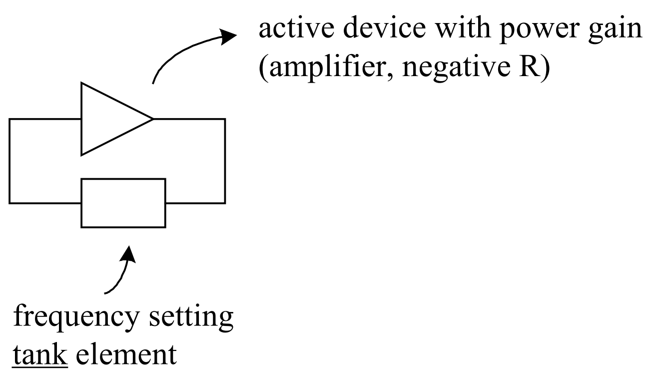
Oscillators and Synthesizers

Oscillators



Why do we need oscillators → to provide a stable frequency output

* useful for timekeepers
* useful for communications → mixing with local oscillator (LO)

Important attributes of oscillators

* long term stability
  - temp. coefficient of oscillation frequency
  - aging and drift

Types of active devices used for oscillators

* 3-terminal : MOSFET, BJT
* 2-terminal : negative resistance diodes
  (tunnel diode, Gunn, IMPATT, ⋯)

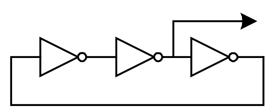
Types of tank element

* Quarter-wave Resonators
  Distributed elements with high $\frac{Volume}{Surface} = Q$
* Lumped LC tank
* Xtal
* SAW (surface acoustic wave devices)

Classification of ociillators

* near sinusoidal
  → low phase noise
  (e.g. high Q LC or Xtal Oscillators)
* Relaxation Oscillators
  → poor stability, but can have
  a very large tunning range
  (application → VCO)

  (e.g. multivibrator, Ring Oscillator)



Approaches to Oscillator Design

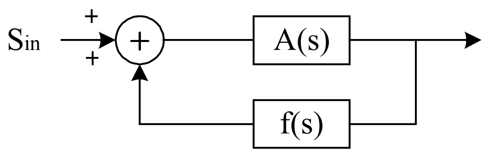
* negative R ⟶ 2-terminal active element oscillator

  used at very high microwave frequencies

  > 50 GHz

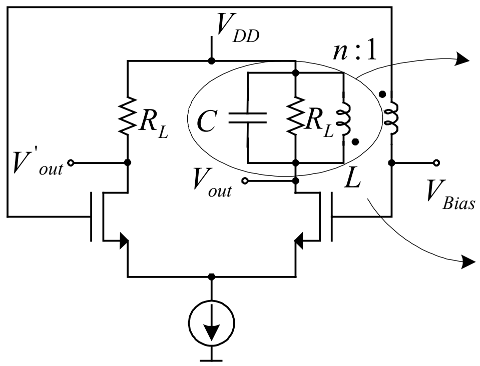
* feedback ⟶ 3-terminal active device

We will look at the feedback approach when dealing with MOS oscillators



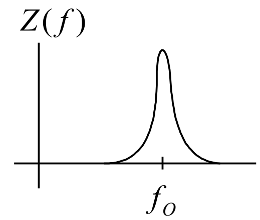
assuming linear for now

$$\frac{S_{out}}{S_{in}} = \frac{A(s)}{1 - A(s)f(s)}$$

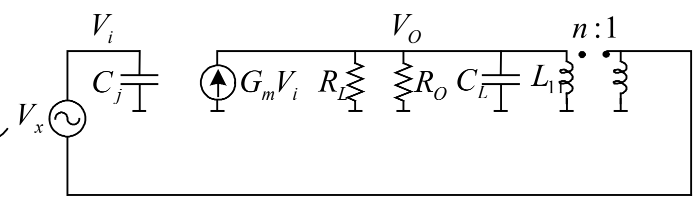


high Q tank ckt (very freq. selective impedance)

transformer is used for matching



AC equivalent



presence of noise starts the oscillation

source coupled pair

$$\begin{cases} C_i = C_{gs}/2 \\ G_m = g_m/2 \\ R_O = r_O \end{cases}$$

Let's calculate the overall transfer function :

$$\frac{V_O(s)}{V_i(s)}$$

* Since it is a feedback system

1) find open loop gain with feedback loading (A_l)

