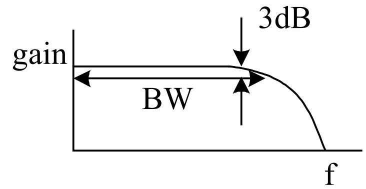
Amplifiers

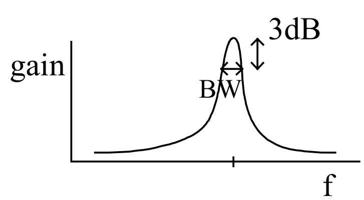
- Gain
- BW
- Noise
- Stability
- Linearity

…

High-frequency Linear Amplifier

High-freq linear amps
- widebandwidth Amps
- Tuned Amps (Narrow band)





* Characteristics of wideband Amps

- useful for * opto-electronic receivers → constant gain vs frequency
  - * instrumentation amplifiers
    - network analyzer / oscilloscope: input → constant gain +constant delay vs frequency → linear phase vs frequency → no phase distortion

- low efficiency
  (high power supply consumption)

- $gain \times Bandwidth = f_T$

  Therefore

  $\boxed{\text{low gain}}$ True only for first order systems (one pole)

- high input noise power $\longrightarrow$ $\overline{v_n^2} = 4kTR\,\Delta f \quad \left(\overline{v_n^2} \propto \Delta f\right)$

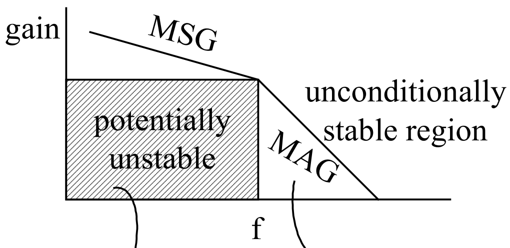
* Characteristics of narrowband Amps

- high gain $\Leftarrow$ $\left(gain \times bandwidth = f_T\right)$
- high efficiency (lower power consumption)
- useful for low noise mobile receivers

good for mobile application

$\longrightarrow$ Stability is of concern in both wideband and turned Amps, but more importantly in wideband Amps

$\boxed{\text{typical RF transistor}}$



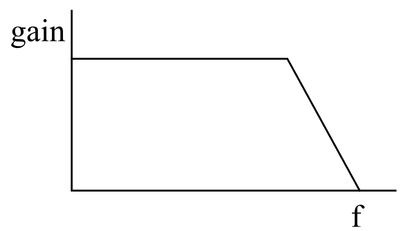
So for a good RF transistor we always have too much gain at very low-frequency + feedback through C_{gd} is always there

transistor can oscillate without any external positive feedback

you need positive feedback to make the transistor oscillate

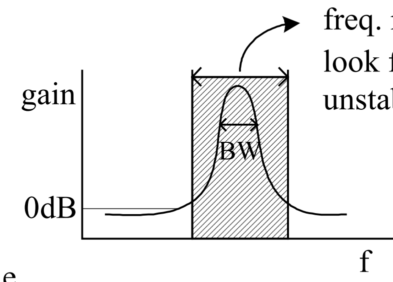
(how?) ⟶ Internal feedback through C_{gd}+ high gain ⟶ oscillator

* So for widebandwidth Amps since we have gain at lower frequency, the chance of instability at lower frequency is higher



* for tuned Amps, we generally do not have high gain at lower frequencies

as far as the amplifier is stable at around the Bandwidth we are fine



freq. range to look for unstable condition

- How do we design / analyze linear amps.

  * let's say you have an RF transistor and you want to design a linear amp with this transistor. How to proceed?

1st measure transistor for its RF performance

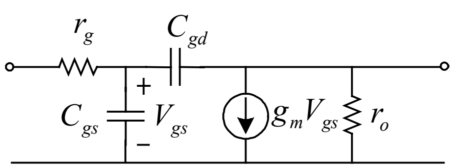
(S-parm) ⟵ use network analyzer

f	S_{11}	S_{12}	S_{21}	S_{22}
$0.1GHz$	•	•	•	•
1	•	•	•	•
2	•	•	•	•
3	•	•	•	•

Try to model the transistor

Design using S-parm of the transistor

Small-Signal Model



* only need S-parm measurement + insight to the physics of device for initial values

Large-Signal Model

-Spice model

-BSIM model

need DC $\begin{cases} V_g - I_d \\ I_d - V_{ds} \end{cases}$

+ RF ⟶ $\begin{cases} C-V \\ S-parm \end{cases}$

<u>+insight to device physics</u>

older technique that measures Caps vs. supplied bias

need S-parm for different biases

So large-signal modeling is a difficult task often your transistor supplier provides that

