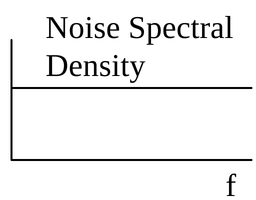
Noise in Integrated Circuits

Fundamental Noise Sources

Thermal Noise (Johnson Noise, Nyquist Noise)

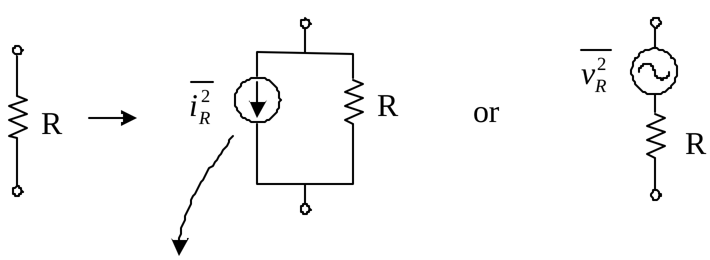
arises from thermally excited random motion of electrons in a conductive medium

Properties of thermal noise

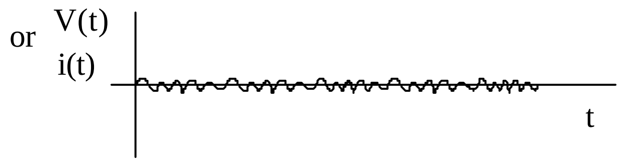


1) noise is white (?)
2) noise is proportional to temperature
3) not associated with DC current
4) can find it in any real physical resistor

Representation of thermal noise

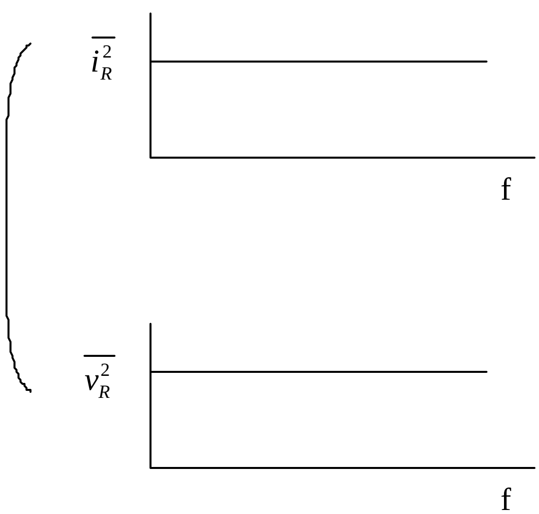


does not mean the current is in certain direction in fact average current (or voltage) is zero



but you can go to frequency domain and plot mean-square value of the noise signal

noise spectral density



$$\overline{i_R^2} = \frac{4kT}{R}\Delta f$$

bandwidth of interest (ckt or measurement)

$$\overline{v_R^2} = 4kTR\Delta f$$

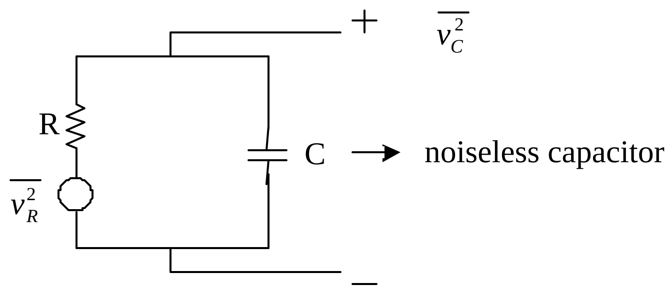
$$\overline{i_R} = \overline{v_R} = 0$$

noise average is zero

but average noise power is not?!

why $\overline{v_R^2} = 4kTR\Delta f$

assume a simple ckt of parallel RC



An equipartition theorem of statistical thermodynamic

for each degree of freedom (or mode) in a given system,

there is a thermal energy of $\frac{1}{2}kT$

k: Boltzmann constant

$1.38\times10^{-23}\ J/K$

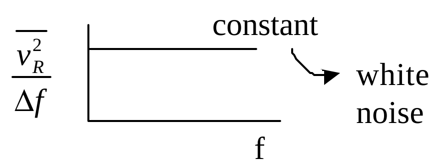
Total energy of the system $= \frac{1}{2}kT = \frac{1}{2}C\,\overline{v_C^2}$

