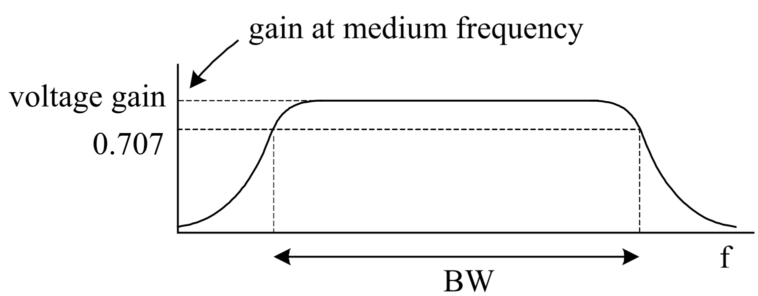
Bandwidth Estimation

A lot of amplifiers that we design are wide bandwidth amplifiers

→ they show gain over a wide frequency range

→ a few octave sometimes a few decade



The 1st question is how to estimate the bandwidth

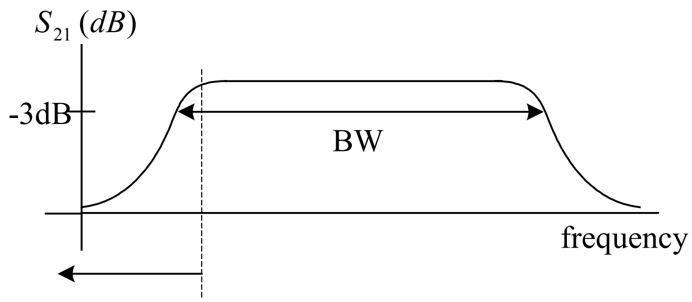
1. from measurement ⟶ frequency measurement / time domain measurement
2. from hand calculation ⟶ of course we can do spice simulation (in our case spectre) and find the predicted performance but that does not give us insight into the design

⟶ So we are looking into a better way of understanding limits of our design

Measurement of bandwidth

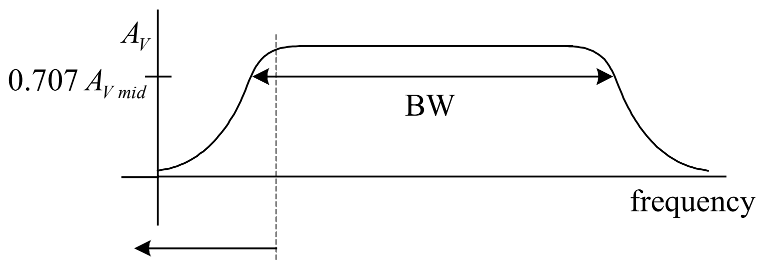
Take a network analyzer and measure S-parameters

$plot\ \ S_{21}\ \ \omega rt\ \ frequency$



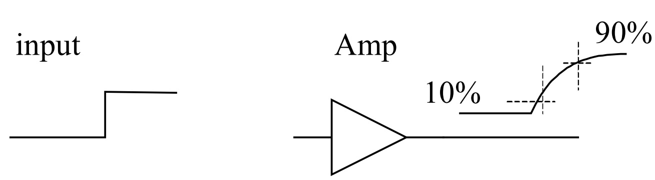
often network analyzer do not work at very low frequency so we may not see low-frequency roll-off regime

from S-parm → calculate h-parm → calculate voltage gain



Question: Can we measure bandwidth with an oscilloscope ? → time domain measurement
The answer is yes you can but how?

