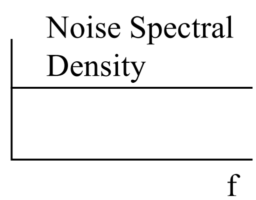
Noise in Integrated Circuits

Fundamental Noise Sources

Thermal Noise (Johnson Noise, Nyquist Noise)

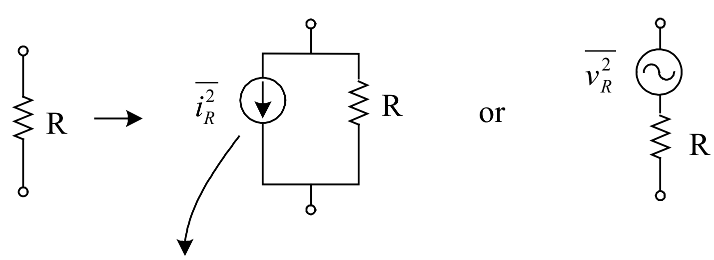
arises from thermally excited random motion of electrons in a conductive medium

Properties of thermal noise



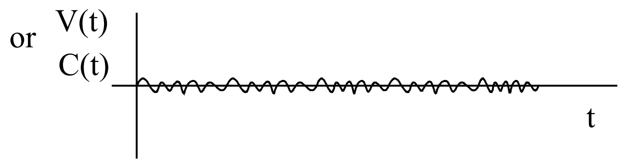
1) noise is white (?)
2) noise is proportional to temperature
3) not associated with DC current
4) can find it in any real physical resistor

Representation of thermal noise

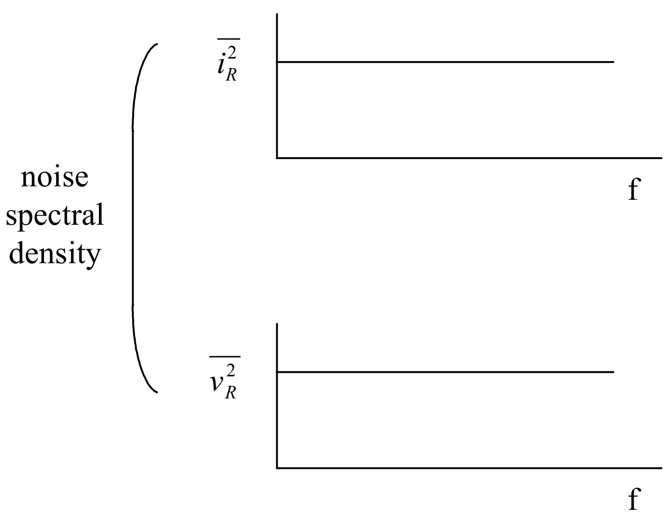


does not mean the current
is in certain direction

in fact average current (or voltage) is zero



but you can go to frequency domain and plot mean-square value of the noise signal



$$\overline{i_R^2} = \frac{4kT}{R}\Delta f$$

bandwidth of interest (ckt or measurement)

$$\overline{v_R^2} = 4kTR\Delta f$$

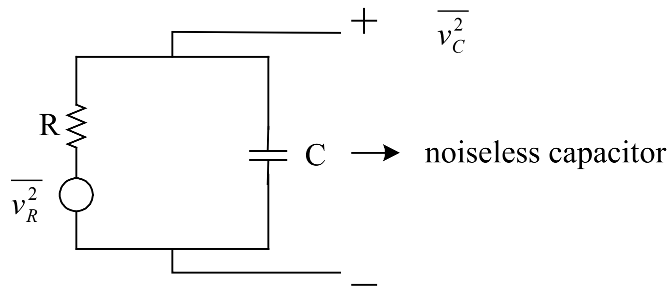
$$\overline{i_R} = \overline{v_R} = 0$$

noise average is zero

but average noise power is not?!

why $\overline{v_R^2} = 4kTR\Delta f$

assume a simple ckt of parallel RC



An equipartition theorem of statistical thermodynamic
for each degree of freedom (or mode) in a given system,
there is a thermal energy of $\frac{1}{2}kT$

k: Boltzmann constant
$1.38\times10^{-23}\ J/K$

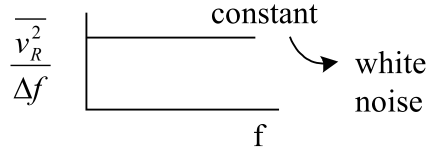
Total energy of the system $= \frac{1}{2}kT = \frac{1}{2}C\overline{v_C^2}$

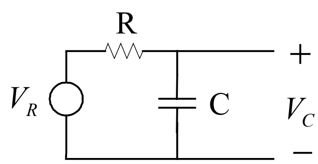
$$\overline{v_C^2} = \frac{kT}{C}$$

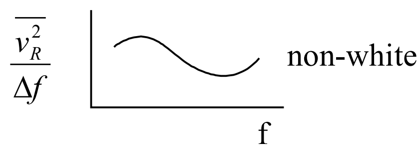
total mean square voltage density
integrated over all frequencies

$$\overline{v_C^2} = \int_0^\infty \left| \frac{1}{1 + j\omega RC} \right|^2 \frac{\overline{v_R^2}}{\Delta f} df = \frac{kT}{C}$$

filtering function







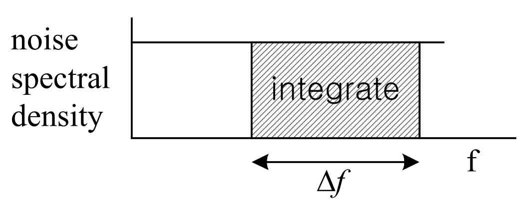
Assuming white noise

$$\frac{\overline{v_R^2}}{R} = \text{constant}$$

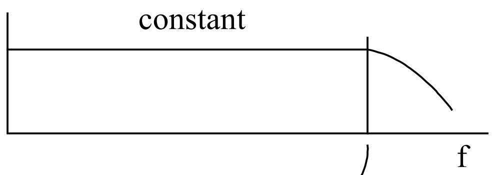
$$\rightarrow \quad \frac{kT}{C} = \frac{\overline{v_R^2}}{2\pi\Delta f} \int_0^\infty \frac{{\omega_0}^2}{\omega^2 + {\omega_0}^2} d\omega \qquad \omega_0 = \frac{1}{RC}$$

