Smith Chart

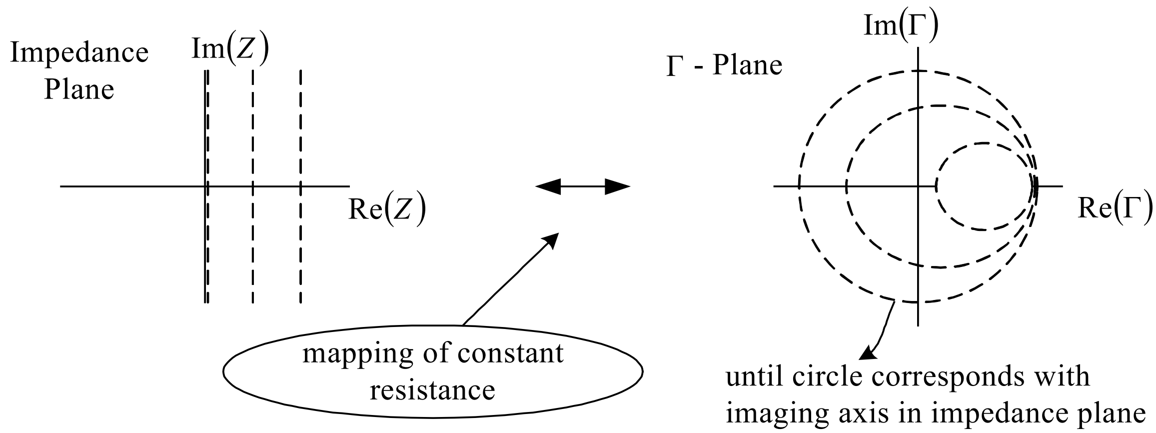
Smith chart is the plot of Γ in complex plane

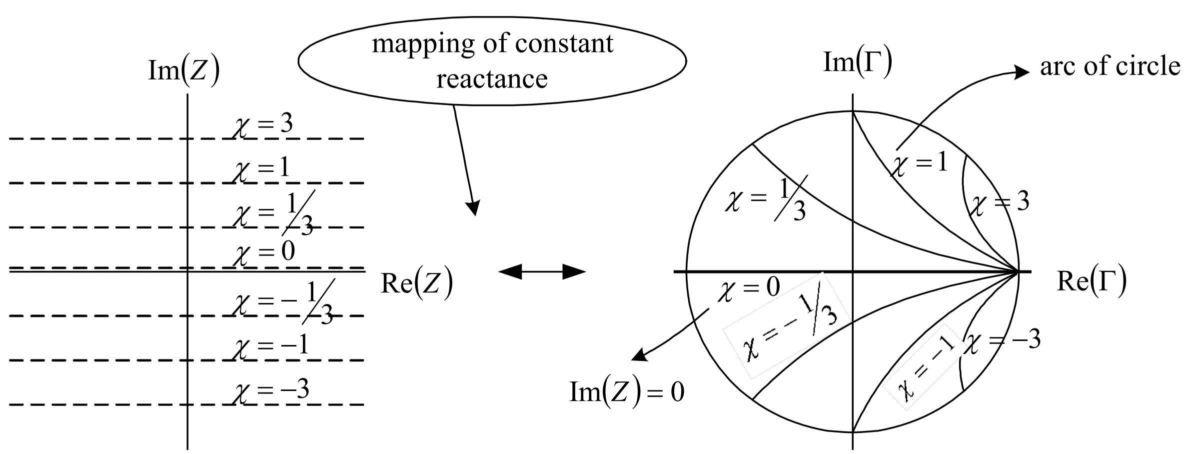
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{Z_{nL} - 1}{Z_{nL} + 1}$$

