$$A(j\omega) = \frac{A_{v_0}}{\left(1 + \frac{j\omega}{\omega_p}\right)} \qquad\qquad A_{v_0} = 14000 \text{ in our case}$$

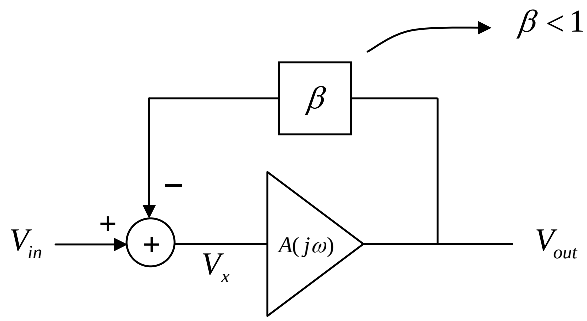
$$\left| A(j\omega)\big|_{\omega=\omega_{ta}} \right| = 1 \;\Rightarrow\; \frac{A_{v_0}}{\left|1 + \frac{j\omega}{\omega_p}\right|} = 1 \;\rightarrow\; \omega_{ta} = A_{v_0}\omega_p$$

$$\left.\begin{aligned} \omega_{ta} &= 2\pi \times 25MHz \\ A_{v_0} &= 14000 \end{aligned}\right\} \; \omega_p = 2\pi f_p \rightarrow \quad f_p \approx 1.8KHz \quad ?$$

For $\omega_p << \omega << \omega_{ta}$ we can write:

$$A(j\omega) = \frac{\omega_{ta}}{j\omega}$$

When you put the opamp in a feedback loop:

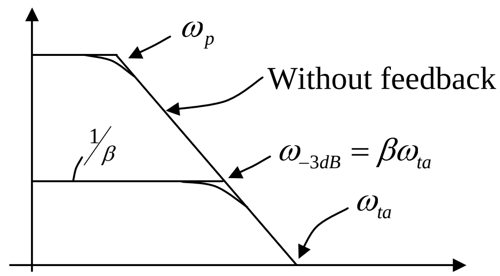


$$\left.\begin{aligned} V_x &= V_{in} - \beta V_{out} \\ V_{out} &= V_x . A(j\omega) \end{aligned}\right\} \;\rightarrow\; \frac{V_{out}}{A(j\omega)} = V_{in} - \beta V_{out} \quad \rightarrow V_{out}\left(\frac{1}{A(j\omega)} + \beta\right) = V_{in}$$

$$\rightarrow \quad \frac{V_{out}}{V_{in}} = \frac{A(j\omega)}{1 + \beta A(j\omega)}$$

Use $A(j\omega) = \frac{\omega_{ta}}{j\omega}$ in the above equation $\rightarrow A_{closed-loop} = \frac{V_{out}}{V_{in}} = \frac{\omega_{ta}/j\omega}{1 + \beta\, \omega_{ta}/j\omega} = \frac{1}{\beta} . \frac{1}{\left(1 + \frac{j\omega}{\beta\omega_{ta}}\right)}$

Gain at low frequency = $\frac{1}{\beta}$ -3dB: $\omega_{-3dB} = \beta\omega_{ta}$.



So both gain and BW change with feedback

$Gain \times BW = \omega_{ta}$ $\leftarrow$ always constant (gain bandwidth product)

Linear settling time

Setting time: time it takes for an OPAMP to reach a specified percentage of its final value when step input is applied.

Settling time
- Linear → due to finite ω_{ta} of OPAMP (in small signal)
- Non-linear → due to slew-rate of OPAMP under large signal input

→ for small step sizes in the output signal, OPAMP may not reach a slew-rate limit at all.

The time constant of OPAMP response in closed-loop system:

$$\tau = \frac{1}{\omega_{-3dB}} = \frac{1}{\beta\omega_{ta}}$$

Since we are assuming a first-order system, transient response is:

$$V_{out}(t) = \overset{\text{input}}{V_{step}}\left(1 - e^{-t/\tau}\right)$$

Time it takes for signal to reach its 99% final value is:

$$e^{-t/\tau} = 0.01 \;\; \rightarrow \;\; t_{settling} = 4.6\tau = \frac{4.6}{\beta\omega_{ta}}$$

For 90% is:

$$e^{-t/\tau} = 0.1 \;\; \rightarrow \;\; t_{settling} = 2.3\tau = \frac{2.3}{\beta\omega_{ta}}$$

Slope of the output:

$$\left.\frac{dV_{out}}{dt}\right|_{t=0} = \frac{V_{step}}{\tau}$$

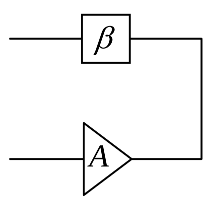
→ If slew-rate is larger than this value, there is no slew-rate limiting mechanism.