<u>For integrated circuit technologies</u>

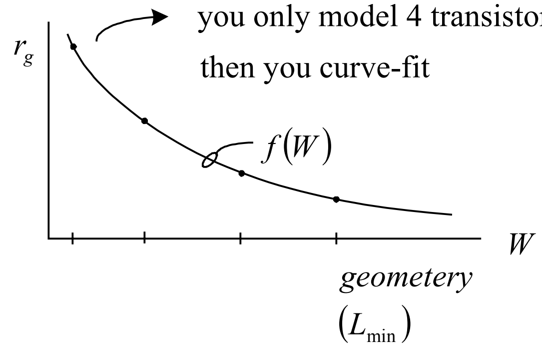
* you cannot possibly measure every given geometery

you measure certain geometery (RF +DC)

generate a scalable large-signal model
that fits all possible geometeries

for instant
your model r_g
for different
geometery transistors
that you measured

you only model 4 transistors
then you curve-fit



- Scalable models are not very accurate, gain may be off by a couple of dB
  but you cannot afford not knowing the stability situation

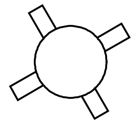
that is why fab foundries often supply
S-parm data along with the model

→ But raw S-parm data is useless
when you deal with an integrated transistor

For packaged transistor

you can measure S-parm of

inherent transistor + package

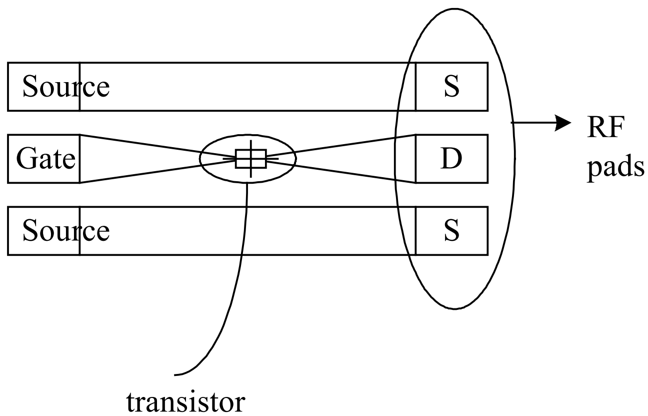


you can use the same S-parm for your ckt design + stability analysis

For integrated transistor

you measure

S-parm of this structure



but in ckt design

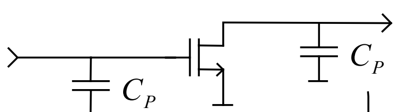
you only use the transistor without its pads

So raw S-parm is useless

you need to <u>de-embed</u> the effect of pads

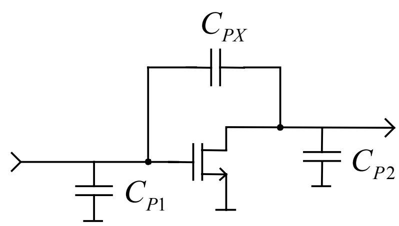
1st degree approximation

Pads are only capacitive

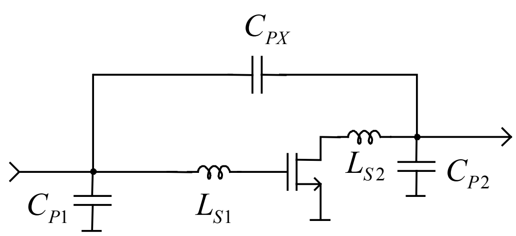


typically the same
since there is often
a symmetry in your pad design

But such large pads may induce capacitive coupling



may also have inductive term especially as interconnects become narrower as you get closer to the transistor



Therefore you need to put embedding structues and measure them so that you can model the effect of pads + interconnects

Summary : to design RF ckts

- measure S-parm of transistor + pads

↓

- de-embed the effect of pads

↓

- find the S-parm of the transistor ← construct small-signal model

↓

- repeat for different bias

↓

- measure DC

↓

- construct large-signal model

Design / stability analysis using

S-parm
of transistor

small-signal
model

large-signal
model

for DC analysis
you still need
a DC model

1st order model
(ideal MOS equation)
→ do hand calculation

higher order model

computer simulation

useful
for hand
calculation

one analysis
for all
the frequencies

you find V_{ds} and I_d of each transistor

make sure the small-signal / S-parm
is extracted under this bias

you are using

using large-signal model you can design
DC and RF performance at the same time

but you can only use CAD software

you can do hand calculation for one or two frequencies,

but to analyze over a range of frequencies
you simulate using a CAD software

→ <u>if you do hand-calculation</u>

- for DC analysis use 1st order DC models (ideal MOS equations)
- for RF analysis use small-signal model find gain / bandwidth etc.
- for stability analysis use S-parms as they are more accurate

→ <u>if you do hand-calculation for only one or two frequencies</u>

small-signal model S-parms

→ <u>if you do computer simulation</u>

- use large-signal model for DC+RF+stability
- use S-parm tables to do stability analysis provided by foundry

(RF analysis)