$$\frac{kT}{C} = \frac{\overline{v_R^2}}{2\pi\Delta f} \times \left(\frac{\pi}{2RC} \right) \quad \longrightarrow \quad \boxed{\overline{v_R^2} = 4kTR\Delta f}$$

$$\frac{\overline{v_R^2}}{\Delta f} = 4kTR$$



* Actual power spectral density of thermal noise



$\frac{kT}{h} \approx 6\,THz$ ⟶ for all practical purpose

$1.38\times10^{-23}\ J/K$

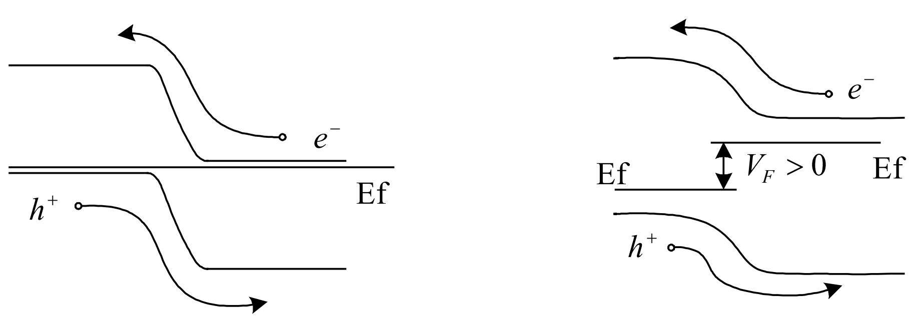
Plank constant $= 6.625\times10^{-34}\ J\cdot\sec$

⟶ Thermal noise is a white noise

Shot noise

* associated with DC current flow across a junction

* arises from random nature of electrons and holes
  surmounting a potential barrier



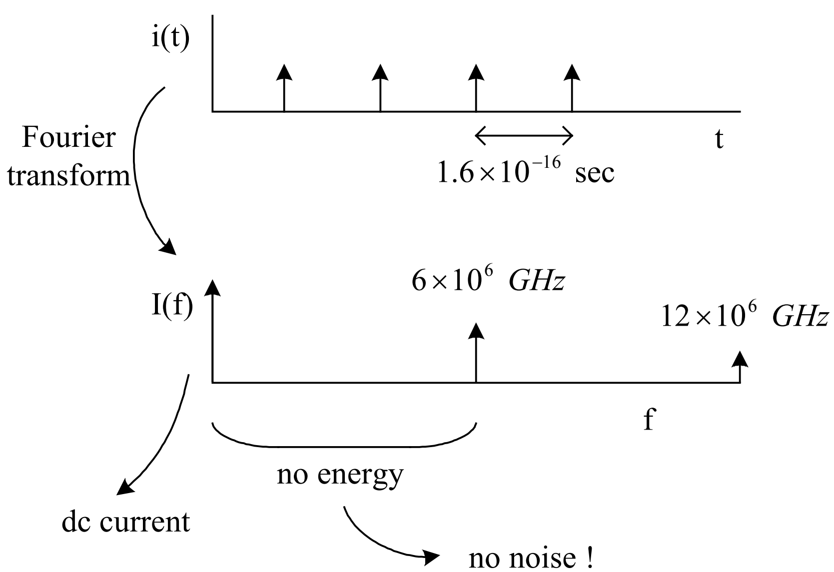
$V_F = 0$ $\qquad\qquad$ $V_F > 0$ ⟶ current flow special case

assume that electrons are very well controlled / behaved and cross the junction in a very uniform manner

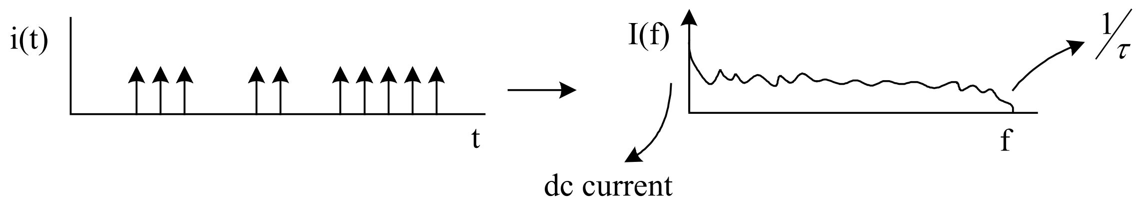
1 mA current → uniform current pulses

$$I = \frac{Q}{t} \quad \rightarrow \quad t = \frac{Q}{I} = \frac{1.6 \times 10^{-19}}{1 \times 10^{-3}} = 1.6 \times 10^{-16} \text{ sec}$$

every so many second one electron passes



in reality carriers surmount the barrier in a random fashion



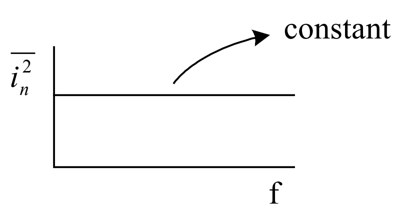
τ : current transit time in the depletion region

$1/\tau \rightarrow$ hundreds of GHz

$\longrightarrow$ So for all practical purposes shot noise is a white noise

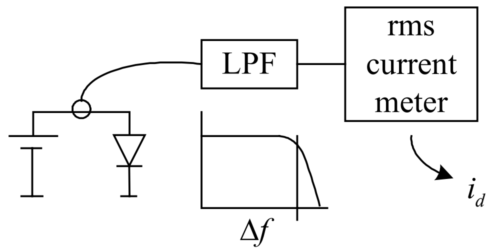
again it is better to talk about noise spectral density

$$\overline{i_n^2} = 2\,q\,I_D\,\Delta f$$



shot noise :
1) related to DC current over a potential barrier
2) independent of T
3) white
4) noise power $\propto \Delta f$, I_D