$$A_l = \frac{V_O}{V_i} = G_m \underbrace{Z_T}_{\text{tank ckt impedance}}$$

$$Z_T = \frac{1}{R} \quad \frac{\frac{1}{R}s}{1+\frac{L}{R}s+s^2LC}$$

where

$$L = L_{11}$$

$$C = C_L + \frac{C_i}{n^2}$$

$$R = R_O \| R_L$$

2) find feedback factor $f(s) = \dfrac{1}{n}$

$$\boxed{\frac{V_O(s)}{V_x(s)} = \frac{G_m L \cdot s}{1+\frac{L}{R}\left(1-\frac{G_m R}{n}\right)s+s^2LC}}$$

$\longrightarrow$ response of osc. Ckt to an input noise signal V(x)

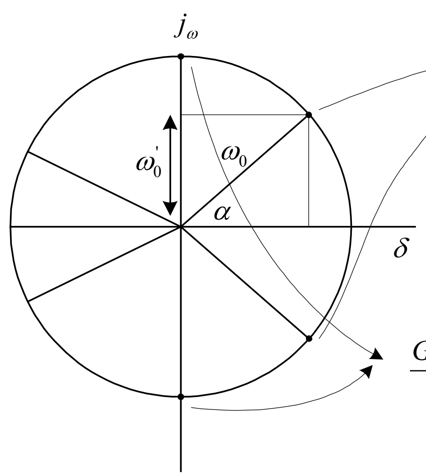
$$\frac{V_O(s)}{V_x(s)} = \frac{N(s)}{D(s)} = \frac{N(s)}{(s-P_1)(s-P_2)}$$

poles of the transfer function

$\dfrac{G_m R}{n} < 1$

poles in left-hand side

stable condition



$\dfrac{G_m R}{n} > 1$

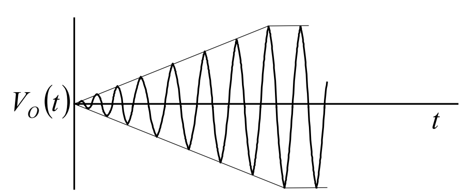
poles in RHS

unstable condition

$$V_O(t) = ke^{\alpha t}\cos(\omega_0' t + \beta)$$

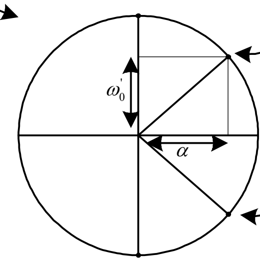
depend on initial condition

$\dfrac{G_m R}{n} = 1$



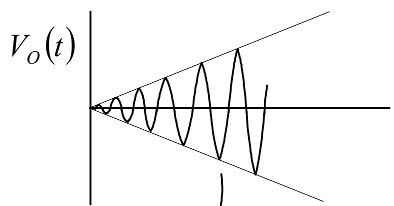
Oscillator response (Transfer function)

$$\frac{V_O}{V_x} = \frac{G_m L \cdot s}{1 + \frac{L}{R}\left(1 - \frac{G_m R}{n}\right)s + s^2 LC}$$



poles of the transfer function

for small signal $G_m = \frac{g_m}{2}$



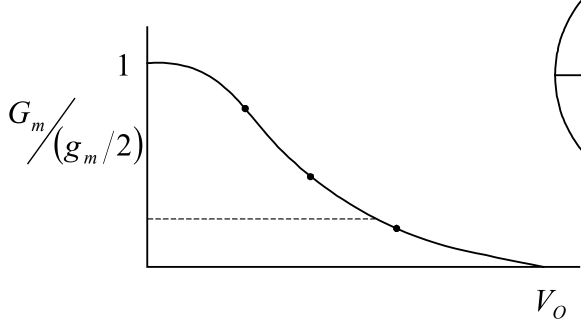
$$V_O(t) = ke^{\alpha t}\cos(\omega_0' t + \beta)$$