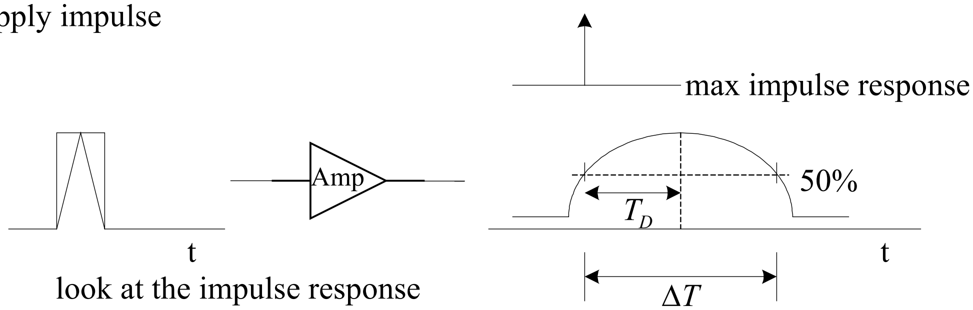
from tome domain, measure rise time to a step input



t_r : rise time → time it take for the signal to go from 10% to 90% of its final value
of course this definition is completely arbitrary

Another rise time definition (Elmore rise time)

Apply impulse

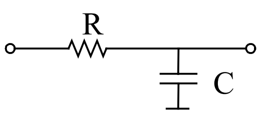


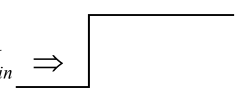
look at the impulse response

time duration that impulse response is greater than 50% of its maximum value

→ this is called Elmore time delay

Take a simple RC network



$t_{rise\,10\%\to 90\%}$ $\qquad V_{in} \Rightarrow$ 

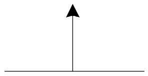
$$V_{out} = 1 - \exp\left(-t/RC\right)$$

$$V_{out\ final} = 1$$

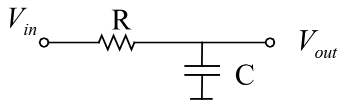
$$t_{10\%} = RC\ \ln(0.1)$$
$$t_{90\%} = RC\ \ln(0.9)$$

$$\longrightarrow \quad t_{rise\,10\%\to 90\%} = t_{90\%} - t_{10\%} = RC\ln\left(\frac{0.9}{0.1}\right) = 2.2RC$$

$$\boxed{t_{rise\,10\%\to 90\%} = 2.2RC}$$



$V_{in} = \delta(t)$



$$\frac{V_{out}}{V_{in}}(f) = \frac{1}{1 + jRC \times 2\pi f} = H(f)$$

now calculate Elmore rise time

$$t_{rise}^{\;2} = \frac{4}{(2\pi)^2 H(0)}\left[-\frac{d^2}{df^2} H(f)\bigg|_{f=0} - \frac{1}{H(0)}\left(\frac{d}{df} H(f)\bigg|_{f=0} \right)^2 \right]$$

Elmore' s
rise time

approximation

$$H(f) = \frac{1}{1 + j2\pi RCf}$$

$$\frac{dH}{df} = \frac{-j2\pi RC}{\left|1 + j2\pi RC\right|^2} \quad \rightarrow \quad \frac{dH}{df}\bigg|_{f=0} = -j2\pi RC \quad \rightarrow \quad \left(\frac{dH}{df}\bigg|_{f=0} \right)^2 = -4\pi^2 R^2 C^2$$

$$\frac{d^2 H}{df^2} = \frac{-2(2\pi RC)^2}{\left|1 + j2\pi RCf\right|^3} \quad \rightarrow \quad \frac{d^2 H}{df^2}\bigg|_{f=0} = -2(2\pi RC)^2$$

$$t_{rise}^{\;2} = \frac{4}{(2\pi)^2}\left(2(2\pi RC)^2 - (2\pi RC)^2\right) = (2RC)^2$$

$$\boxed{t_{rise} = 2RC} \quad \longleftarrow \quad \text{Elmore rise time}$$

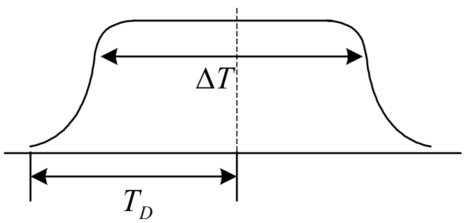
* one more thing: Elmore rise time for cascaded system
use approximation formula