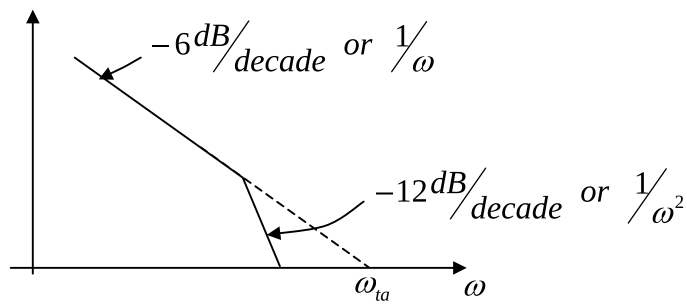
OPAMP compensation

In order to analyze OPAMP stability under feedback, we have to consider OPAMP as a 2-pole system.

$$A(j\omega) = \frac{\omega_{ta}}{j\omega\left(1+\dfrac{j\omega}{\omega_{eq}}\right)} \quad \leftarrow \text{ open-loop OPAMP transfer function}$$

2nd pole that takes into account all of OPAMPs poles and zeros.





$$loop\ \ gain = \beta A(j\omega) = \frac{\beta\omega_{ta}}{j\omega\left(1+\dfrac{j\omega}{\omega_{eq}}\right)}$$

Frequency at which loop gain becomes 1 = ω_{ta}

$$\left|\beta A(j\omega_t)\right| = 1 \quad \rightarrow \quad \left|\frac{\beta\omega_{ta}}{j\omega_t\left(1+\dfrac{j\omega_t}{\omega_{eq}}\right)}\right| = 1$$

$$\rightarrow \quad \beta\omega_{ta} = \omega_t\sqrt{1+\frac{\omega_t^2}{\omega_{eq}^2}} \quad \rightarrow \quad \omega_{ta} = \frac{\omega_t}{\beta}\sqrt{1+\frac{\omega_t^2}{\omega_{eq}^2}}$$

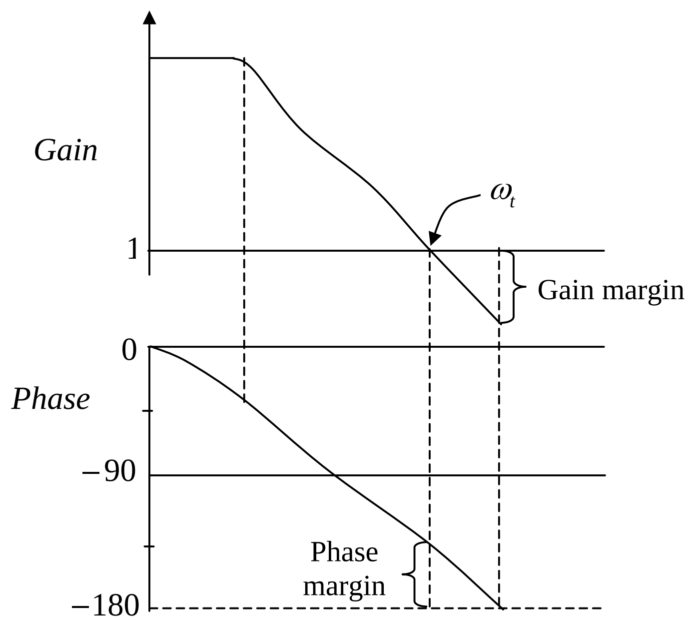
Assuming that the second pole of T.F. (ω_{eq}) is very high

$$\rightarrow \quad \omega_{ta} \approx \frac{\omega_t}{\beta}$$

When unity gain freq. is smaller than dominant pole without comp. → $\beta\omega_{ta} = \omega_{-3dB}$

Gain margin / Phase margin concept

PM/GM are measures of stability.