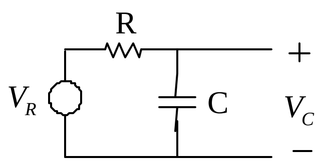
$$\overline{v_C^2} = \frac{kT}{C}$$

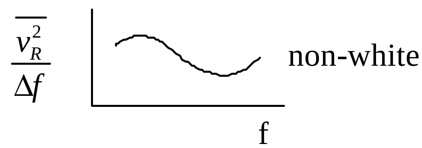
total mean square voltage density
integrated over all frequencies

$$\overline{v_C^2} = \int_0^\infty \left| \frac{1}{1 + jwRC} \right|^2 \frac{\overline{v_R^2}}{\Delta f} df = \frac{kT}{C}$$

filtering function







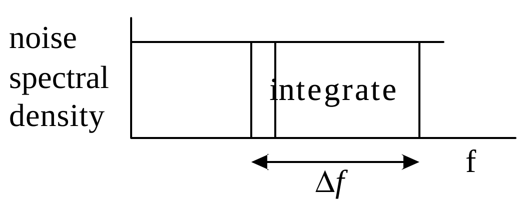
Assuming white noise

$$\frac{\overline{v_R^2}}{R} = \text{ constant}$$

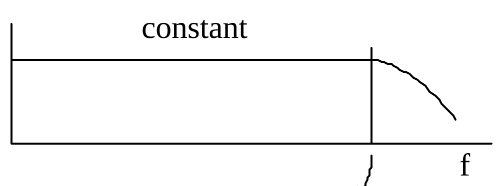
$$\rightarrow \quad \frac{kT}{C} = \frac{\overline{v_R^2}}{2p\Delta f} \int_0^\infty \frac{w_0{}^2}{w^2 + w_0{}^2} dw \qquad w_0 = \frac{1}{RC}$$

$$\frac{kT}{C} = \frac{\overline{v_R^2}}{2p\Delta f} \times \left(\frac{p}{2RC} \right) \longrightarrow \boxed{\overline{v_R^2} = 4kTR\Delta f}$$

$$\frac{\overline{v_R^2}}{\Delta f} = 4kTR$$



* Actual power spectral density of thermal noise



$$\frac{kT}{h} \approx 6THz \longrightarrow \text{for all practical purpose}$$

k: $1.38\times10^{-23}\ J/K$

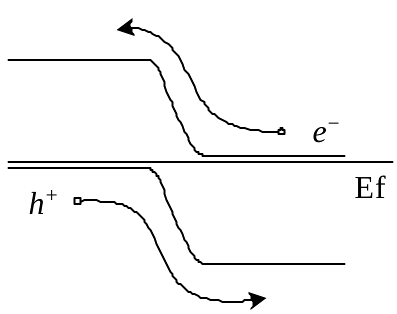
h: Plank constant $= 6.625\times10^{-34}\ J\cdot\sec$

$\longrightarrow$ Thermal noise is a white noise

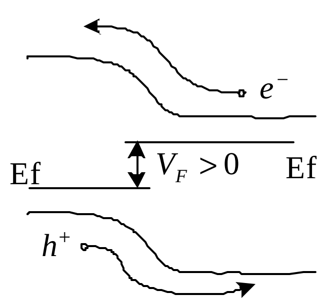
Shot noise

* associated with DC current flow across a junction

* arises from random nature of electrons and holes
  surmounting a potential barrier



$V_F = 0$



$V_F > 0 \longrightarrow$ current flow

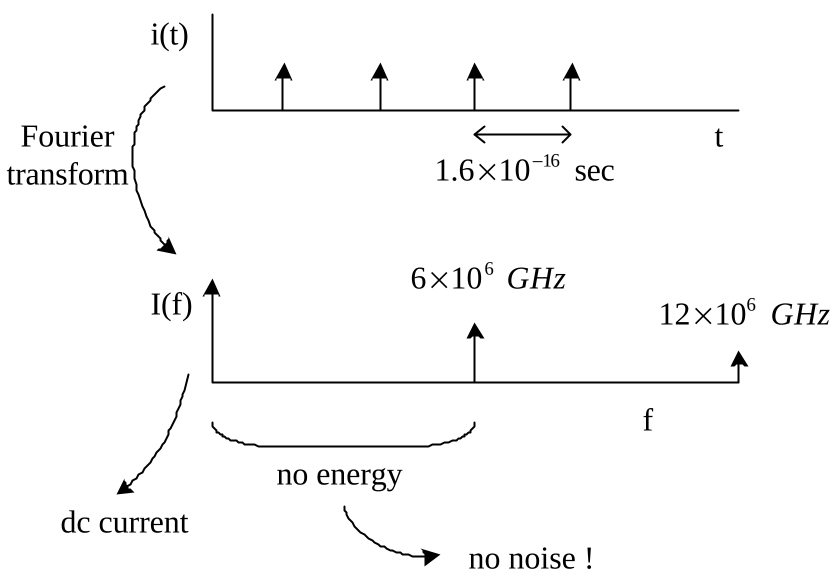
special case

assume that electrons are very well controlled / behaved
and cross the junction in a very uniform manner

