ECE 654: Solid State Devices II

Chapter I
Power Gains, and Stability Analysis of Microwave Transistors

Introduction

We start our discussion by introducing Smith Chart which is the plot of reflection coefficient in the complex plain. Then we will discuss the importance of interconnects and the parasitics they introduce on the performance of a microwave transistor. Finally we will discuss various gain definitions as well as the concept of stability, device cut off frequency and maximum oscillation frequency.

Smith Chart

Smith chart is the plot of Γ in complex plane

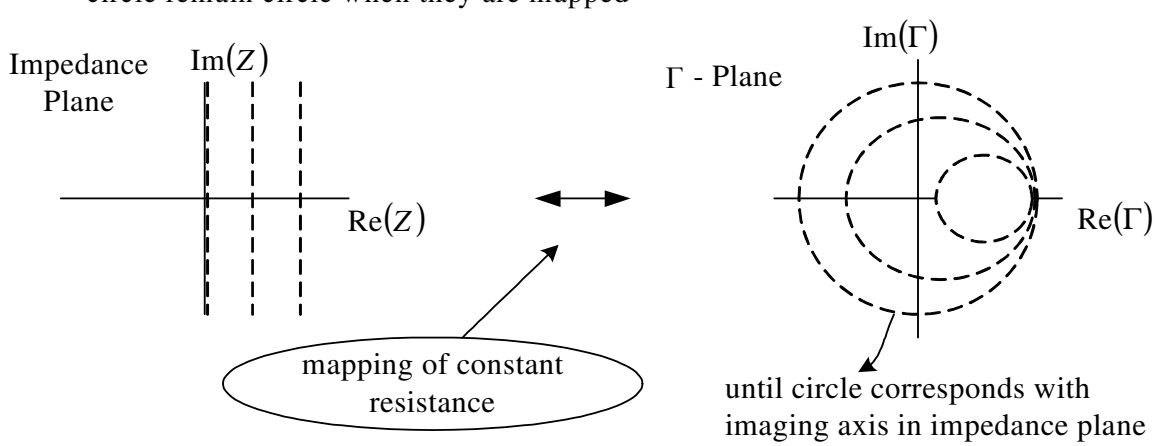
Note: Γ (the reflection coefficient) is defined as the ratio of reflected to incident signals.

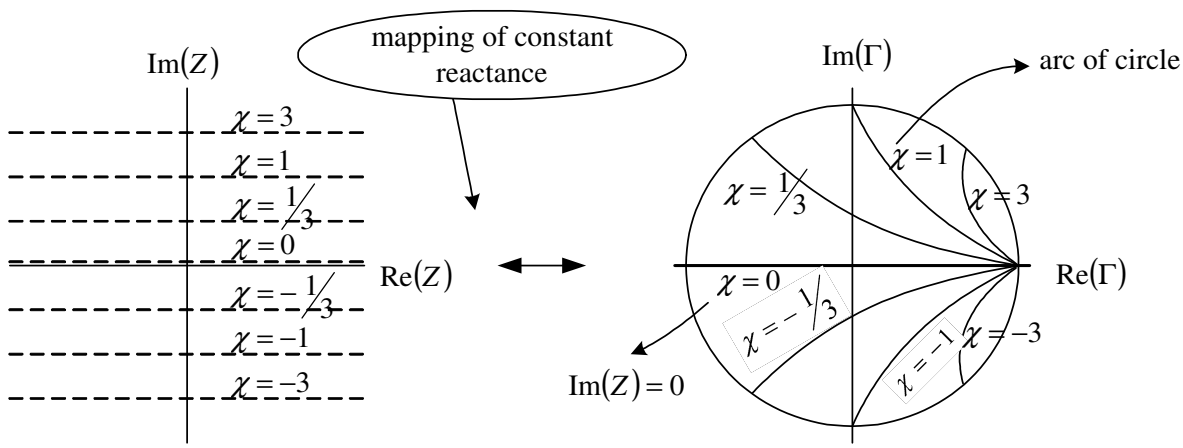
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{Z_{nL} - 1}{Z_{nL} + 1}$$

normalized impedance
with respect to characteristic impedance

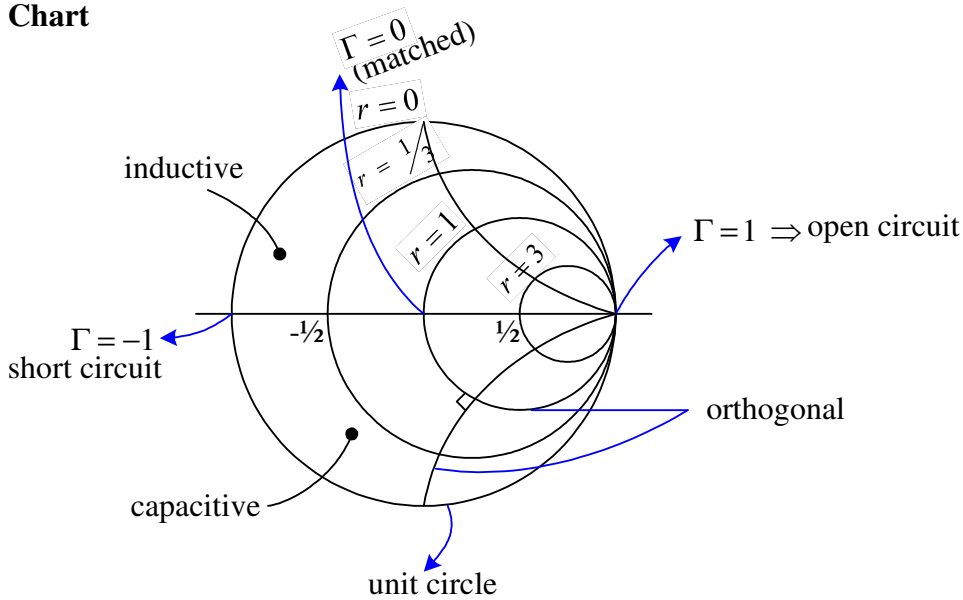
relationship between Γ and Z is a bilinear transformation

circle remain circle when they are mapped





Smith Chart



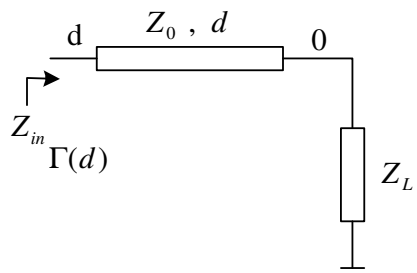
Note that for Γ=0, there is no reflected wave (matching condition to Z_0). For an impedance Smith chart, the bottom half of the circle represents capacitive impedances while the top half is associated with inductive impedances. Positive real values of impedance (positive resistances) reside inside the unity circle while negative resistances go outside of the circle.

It is also possible to draw Smith chart for admittances. In this case the locations of open and short and capacitive and inductive areas are reversed.

Smith chart can be used to calculate impedance transformation

↓

You can graphically find the transformed impedance



$\Gamma(x) = \Gamma_L e^{-2\gamma x}$

loss less → $\Gamma(d) = \Gamma_L e^{-2j\beta d}$

(βd) : *electrical length*

<u>to do so</u>

* *normalize* Z_L wrt Z_0 (Z_{nL})
* *locate* Z_{nL} *on the smith chart*
* *identify* $\Gamma_L = |\Gamma| e^{-j\theta}$
* *(clockwise) rotate* Γ_L *by twice the electrical length* $(2\beta d)$ *to find* Γ_{in}
* *find* Z_{nin}
* *convert it to* Z_{in}

* example

$$Z_L = 30 + j60\Omega$$

connected to 50Ω TRL impedance

TRL : ⟶ 2cm length @ 2GHz

⟶ assume phase velocity 50% of the speed of light

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 + j60 - 50}{30 + j60 + 50} = 0.2 + j0.6 = \sqrt{0.4} \cdot e^{j71°}$$

$$\Gamma(d = 2cm) = ? \qquad \beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_P} = \frac{2\pi f}{0.5c_0} = 83.7m^{-1}$$

$$2\beta d = 192°$$

$$\Gamma_L = \Gamma_0 e^{-j2\beta d} = -0.32 - j0.55 = \sqrt{0.4} \cdot e^{-j120°}$$