PM/GM are measured on the loop gain of an open loop system. The idea is if you have gain in a system that has an overall phase of less than -180˚, by applying feedback the system can become unstable.

$$PM = \angle loop - gain(j\omega)\Big|_{\omega=\omega_t} - (-180°) \quad \rightarrow$$

$$PM = \angle \frac{\beta\omega_{ta}}{j\omega_t\left(1+\dfrac{j\omega_t}{\omega_{eq}}\right)} + 180° = -90° - \tan^{-1}\left(\frac{\omega_t}{\omega_{eq}}\right) + 180° = 90° - \tan^{-1}\left(\frac{\omega_t}{\omega_{eq}}\right)$$

$$\rightarrow \quad \frac{\omega_t}{\omega_{eq}} = \tan(90° - PM) \quad \rightarrow \quad \omega_t = \omega_{eq} \times \tan(90° - PM)$$

For a given PM unity gain frequency (ω_t) is independent of the amount of feedback (β).

Example: A closed loop amplifier is compensated to have PM = 75˚.

$$\left.\begin{aligned} & f_{eq} = \frac{\omega_{eq}}{2\pi} = 50MHz \\ & \text{Find } \omega_t,\ \omega_{ta}. \end{aligned}\right\} \qquad \begin{aligned} & \omega_t = \omega_{eq} \times \tan(90° - PM) \\ & \rightarrow f_T = \frac{\omega_t}{2\pi} = 13.4MHz \end{aligned}$$

$$\omega_{ta} = \frac{\omega_t}{\beta}\sqrt{1+\frac{\omega_t^2}{\omega_{eq}^2}} = \frac{1.03}{\beta}\omega_t$$

If $\beta = 1 \rightarrow \omega_{ta} = 1.03\omega_t \rightarrow f_{ta} = 13.9MHz$

We can calculate closed loop gain using second-order model for TF of OPAMP →

$$\rightarrow A_{closed-loop}(j\omega) = \frac{V_{out}}{V_{in}} = \frac{A(j\omega)}{1+\beta A(j\omega)} = \frac{\dfrac{\omega_{ta}}{j\omega\left(1+\dfrac{j\omega}{\omega_{eq}}\right)}}{1+\beta\dfrac{\omega_{ta}}{j\omega\left(1+\dfrac{j\omega}{\omega_{eq}}\right)}}$$

$$A(j\omega) = \frac{\omega_{ta}}{j\omega\left(1+\dfrac{j\omega}{\omega_{eq}}\right)}$$

$$\left.\begin{aligned} & \rightarrow A_{closed-loop}(j\omega) = \frac{\omega_{ta}}{\beta\omega_{ta} + j\omega - \dfrac{\omega^2}{\omega_{eq}}} = \frac{\dfrac{1}{\beta}}{1+\dfrac{j\omega}{\beta\omega_{ta}} - \dfrac{\omega^2}{\beta\omega_{ta}\omega_{eq}}} \\ & A_{closed-loop}(j\omega) = \frac{K}{1+\dfrac{j\omega}{\omega_0 Q} - \dfrac{\omega^2}{\omega_0}} \end{aligned}\right\} \rightarrow$$

$\omega_0^2 = \beta\omega_{ta}\omega_{eq} \rightarrow \boxed{\omega_0 = \sqrt{\beta\omega_{ta}\omega_{eq}}}$ → resonance frequency in a a2-pole system.

$\boxed{Q = \frac{\beta\omega_{ta}}{\omega_0} = \frac{\beta\omega_{ta}}{\sqrt{\beta\omega_{ta}\omega_{eq}}} = \sqrt{\frac{\beta\omega_{ta}}{\omega_{eq}}}}$ → Q-factor in a 2-pole system

Q determines how much overshoot you have in time domain.

Response: $\%\ overshoot = 100.e^{\frac{-\pi}{\sqrt{4Q^2-1}}}$

$Q \leq 0.5 \rightarrow$ no overshoot

$Q \leq \sqrt{\frac{1}{2}} = 0.707 \rightarrow$ widest BW for magnitude of the response.

One can relate Q (and thus overshoot) to PM.