Experiment



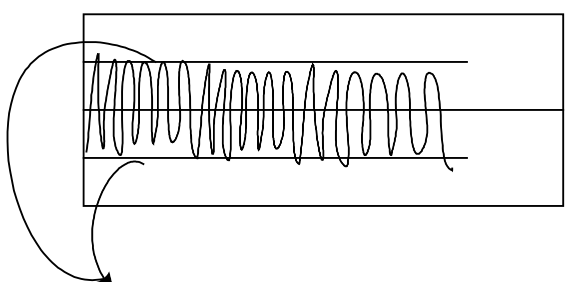
$$i_d = \sqrt{\overline{i_d^2}} = \sqrt{2qI_D\Delta f}$$

$$\left.\begin{array}{l} I_D = 1\,mA \\ \Delta f = 1\,MHz \end{array}\right\} \quad \underbrace{\overline{i_d^2} = 3.2\times10^{-16}\,A^2}_{2\,q\,I_D\,\Delta f}$$

$$i_d = \sqrt{\overline{i_d^2}} = 17\,nA$$

Experiment 2

if you look at a noisy signal on the oscilloscope



$V_{PP} = 100\,\mu V \longrightarrow$ what would be the rms noise value

$\downarrow$

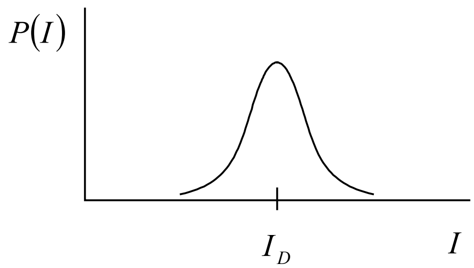
$$v_{noise} \approx \frac{V_{PP}}{6} = 16\,\mu V_{rms}$$

$\downarrow$

why?

both thermal noise and shot noise have Gaussian distribution

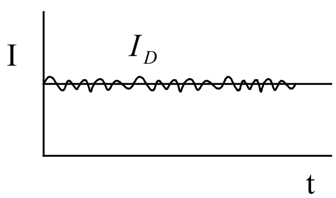
plot probability density function of the diode current



$P(I)dI \rightarrow$ probability that diode current

$$I < \quad < I + dI$$

σ: standard deviation
of the Gaussian distribution



turnsout $\rightarrow$ $\sigma^2 = \overline{i^2}$

$$\sigma = \sqrt{2qI_D\Delta f}$$

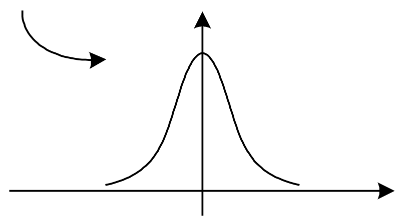
$\pm 3\sigma \longrightarrow$ limits of the noise amplitude within 99.7% of the time

$\longrightarrow$ what you see on the scope is usually $\pm 3\sigma$

$\longrightarrow$ divide by 6 $\longrightarrow$ gives you σ

which is the rms value of noise

Resistor noise (thermal noise) $\longrightarrow$ any voltage / current around 0



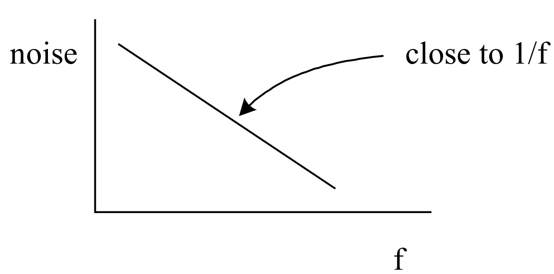
Other types of noise

Flicker Noise (1/f noise)

* associated with random trapping and release of carriers from slow states

such as interface state between Si/SiO_2 in MOS

Flicker noise has a non-white spectral density



$$\underbrace{\overline{i_n^2} = k\frac{I_D^a}{f^b}\Delta f}_{}$$

general form of flicker noise

noise depends on current $\longrightarrow$ (no current $\longrightarrow$ no noise)

k $\longrightarrow$ constant for a particular device

unlike shot noise and thermal noise, this constant varies from device to device and from technology to technology

a : 0.5 ~ 2

current dependence of noise

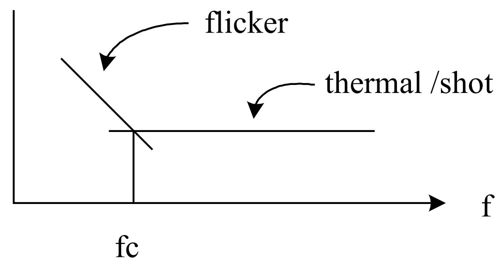
b = 1

flicker noise $\longrightarrow$ very high in MOS transistors

$\longrightarrow$ exists in bipolar transistors (especially the base current)

$\longrightarrow$ also exists in carbon resistors

thin film wire wound resistors have much less flicker noise flicker noise is often characterized with noise corner frequency



noise corner frequency ~ 10 Hz ~ kHz for bipolar

~ 10 kHz ~ MHz for MOS

Burst noise

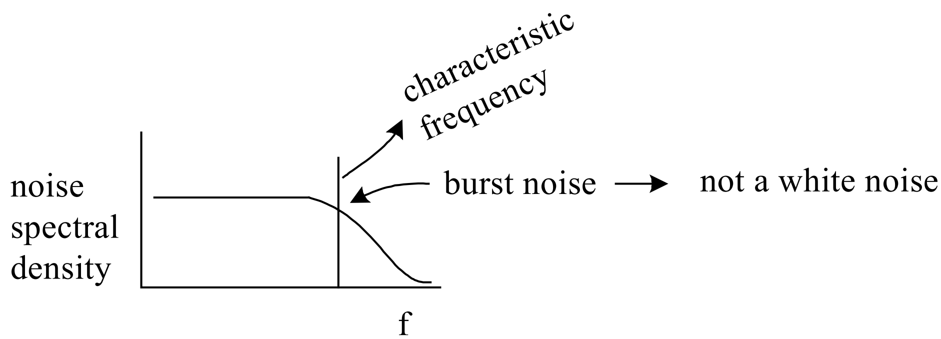
also a low-frequency noise

due to a single trap

1/f noise → due to an ensemble of traps at different frequency/ energy

burst noise → has a certain energy

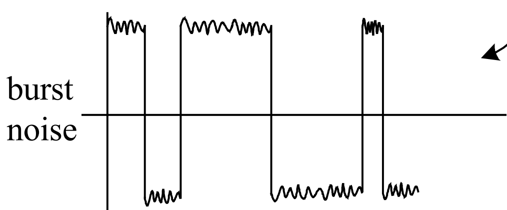
certain time constant



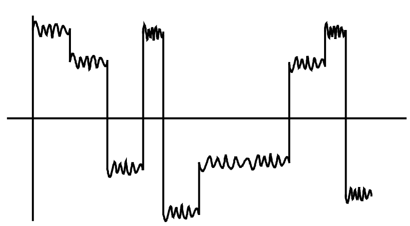
noise amplitude does not follow Gaussian distribution