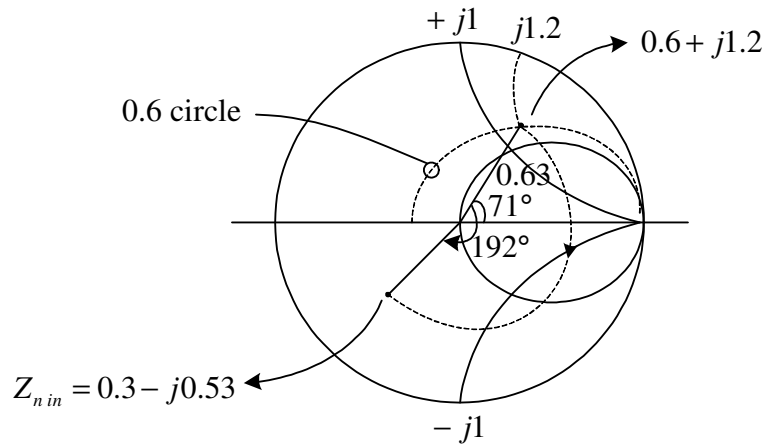
$$Z_{in} = Z_0 \frac{1+\Gamma}{1-\Gamma} = 14.7 - j26.7\Omega$$

difficult to calculate

now graphical solution using Smith Chart

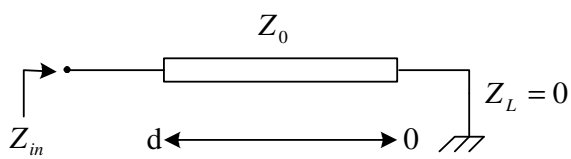
$$Z_L = 30 + j60 \quad \rightarrow \quad Z_{nL} = \frac{30 + j60}{50} = 0.6 + j1.2$$

$$2\beta d = 192°$$



$$Z_{in} = 50 \times (0.3 - j0.53)$$
$$= 15 - j26.5\,\Omega$$

<u>Standing wave</u>



assume a loss-less transmission line

↓

propagation constant is purely imaginary

incident reflected $\gamma = -j\beta$

$$V(x) = V^+\left(e^{-j\beta x} + \Gamma_0 e^{j\beta x}\right) = V^+ e^{-j\beta x}\left[1 + \Gamma_0 e^{+j2\beta x}\right] = V_{inc}\left(1 + \Gamma(x)\right)$$

incident voltage reflected voltage

$$I(x) = \frac{V^+}{Z_0}\left(e^{-j\beta x} - \Gamma_0 e^{j\beta x}\right) = \frac{V^+}{Z_0} e^{-j\beta x}\left[1 - \Gamma_0 e^{j2\beta x}\right] = \frac{V_{inc}}{Z_0}\left[1 - \Gamma(x)\right]$$

* incident and reflected wave make a standing wave along the line

→ at any point you can define standing wave ratio

$$SWR = \left|\frac{V_{\max}}{V_{\min}}\right| = \left|\frac{I_{\max}}{I_{\min}}\right|$$ → shows how bad is the mismatch

very often SWR is referred to us VSWR

↓

voltage standing wave ratio

$$V(x) = V^{+} e^{-j\beta x}\left[1 + \Gamma_0 e^{j\beta x}\right]$$

by varying x you can change the phase to reach max and min of voltage

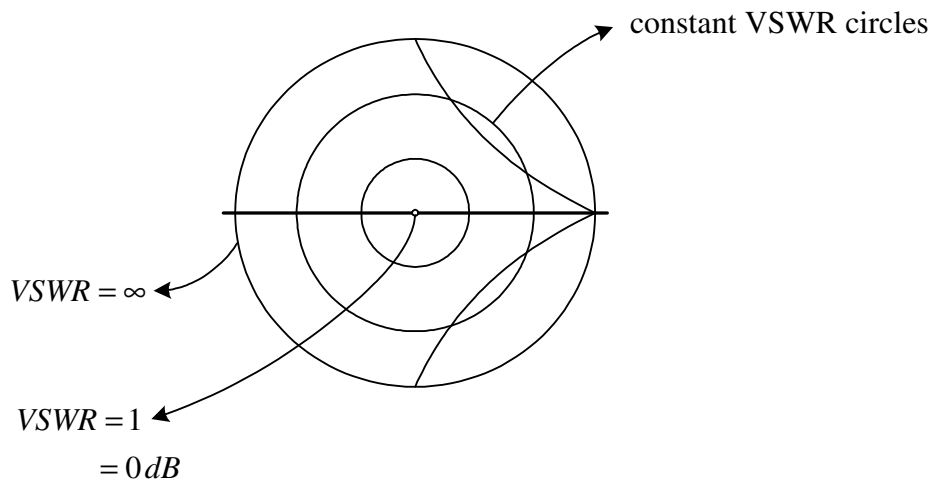
$\pm 1 \rightarrow$ max and min of this exponential function

$$VSWR = \left|\frac{V_{\max}}{V_{\min}}\right| = \frac{1+|\Gamma_0|}{1-|\Gamma_0|}$$

open & short ckts give VSWR $\rightarrow \infty$

matched condition $\Gamma_0 = 0 \rightarrow VSWR = 1$

VSWR is often expressed in dB



Impedance transformation

$\lambda/4$ line $\longrightarrow$ transforms short ckt to open ckt and vice versa
(quarter wave line)

$$\Gamma(x) = \Gamma_L e^{-2j\beta x}$$

$$\beta = \frac{2\pi}{\lambda}$$

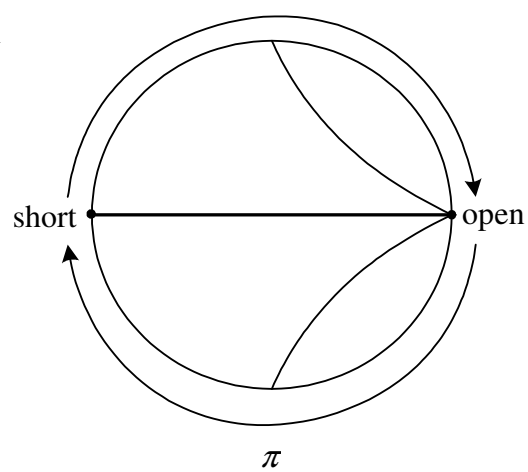
propagation constant

wavelength

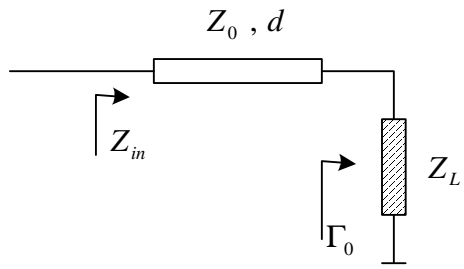
$$\Gamma(x) = \Gamma_L e^{-2j\times\frac{2\pi}{\lambda}x}$$

$$x = \frac{\lambda}{4}$$

$$\Gamma\left(\frac{\lambda}{4}\right) = \Gamma_L e^{-j\pi}$$



any given line:



$$Z_{in} = Z_0 \frac{1+\Gamma(d)}{1-\Gamma(d)}$$

work out the equations

$$\boxed{Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)}}$$

consider special cases

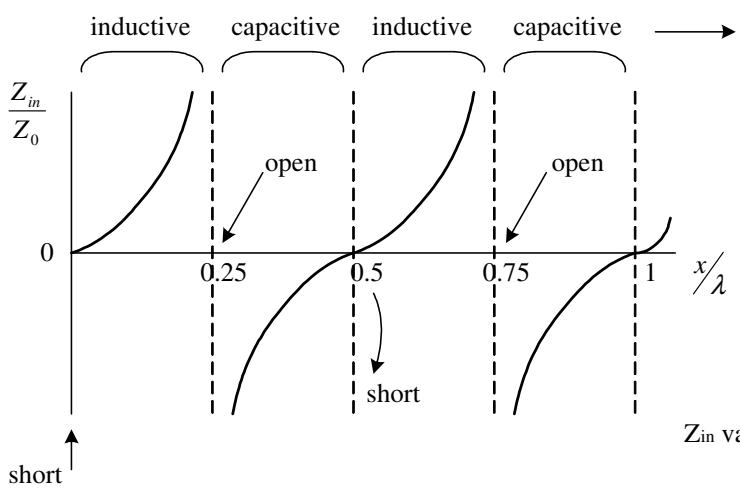
for quarter wave line

$$\boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$$

short ckt line

$$Z_L = 0 \quad \rightarrow \quad Z_{in} = Z_0 \frac{jZ_0 \tan(\beta d)}{Z_0} = jZ_0 \tan(\beta d) = jZ_0 \tan\left(\frac{2\pi d}{\lambda}\right)$$