$$t_{rise_{total}}^{\;2} = \frac{4}{(2\pi)^2 H_1(0) H_2(0)}\left[-\frac{d^2}{df^2} H_1 H_2\bigg|_{f=0} \quad \frac{1}{H_1(0) H_2(0)}\left(\frac{dH_1 H_2}{df}\bigg|_{f=0} \right)^2 \right]$$

$$\Rightarrow \boxed{t_{rise_{total}}^{\;2} = t_{rise1}^{\;2} + t_{rise2}^{\;2}} \quad \longrightarrow \quad \text{applies for Elmore rise times}$$

How about delay time

$$T_D = \frac{1}{j2\pi H(0)} \frac{dH(f)}{df}\bigg|_{f=0}$$



$\Delta T = t_{rise}$

T_D : Elmore delay time

for the case of cascaded systems

$$T_{D\,total} = -\frac{1}{j2\pi H_1(0)H_2(0)} \frac{d}{df} H_1 H_2\bigg|_{f=0}$$

$$= -\frac{1}{j2\pi H_1(0)H_2(0)}\left[H_1(0)\frac{d}{df}H_2\bigg|_{f=0} - H_2(0)\frac{d}{df}H_1\bigg|_{f=0}\right]$$

$$\Rightarrow \quad T_{D\,total} = T_{D1} + T_{D2}$$

* Overall Delay of Cascaded System is the sum of the individual delays

$$T_D = \Sigma T_D$$

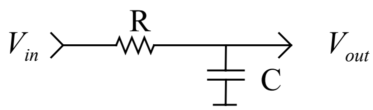
* Overall Rise time of Cascaded System is the root-sum-squared of the individual rise times

$$T_{rise\,total} = \sqrt{\Sigma T_{rise}{}^2}$$

Now that we have measured the rise time

how does it relate to Bandwidth

example RC low pass network



$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + j2\pi fRC}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = 0.707 = \left|\frac{1}{1 + j2\pi RC \times BW}\right| \quad \rightarrow \quad BW = \frac{1}{RC}$$

$$\longrightarrow \quad BW_{-3dB} \cdot t_{rise_{10\% \rightarrow 90\%}} = 2.2$$

$$BW_{-3dB} \cdot t_{rise_{Elmore}} = 2$$

10% error

So if we measure the rise time we can estimate the bandwidth

$\longrightarrow$ but this calculation was just done for 1-pole RC network

what if we have a more complex system

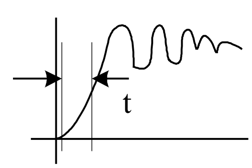
Take a 2-pole system

2-pole transfer function

$$H(s) = \frac{1}{\dfrac{s^2}{\omega_n^{\,2}} + \dfrac{2\xi s}{\omega_n} + 1}$$

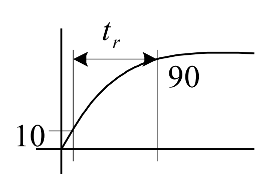
$\xi \longrightarrow$ damping factor

small ξ
undampped
step response



large ξ

over damped



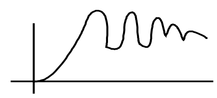
Bandwidth

$$\omega_h = \omega_n \left[1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}\right]^{1/2} \longrightarrow \text{3 dB BW}$$

Output response to step input

$$V_0(t) = 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \times \sin\left(\sqrt{1-\xi^2}\,\omega_n t + \phi\right)$$

$$\phi = \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right]$$

$\xi = 0$ (*zero damping*) $\longrightarrow$ 

$$t_{rise} = \frac{1}{\omega_n}\left[\sin^{-1} 0.9 - \sin^{-1} 0.1\right] \approx \frac{1.02}{\omega_n}$$

$$\omega_h \Big|_{\xi=0} = \omega_n \left(1 + \sqrt{2}\right)^{1/2} = 1.55\,\omega_n$$