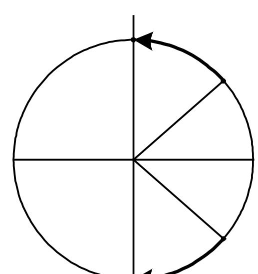
Therefore in the beginning the signal grows exponentially

until ⟶ $V_O(t)$ is no longer a small-signal for the oscillator

⟶ in large-signal operation G_m starts to decrease

$$G_m \Big|_{\max} = \frac{g_m}{2} = \text{ small-signal transconductance}$$



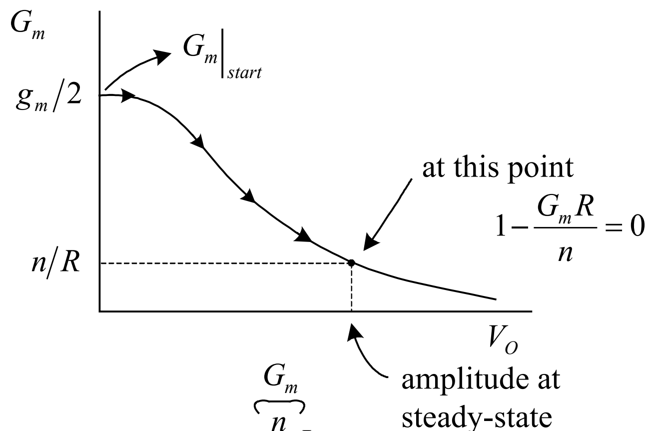


as $G_m \downarrow$ there poles starts to move until

$$1 - \frac{G_m R}{n} = 0$$

poles are now on imaginary access

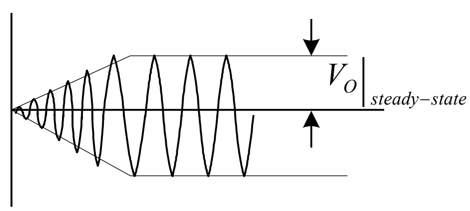
Once poles are on imaginary access the signal amplitude does not grow any more



$$\longrightarrow \quad \frac{V_O}{V_x}=\frac{\overbrace{\frac{n}{R}}^{G_m} L\cdot s}{1+s^2LC} \quad \longrightarrow \quad \text{signal reaches steady state}$$

current may be non-sinusoidal but very selective turned circuit tunes out the fundamental component at ω_0

$$\omega_0=\frac{1}{\sqrt{LC}}$$



typically $\dfrac{G_m\big|_{start}}{G_m\big|_{steady-state}} > 3$ to make sure

1. oscillation starts
2. enough amplitude
   for your near-sinusoidal

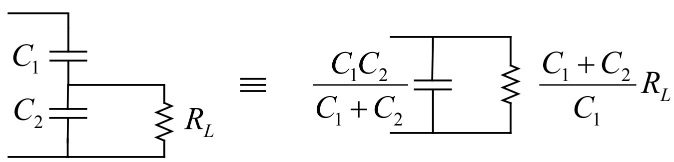
$$\frac{g_m/2}{n/R}>3 \quad \longrightarrow \quad g_m>\frac{6n}{R}$$

a function of device size (W/L) and also gate overdrive $\left(V_{gs}-V_T\right)$

Problem with transformers:

- * cannot be integrated
- * low frequancy
- * low Q

solution $\longrightarrow$ use capacitive transformer

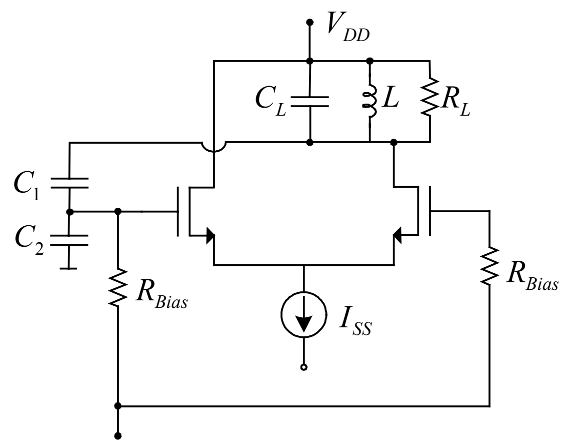


Source-coupled pair oscillator (at higher-freq.)

$$n = \frac{C_2 + C_1}{C_1}$$

$$C_2 = C_2 + C_{gs}$$

varies with
bias point
and process



V_{Bias} $\longrightarrow$ Can change Gm to make sure oscillator starts

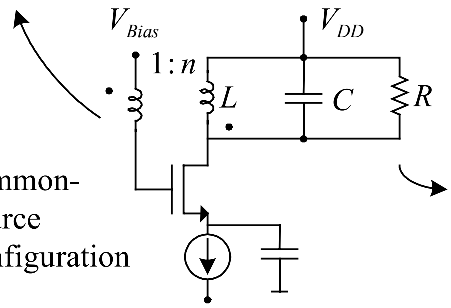
so we often design

$C_{gs} < C_2$ such that oscillator performance becomes independent of transistor parameters