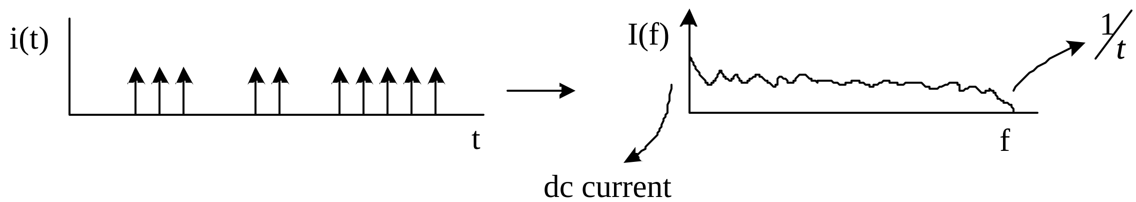
1 mA current → uniform current pulses

$$I = \frac{Q}{t} \quad \rightarrow \quad t = \frac{Q}{I} = \frac{1.6\times10^{-19}}{1\times10^{-3}} = 1.6\times10^{-16} \text{ sec}$$

every so many second one electron passes



in reality carriers surmount the barrier in a random fashion



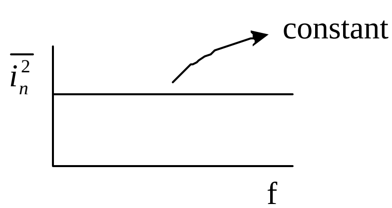
t : current transit time in the depletion region

$1/t \rightarrow$ hundreds of GHz

→ So for all practical purposes shot noise is a white noise

again it is better to talk about noise spectral density

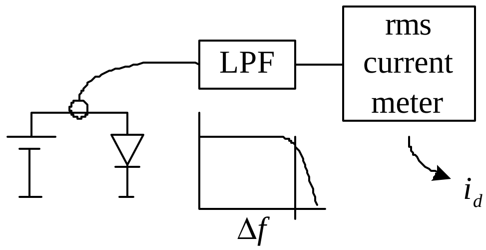
$$\overline{i_n^2} = 2\,q\,I_D\,\Delta f$$



shot noise :

1) related to DC current over a potential barrier
2) independent of T
3) white
4) noise power $\propto \Delta f$, I_D

Experiment



$$i_d = \sqrt{\overline{i_d^2}} = \sqrt{2qI_D\Delta f}$$

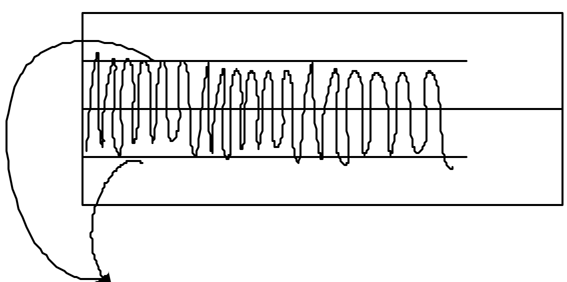
$$\left.\begin{array}{l} I_D = 1\,mA \\ \Delta f = 1\,MHz \end{array}\right\} \quad \overline{i_d^2} = 3.2\times10^{-16}\,A^2$$

$$2\,q\,I_D\,\Delta f$$

$$i_d = \sqrt{\overline{i_d^2}} = 17\,nA$$

Experiment 2

if you look at a noisy signal on the oscilloscope



$V_{PP} = 100\,mV$ ⟶ what would be the rms noise value

↓

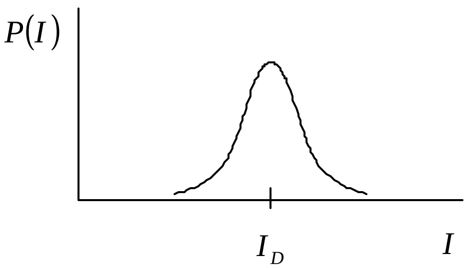
$$v_{noise} \approx \frac{V_{PP}}{6} = 16\,mV_{rms}$$

↓

why?

both thermal noise and shot noise have Gaussian distribution

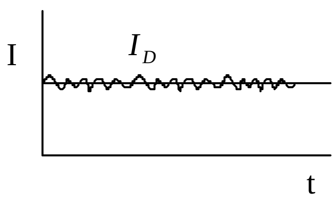
plot probability density function of the diode current