In fact it is a binomial distribution

→ charging / discharging a trap



→ sometimes you have more than 2 levels as there is more than 1 trap

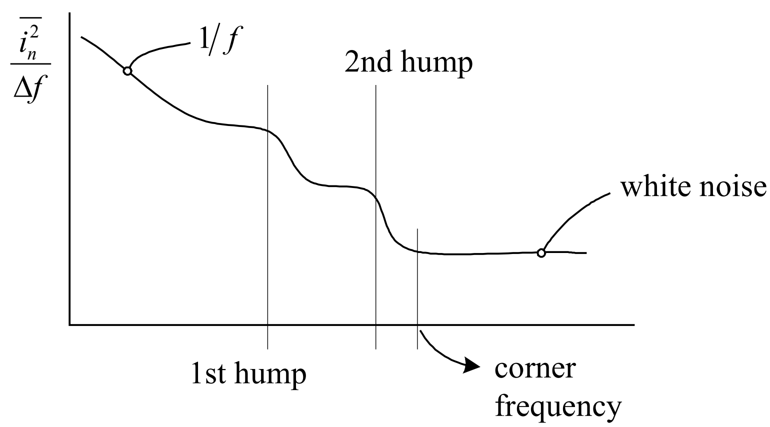


when there is more than one trap you often see

two characteristic frequencies (two humps)

burst and 1/f noise are always combined

so what you may see



<u>Another type of noise</u>

<u>Avalanche Noise</u>

avalanche breakdown is a very noisy process

0.5 mA zener current is equivalent to the thermal noise of $600\,K\Omega$ resistor

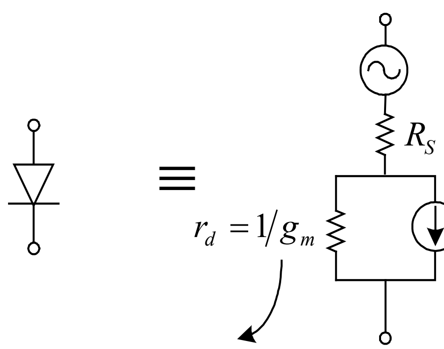
Noise models for devices

Resistor $\longrightarrow$ thermal noise (carbon resistors + flicker noise)

Capacitors / Inductors $\longrightarrow$ noiseless

but they shape the overall noise through the transfer function

Noise in Diodes



$$\overline{v_S^2} = 4kTR_S\Delta f \quad \text{thermal}$$

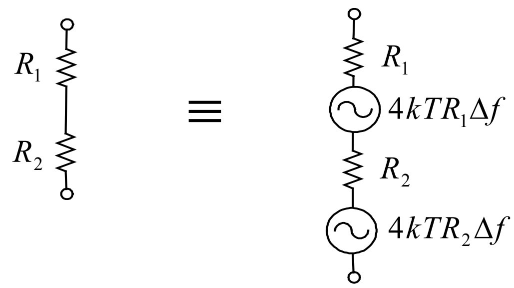
$$\overline{i_d^2} = \underbrace{2qI_D\Delta f}_{\text{shot}} + \underbrace{k\frac{I_D^a}{f}\Delta f}_{\text{flicker}}$$

fictitious resistor (noiseless)

These three noise sources are independent of each other $\longrightarrow$ uncorrelated

Correlation study case

Noise in series resistors

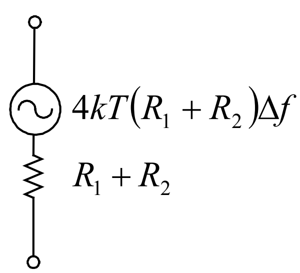


$$v_T(t) = v_1(t) + v_2(t)$$

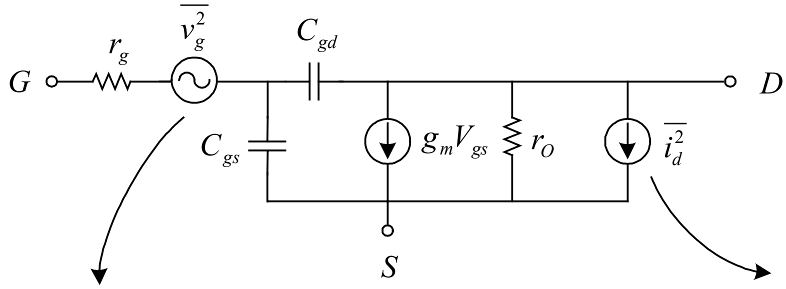
$$\begin{aligned}\overline{v_T(t)^2} &= \overline{[v_1(t)+v_2(t)]^2} \\ &= \overline{v_1(t)^2} + \overline{v_2(t)^2} + \overline{2v_1(t)v_2(t)} \\ &= 4kTR_1\Delta f + 4kTR_2\Delta f + \overline{2v_1(t)v_2(t)} \\ &= \overline{v_1}^2 + \overline{v_2}^2 + 0\end{aligned}$$

uncorrelated $\leftarrow$ since the two resistors are separate from each other, their noise is orthogonal (independent from each other)

$\rightarrow$



<u>Noises in MOS transistor</u>



Thermal noise (correlated + uncorrelated)