<u>Single active device oscillators</u>

$180°$ phase shift

common-source configuration



(pierce configuration)

small-signal loop gain $\quad A_l = \dfrac{g_m R_L}{n}$

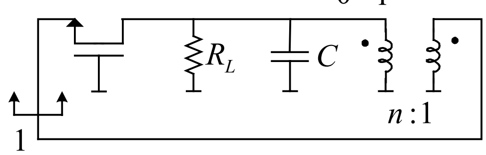
$R_L = R \parallel$ loading

large-signal loop gain $= \dfrac{G_m R_L}{n} = 1$

$$\frac{G_{mL}}{g_m} = \frac{1}{A_l} \quad \text{: describing function for single transistor oscillator}$$

$0°$ phase shift

common-gate configuration



$R_i = \dfrac{1}{g_m}$

small-signal loop gain :

initial loop gain $\longleftarrow$ $\quad A_l = \dfrac{g_m R_L}{n}\left(1 + \dfrac{g_m R}{n^2}\right)^{-1} \quad (1)$

typically > 3

steady-state large-signal loop gain :

$$\frac{G_m R_L}{n}\left(1 + \frac{G_m R_L}{n^2}\right)^{-1} = 1$$

$$\frac{G_m R_L}{n} = 1 + \frac{G_m R_L}{n^2}$$

$$1 = \frac{n}{G_m R_L} + \frac{1}{n} \quad \longrightarrow \quad \left(1 - \frac{1}{n}\right)\frac{G_m R_L}{n} = 1 \qquad (2)$$

$$(1) \longrightarrow \frac{g_m R_L}{n} = \frac{A_l}{1 - A_l/n}$$

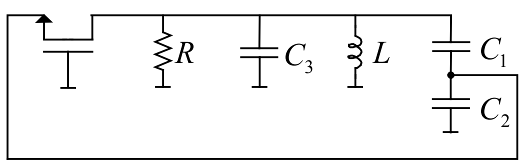
$$(2) \longrightarrow \frac{G_m R_L}{n} = \frac{1}{1 - 1/n}$$

divide $\longrightarrow$ $\dfrac{G_m}{g_m} = \dfrac{1}{A_l}\left(\dfrac{1 - A_l/n}{1 - 1/n}\right)$

<u>Another widely used oscillator :</u>

<u>Colpitts oscillator</u>

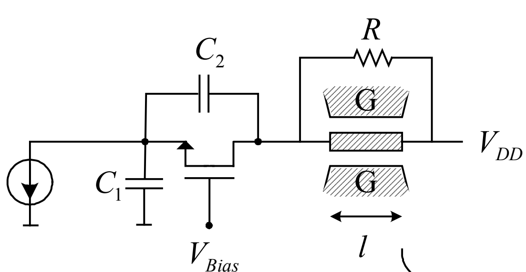
$$n:1 \rightarrow n = \frac{C_1 + C_2}{C_1}$$

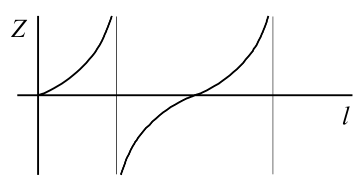


$$\omega_0 = \frac{1}{\sqrt{L\left(C_3 + \dfrac{C_1 C_2}{C_1 + C_2}\right)}}$$

you can use transmission line instead of inductor

(for higher Q and higher freq.)





$$Z = j Z_0 \tan\left(\frac{2\pi l}{\lambda}\right)$$

length of trans. line

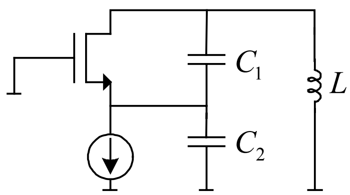
char. impedance

wave length

$\underbrace{\qquad}$
$Q \sim 100$ can be achieved on Si substrate

problem :
l is too long at low freq.