$\Rightarrow \quad t_{rise}\omega_h = 1.6 \quad \rightarrow \quad 37\%$ *error*

still not bad

$\xi = \frac{1}{\sqrt{2}}$ (*well damped system*)

$$\left.\begin{aligned} t_{rise} &\approx \frac{2.14}{\omega_n} \\ \omega_h &= \omega_n \end{aligned}\right\} \longrightarrow \omega_h t_{rise} = 2.14 \quad < \quad 3\% \ \textit{error}$$

pretty good
estimation

in genetal

$$2 < \quad BW \times t_{rise} \quad < 2.2$$

as long as the step response is monotonic $\rightarrow$ not oscillating

from
$$\begin{cases} t_{rise} \cdot BW = 2.2 \\ t_{rise_{total}} = \sqrt{t_{rise_1}{}^2 + t_{rise_2}{}^2 + \ldots} \end{cases}$$

we can calculate BW of cascaded systems

Assume identical amplifiers are cascaded with identical BW=BW_1

$$\Rightarrow \; total\ bandwidth = \frac{BW_1}{\sqrt{N}}$$

# of amplifier stages

Looking ahead

more exact analysis (we will see later)

$$BW_{total} = \frac{\sqrt{\ln 2}BW}{\sqrt{N}} = \frac{0.833\,BW}{\sqrt{N}}$$

Method of Open Circuit Time constant (we will see in a moment)

$$BW_{total} = \frac{BW}{N}$$

$\rightarrow$ Pesimistic estimate
this method is good when one pole
in the transfer function dominates

now how do you actually do rise time measurement using an oscilloscope to estimate BW?

* you need to know the rise time of input signal (step input almost)

* rise time of oscilloscope

→ then measure rise time (10% → 90%)

Actual rise time of your circuit

$$t_{rise_{actual}} = \sqrt{t^2_{rise\ measured} - t^2_{rise\ oscilloscope} - t^2_{rise\ input}}$$

according to Elmore rise time for cascaded systems that we just calculated

example $\longrightarrow$ $t_{rise\ measured} = 10\,n\sec$

65MHz oscope $\longrightarrow$ $t_{rise\ oscope} = 5\,n\sec$

$t_{rise\ input} = 3\ n\sec$

$$t_{rise\ ckt} = \sqrt{100-9-25}$$

$$= \sqrt{64} = 8\,n\sec$$

$$BW_{ckt} = \frac{2.2}{2\times\pi\times 8\,n\sec} = 44\,MHz$$

you can even use a 25 MHz oscope to measure the rise time of a 44 MHz bandwidth amplifier

$\longrightarrow$ <u>in this case</u>

$t_{rise\ oscope} \sim 14\,n\sec$

$t_{rise\ input} = 3\ n\sec$

$t_{rise\ measured}$ $\rightarrow$ would be $\sim 16.2\,n\sec$

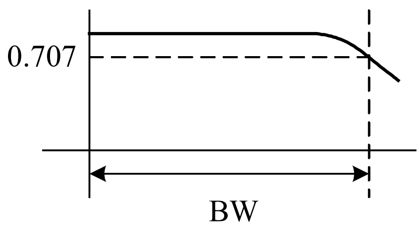
$$t_{rise\ ckt} = 8\,n\sec$$

now that we know how to measure / then estimate the bandwidth,

let's see if we can predict the bandwidth

Method of Open Circuit Time constants (OCT)

* applied to determine high frequency bandwidth



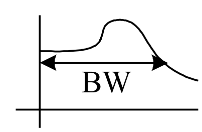
* results in error if we have resonance ckt (narrow band Amps)

* underestimates the bandwidth if we have several dominant poles in the transfer function

$$\left(BW_{total} = \frac{BW}{N} \quad \textit{instead of} \quad BW_{total} = \frac{BW}{\sqrt{N}} \right)$$

* underestimates the bandwidth if we have zero in the transfer function (peaking of gain)