This is basically a step to make sure your model is not too off

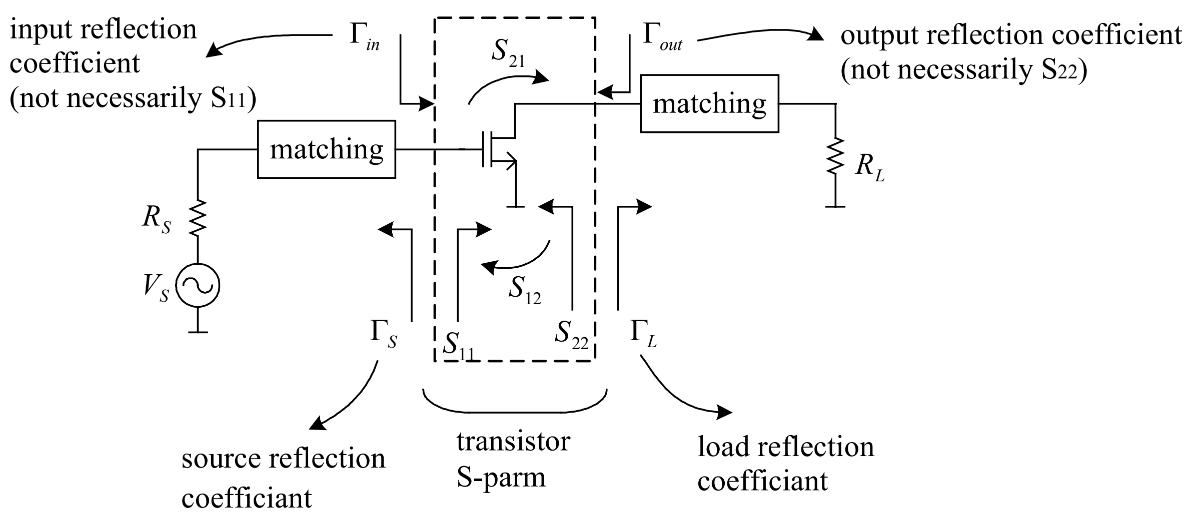
Provided that you have S-parm for that particular size transistor for that particular bias let's start with S-parm analysis

<u>1st.</u> define various gains

$$Transducer\ Gain = G_T = \frac{P_L}{P_{AVS}} = \frac{Power\ delivered\ to\ the\ load}{Power\ available\ from\ the\ source}$$

$$Power\ Gain = G_P = \frac{P_L}{P_{IN}} = \frac{Power\ delivered\ to\ the\ load}{Power\ input\ to\ the\ transistor}$$

$$Available\ Gain = G_A = \frac{P_{AVN}}{P_{AVS}} = \frac{Power\ available\ from\ the\ network}{Power\ available\ from\ the\ source}$$



$$\boxed{\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}} \longrightarrow$$

when you do not put an output matching network

if transistor is matched at output

then $\Gamma_{in} = S_{11}$

$Z_L = 50\Omega \Rightarrow \Gamma_L = 0$

$$\boxed{\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}} \longrightarrow$$

if transistor is matched at input

then $\Gamma_{out} = S_{22}$

$Z_S = 50\Omega \Rightarrow \Gamma_S = 0$

when you do not put an input matching network

you can write different gains in terms of S-parm + Γ_L , Γ_S

Transducer Gain

$$G_T = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2}$$

That means

$$|1-\Gamma_{in}\Gamma_S|^2|1-S_{22}\Gamma_L|^2 = |1-S_{11}\Gamma_S|^2|1-\Gamma_{out}\Gamma_L|^2$$

substitute Γ_{in} and Γ_{out}, you can prove that this is correct

transducer gain is basically whatever power gain you have considering the fact that you could not perfectly match at input and output

Power Gain

$$G_P = \frac{1}{1-|\Gamma_{in}|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

G_P is not a function of Γ_S

for power gain you do not care about input matching network

you are looking at the input power to the transistor

That is why G_P is not related to Γ_S

Available Gain

$$G_A = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} |S_{21}|^2 \frac{1}{1-|\Gamma_{out}|^2}$$

G_A is not a function of Γ_L

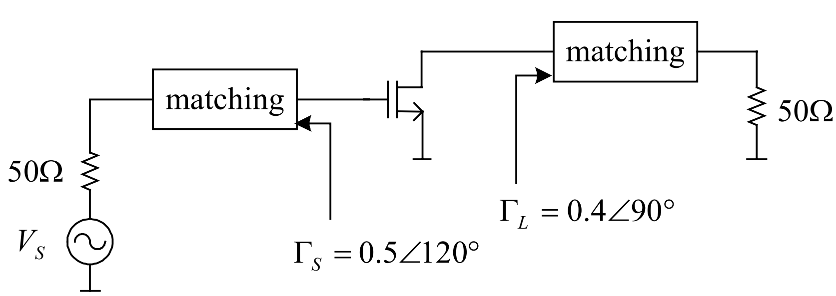
for available gain, you do not care about output matching network

you consider only the output power from the transistor

That is why G_A is not related to Γ_L

Example

For a 2.4 GHz MOS amplifier



Calculate different gain expressions assuming

$$S_{11} = 0.6\angle -160^\circ \qquad \Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.627\angle -164^\circ$$

$$S_{12} = 0.045\angle 16^\circ$$

$$S_{21} = 2.5\angle 30^\circ \qquad \Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} = 0.471\angle -97^\circ$$

$$S_{22} = 0.5\angle -90^\circ$$

$G_T = 9.43$ $(9.75dB)$ $\longrightarrow$ usually lower than the other two gains $(G_P,\ G_A)$, because of mismatch at source and load