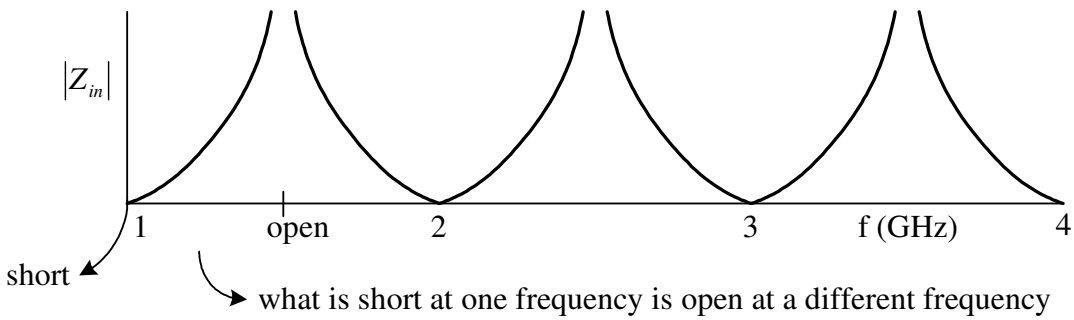
$$\beta = \frac{2\pi}{\lambda}$$



so you can make inductors/capacitors using shorted transmission line (at one frequency!)

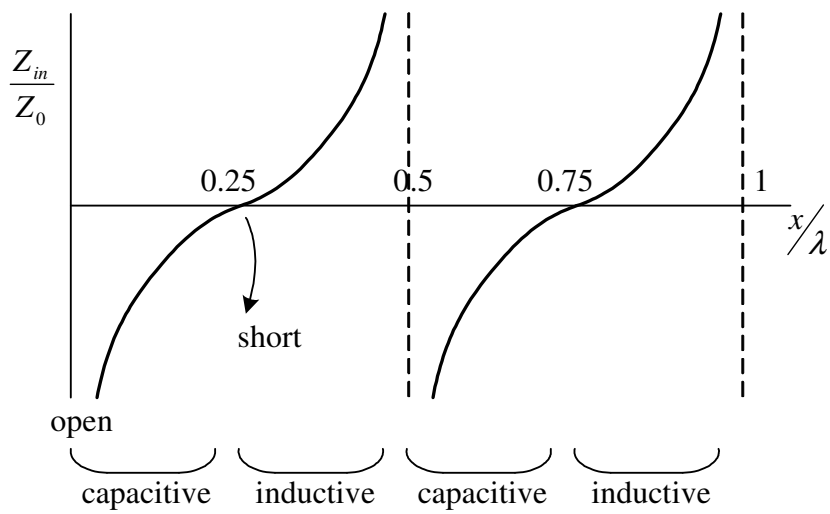
Z_{in} varies with frequency

$$Z_{in} = jZ_0 \tan(\beta d) = jZ_0 \tan\left(\frac{2\pi d}{v_P} f\right)$$



open circuit transmission line

$$Z_{in}(x) = -jZ_0 \frac{1}{\tan(\beta x)}$$

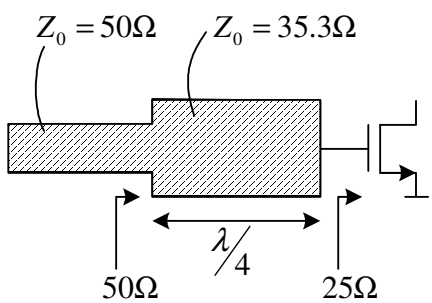


going back to quarter wave line

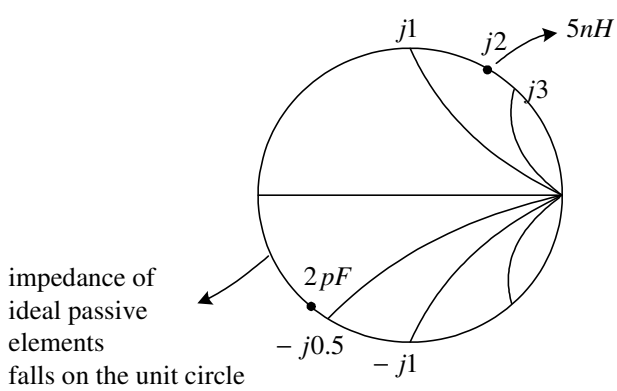
$$Z_{in} = \frac{Z_0^2}{Z_L} \longrightarrow \text{you can do matching}$$

assume $Z_L = 25\Omega$ we want to match it to 50Ω

$$Z_{line}\left(\lambda/4\right) = Z_0 = \sqrt{Z_L Z_{in}} = \sqrt{25\times 50} = 35.3\Omega$$



Smith chart and passive elements



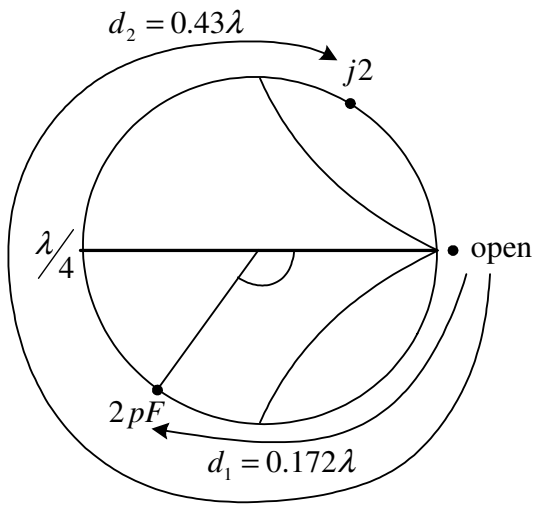
say at 3 GHz

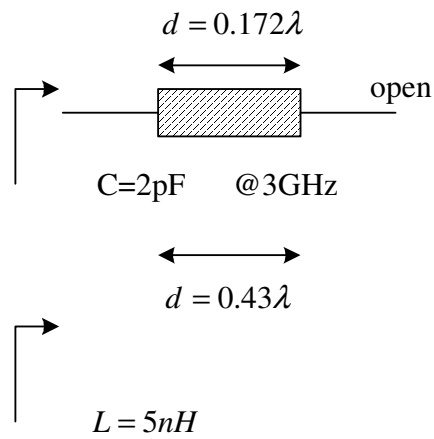
2pF Cap $\longrightarrow$ $X_C = \frac{1}{\omega C} = 26.5\Omega$

$$X_{nC} = -jXc/Z_0 = -j0.53$$

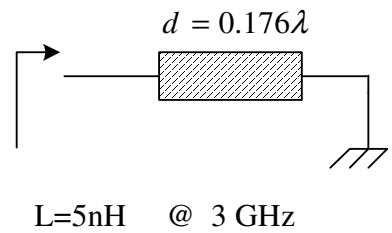
$5nH$ *inductor* $X_{nL} = \omega L = j2$

how to realize 2pF cap and 5nH inductor using open ckt line (let say at 3GHz)



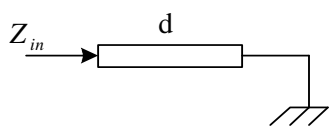


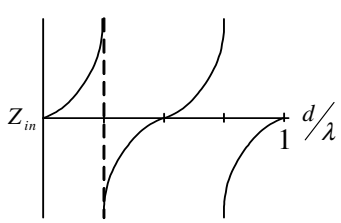
you can use short ckt line



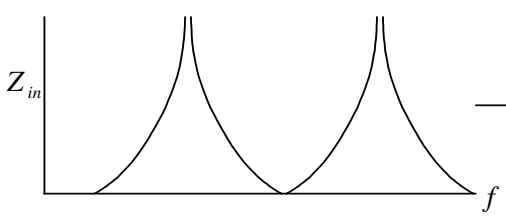
* you can also use Smith Chart for admittance instead of impedance
we will have some homework on impedance transformation