* underestimates the bandwidth if we have complex pole



actual BW is increased by gain peaking
(adding zero to transfer function but
this method cannot detect it)

So this method is always pessimistic but becomes accurate
when we have one dominant pole

Advantage of this method is that it gives a very good
insight of what limits the bandwidth

so you can fix you design before you even
start doing blind CAD simulation

OCT method

consider all-pole transfer function

$$\frac{V_0(s)}{V_i(s)} = \frac{a_0}{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1), \ldots (\tau_n s + 1)}$$

multiplying out the denominator

$$b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + 1$$

do the following approximation

$$\frac{V_0(s)}{V_i(s)} \approx \frac{a_0}{b_1 s + 1} = \frac{a_0}{(\tau_1 + \tau_2 + \ldots + \tau_n)s + 1}$$

in this case the estimated bandwidth would be

$$\omega_h \approx \frac{1}{b_1} = \frac{1}{\tau_1 + \tau_2 + \ldots + \tau_n}$$

to find τ_1, , … τ_n you need to calculate the transfer function which is impossible for a complex amplifier design you can prove that the sum of these time constants is equal to the sum of $R_{jo}C_j$

when you consider one capacitance at a time and open-circuit the rest of them

so how this method works?

1. find the amplified equivalent ckt of your amplifier

2. get rid of capacitors that are not gain limiting at higher frequency
   (often coupling capacitors or caps $\parallel$ degenerate resistors)
   → bypass caps

3. for each capacitor calculate the effective resistance facing the capacitor
   while the rest of caps are open ckts (non-existent) R_{jo}

4. calculate $$\omega_h = \frac{1}{R_{1o}C_1 + R_{2o}C_2 + \cdots R_{no}C_n}$$

⟶ if you have inductors you can use L/R as the time constant

→ not always accurate

<u>note</u> you need to short ckt inductors

when you want to remove them from your calculation how accurate is this method when you have a dominant complex pole instead of a real pole

$$H(s) = \frac{1}{\frac{s^2}{\omega_n^{\ 2}} + \frac{2\xi s}{\omega_n} + 1}$$

OCT $\xrightarrow[\text{you}]{\text{gives}}$ $\omega_h = \frac{\omega_n}{2\xi}$

actual BW $\omega_h = \omega_n \left[1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}\right]^{1/2}$

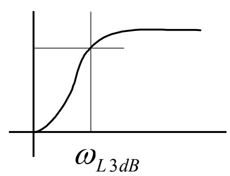
the accuracy depends on ξ

$\xi \to 0$ OCT $BW \uparrow\uparrow$
actual $BW = 1.55\,\omega_n$
$\to$ very wrong estimate for undampped transfer function

$\xi = 0.35$ OCT $\omega_h = 1.4\omega_n$
actual $BW \to \omega_h = 1.4\,\omega_n$
exactly the same

$\xi > 0.35$ $\to$ pessimistic results

so for good damped complex pole we get good/pessimistic estimates you can apply the same concept for estimating low frequency bandwidth (ω_L)