$G_P = 13.51$ $(11.31dB)$

$G_A = 9.55$ $(9.8dB)$

There are still three more gain expressions:

Maximum Available Gain (MAG)

$$\left.\begin{aligned} \Gamma_S &= \Gamma_{in}^* \\ \Gamma_L &= \Gamma_{out}^* \end{aligned}\right\} \text{ simultaneous input / output matching}$$

valid as long as it is stable

$$MAG = G_{T\max} = G_{A\max} = G_{P\max} = \frac{|S_{21}|}{|S_{12}|}\left(k - \sqrt{k^2 - 1}\right)$$

where $$k = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$$

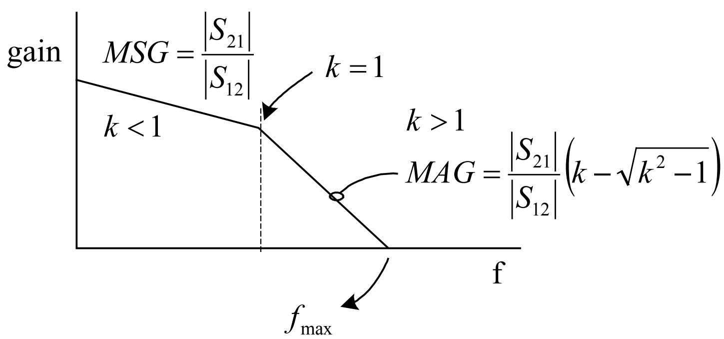
k is stability factor as we shall use soon

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

Maximum Stable Gain (MSG)

$$MSG = MAG\big|_{k=1} = \frac{|S_{21}|}{|S_{12}|}$$

you use this gain definition instead of MAG when transistor is potentially unstable



Unilateral Gain (G_U)

set $S_{12} = 0$

$$\Gamma_{in} = S_{11}\ , \quad \Gamma_{out} = S_{22}$$

$$\longrightarrow \quad G_{TU} = \underbrace{\frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}}_{\text{input matching gain}} \underbrace{|S_{21}|^2}_{\text{inherent gain of the transistor}} \underbrace{\frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}}_{\text{output matching gain}}$$

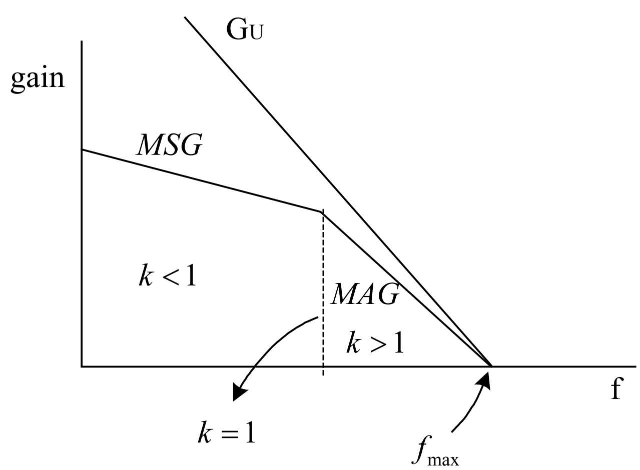
→ unilateral transistor is always stable

→ maximum G_{TU} is called maximum unilateral gain or sometimes unilateral gain (G_U)

$$G_{TU} = G_{TU,\max} = G_{TU}\Big|_{\substack{S_{11}^*=\Gamma_S \\ S_{22}^*=\Gamma_L}}$$

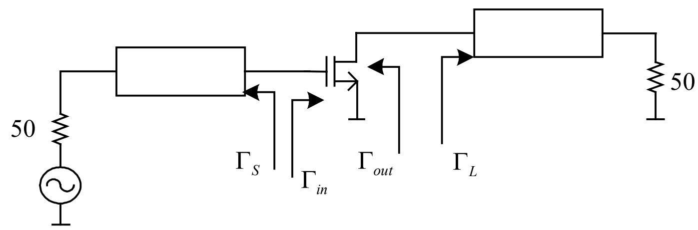
$$G_{TU} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2}$$

