$