Thermal +flicker

$$\overline{v_g^2} = 4kT\delta\left(r_g + \frac{1}{5g_m}\right)\Delta f$$

correction factor

long channel $\delta = 4/3$

short channel $\delta = 4 \sim 6$

uncorrected thermal noise

correlated noise → coming from <u>voltage fluctuation in the channel</u> coupled to gate

due to drain thermal noise

correlation factor ~ 0.4 but varies from technology to technology

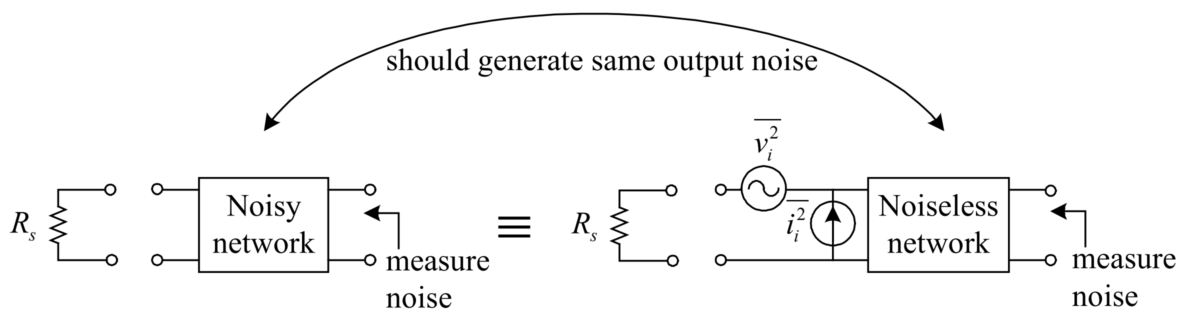
$$\overline{i_d^2} = \overbrace{4kT\gamma g_m \Delta f}^{\text{thermal}} + \overbrace{K\frac{g_m^2}{WLC_{OX}^2}\frac{\Delta f}{f}}^{1/f}$$

correction factor
long channel $\gamma = 2/3$
short channel $\gamma = 2 \sim 3$

technology / device size dependent

experimentally, they have found that 1/f noise
in MOS is not a function of drain current

- Now that we have correlated / uncorrelated noise sources
how do we calculate total noise?



we can represent any noisy two-part network
(with correlated /uncorrelated noise sources)
with a noise-less network and two input current and voltage noise sources
(there might be some correlation between the two)

* why do we need both $\overline{v_i^2}$ and $\overline{i_i^2}$

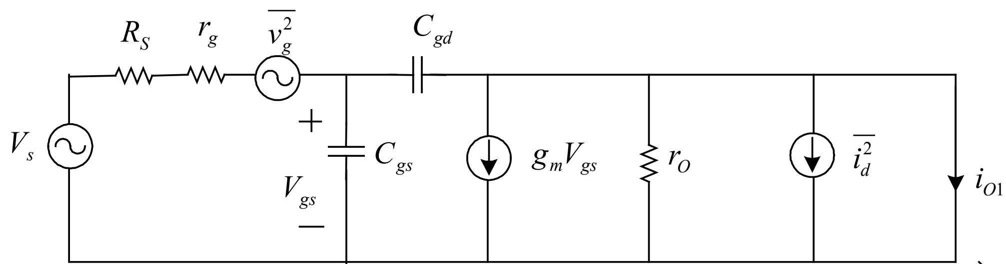
$R_s \rightarrow 0 \rightarrow \overline{i_i^2}$ shorts out but $\overline{v_i^2}$ still represents our noise

$R_s \rightarrow \infty \rightarrow \overline{v_i^2}$ cannot generate any noise at output but $\overline{i_i^2}$ does

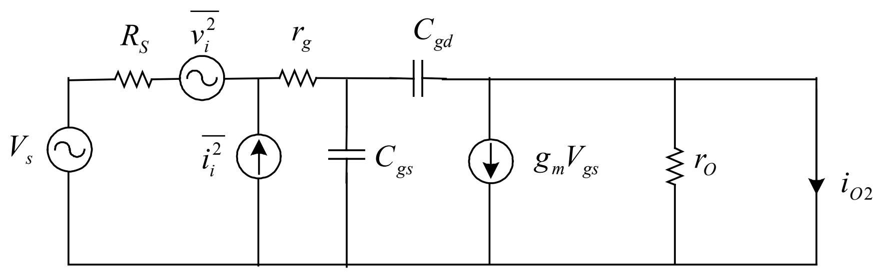
Consider MOS transistor

to find $\overline{v_i^2}$ short-ckt input and measure output noise

to find $\overline{i_i^2}$ open ckt input and measure output noise

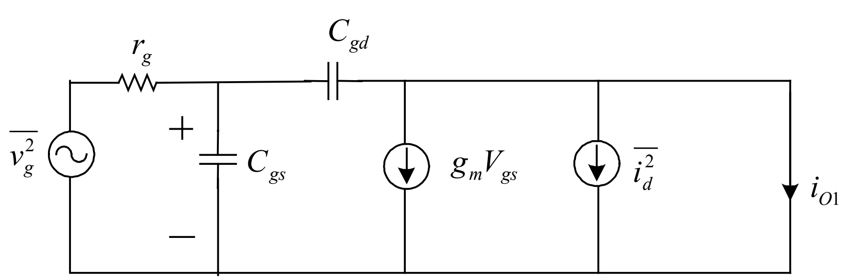


short-ckt output current noise



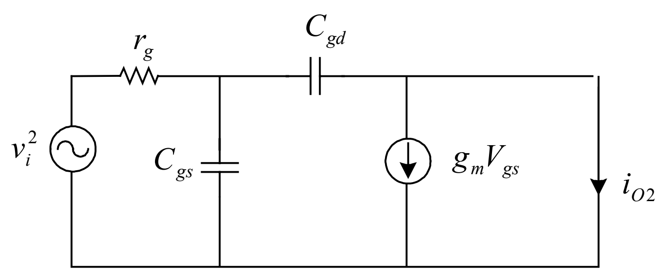
to find $\overline{v_i^2}$ and $\overline{i_i^2}$

first short-ckt input $\left(R_s = \overline{v_i^2} = 0\right) \rightarrow \overline{i_i^2} \rightarrow$ short-ckted