normalized impedance
ωrt characteristic impedance

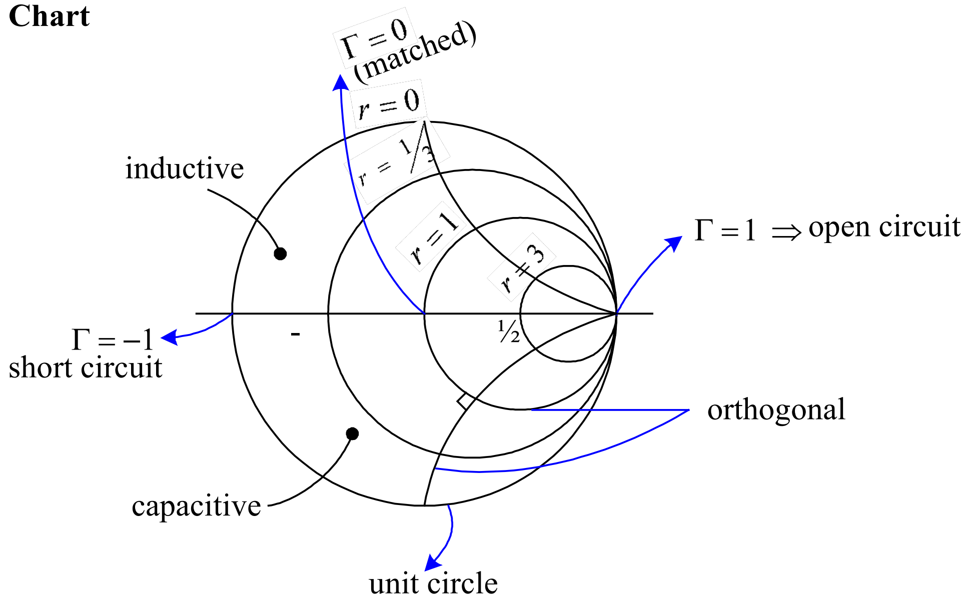
relationship between Γ and Z is a bilinear transformation

circle remain circle when they are mapped





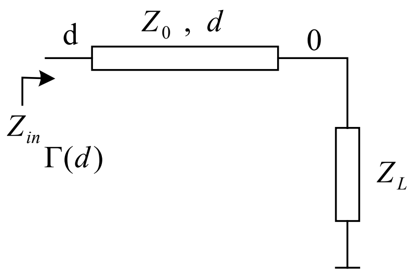
Smith Chart



Smith chart can be used to calculate impedance transformation

↓

You can graphically find the transformed impedance



$\Gamma(x) = \Gamma_L e^{-2\gamma x}$

loss less → $\Gamma(d) = \Gamma_L e^{-2j\beta d}$

(βd) : *electrical length*

<u>to do so</u>

* *normalize* Z_L *ωrt* Z_0 (Z_{nL})
* *locate* Z_{nL} *on the smith chart*
* *identify* $\Gamma_L = |\Gamma| e^{-j\theta}$
* *(clockwise) rotate* Γ_L *by twice the electrical length* $(2\beta d)$ *to find* Γ_{in}
* *find* Z_{nin}
* *convert it to* Z_{in}

* example

$Z_L = 30 + j60\Omega$

connected to 50Ω TRL impedance

TRL : $\longrightarrow$ 2cm length @ 2GHz

$\longrightarrow$ assume phase velocity 50% of the speed of light

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 + j60 - 50}{30 + j60 + 50} = 0.2 + j0.6 = \sqrt{0.4} \cdot e^{j71^\circ}$$

$$\Gamma(d = 2cm) = ? \qquad \beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_P} = \frac{2\pi f}{0.5c_0} = 83.7m^{-1}$$

$$2\beta d = 192^\circ$$

$$\Gamma_L = \Gamma_0 e^{-j2\beta d} = -0.32 - j0.55 = \sqrt{0.4} \cdot e^{-j120^\circ}$$