<u>Colpitts oscillator</u>

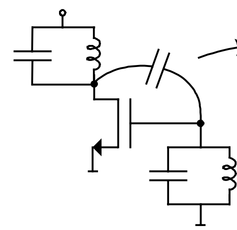


→ very good phase noise

source-drain feedback

Also Tito
tuned in, tuned out

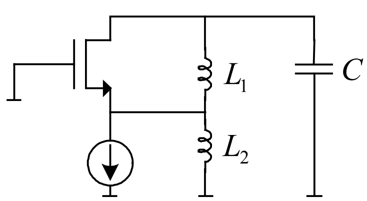
not
very
popular



a lot of cases
this feedback
is provided by
transistor

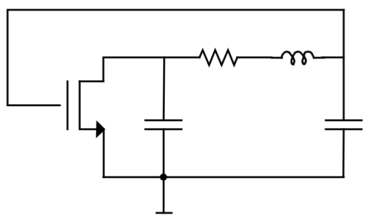
∵ need 2 L's

<u>Hartley Oscillator</u>



→ disadvantage
needs two
inductors

Pierce Oscillator

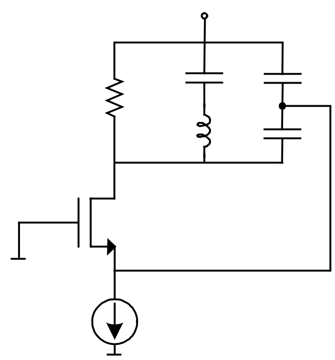


→ often used
for Xtal Oscillator

Clapp Oscillator

→ replace L with series LC

basically colpitts
with additional
capacitor tap



→ voltage across L can be higher than device breakdown voltage

→ better spectral purity

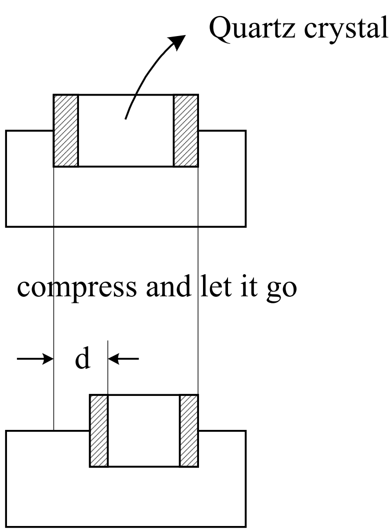
Quartz Crystals

Quartz : single cystal SiO_2

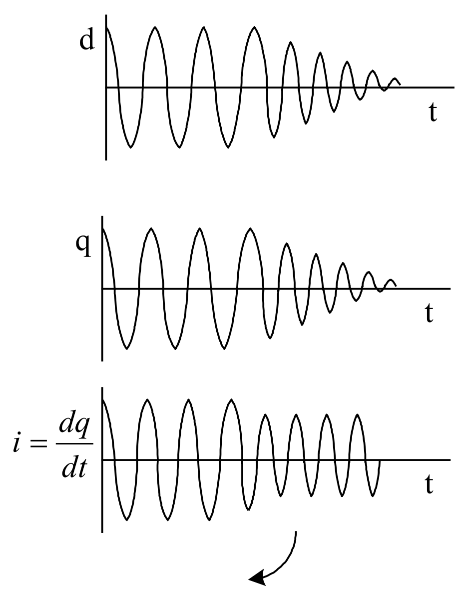
It is a piezoelectric material :

displacement d generates a charge q $\longrightarrow$ $q \alpha d$

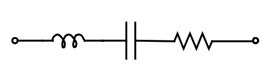
Experiment



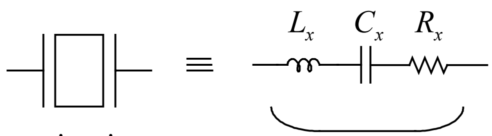
Quartz growth
takes several months
Quartz cannot be
integrated with Si
Process