in this case we use the method of <u>Short Ckt Time constants (SCT)</u>

→ similar to OCT you can look at derivation in the book final results is

for SCT $$\omega_L = \frac{1}{R_{1s}C_1} + \frac{1}{R_{2s}C_2} + \cdots$$

so to measure R_{1s} which is facing C_1

short ckt all other capacitance, then calculate R_{1s}

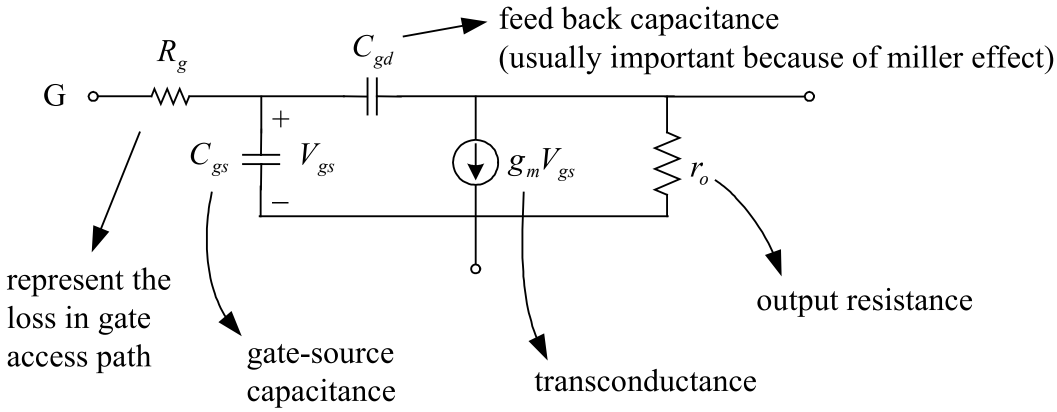
in this case you also need to simplify your equivalent ckt before you do the analysis

→ remove all of the internal caps of transistors as they are open ckt at very low frequency analysis

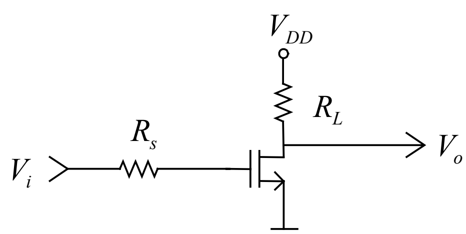
→ so only consider bypass, coupling caps

let's look at some examples

A simplified MOS model :



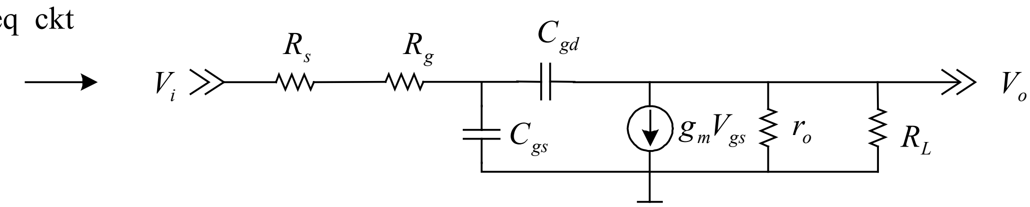
let' s look at a simple ckt such as the following



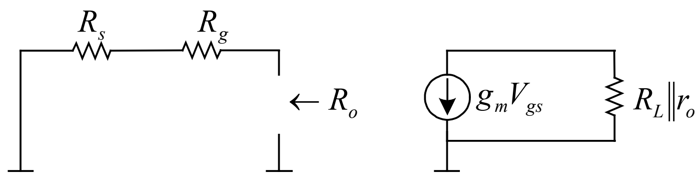
$$\omega_h = \frac{1}{\tau_1 + \tau_2 + \cdots}$$