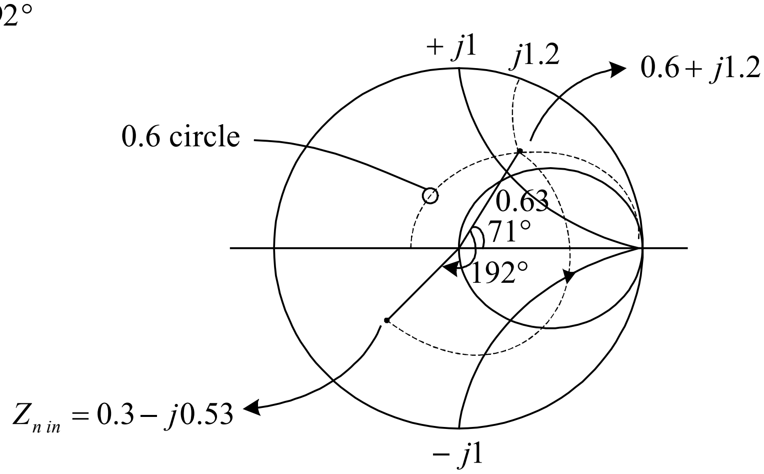
$$Z_{in} = Z_0 \frac{1+\Gamma}{1-\Gamma} = 14.7 - j26.7\Omega$$

difficult
to calculate

now graphical solution using Smith Chart

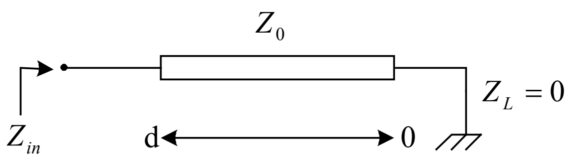
$$Z_L = 30 + j60 \quad \rightarrow \quad Z_{nL} = \frac{30 + j60}{50} = 0.6 + j1.2$$

$$2\beta d = 192^\circ$$



$$Z_{in} = 50 \times (0.3 - j0.53)$$
$$= 15 - j26.5\,\Omega$$

Standing wave



assume a loss-less transmission line

$\downarrow$

propagation constant is purely imaginary

incident reflected $\gamma = -j\beta$

$$V(x) = V^+\left(e^{-j\beta x} + \Gamma_0 e^{j\beta x}\right) = V^+ e^{-j\beta x}\left[1 + \Gamma_0 e^{+j2\beta x}\right] = V_{inc}\left(1 + \Gamma(x)\right)$$

incident voltage reflected voltage

$$I(x) = \frac{V^+}{Z_0}\left(e^{-j\beta x} - \Gamma_0 e^{j\beta x}\right) = \frac{V^+}{Z_0} e^{-j\beta x}\left[1 - \Gamma_0 e^{j2\beta x}\right] = \frac{V_{inc}}{Z_0}\left[1 - \Gamma(x)\right]$$

* incident and reflected wave make a standing wave along the line

$\longrightarrow$ at any point you can define standing wave ratio

$$SWR = \left|\frac{V_{\max}}{V_{\min}}\right| = \left|\frac{I_{\max}}{I_{\min}}\right| \longrightarrow \text{shows how bad is the mismatch}$$

very often SWR is referred to us VSWR

voltage standing wave ratio

$$V(x) = V^{+} e^{-j\beta x}\left[1 + \Gamma_0 e^{j\beta x}\right]$$

by varying x you can change the phase to reach max and min of voltage

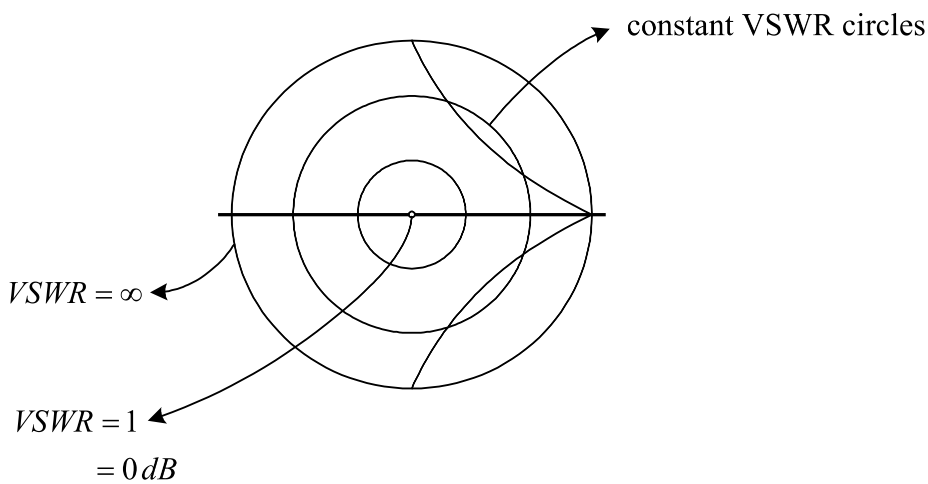
$\pm 1 \rightarrow$ max and min of this exponential function