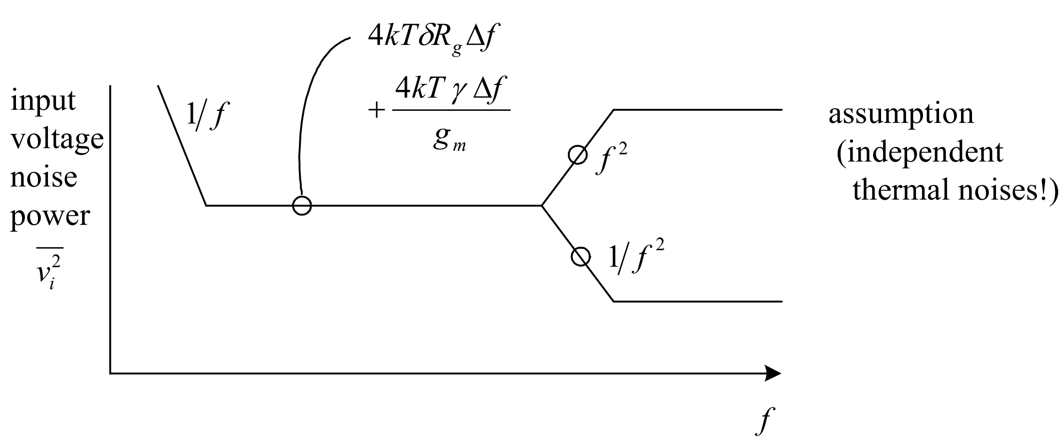
assumption: ignore that noise sources are correlated

$$\overline{i_{O1}^2} = \left(4kT\delta R_g \Delta f\right)\left(g_m^2 + \omega^2 C_{gd}^2\right) \times \frac{1}{1+\omega^2\left(C_{gs}+C_{gd}\right)^2 r_g^2} + 4kT\gamma g_m \Delta f + \frac{K\, g_m^2 \Delta f}{WL\, C_{OX}^2 f}$$



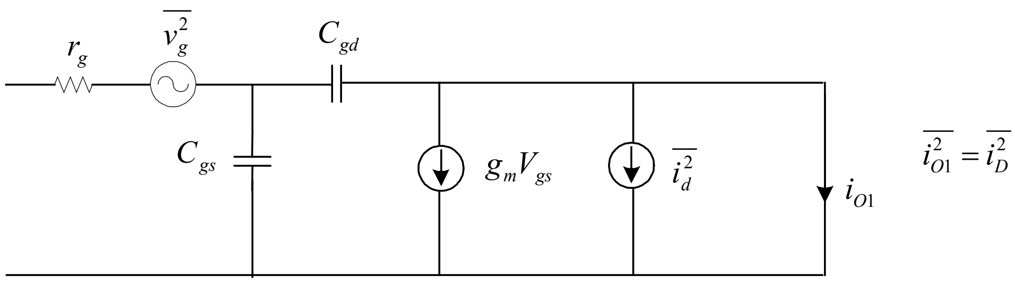
$$\overline{i_{O2}^2} = \left(g_m^2 + \omega^2 C_{gd}^2\right)\overline{v_i^2} \times \frac{1}{1+\omega^2\left(C_{gs}+C_{gd}\right)^2 r_g^2}$$

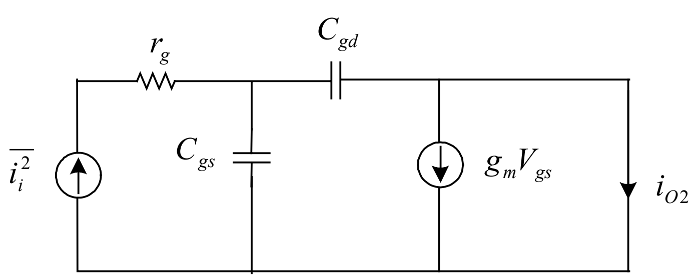
$$\rightarrow \boxed{\overline{v_i^2} = 4kT\delta R_g \Delta f + \frac{1+\omega^2\left(C_{gs}+C_{gd}\right)^2 r_g^2}{1+\omega^2 \dfrac{C_{gd}^2}{g_m^2}}\left[\frac{4kT\gamma\Delta f}{g_m} + \frac{K\Delta f}{WL\, C_{OX}^2 f}\right]}$$



to calculate equivalent input current noise source $\overline{i_i^2}$

open- ckt input $\longrightarrow$ v_i^2 in series with ∞ resistor $\rightarrow$ has no effect on output noise

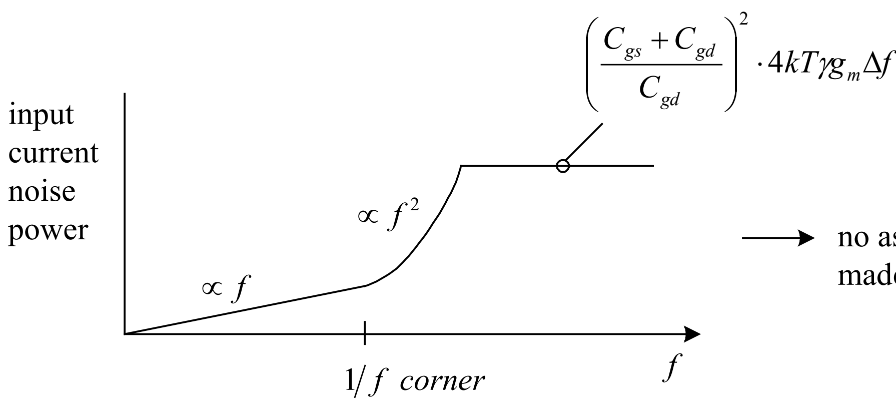




$$\overline{i_{O2}^2} = \overline{i_i^2}\left|\frac{-g_m}{j\omega\left(C_{gs}+C_{gd}\right)}+\frac{C_{gd}}{C_{gs}+C_{gd}}\right|^2$$

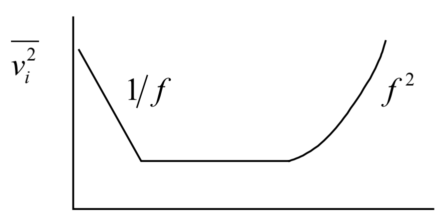
$$= \overline{i_i^2}\left|\frac{j\omega C_{gd}-g_m}{j\omega\left(C_{gs}+C_{gd}\right)}\right|^2$$

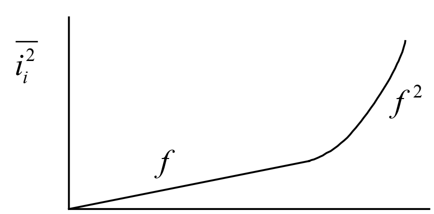
$$\overline{i_{o1}^2} = \overline{i_{o2}^2} \quad \Rightarrow \quad \boxed{\overline{i_i^2} = \frac{\omega^2 (C_{gs} + C_{gd})^2}{g_m^2 + \omega^2 C_{gd}^2} \left[4kT\gamma g_m \Delta f + \frac{K\, g_m^2 \Delta f}{WL\, C_{OX}^2 f} \right]}$$



→ no assumption made!