open ckt time constants

eq ckt

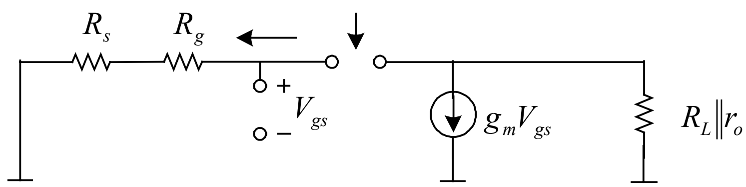


eq resistance for C_{gs} : → open ckt C_{gd}



$$R_o = R_s + R_g$$

eq ckt for C_{gd} : $R_L + R_s + R_g + g_m R_L (R_s + R_g)$



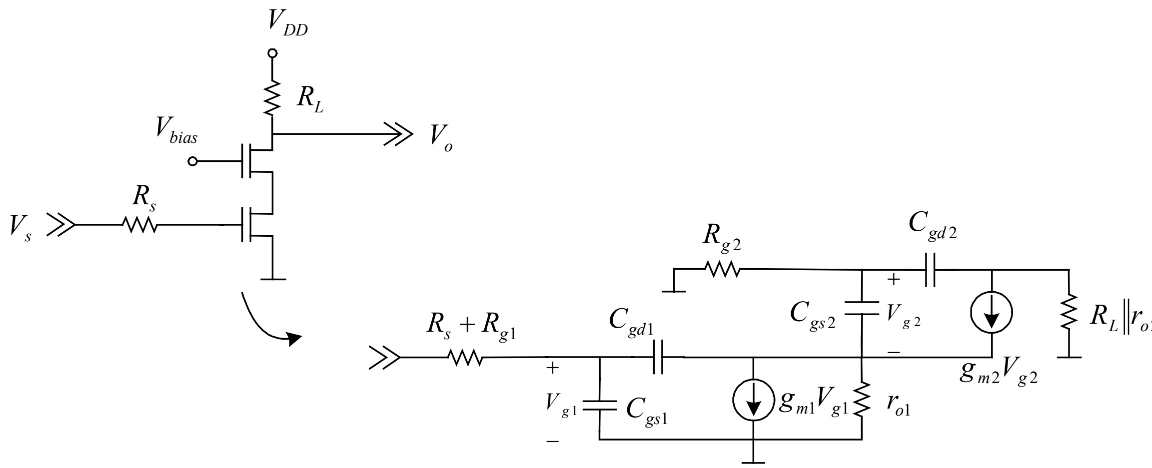
KVL, KCL → $V_{test} = I_{test}(R_s + R_g) + (I_{test} + g_m V_{gs}) R_L \| r_o$

$$V_{gs} = I_{test}(R_s + R_g)$$

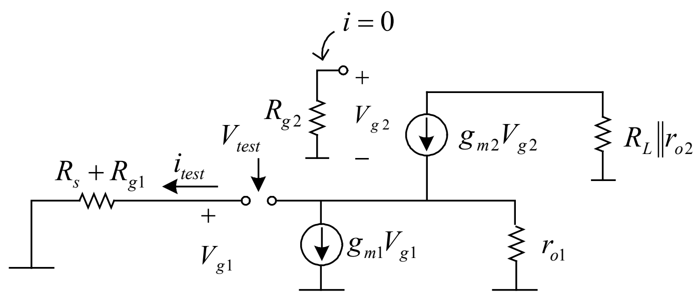
$$\longrightarrow \quad V_{test} = I_{test}(R_s + R_g) + I_{test}(1 + g_m (R_s + R_g)) R_L \| r_o$$

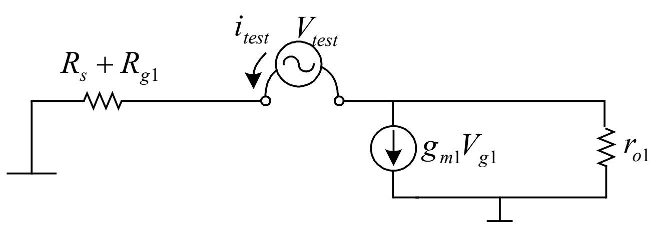
$$R_{o\ gd} = \frac{V_{test}}{I_{test}} = R_s + R_g + R_L \| r_o + (R_L \| r_o) g_m (R_s + R_g)$$

Cascode design



for C_{gd1} :





$$\| \frac{1}{g_{m2}} \approx \frac{1}{g_{m2}}$$

$$V_{test} = i_{test}\left(R_s + R_{g1}\right) + \left(i_{test} + g_{m1}V_{g1}\right)\frac{1}{g_{m2}}$$

$$V_{g1} = i_{test}\left(R_s + R_{g1}\right)$$

$$\longrightarrow \quad \frac{V_{test}}{i_{test}} = R_s + R_{g1} + \frac{1}{g_{m2}} + \frac{g_{m1}}{g_{m2}}\left(R_s + R_{g1}\right) = R_{ogd1}$$