$$VSWR = \left|\frac{V_{\max}}{V_{\min}}\right| = \frac{1+|\Gamma_0|}{1-|\Gamma_0|}$$

$$\textit{open \& short ckts give } VSWR \rightarrow \infty$$

$$\textit{matched condition} \quad \Gamma_0 = 0 \rightarrow VSWR = 1$$

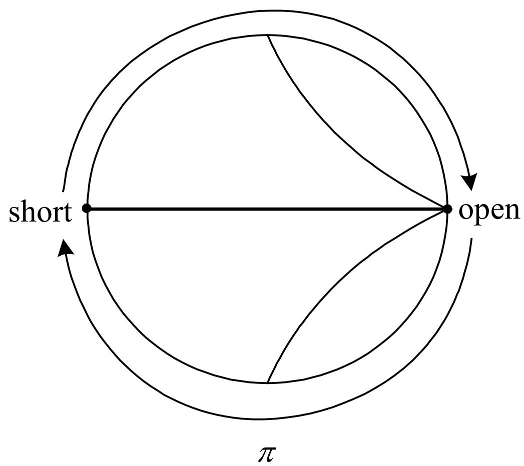
VSWR is often expressed in dB



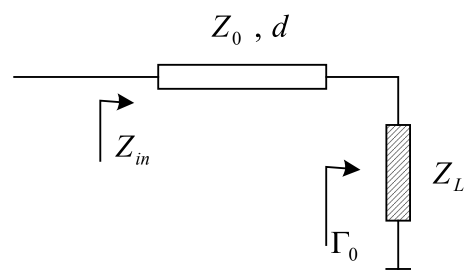
Impedance transformation

$\lambda/4$ line $\longrightarrow$ transforms short ckt to open ckt and vice versa
(quarter wave line)

$$\left.\begin{array}{l}\Gamma(x)=\Gamma_L e^{-2j\beta x} \\ \beta=\dfrac{2\pi}{\lambda}\end{array}\right\} \longrightarrow \left.\begin{array}{l}\Gamma(x)=\Gamma_L e^{-2j\times\frac{2\pi}{\lambda}x} \\ x=\dfrac{\lambda}{4}\end{array}\right\} \longrightarrow \Gamma\left(\frac{\lambda}{4}\right)=\Gamma_L e^{-j\pi}$$



any given line:



$$Z_{in}=Z_0\frac{1+\Gamma(d)}{1-\Gamma(d)}$$

work out the equations

$$\boxed{Z_{in}=Z_0\frac{Z_L+jZ_0\tan(\beta d)}{Z_0+jZ_L\tan(\beta d)}} \longrightarrow \text{for quarter wave line}$$

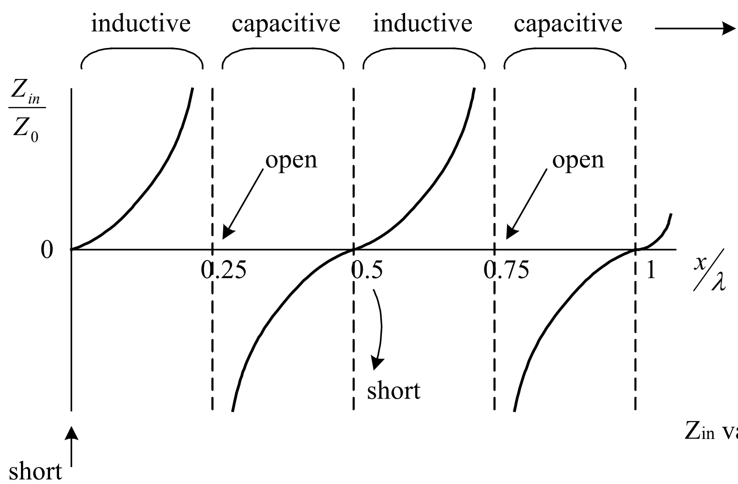
consider special cases

$$\boxed{Z_{in}=\frac{Z_0^2}{Z_L}}$$

short ckt line

$$Z_L = 0 \quad \rightarrow \quad Z_{in} = Z_0 \frac{jZ_0 \tan(\beta d)}{Z_0} = jZ_0 \tan(\beta d) = jZ_0 \tan\left(\frac{2\pi d}{\lambda}\right)$$