<u>if you ignore C_{gd}</u>

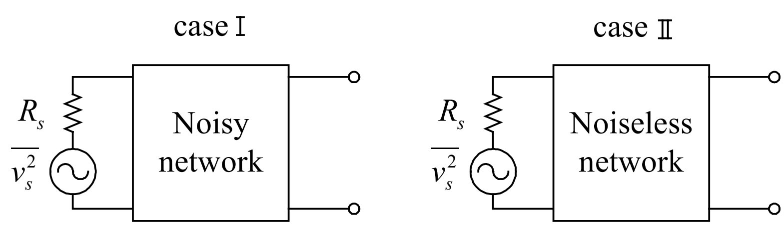




→ C_{gd} actually reduces the equivalent input noise at very high frequency

Noise Figure and Noise Temperature

Noise Figure → shows how noisy a device / amplifier is

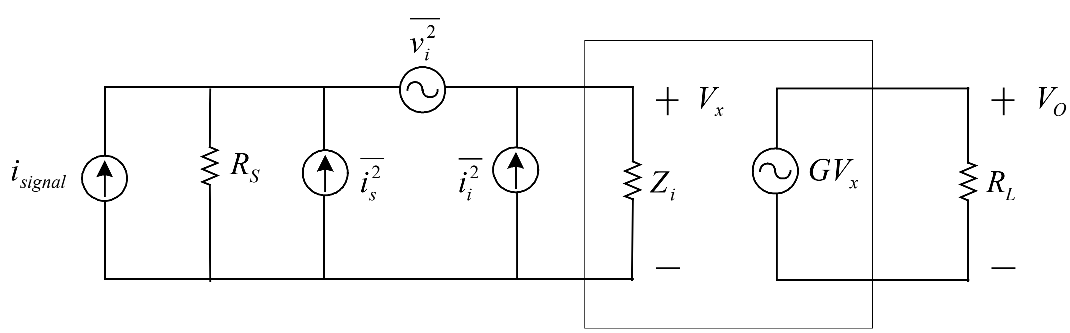


$$\text{noise figure} = \frac{\text{available noise power at the output in case Ⅰ}}{\text{available noise power at the output in case Ⅱ}}$$

you can also show noise figure in terms of S/N ratio

$$\text{noise figure} = F = \frac{\text{input S/N ratio}}{\text{output S/N ratio}}$$

we can calculate the noise figure for a noisy network with equivalent input noise current and voltage



$$V_x = V_i \frac{Z_i}{Z_i + R_S} + i_i \frac{R_S Z_i}{R_S + Z_i} + i_s \frac{R_S Z_i}{R_S + Z_i}$$

assuming uncorrelated current / voltage noise source noise power in R_L :

$$N_{out\ noisy} = \frac{|G|^2}{R_L}\overline{v_x^2} = \frac{|G|^2}{R_L}\left(\overline{v_i^2}\frac{|Z_i|^2}{|Z_i + R_S|^2} + \left(\overline{i_i^2} + \overline{i_s^2}\right)\frac{|R_S Z_i|^2}{|R_S + Z_i|^2}\right)$$

$$N_{out\ noiseless\ \ network} = \frac{|G|^2}{R_L}\overline{i_S^2}\frac{|R_S Z_i|^2}{|R_S + Z_i|^2} \quad , \quad \overline{i_s^2} = \frac{4kT\Delta f}{R_S}$$

$$\text{noise figure} = F = \frac{N_{out\ noisy}}{N_{out\ noiseless\ network}} \longrightarrow$$

$$F = 1 + \frac{\overline{v_i^2}}{4kTR_S\Delta f} + \frac{\overline{i_i^2}}{4kT\frac{1}{R_S}\Delta f}$$

$\longrightarrow$ noise figure depends on Rs and input current / voltage noise sources

There is an optimum Rs that results in a minimum F

$$\frac{\partial F}{\partial R_S} = 0 \quad \rightarrow \quad \frac{\overline{v_i^2}}{4kT\Delta f}\left(\frac{-1}{R_{S\ opt}^2}\right) + \frac{\overline{i_i^2}}{4kT\Delta f} = 0$$

$$R_{S\ opt}^2 = \frac{\overline{v_i^2}}{\overline{i_i^2}}$$ optimum source impedance for minimum noise

for MOS transistor (at very high frequency)

$$R_{S\,opt}^2 = \frac{4kT\delta R_g \Delta f + \left(1+\dfrac{C_{gs}}{C_{gd}}\right)^2 r_g^2 \times 4kT\gamma g_m \Delta f}{\left(1+\dfrac{C_{gs}}{C_{gd}}\right)^2 \times 4kT\gamma g_m \Delta f}$$

$$R_{S\,opt}^2 = r_g^2 + \frac{\delta}{\gamma}\cdot\frac{R_g}{g_m}\times\frac{1}{\left(1+\dfrac{C_{gs}}{C_{gd}}\right)^2}$$

$$\boxed{R_{S\,opt} = \sqrt{r_g^2 + \frac{\delta}{\gamma}\cdot\frac{R_g}{g_m\left(1+\dfrac{C_{gs}}{C_{gd}}\right)^2}}} \longrightarrow$$

optimum source res. for min. noise figure

in reality there is a source impedance (and not just a resistance) that results in a minimum noise

$$F = F_{\min} + \frac{r_n \left| y_s - y_{opt} \right|^2}{g_s}$$

F = noise figure

F_{min} = minimum noise figure

$r_n = \frac{R_n}{Z_0}$ normalized noise resistance

$\longrightarrow$ characteristic impedance of the system (usually 50Ω)