<u>S-parameters → scattering parameters</u>

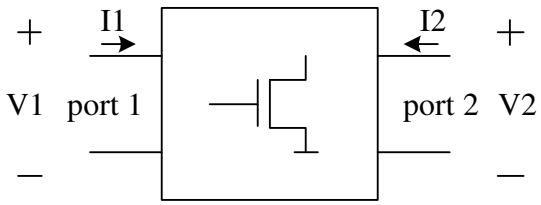




→ short ckt impedance seen through a short piece of transmission line can be anything



→ it is also frequency dependant



when we do not know anything about device physics & modeling we represent it as a black box with ports attached to it

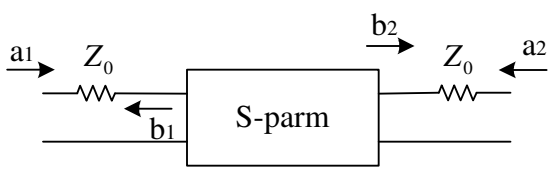
There are many representation of this black box

Impedance Parameters → $V_1 = Z_{11}I_1 + Z_{12}I_2$
Admittance Parameters
Hybrid Parameters $V_2 = Z_{21}I_1 + Z_{22}I_2$

to find Z_{11} → open circuit output → $I_2 = 0$ → OK for low frequency

so for high frequency we are looking at certain parameters that are not determined by short / open ckts rather than by matching

→ under matched condition → no reflection



a_1, a_2, b_1, b_2 are power waves → $|a_1|^2$ → incident power

$$b_1 = S_{11}a_1 + S_{12}a_2 \qquad a_1 = \frac{E_{inc1}}{\sqrt{Z_0}} \qquad b_1 = \frac{E_{ref1}}{\sqrt{Z_0}}$$ → voltage waveform

$$b_2 = S_{21}a_1 + S_{22}a_2 \qquad a_2 = \frac{E_{inc2}}{\sqrt{Z_0}} \qquad b_2 = \frac{E_{ref2}}{\sqrt{Z_0}}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \Gamma_1$$ → S_{11}, S_{22} are input, output reflection coefficient

S_{21}, S_{12} are transmission and reverse transmission

S_{11}, S_{12}, S_{21}, S_{22} are often presented in dB form

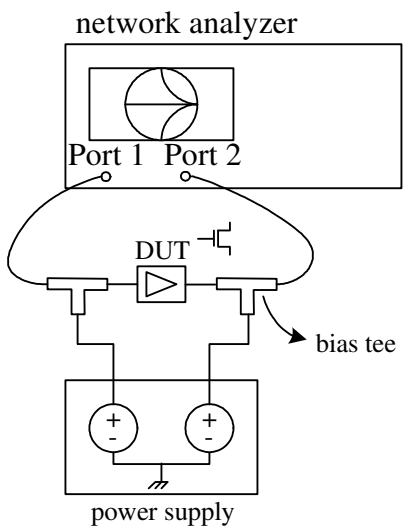
$$S_{11} = \Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \qquad -20\log|S_{11}| \Rightarrow$$ Return loss when port 2 is terminated with 50Ω

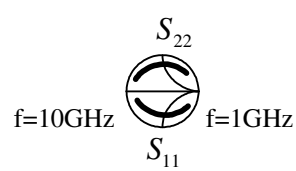
$$\downarrow$$

$$a_2 = 0$$

no reflection from termination

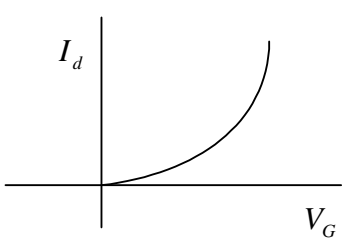
Measurement of S-parm

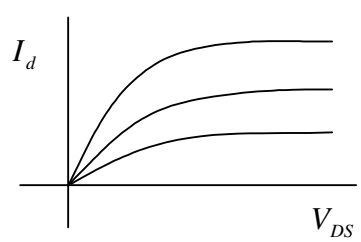




you often measure S-parm under different bias conditions (voltage & current) and over a frequency range of interest

you also have DC measurement of your device





using all these data you can find the device model (non-linear: spice) that represents the device under DC and RF under different bias conditions

Circuit Design and Analysis using a microwave transistor

Assume that you have a microwave transistor and you would like to design a circuit (or an integrated circuit) using this transistor. How do you proceed?

1st measure transistor for its RF performance

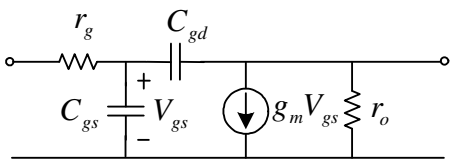
(S-parm) ← use network analyzer

f	S_{11}	S_{12}	S_{21}	S_{22}
$0.1GHz$	•	•	•	•
1	•	•	•	•
2	•	•	•	•
3	•	•	•	•

Try to model the transistor

Design using S-parm of the transistor

Small-Signal Model



* only need S-parm measurement
+ insight to the physics of device for initial values

Large-Signal Model

-Spice model

-BSIM model

need DC $V_g - I_d$, $I_d - V_{ds}$

+ RF → $C-V$, $S-parm$

+insight to device physics

older technique that measures Caps vs. supplied bias

need S-parm for different biases

So large-signal modeling is a difficult task

often your transistor supplier provides that

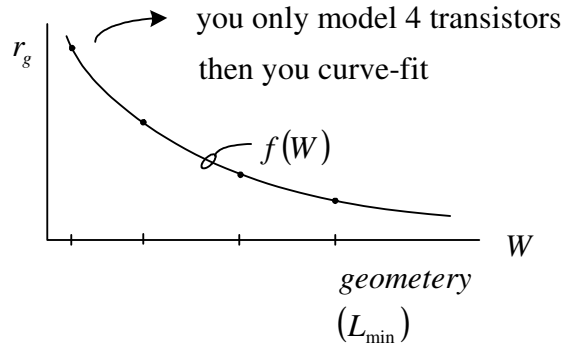
For integrated circuit technologies

* you cannot possibly measure every given geometery

you measure certain geometery (RF +DC)

generate a scalable large-signal model
that fits all possible geometeries

for instant
your model r_g
for different
geometery transistors
that you measured



- Scalable models are not very accurate, gain may be off by a couple of dB
but you cannot afford not knowing the stability situation

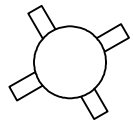
that is why fab foundries often supply
S-parm data along with the model

→ But raw S-parm data is useless
when you deal with an integrated transistor

For packaged transistor

you can measure S-parm of

inherent transistor + package

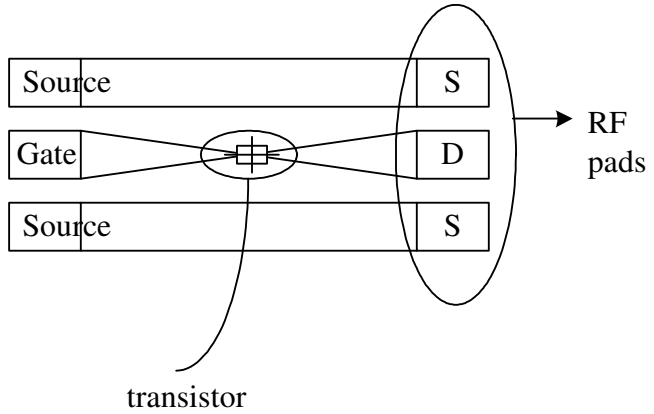


you can use the same S-parm for your ckt design + stability analysis

For integrated transistor

you measure

S-parm of this structure



but in ckt design

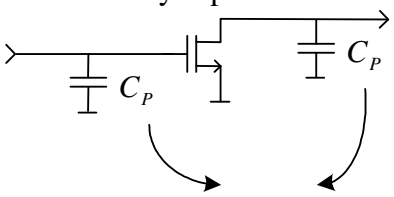
you only use the transistor without its pads