you can get the
same response
from RLC



Equivalent ckt for a quartz crystal

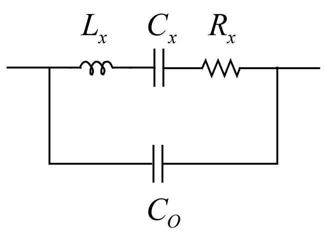


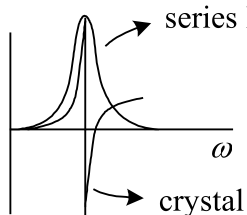
circuit
symbol

motional
element → subscript x shows
that they are motional

There is also a parallel
intrinsic capacitance C_O

very high Q can
be achieved



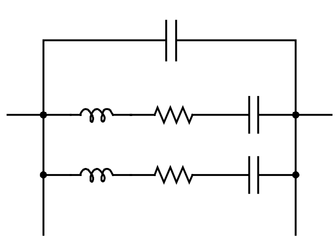


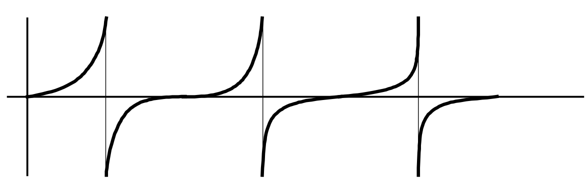
series LRC

crystal

the actual
response
you get

due to motional effects
in practice there are many
mechanical mode and
they are not necessarily
harmonics of each other



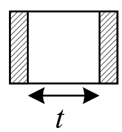


Xtal resonance frequency

Set by geometery and the speed of acoustic waves
in the material

$f_0 = \dfrac{1}{\eta}$, $\tau = \dfrac{2t}{v}$ ⟶ thickness of quartz blank

acoustic wave propagation velocity $\sim 3 \times 10^3 \, m/\text{sec}$



for $t = 1 \, mm$

$\eta = 0.67 \, \mu\text{sec}$

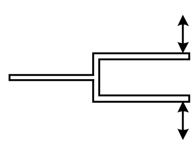
$f_0 = 1.5 \, MHz$

Practical limitation $\sim 200 \, MHz$

since it gets too thin and fragile

* for $f_0 > 50$ MHz use overtones

* for very low freq. use tunning forks



like watches
16 kHz

How accurately must they be cut?

Example : watch application

1 sec /month is acceptable

$$\because \quad \frac{1 \text{ sec}}{60 \times 60 \times 24 \times 31 \text{ sec}} = 0.4 \, ppm \quad \text{accuracy}$$

$f_0 = 1.5 \, MHz$ *need* $0.6 \, ppm$ accuracy in thickness

if $t = 1 \, mm \longrightarrow \Delta t = 0.6 \, ppm = 6 \, \mathring{A} \sim 2 \, atoms$

sequence of xtal fabrication process :

1. Polish them down to the ~ right thickness
2. Chemical etch
3. Deposit gold to get a fixed mass

This affects the freq.

Temperature coefficient of f_0

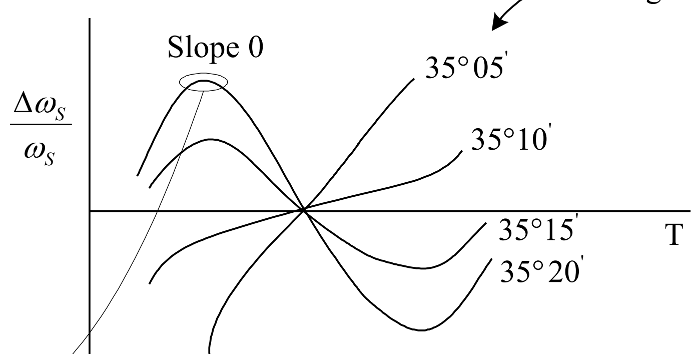
$$f_0 = \frac{1}{\eta} = \frac{v}{2t}$$ both have TCF ~ 14 $ppm/°C$

but not complete cancellation
in the end we get
certain cut angle

TCF/f_0 ~ 0.5 $ppm/°C$ average

for AT cut crystal

cut angle

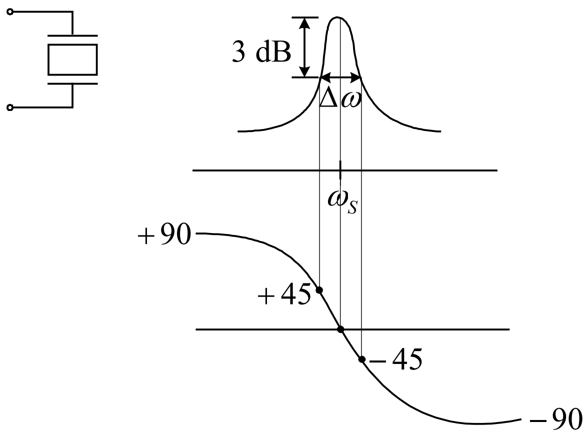


oven controlled OSC : used for very accurate synthesizer

you wait 15 min to warm up

what controls the overall temp coeff. of a high-Q oscillator

$$\Delta\omega = \frac{\omega_S}{Q} \longrightarrow \text{give } 90^\circ \text{ phase shift}$$



if $Q = 50{,}000$

$20\,ppm$ *freq. shift*
for 90° *phase shift*

Suppose active device in osc. has $\Delta\phi = 25^\circ$ phase shift
over $100^\circ C$ ΔT