frequency (G_U at zero dB) is also f_{max}



k : stability factor

Stability



$$\Gamma_S = \frac{Z_S - Z_O}{Z_S + Z_O} = \frac{Z_S - 50}{Z_S + 50}$$

when the real part of Zs is positive then

resistance part

$$R_S \geq 0 \rightarrow |\Gamma_S| \leq 1 \quad (1)$$
$$R_L \geq 0 \rightarrow |\Gamma_L| \leq 1 \quad (2)$$

means that both input and output matchings provide resistances (positive resistance)

in addition if the real part of Zin and Zout is also positive that means

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| \leq 1 \quad (3)$$

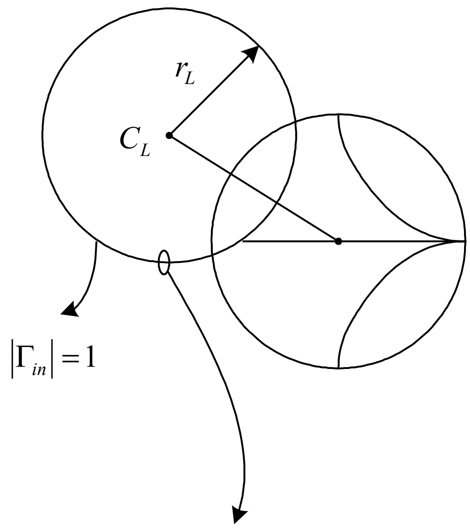
$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right| \leq 1 \quad (4)$$

* if eqns (1) ~ (4) are all satisfied your amplifier is stable
(provided that you only have one transistor
we will look at multiple transistor case)

find $|\Gamma_{in}| = 1$ and $|\Gamma_{out}| = 1$ $\longrightarrow$ solve for Γ_L and Γ_S

$\longrightarrow$ These equations give circles on Smith Chart

$|\Gamma_{in}| = 1 \longrightarrow$ find $\Gamma_L \longrightarrow$ it gives a circle that has a radius and center as following :



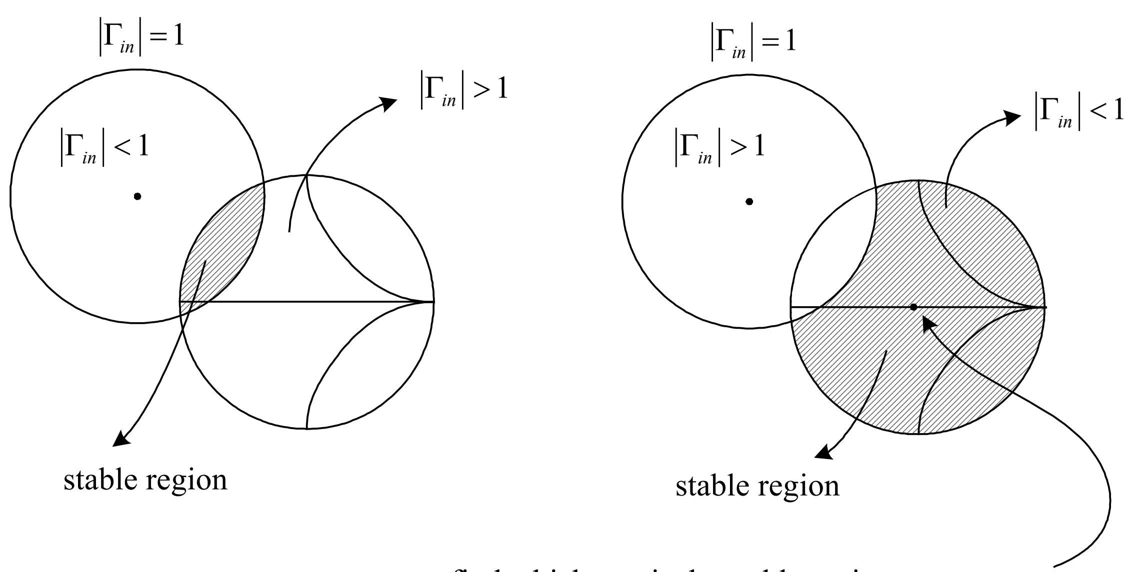
$$r_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|$$

$$C_L = \frac{\left(S_{22} - \Delta S_{11}^{\ *}\right)^*}{|S_{22}|^2 - |\Delta|^2}$$

we have defined Δ before

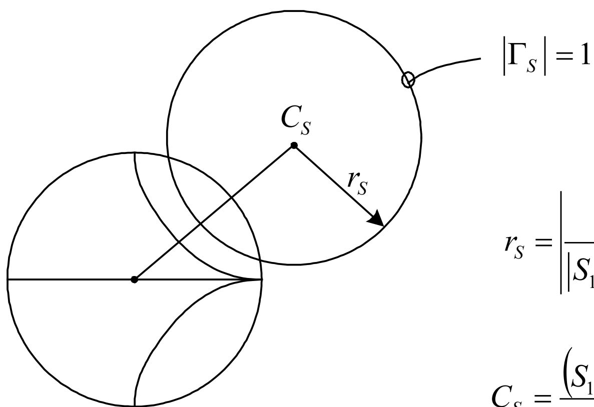
$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