So raw S-parm is useless

you need to de-embed the effect of pads

1st degree approximation

Pads are only capacitive

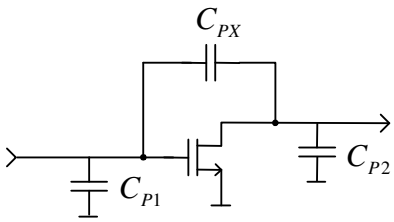


typically the same

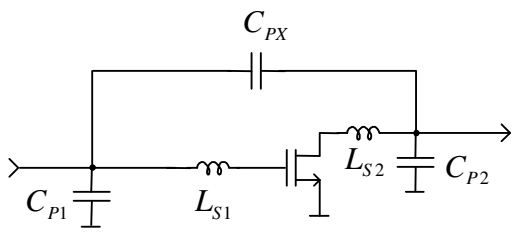
since there is often

a symmetry in your pad design

But such large pads may induce capacitive coupling



may also have inductive term especially as interconnects become narrower as you get closer to the transistor

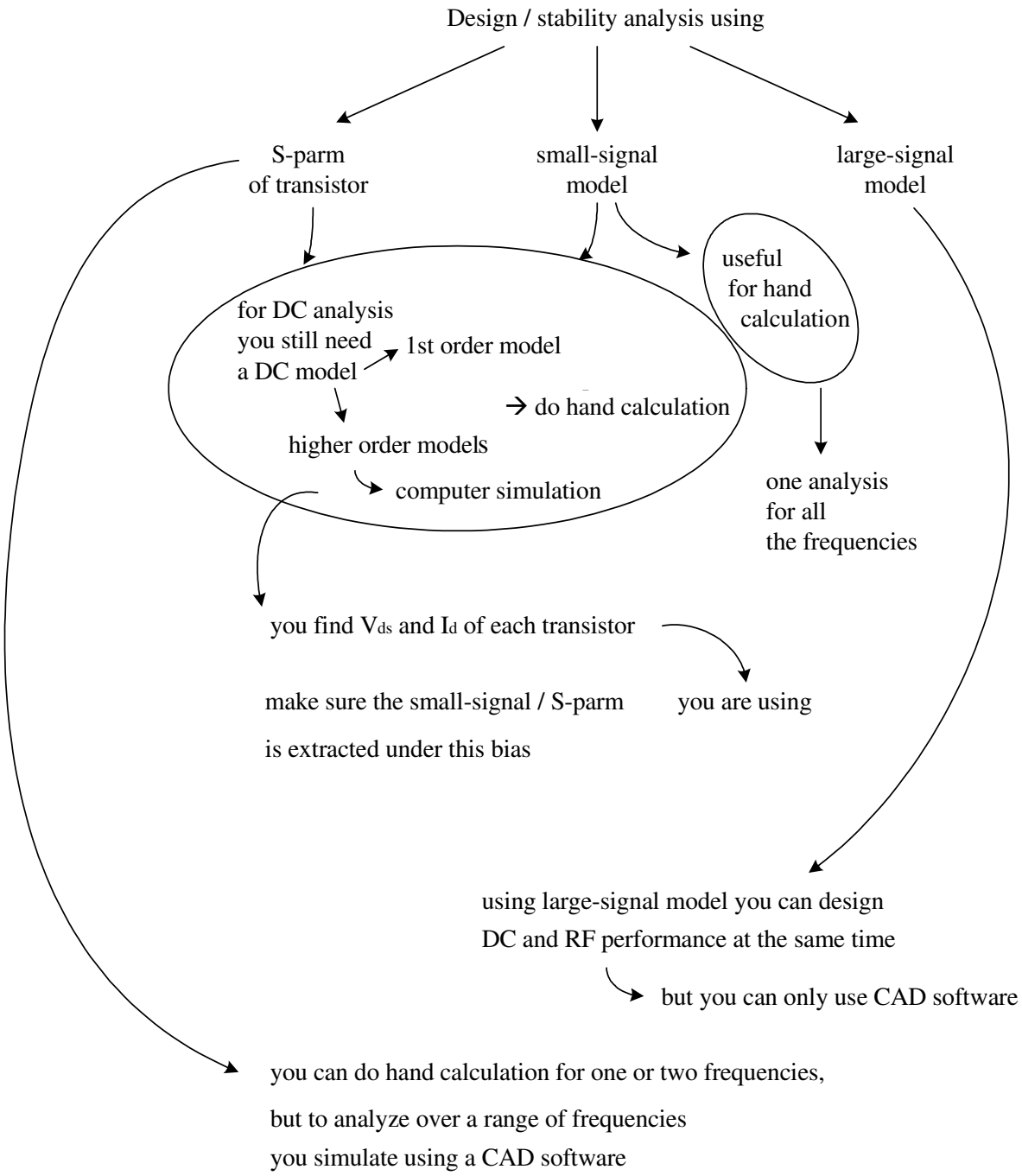


Therefore you need to put embedding structues and measure them so that you can model the effect of pads + interconnects

Summary : to design circuits based on a microwave transistor:

- measure S-parm of transistor + pads
- de-embed the effect of pads
- find the S-parm of the transistor
- repeat for different bias
- measure DC
- construct large-signal model

construct small-signal model



→ <u>if you do hand-calculation</u>

- for DC analysis use 1st order DC models (ideal MOS equations)
- for RF analysis use small-signal model find gain / bandwidth etc.
- for stability analysis use S-parms as they are more accurate

→ <u>if you do hand-calculation for only one or two frequencies</u>

small-signal model S-parms

→ <u>if you do computer simulation</u>

- use large-signal model for DC+RF+stability
- use S-parm tables to do stability analysis provided by foundry (RF analysis)

Provided that you have S-parm for that particular size transistor for that particular bias

This is basically a step to make sure your model is not too off

let's start with S-parm analysis

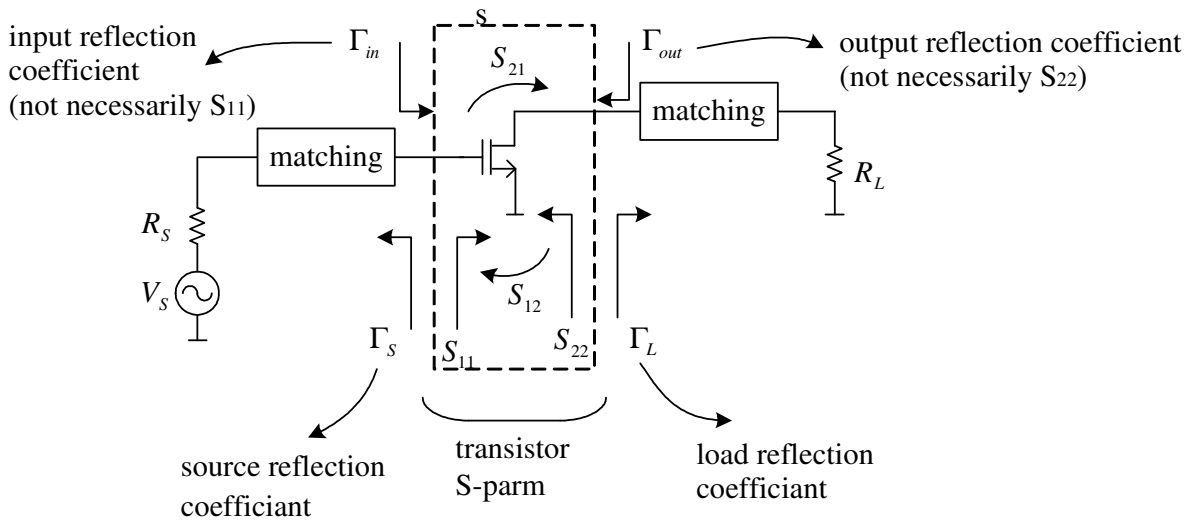
<u>Various Power Gain Definition of a Microwave Transistor</u>