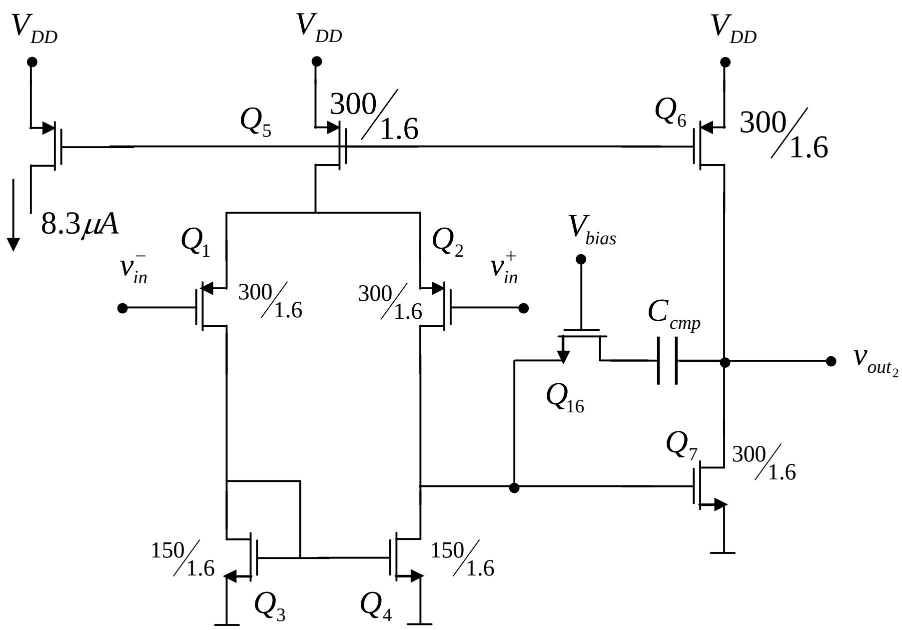
Remember that $\frac{\omega_t}{\omega_{eq}} = \tan(90° - PM)$.

PM	$\frac{\omega_t}{\omega_{eq}}$	$Q = \sqrt{\frac{\beta\omega_{ta}}{\omega_{eq}}}$ Assuming $\beta = 1$	% overshoot
55°	0.7	0.925	13.3%
60°	0.58	0.817	8.7%
65°	0.47	0.717	4.7%
70°	0.36	0.622	1.4%
75°	0.27	0.527	0.008%

If OPAMP is stable for $\beta = 1 \;\rightarrow$ it is stable for all sort of passive freq-independent feedback ($0 \le \beta \le 1$).

Problem is the OPAMP is overcompensated if $\beta = 1$ is not used → response is slower.

<u>Compensating our OPAMP</u>



C_{cmp} → controls ω_{p_1} → dominant pole of the OPAMP (at 1.8KHz)

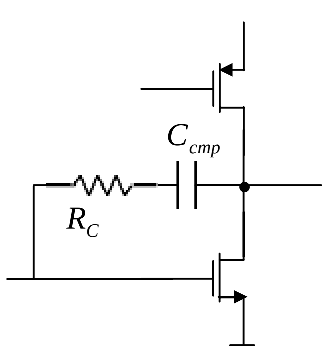
$$\omega_{ta} = A_0 \omega_{p_1} \;\rightarrow \omega_{p_1} = \frac{\omega_{ta}}{A_0}$$

$$\rightarrow f_{p_1} = \frac{25MHz}{14000} = 1.8KHz$$

Q_{16} is series connected to a capacitor → $V_{DS_{16}} = 0V$ → Q_{16} is in triode region

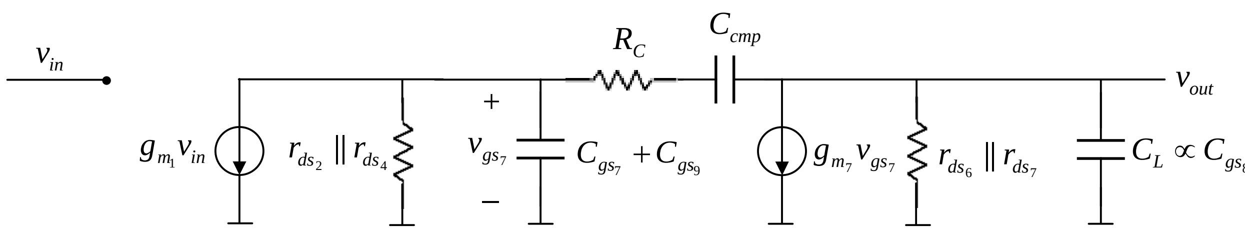
$$R_C = r_{ds_{16}} = \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_{16}\left(V_{GS_{16}} - V_{tn}\right)} \rightarrow$$

Note:
Since it is in triode, it is a real resistor with noise (unlike r_{ds} in saturation which is noiseless)



The RC compensation seen here is called “lead compensation”.