<u>There are two possibilities</u>



to find which one is the stable region, try $\Gamma_L = 0$

→ similarity you can find input stability circle by setting $|\Gamma_{out}| = 1$ and find Γ_S

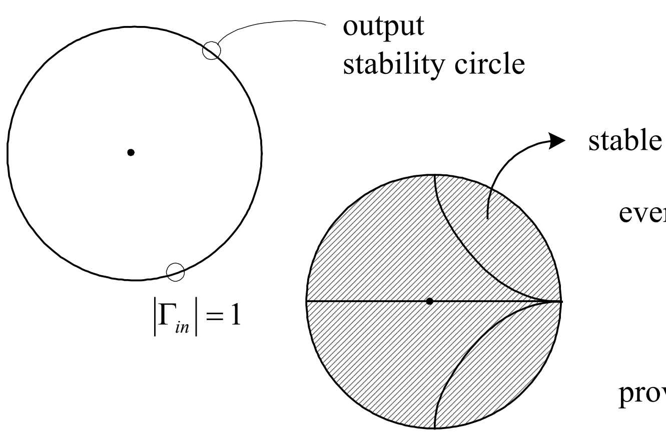


$$r_S = \left| \frac{S_{12} S_{21}}{|S_{11}|^2 - |\Delta|^2} \right|$$

$$C_S = \frac{\left(S_{11} - \Delta\, S_{22}^{\ *}\right)^*}{|S_{11}|^2 - |\Delta|^2}$$

<u>You can have two cases</u> → consider output stability circle

<u>Case 1</u>



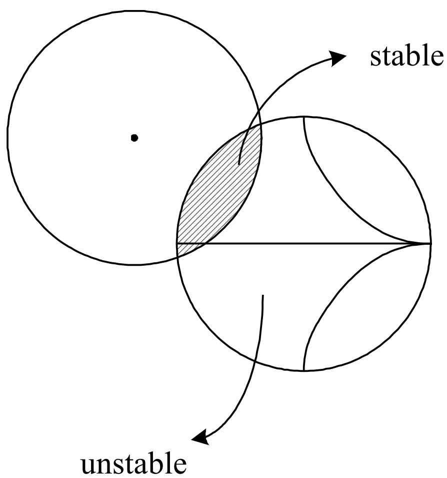
every where on the Smith Chart

$\left(|\Gamma_L| \leq 1\right)$

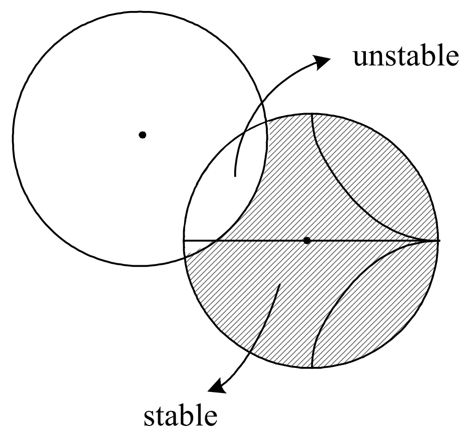
provides stable operation

we refer to this as <u>unconditionally stable</u>

Case 2



or



in each of the above conditions you can find load impedances (Γ_L) that result in unstable condition

→ potential unstable

→ does not mean it is unstable but you have to be careful providing right load impedance

you can define these two different cases using stability factor

work out equations → $k = \dfrac{1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2}{2|S_{12}S_{21}|}$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

if $k > 1$, $|\Delta| < 1$ ⟶ unconditional stability

$0 < k < 1$, $|\Delta| < 1$ ⟶ potentially unstable

(true for most RF transistors at low frequency)