<u>1st.</u> define various gains

$$Transducer\ Gain = G_T = \frac{P_L}{P_{AVS}} = \frac{Power\ delivered\ to\ the\ load}{Power\ available\ from\ the\ source}$$

$$Power\ Gain = G_P = \frac{P_L}{P_{IN}} = \frac{Power\ delivered\ to\ the\ load}{Power\ input\ to\ the\ transistor}$$

$$Available\ Gain = G_A = \frac{P_{AVN}}{P_{AVS}} = \frac{Power\ available\ from\ the\ network}{Power\ available\ from\ the\ source}$$



$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

if transistor is matched at output

then $\Gamma_{in} = S_{11}$

$Z_L = 50\Omega \Rightarrow \Gamma_L = 0$

when you do not put an output matching network

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

if transistor is matched at input

then $\Gamma_{out} = S_{22}$

$Z_S = 50\Omega \Rightarrow \Gamma_S = 0$

when you do not put an input matching network

you can write different gains in terms of S-parm + Γ_L, Γ_S

<u>Transducer Gain</u>

$$G_T = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2}$$

That means

$$|1-\Gamma_{in}\Gamma_S|^2|1-S_{22}\Gamma_L|^2 = |1-S_{11}\Gamma_S|^2|1-\Gamma_{out}\Gamma_L|^2$$

substitute Γ_{in} and Γ_{out}, you can prove that this is correct

transducer gain is basically whatever power gain
you have considering the fact that you could not
perfectly match at input and output

Power Gain

$$G_P = \frac{1}{1-|\Gamma_{in}|^2}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

G_P is not a function of Γ_S

for power gain you do not care about input matching network

you are looking at the input power to the transistor

That is why G_P is not related to Γ_S

Available Gain

$$G_A = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} |S_{21}|^2 \frac{1}{1-|\Gamma_{out}|^2}$$

G_A is not a function of Γ_L

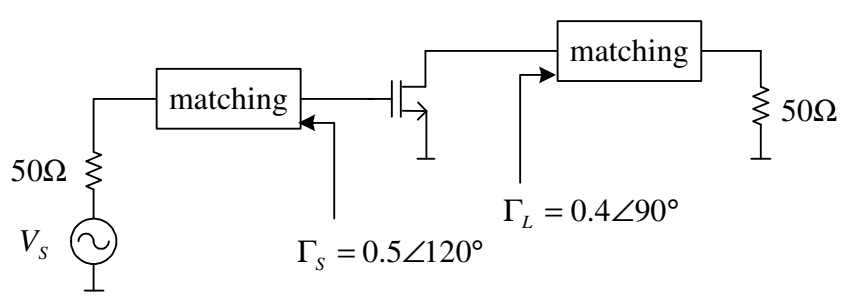
for available gain, you do not care about output matching network

you consider only the output power from the transistor

That is why G_A is not related to Γ_L

Example

For a 2.4 GHz MESFET Amplifier



Calculate different gain expressions assuming

$S_{11} = 0.6\angle -160°$

$S_{12} = 0.045\angle 16°$

$S_{21} = 2.5\angle 30°$

$S_{22} = 0.5\angle -90°$

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.627\angle -164°$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} = 0.471\angle -97°$$

$G_T = 9.43$ $(9.75dB)$ → usually lower than the other two gains $(G_P,\ G_A)$, because of mismatch at source and load

$G_P = 13.51$ $(11.31dB)$

$G_A = 9.55$ $(9.8dB)$

There are still three more gain expressions:

<u>Maximum Available Gain (MAG)</u>

$\Gamma_S = \Gamma_{in}^*$

$\Gamma_L = \Gamma_{out}^*$

simultaneous input / output matching

valid as long as it is stable

$$MAG = G_{T\,\max} = G_{A\,\max} = G_{P\,\max} = \frac{|S_{21}|}{|S_{12}|}\left(k - \sqrt{k^2 - 1}\right)$$

where $$k = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$$

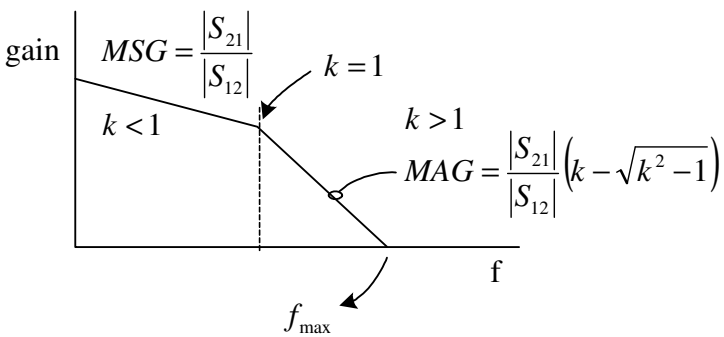
k is stability factor as we shall use soon

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

Maximum Stable Gain (MSG)

you use this gain definition instead of MAG when transistor is potentially unstable

$$MSG = MAG\big|_{k=1} = \frac{|S_{21}|}{|S_{12}|}$$



Unilateral Gain (G_U)

set $S_{12} = 0$

$\Gamma_{in} = S_{11}$, $\Gamma_{out} = S_{22}$

$$\longrightarrow \quad G_{TU} = \underbrace{\frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}}_{\text{input matching gain}} \underbrace{|S_{21}|^2}_{\text{inherent gain of the transistor}} \underbrace{\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}}_{\text{output matching gain}}$$

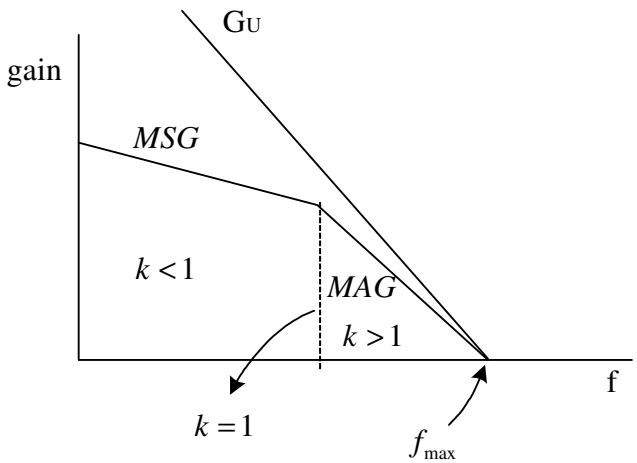
$\longrightarrow$ unilateral transistor is always stable

$\longrightarrow$ maximum G_{TU} is called maximum unilateral gain or sometimes unilateral gain (G_U)

$$G_{TU} = G_{TU,\max} = G_{TU}\Big|_{\substack{S_{11}^* = \Gamma_S \\ S_{22}^* = \Gamma_L}}$$

$$G_{TU} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2 \frac{1}{1-|S_{22}|^2}$$

frequency (G_U at zero dB) is also f_{max}

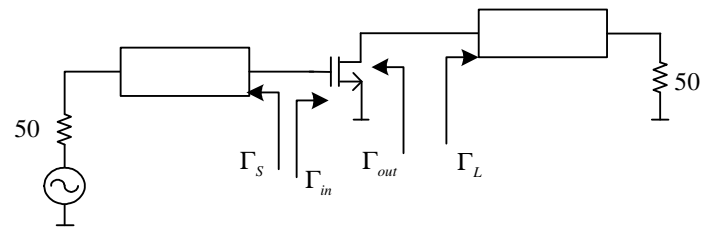


k : stability factor

Note: If you plot the short circuit current gain (H_{21}) of the device as a function of frequency, it crosses the 0dB (H_{21}=1) at a frequency known as cut-off frequency (f_T) or unity current gain frequency. Of course you cannot measure H_{21} directly as it is impossible to maintain a good short circuit impedance at the device terminal. Therefore, you calculate H_{21} from measured S-parameters and plot it along maximum unilateral gain or MSG/MAG combination shown in the above figure.

f_T and f_{max} are two important figure of merit that describe the small signal behavior of a microwave transistors. However, you need to know more about the transistor than just its f_T and f_{max}. Parameters that are important for you in addition to these two figures of merits are stability condition, noise figure, linearity, power efficiency and maximum output power as shall be described later.