$P(I)dI$ $\rightarrow$ probability that diode current

$I < \quad < I + dI$

s : standard deviation
of the Gaussian distribution



turnsout $\rightarrow$ $s^2 = \overline{i^2}$

$s = \sqrt{2qI_D \Delta f}$

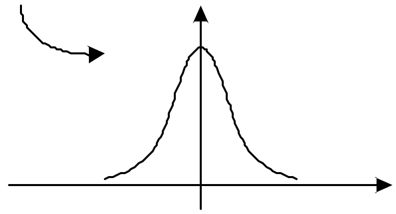
$\pm 3s$ ⟶ limits of the noise amplitude within 99.7% of the time

⟶ what you see on the scope is usually $\pm 3s$

⟶ divide by 6 ⟶ gives you s

which is the rms value of noise

Resistor noise (thermal noise) ⟶ any voltage / current around 0



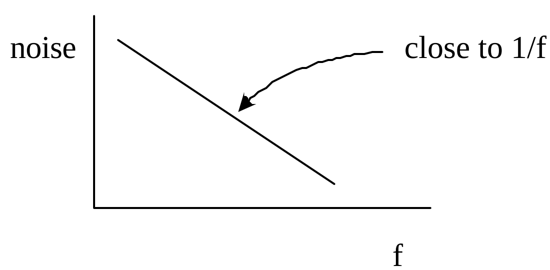
Other types of noise

Flicker Noise (1/f noise)

* associated with random trapping and
release of carriers from slow states

such as interface state between Si/SiO_2 in MOS

Flicker noise has a non-white spectral density



$$\underbrace{\overline{i_n^2} = k\frac{I_D^a}{f^b}\,\Delta f}_{}$$

general form of flicker noise

noise depends on current $\longrightarrow$ (no current $\longrightarrow$ no noise)

k $\longrightarrow$ constant for a particular device

$\longrightarrow$ unlike shot noise and thermal noise, this constant varies from device to device and from technology to technology

a : 0.5 ~ 2

$\longrightarrow$ current dependence of noise

b = 1

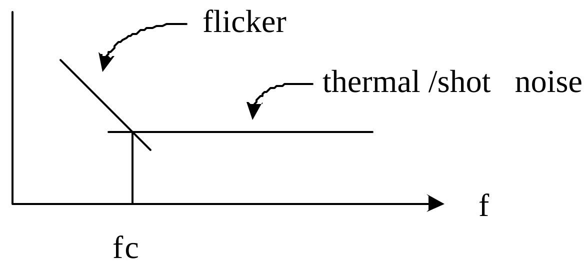
flicker noise $\longrightarrow$ very high in MOS transistors

$\longrightarrow$ exists in bipolar transistors
(especially the base current)

$\longrightarrow$ also exists in carbon resistors

$\downarrow$

thin film wire wound resistors have much less flicker noise flicker noise is often characterized with noise corner frequency



noise corner frequency ~ 10 Hz ~ kHz for bipolar

~ 10 kHz ~ MHz for MOS

<u>Burst noise</u>

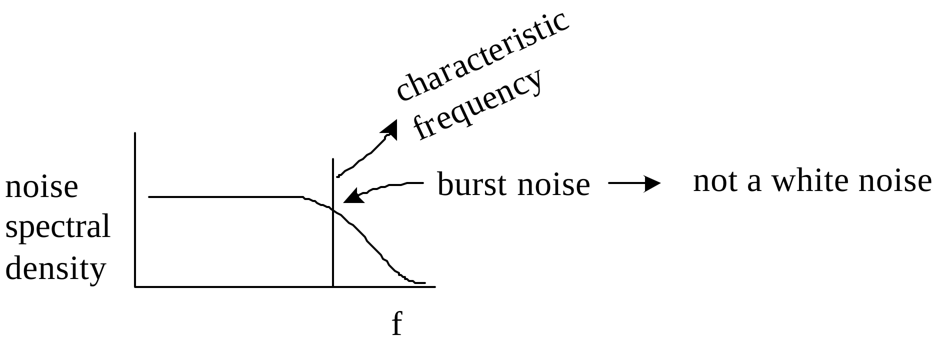
also a low-frequency noise

due to a single trap

1/f noise → due to an ensemble of traps at different frequency/ energy

burst noise → has a certain energy

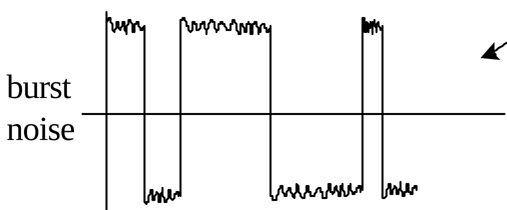
certain time constant



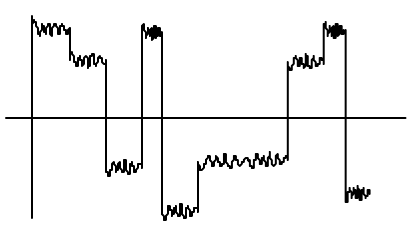
noise amplitude does not follow Gaussian distribution