$-1 < k < 0$, $|\Delta| < 1$ ⟶ potentially unstable

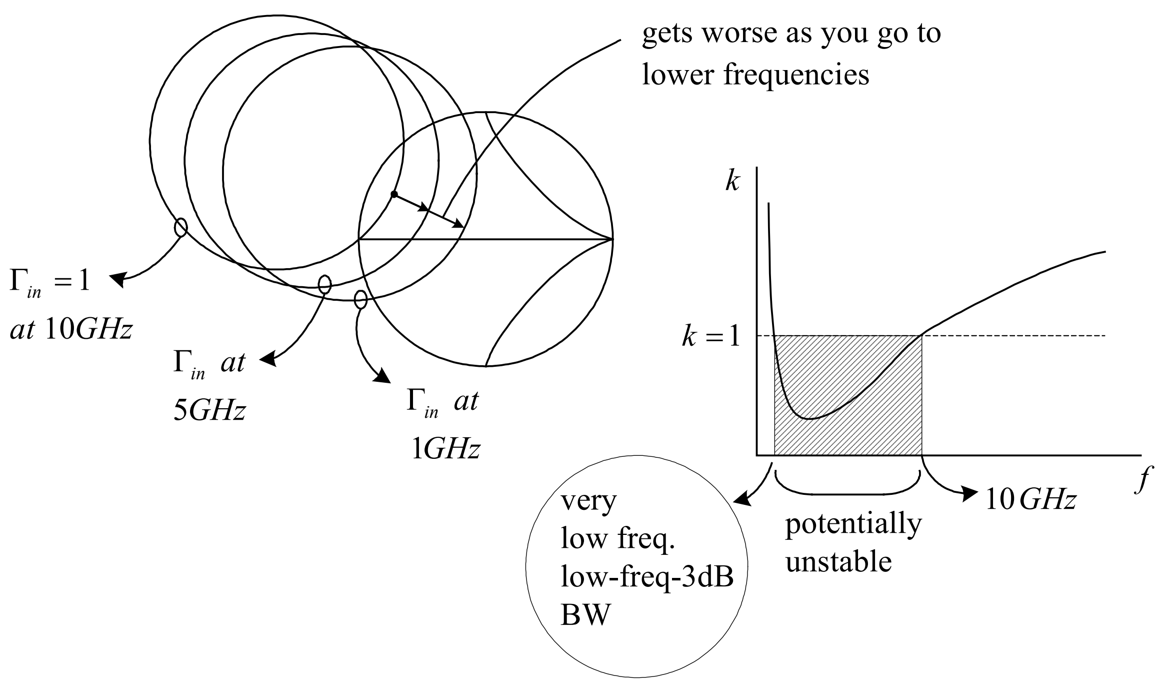
you often use this condition
to make an oscillator

in this case most of Smith Chart results in unstable operation
that is why it is good for oscillators

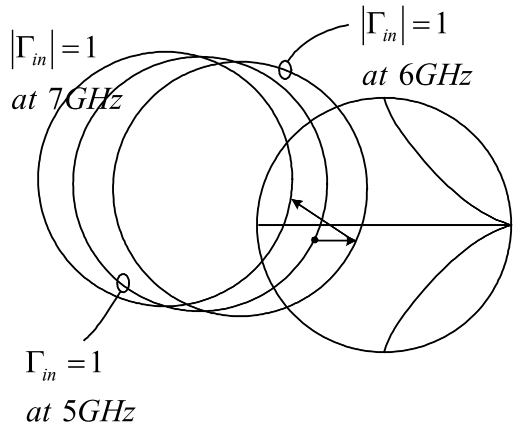
<u>* stability circles vs. frequency</u>

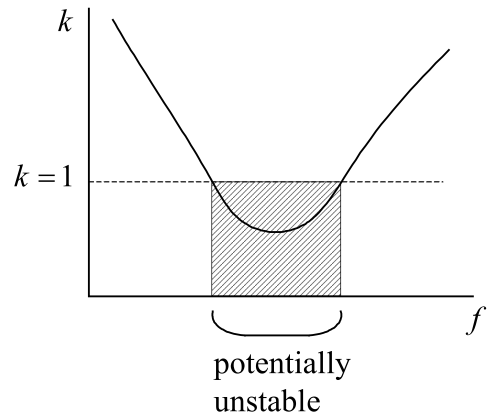
typically you have the following situation

for widebandwidth Amps



for narrowband Amps





unconditionally stable design

* check stability for all frequencies, not just BW of interest
* to be safe, design such that k>2 for all frequencies
* if you have multi-stage amplifiers, need to consider stability within the transistors

multiple transistor ckt

$\longrightarrow$ is the stability criteria different?

example

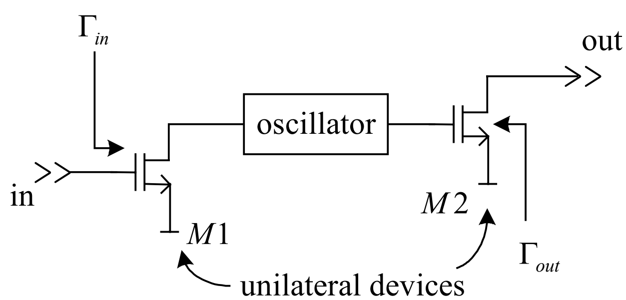
consider a unilateral device $(S_{12} = 0)$

assume $|S_{11}|,\ |S_{22}| < 1$

$$\left.\begin{aligned} \longrightarrow \quad \Delta = S_{11}S_{22} \quad &\rightarrow \quad |\Delta| < 1 \\ k = \frac{1-|S_{11}|^2-|S_{22}|^2+|S_{11}S_{22}|^2}{2|S_{12}S_{21}|} \quad &\rightarrow \quad +\infty \end{aligned}\right\} \longrightarrow$$

unilateral device is stable

now consider the following ckt :