Stability



$$\Gamma_S = \frac{Z_S - Z_O}{Z_S + Z_O} = \frac{Z_S - 50}{Z_S + 50}$$

when the real part of Zs is positive then

resistance part

$$R_S \geq 0 \;\rightarrow\; |\Gamma_S| \leq 1 \quad ①$$
$$R_L \geq 0 \;\rightarrow\; |\Gamma_L| \leq 1 \quad ②$$

means that both input and output matchings provide resistances (positive resistance)

in addition if the real part of Z_{in} and Z_{out} is also positive that means

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| \leq 1 \qquad ③$$

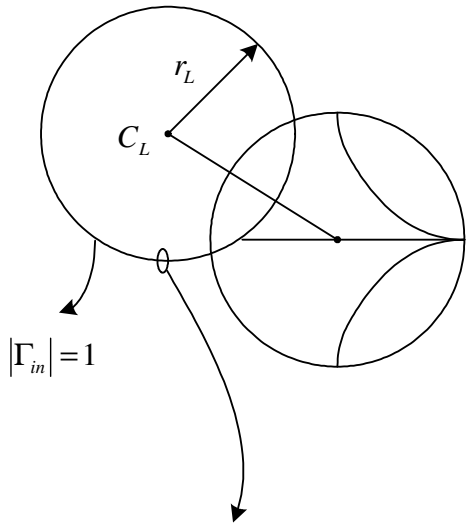
$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right| \leq 1 \qquad ④$$

* if eqns ① ~ ④ are all satisfied your amplifier is stable
(provided that you only have one transistor
we will look at multiple transistor case)

find $|\Gamma_{in}| = 1$ and $|\Gamma_{out}| = 1$ $\longrightarrow$ solve for Γ_L and Γ_S

$\longrightarrow$ These equations give circles on Smith Chart

$|\Gamma_{in}| = 1$ $\longrightarrow$ find Γ_L $\longrightarrow$ it gives a circle that has a radius and center as following :



output stability circle

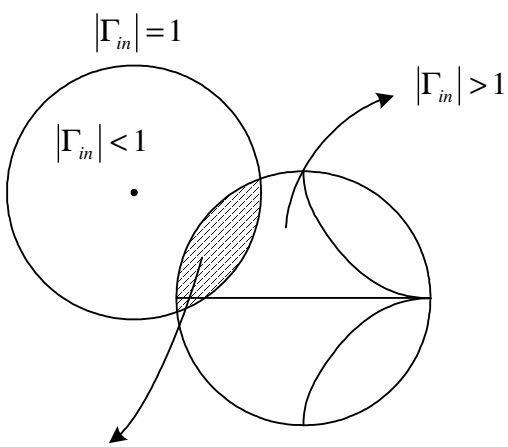
$$r_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|$$

$$C_L = \frac{\left(S_{22} - \Delta S_{11}^{\ *}\right)^*}{|S_{22}|^2 - |\Delta|^2}$$

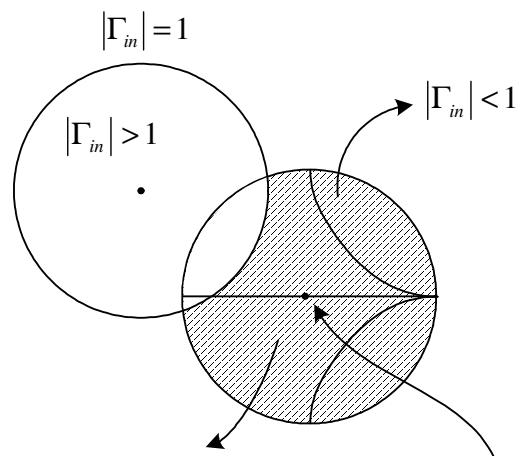
we have defined Δ before

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

<u>There are two possibilities</u>



stable region

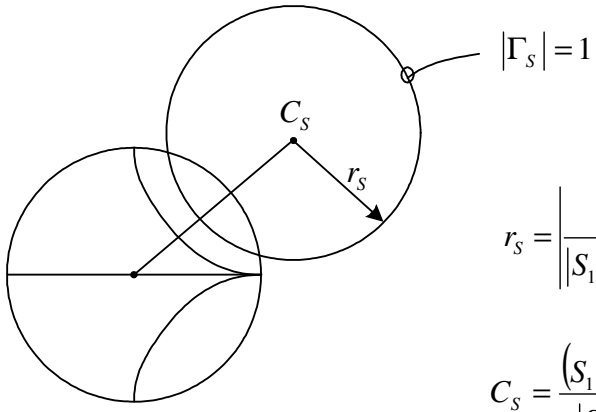


stable region

to find which one is the stable region, try $\Gamma_L = 0$

$\longrightarrow$ similarity you can find input stability circle by setting

$|\Gamma_{out}| = 1$ and find Γ_S

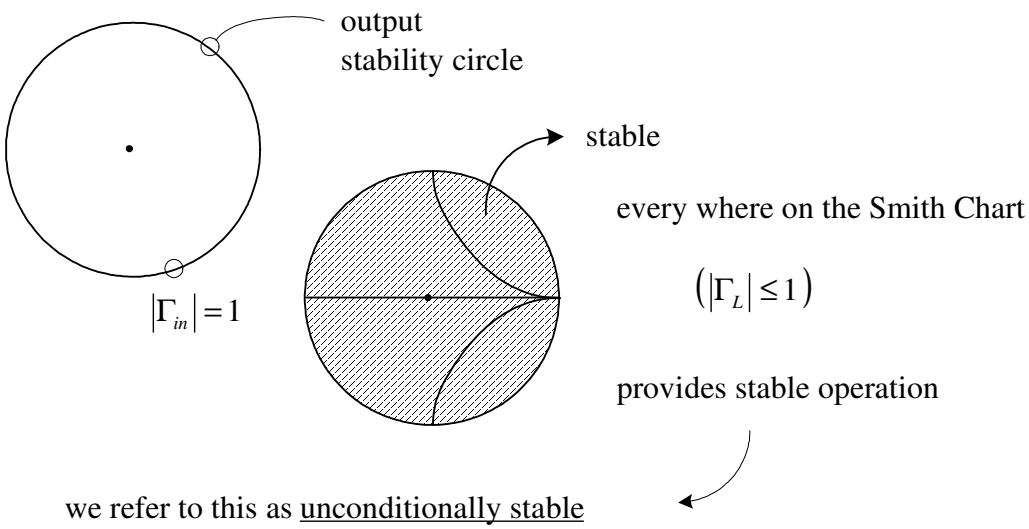


$$r_S = \left| \frac{S_{12} S_{21}}{|S_{11}|^2 - |\Delta|^2} \right|$$

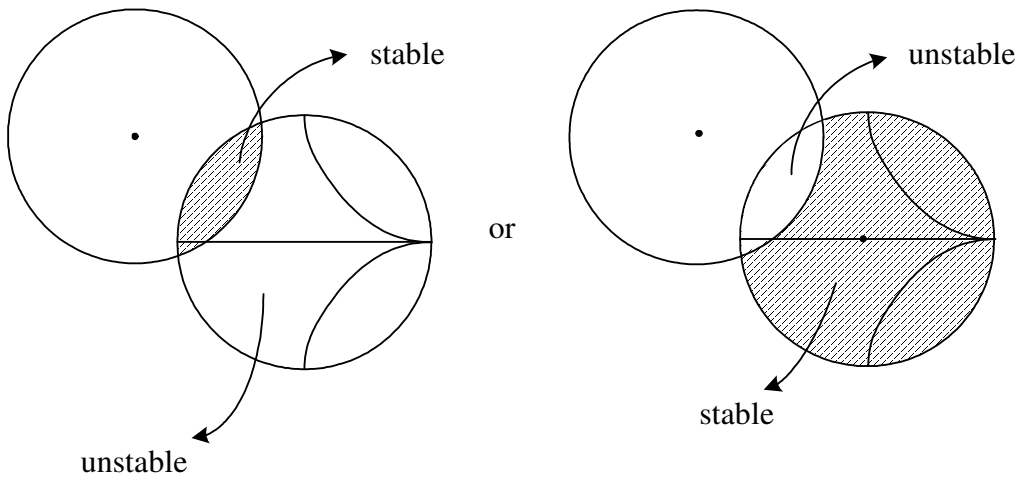
$$C_S = \frac{\left(S_{11} - \Delta S_{22}^*\right)^*}{|S_{11}|^2 - |\Delta|^2}$$