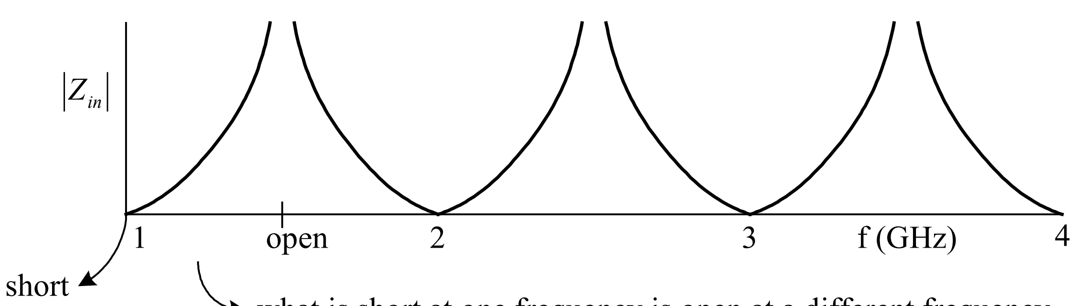
$$\beta = \frac{2\pi}{\lambda}$$



so you can make inductors/capacitors using shorted transmission line (at one frequency!)

Zin varies with frequency

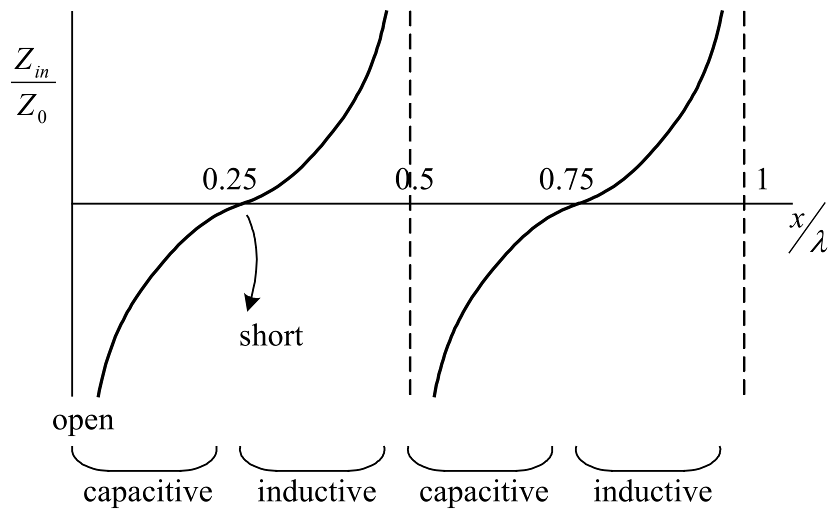
$$Z_{in} = jZ_0 \tan(\beta d) = jZ_0 \tan\left(\frac{2\pi d}{v_P} f\right)$$