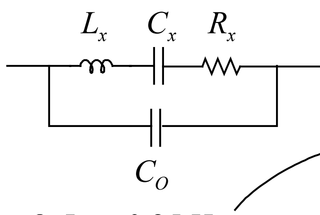
$\because$ Oscillator loop phase-shift has $\Delta\phi = 0.25^\circ/^\circ C$

This $\Delta\phi$ in loop phase-shift must be
compensated by xtal phase shift

$$\Delta\omega = \frac{20\,ppm}{90^\circ} \times 0.25^\circ/^\circ C = 0.06\,ppm/^\circ C$$

shift in freq. due to variation
in the phase of the active device

Conclusion : Oscillator temperature stability is set by crystal, not the active device



in micromechanical resonator
$L_x \sim 10^5\,H$!!

typical $\begin{cases} L_x = 0.25\,H \\ C_x = 0.04\,PF \\ R_x = 50\Omega \end{cases}$

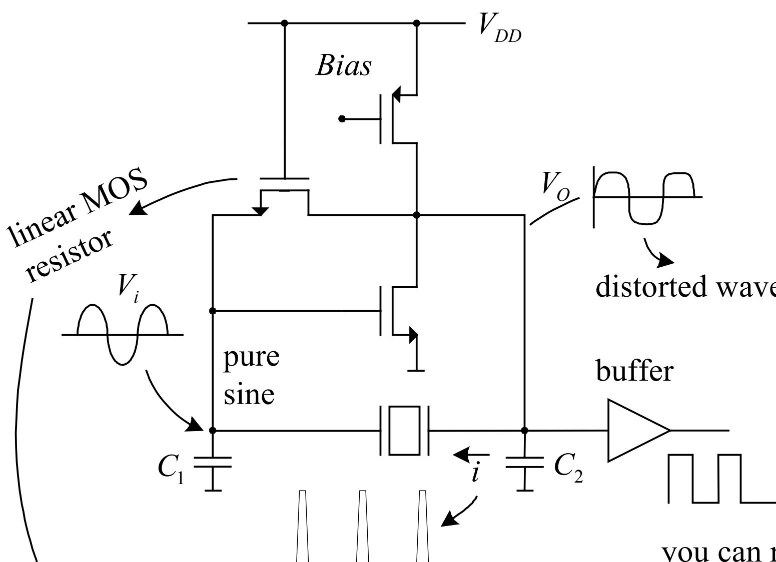
$$\omega_S = \frac{1}{\sqrt{L_x C_x}} \qquad Q = \frac{\omega_x L_x}{R_x}$$

Crystal Oscillator

Pierce, Colpitts, Clapp ...

→ best configuration for xtal osc.

Pierce xtal oscillator

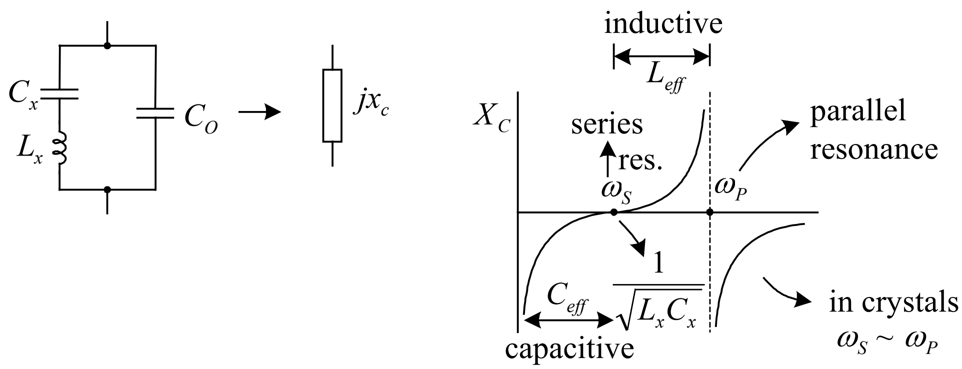


you can ratio C_1 and C_2 to get V_i, V_O with the same amplitude

want to be large
typical values
you get $R \sim 200\,k\Omega$ limited by geometery of the transistor → can get higher R if operates in weak inversion (subthreshold)