Small-signal model



* You can calculate V_{out}/V_{in}. Here we consider the effect of current through $R_C.C_{cmp}$:

What you will find is:

$$\frac{V_{out}}{V_{in}} = \frac{A_0\left(1-\dfrac{j\omega}{\omega_z}\right)}{\left(1+\dfrac{j\omega}{\omega_{p_1}}\right)\left(1+\dfrac{j\omega}{\omega_{p_2}}\right)}$$

Zero in transfer function due to $R_C.C_{cmp}$ path

Poles of the transfer function due to C_{cmp} and transistor capacitors

$$\omega_{p_1} = \frac{1}{r_{ds_2} \parallel r_{ds_4}\left(C_{gs_7} + C_{gs_9} + C_{cmp}(1+g_{m_7}.r_{ds_6} \parallel r_{ds_7})\right) + r_{ds_6} \parallel r_{ds_7} \times (C_L + C_{cmp})}$$

$$\omega_{p_1} \approx \frac{1}{(r_{ds_2} \parallel r_{ds_4})\left(C_{cmp}(1+g_{m_7}.r_{ds_6} \parallel r_{ds_7})\right)}$$

$$\boxed{\omega_{p_1} \approx \frac{1}{C_{cmp} g_{m_7} (r_{ds_2} \parallel r_{ds_4})(r_{ds_6} \parallel r_{ds_7})}} \leftarrow \text{dominant pole}$$

$$\omega_{p_2} = \frac{g_{m_7} C_{cmp}}{(C_{gs_7} + C_{gs_9}) C_L + C_L . C_{cmp} + (C_{gs_7} + C_{gs_9}) C_{cmp}}$$

$$C_{cmp} >> C_L ,\ (C_{gs_7} + C_{gs_9})$$

$$\Rightarrow \boxed{\omega_{p_2} \approx \frac{g_{m_7}}{C_{gs_7} + C_{gs_9} + C_L}}$$

$\leftarrow$ second pole, which is typically much higher in frequency proportional to the transconductance of the 2^{nd} stage.

$$\boxed{\omega_z = \frac{-1}{C_{cmp}\left(\dfrac{1}{g_{m_7}} - R_C\right)}} \leftarrow \text{right half-plane zero can cause instability}$$

You can choose ω_z for optimum performance.

* <u>If $R_C = 0\Omega$</u> → $\omega_z = -\dfrac{g_{m_7}}{C_{cmp}}$

Let's calculate $\dfrac{\omega_{p_1}}{\omega_{p_2}} = g_{m7}^2 . C_{cmp} \left(\dfrac{(r_{ds_2} \parallel r_{ds_4})(r_{ds_6} \parallel r_{ds_7})}{C_{gs_7} + C_{gs_9} + C_L} \right)$

If you separate the two poles → $\boxed{\text{more like one pole system}}$

You get better stability.

$\rightarrow$ to separate the two poles you want both g_{m_7} and C_{cmp} to be large.

$$\begin{cases} C_{cmp} \uparrow \quad \rightarrow \quad \omega_{p_1} \text{ decreases but } \omega_{p_2} \text{ is not changes} \rightarrow \text{OPAMP slower, but stable.} \\ \\ g_{m_7} \uparrow \quad \rightarrow \quad \omega_{p_1} \downarrow \text{ and } \omega_{p_2} \uparrow \end{cases}$$

Problem is with the zero that has a negative phase shift (phase lag), current in C_{cmp} is in the opposite phase with $g_{m_7}V_{gs_7}$ current that can cause instability.

To remove the phase lag you would like to reduce ω_z $\rightarrow$ make C_{cmp} large $\rightarrow$ not good because OPAMP becomes very slow.

g_{m_7} smaller $\rightarrow$ not good since the two poles become closer.

That's why we often use R_C (transistor Q_{16}) to move ω_z to smaller values without changing ω_{p_1}, ω_{p_2}.

* <u>If $R_C = 1/g_{m_7}$</u>

→ You can eliminate the zero of the TF all together.

* <u>If R_C is even higher</u>

→ You can move the zero from right half-plane to left half-plane (phase lead instead of phase lag) and ideally if:

$$\omega_z = \omega_{p_2}$$

Then you can cancel the non-dominant pole of the TF → one pole system → always stable.

$$\omega_z = \omega_{p_2} \quad \Rightarrow \quad -\frac{1}{C_{cmp}\left(\frac{1}{g_{m_7}} - R_C\right)} = \frac{g_{m_7}}{C_{gs_7} + C_{gs_9} + C_L}$$

$$\Rightarrow \quad -\frac{1}{g_{m_7}} + R_C = \frac{1}{g_{m_7}}\left(\frac{C_{gs_7} + C_{gs_9} + C_L}{C_{cmp}}\right)$$

$$\Rightarrow \quad R_C = \frac{1}{g_{m_7}}\left(\frac{C_{cmp} + C_{gs_7} + C_{gs_9} + C_L}{C_{cmp}}\right) \quad , \quad C_L \propto C_{gs_8}$$