y_{opt} : optimum source admittance for minimum noise

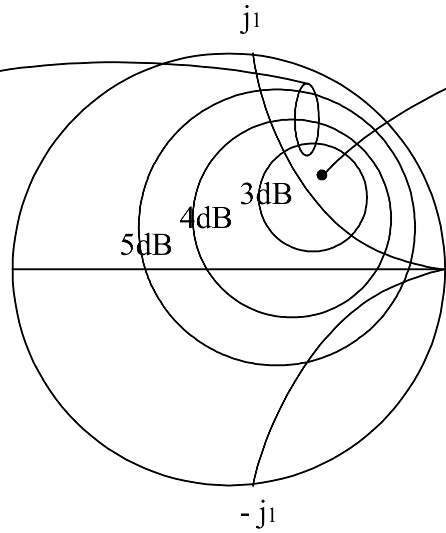
y_s : actual source admittance

g_s : real part of y_s

use $y_s = \frac{1-\Gamma_S}{1+\Gamma_S}$ *then*

$$F = F_{\min} + \frac{4 r_n \left| \Gamma_S - \Gamma_{opt} \right|^2}{\left(1 - \left| \Gamma_S \right|^2\right)\left(1 + \Gamma_{opt}\right)^2} \longrightarrow \text{equation of a circle}$$

constant
noise circles



$\Gamma_S = \Gamma_{opt}$ *where you get* $F = F_{\min}$ (*let's say* $2dB$)

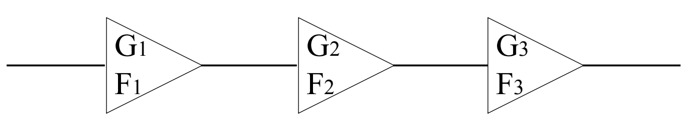
r_n : shows the rate of degradation of noise figure as your source matching deviates from optimum point

if you are at Γ_{opt}
you don't care what r_n is
but if you cannot mater to Γ_{opt}
you want small r_n match

you can find noise figure of cascaded amplifiers

$$F_{total} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots$$

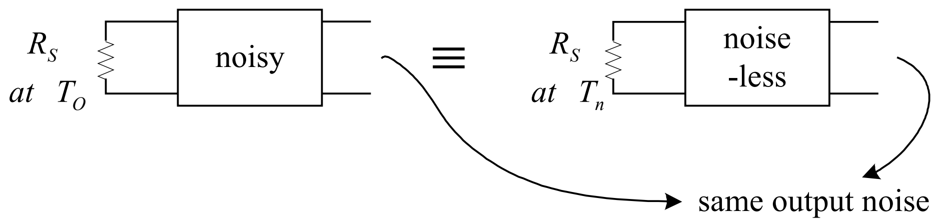
available gain (we will define this later)



Noise temperature

is an alternative way of noise representation

Noise Temp T_n :



Turns out that noise temp. is related to noise figure

$$\frac{T_n}{T} = F - 1 \longrightarrow$$

ratio not dB

noise factor = F - 1

3dB noise figure → F=2
T_n = 290k

0.4dB noise figure → F=1.1
T_n = 29k

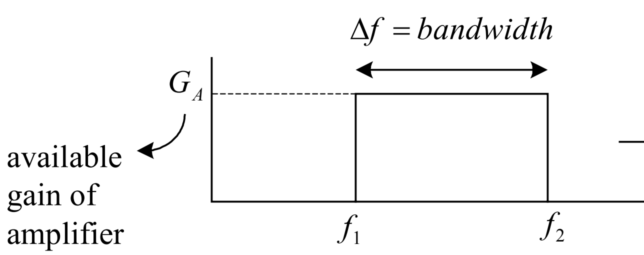
Minimum Detectable Signal (MDS)

also known as noise floor of the system

Important observation

An input signal is detectable only if its output is above noise level

To calculate MDS you assume your amplifier / device has an ideal box shape transfer function



$$S_{out} = G_A \cdot S_{in}$$

output signal

gain

input signal

$$N_{out} = F \cdot G_A \cdot N_{in}$$

noise figure since amplifier adds noise

We know that for thermal noise

$$(v_n \ \ noise \ rms \ value)$$

$$\overline{v_n^2} = 4kTR_g\Delta f \qquad \rightarrow \qquad \frac{\overline{v_n^2}}{4R_S} = kT\Delta f = N_{in} \ : thermal \ noise \ power \ at \ input$$

$$\frac{\overline{v_n^2}}{2R_S} : total \ noise$$

$$\frac{v_n^2}{4R_S} : available \ noise$$

$$N_{out} = FG_A \underbrace{kT\Delta f}_{N_{in}}$$

Now to be able to detect your output power has to be above your output noise by X_ dB typical value for X is 3dB

$$\frac{P_{out}}{N_{out}} = 2 \ (3dB)$$

$$\longrightarrow \quad P_{out} = 2FG_AkT\Delta f$$

$$P_{in} = \frac{P_{out}}{G_A} = 2FkT\Delta f$$

$$10\log P_{in} = F(dB) + 10\log kT + 10\log \Delta f + 3$$

$$T = 290\,k$$

$$10\log P_{in} = F - 174\,dBm + 10\log\ \Delta f(Hz) + 3$$

$$MDS = v_{in} = \sqrt{4R_S \times P_{in}} \qquad\qquad R_S\ \ tpically\ 50\Omega$$

Example

Determine MDS of an amplifier with 8 dB noise figure

and bandwidth of 2.1 KHz

$$\begin{aligned}10\log P_{in} &= 8 - 174\,dBm + 10\log\left(2.1\times10^{3}\right) + 3\\ &= -130\ dBm\end{aligned}$$

$$P_{in} = 10^{-13}\ \ mW = 10^{-16}\ W$$

$$\begin{aligned}MDS &= \sqrt{4\times R_S \times P_{in}} = \sqrt{200\times10^{-16}}\\ &= \sqrt{2\times10^{-14}} \quad = 1.41\times10^{-7}\ V\ = 0.14\ \mu V\end{aligned}$$

MDS is the minimum limit of detected signal

What is the maximum limit of the signal

→ distortion determines the maximum level permitted in your signal

→ Signal distortion depends on the nonlinearity of your amplifier