what is short at one frequency is open at a different frequency

open circuit transmission line

$$Z_{in}(x) = -jZ_0 \frac{1}{\tan(\beta x)}$$

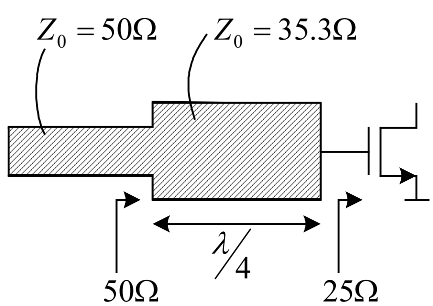


going back to quarter wave line

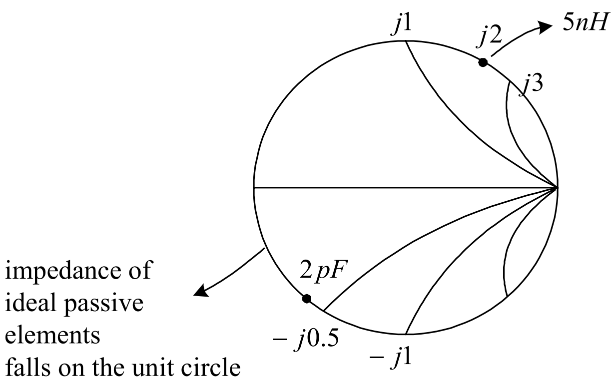
$$Z_{in} = \frac{Z_0^2}{Z_L} \longrightarrow \text{you can do matching}$$

assume $Z_L = 25\Omega$ we want to match it to 50Ω

$$Z_{line}\left(\lambda/4\right) = Z_0 = \sqrt{Z_L Z_{in}} = \sqrt{25 \times 50} = 35.3\Omega$$



Smith chart and passive elements



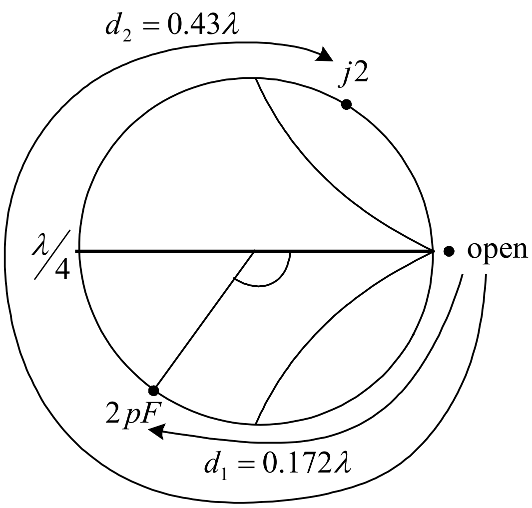
say at 3 GHz

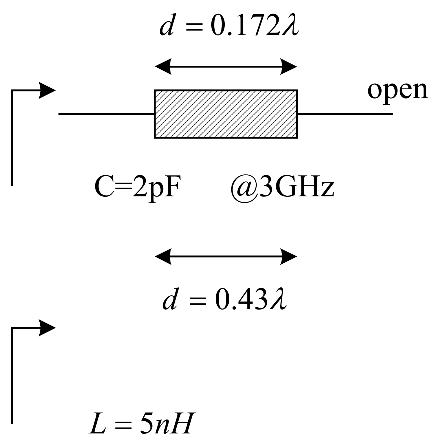
2pF Cap $\longrightarrow$ $X_C = \dfrac{1}{\omega C} = 26.5\Omega$

$X_{nC} = {-jXc}/{Z_0} = -j0.53$

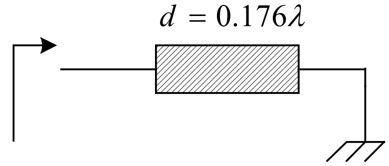
$5nH \quad inductor \quad X_{nL} = \omega L = j2$

how to realize 2pF cap and 5nH inductor using open ckt line (let say at 3GHz)





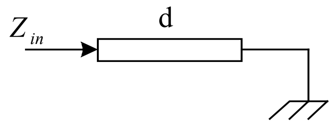
you can use short ckt line

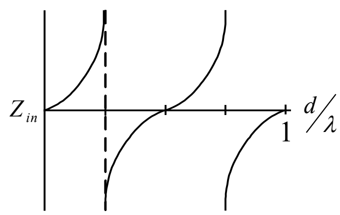


L=5nH @ 3 GHz

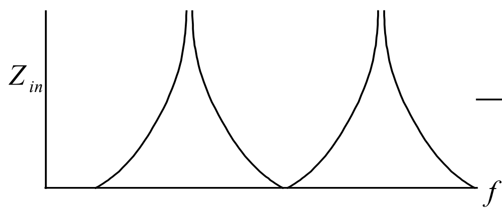
* you can also use Smith Chart for admittance instead of impedance
we will have some homework on impedance transformation