In fact it is a binomial distribution

charging / discharging a trap



sometimes you have more than 2 levels
as there is more than 1 trap

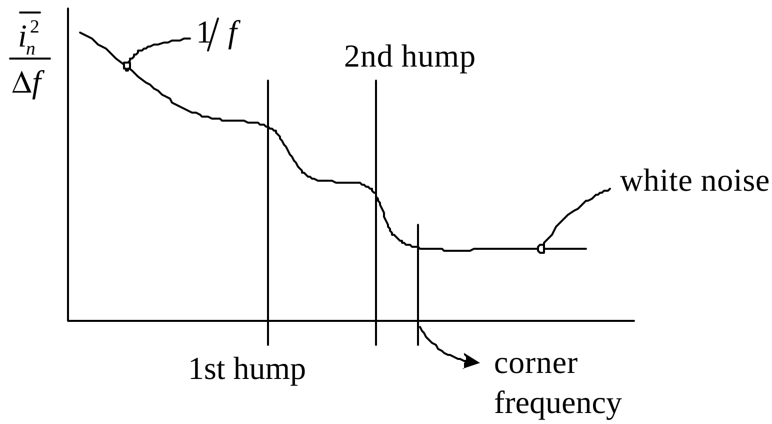


when there is more than one trap you often see

two characteristic frequencies (two humps)

burst and 1/f noise are always combined

so what you may see



Another type of noise

Avalanche Noise

avalanche breakdown is a very noisy process

0.5 mA zener current is equivalent to the thermal noise of $600\,K\Omega$ resistor

Noise models for devices

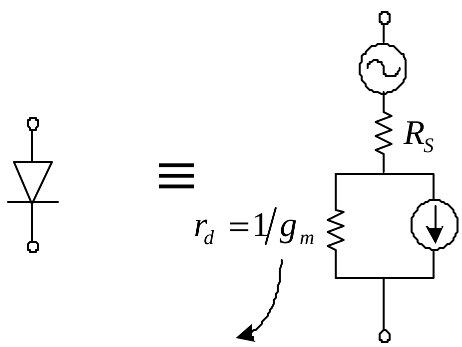
Resistor $\longrightarrow$ thermal noise (carbon resistors

+ flicker noise)

Capacitors / Inductors $\longrightarrow$ noiseless

but they shape the overall noise through the transfer function

Noise in Diodes



$$\overline{v_S^2} = 4kTR_S\Delta f$$ thermal

$$\overline{i_d^2} = \underbrace{2qI_D\Delta f}_{\text{shot}} + \underbrace{k\frac{I_D^a}{f}\Delta f}_{\text{flicker}}$$

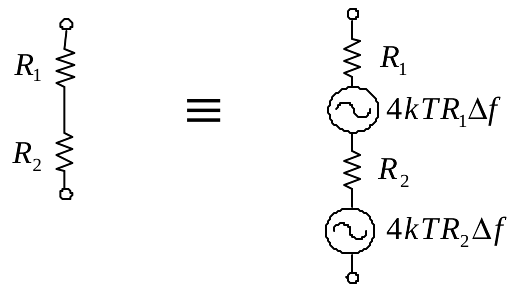
$r_d = 1/g_m$

fictitious
resistor
(noiseless)

These three noise sources are independent of each other $\longrightarrow$ <u>uncorrelated</u>

<u>Correlation study case</u>

Noise in series resistors



$$v_T(t) = v_1(t) + v_2(t)$$

$$\begin{aligned}\overline{v_T(t)^2} &= \overline{[v_1(t) + v_2(t)]^2} \\ &= \overline{v_1(t)^2} + \overline{v_2(t)^2} + \overline{2v_1(t)v_2(t)} \\ &= 4kTR_1\Delta f + 4kTR_2\Delta f + \overline{2v_1(t)v_2(t)} \\ &= \overline{v_1}^2 + \overline{v_2}^2 + 0\end{aligned}$$

uncorrelated ⟵ since the two resistors are separate from each other, their noise is orthogonal (independent from each other)

$\longrightarrow$