you can get substitute crystal with L_{eff}

<u>Closer look at crystal</u>

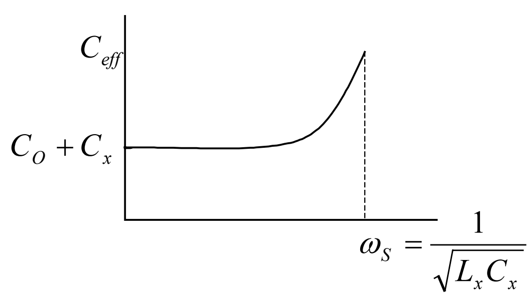


$$\textit{below} \quad \omega_S \longrightarrow \quad jx_c = \frac{1}{j\omega C_{eff}}$$

$$= \frac{1}{j\omega C_O} \parallel \left(\frac{1}{j\omega C_x} + j\omega L_x \right)$$

$$= \frac{1}{j\omega C_O} \parallel \left(\frac{1}{j\omega C_x} \left(1 - \omega^2 L_x C_x\right) \right)$$

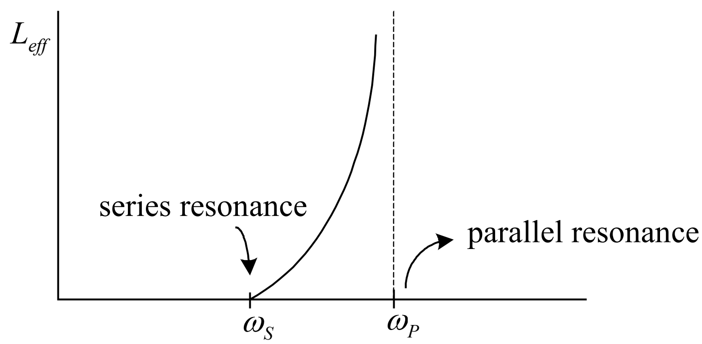
$$\boxed{C_{eff} = C_O + \frac{C_x}{1 - \omega^2 L_x C_x}}$$



between ω_S and ω_P

$$jx_c = j\omega L_{eff}$$

$$j\omega L_{eff} = \frac{1}{j\omega C_O} \,\Big\|\, j\omega L_x\left(1 - \frac{1}{\omega^2 L_x C_x}\right)$$



in crystals $\omega_S \sim \omega_P$

For pierce oscillator the oscillator freq. $\sim \omega_P$

in order to get 90° phase-shift

across the crystal

@ parallel resonance

$$\frac{1}{j\omega C_O} = -j\omega_P L_x\left(1 - \frac{1}{\omega_P^2 L_x C_x}\right)$$

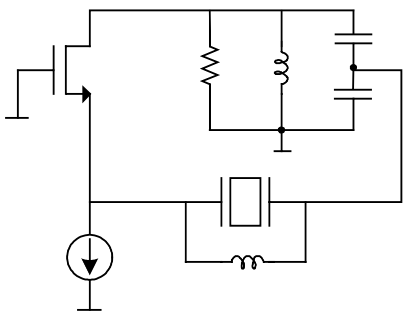
$$\boxed{\omega_P = \omega_S\sqrt{1 + \frac{C_x}{C_O}}}$$

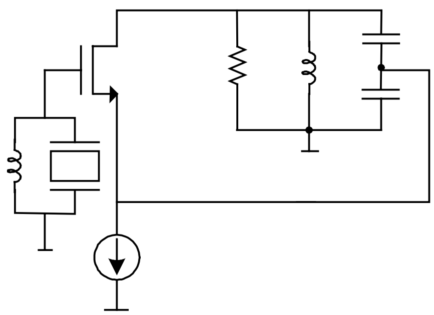
typical values : $C_x = 0.04\,pF \longrightarrow \omega_P = 1.005\,\omega_S$

$C_O = 4\,pF$

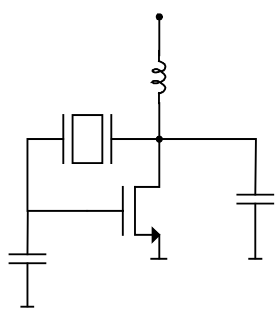
MOS Xtal oscillator

Colpitts





Pierce



Miller