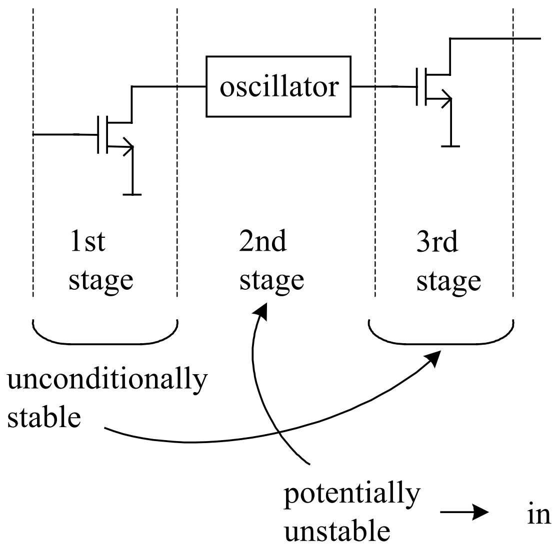


$$\left.\begin{aligned} \Gamma_{in} &= S_{11\,M1} \\ \Gamma_{out} &= S_{22\,M2} \\ S_{12\,ckt} &= 0 \end{aligned}\right\} \longrightarrow \left.\begin{aligned} &|\Delta| < 1 \\ &k \rightarrow +\infty \end{aligned}\right\} \longrightarrow$$

stability criteria confirms that this is a stable ckt but in actuality the circuit is oscillating

what to do?

$\longrightarrow$ need to analyze ckt stability for each stage
and make sure that each and every stage is stable



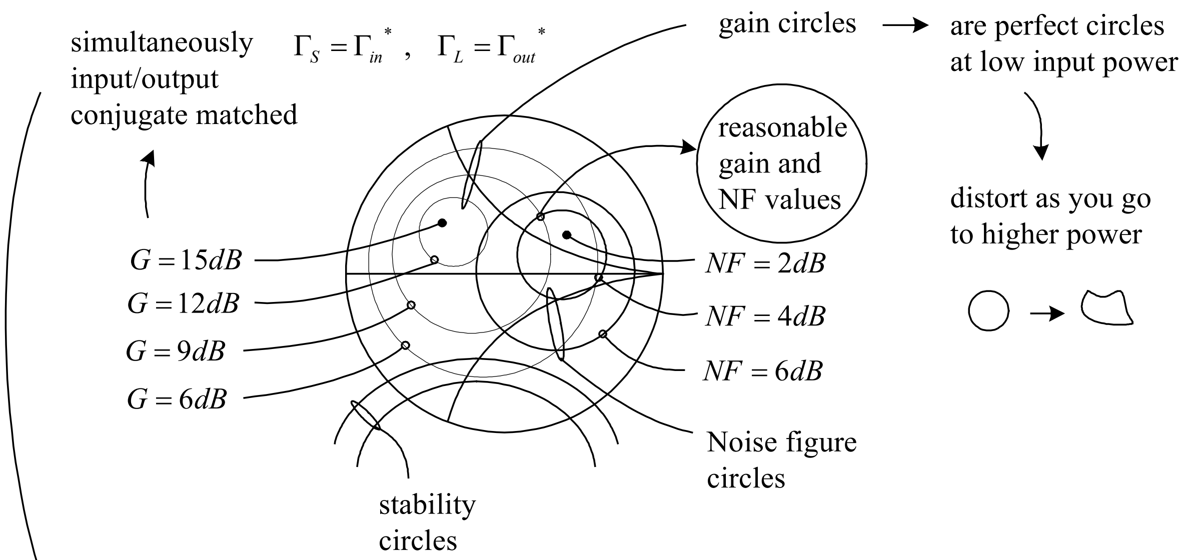
→ in this case definitely unstable

<u>typically $|\Delta| < 1$ for RF transistors</u>

we can look at stability factor <u>k</u> ,

plot input/output stability circles in cadence

last thing on gain/stability → most of the gain equations we saw are also circle equations on Smith Chart



how to find this optimum point for gain?

$$\left.\begin{aligned} \Gamma_S^* &= S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \\ \Gamma_L^* &= S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \end{aligned}\right\} \quad \text{find} \quad \Gamma_{L\,opt} \;,\quad \Gamma_{S\,opt}$$

$$\Gamma_{S\,opt} = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \qquad \Gamma_{L\,opt} = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \qquad B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$C_1 = S_{11} - \Delta\, S_{22}^* \qquad C_2 = S_{22} - \Delta\, S_{11}^*$$

$$if \quad |\Gamma_{S\,opt}| \,,\; |\Gamma_{L\,opt}| < 1 \quad \rightarrow \quad k > 1 \quad stable\,(probably)$$

So that optimum point for $|\Gamma_{S\ opt}|$, $|\Gamma_{L\ opt}|$ exist

if you have unconditionally stable case

Gain at that optimum input/output conjugate

matched is MAG $= \dfrac{|S_{21}|}{|S_{12}|}\left(k - \sqrt{k^2 - 1}\right)$

* now that we know enough about gain and stability,

let's see how we can make RF linear amplifier