S-parameters → scattering parameters

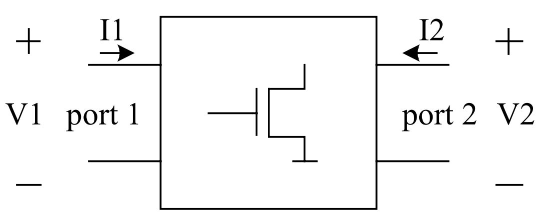




→ short ckt impedance seen through a short piece of transmission line can be anything



→ it is also frequency dependant



when we do not know anything about device physics & modeling we represent it as a black box with ports attached to it

There are many representation of this black box

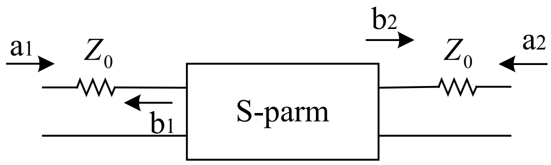
Impedance Parameters →
Admittance Parameters
Hybrid Parameters

$$\begin{cases} V_1 = Z_{11}I_1 + Z_{12}I_2 \\ V_2 = Z_{21}I_1 + Z_{22}I_2 \end{cases}$$

to find Z_{11} → open circuit output → $I_2 = 0$ → OK for low frequency

so for high frequency we are looking at certain parameters that are not determined by short / open ckts rather than by matching

→ under matched condition → no reflection



a1, a2, b1, b2 are power waves $\longrightarrow$ $|a_1|^2$ $\longrightarrow$ incident power

$$b_1 = S_{11}a_1 + S_{12}a_2 \qquad a_1 = \frac{E_{inc1}}{\sqrt{Z_0}} \qquad b_1 = \frac{E_{ref1}}{\sqrt{Z_0}}$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \qquad a_2 = \frac{E_{inc2}}{\sqrt{Z_0}} \qquad b_2 = \frac{E_{ref2}}{\sqrt{Z_0}}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \Gamma_1$$

$\longrightarrow$ S11, S22 are input, output reflection coefficient

S21, S12 are transmission and reverse transmission

S11, S12, S21, S22 are often presented in dB form

$$S_{11} = \Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

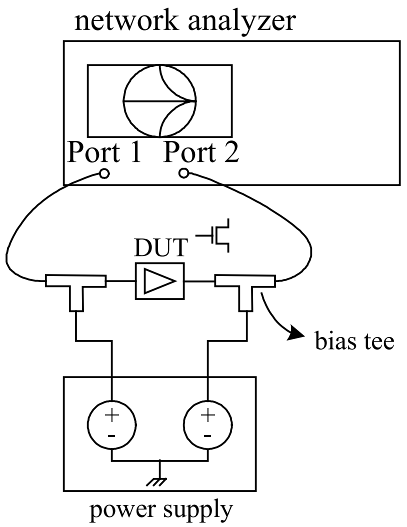
$-20\,\log|S_{11}| \Rightarrow$ Return loss when port 2 is terminated with 50Ω

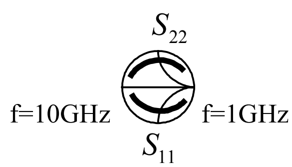
$\downarrow$

$a_2 = 0$

no reflection from termination

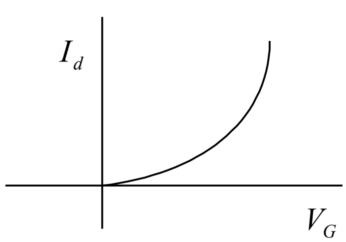
Measurement of S-parm

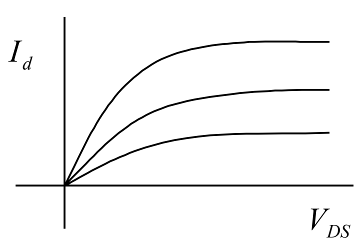




you often measure S-parm under different bias conditions (voltage & current) and over a frequency range of interest

you also have DC measurement of your device





using all these data you can find the device model (non-linear: spice) that represents the device under DC and RF under different bias conditions