<u>You can have two cases</u> → consider output stability circle

<u>Case 1</u>



Case 2



in each of the above conditions you can find load impedances (Γ_L) that result in unstable condition

→ potential unstable

→ does not mean it is unstable but you have to be careful providing right load impedance

you can define these two different cases using stability factor

work out equations → $k = \dfrac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$

$\Delta = S_{11}S_{22} - S_{12}S_{21}$

if $k > 1,\ |\Delta| < 1$ → unconditional stability

$0 < k < 1\ ,\ |\Delta| < 1$ → potentially unstable

(true for most RF transistors at low frequency)

$-1 < k < 0$, $|\Delta| < 1$ ⟶ potentially unstable

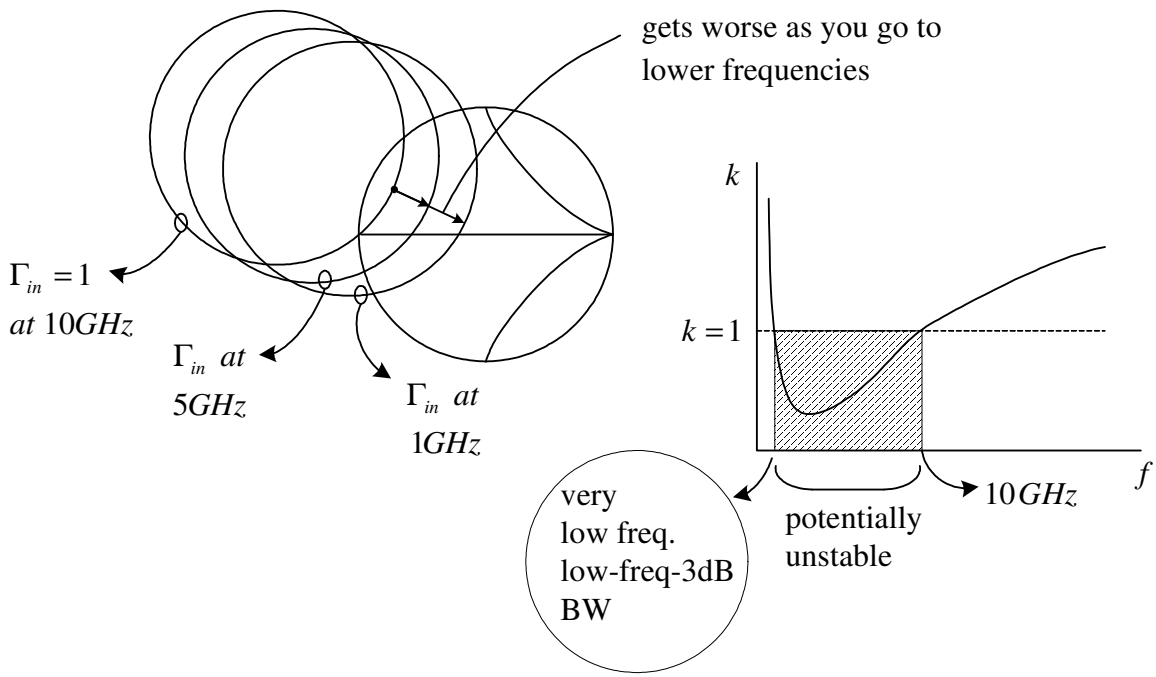
you often use this condition
to make an oscillator

in this case most of Smith Chart results in unstable operation
that is why it is good for oscillators

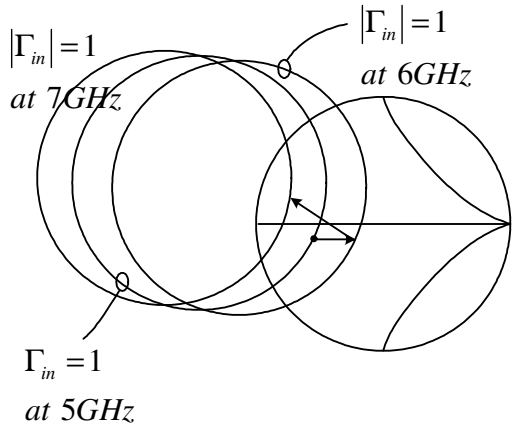
<u>* stability circles vs. frequency</u>

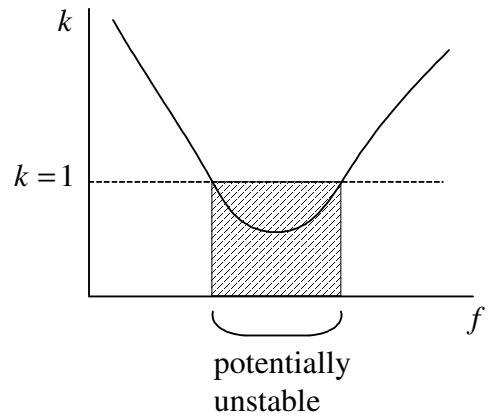
typically you have the following situation

for widebandwidth Amps



for narrowband Amps





<u>unconditionally stable design</u>

* check stability for all frequencies, not just BW of interest
* to be safe, design such that k>2 for all frequencies
* if you have multi-stage amplifiers, need to consider stability within the transistors

<u>typically $|\Delta| < 1$ for RF transistors</u>

we can look at stability factor <u>k</u> ,

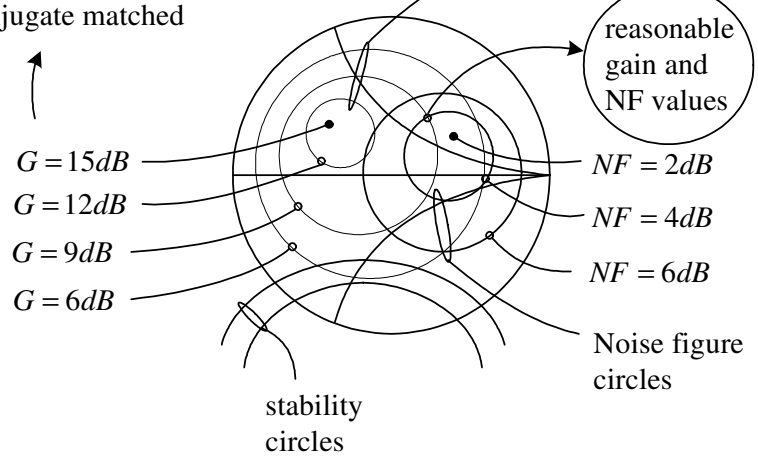
plot input/output stability circles in cadence

last thing on gain/stability $\longrightarrow$ most of the gain equations we saw are also circle equations on Smith Chart

simultaneously input/output conjugate matched $\Gamma_S = \Gamma_{in}^*$, $\Gamma_L = \Gamma_{out}^*$

gain circles → are perfect circles at low input power

distort as you go to higher power



how to find this optimum point for gain?

$$\left.\begin{aligned} \Gamma_S^* &= S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \\ \Gamma_L^* &= S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \end{aligned}\right\} \text{ find } \Gamma_{L\,opt} \ , \ \Gamma_{S\,opt}$$

$$\Gamma_{S\,opt} = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \qquad \Gamma_{L\,opt} = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \qquad B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$C_1 = S_{11} - \Delta S_{22}^* \qquad C_2 = S_{22} - \Delta S_{11}^*$$

$$if \quad |\Gamma_{S\,opt}| \ , \ |\Gamma_{L\,opt}| < 1 \quad \rightarrow \quad k > 1 \quad stable\,(probably)$$

So that optimum point for $|\Gamma_{S\,opt}|$, $|\Gamma_{L\,opt}|$ exist

if you have unconditionally stable case

$$= \frac{|S_{21}|}{|S_{12}|}(k - \sqrt{k^2 - 1})$$

Gain at that optimum input/output conjugate- matched is MAG