* <u>Even higher R_C</u>

$$R_C = \frac{1}{1.2 g_{m_1}} \quad \Rightarrow \quad \omega_z = 1.2\omega_t$$

$$\omega_z = 1.2\frac{g_{m_1}}{C_{cmp}} = \frac{-1}{C_{cmp}\left(\frac{1}{g_{m_7}} - R_C\right)}$$

$$\rightarrow\; R_C - \frac{1}{g_{m_7}} = \frac{1}{1.2g_{m_1}} \;\rightarrow\; R_C = \frac{1}{1.2\underbrace{g_{m_1}}_{0.78\,mA/V}} + \frac{1}{\underbrace{g_{m_7}}_{1.9\,mA/V}}$$

→ This would give optimum lead compensation.

$R_C \propto \frac{1}{g_m}$ is an interesting concept from the following point of view. Let's look at the OPAMP high frequency performance:

$\omega_{ta} = \frac{g_{m_1}}{C_{cmp}} \propto g_{m_1}$: Unity gain frequency

$\omega_{p_2} = \frac{g_{m_7}}{C_{gs_7} + C_L} \propto g_{m_7}$: Second pole at high frequency

$$\omega_z = \frac{-1}{C_{cmp}\left(\frac{1}{g_{m_7}} - R_C\right)} \propto g_{m_7}$$

If $R_C \propto 1/g_{m_7}$ then

So high frequency performance of the OPAMP is proportional to g_{m_1} and g_{m_7}. As the temperature or process varies, g_{m_1} and g_{m_7} vary at the same rate, so ω_{ta}, ω_{p_2} and ω_z all vary at the same rate. But relative to each other they stay constant so OPAMP stays stable as temperature and process vary.

To get this, you need $R_C \propto {}^{1}\!/_{g_{m_7}}$.

What we know is:

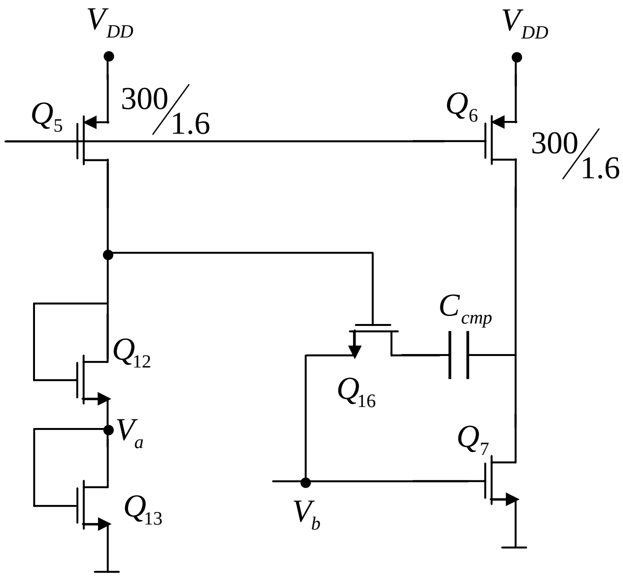
$$R_C = r_{ds_{16}(triode)} = \frac{1}{\mu_{nC_{ox}}\left(\frac{W}{L}\right)_{16}(V_{GS_{16}} - V_{tn})}$$

$$g_{m_7} = \mu_{nC_{ox}}\left(\frac{W}{L}\right)_{7}(V_{GS_7} - V_{tn})$$

$$R_C g_{m_7} = \frac{\mu_{nC_{ox}}\left(\frac{W}{L}\right)_{7}(V_{GS_7} - V_{tn})}{\mu_{nC_{ox}}\left(\frac{W}{L}\right)_{16}(V_{GS_{16}} - V_{tn})}$$

To be independent of process and temperature we need $\frac{(V_{GS_7} - V_{tn})}{(V_{GS_{16}} - V_{tn})}$ to be independent of process and temperature.

Let's look at the circuit:



Let's look at Q_{13} and Q_7 and find the ratio:

$$\frac{I_{D_7}}{I_{D_{13}}} = \frac{\frac{\mu_{nC_{ox}}}{2}\left(\frac{W}{L}\right)_7 (V_{GS_7} - V_{tn})^2}{\frac{\mu_{nC_{ox}}}{2}\left(\frac{W}{L}\right)_{13} (V_{GS_{13}} - V_{tn})^2} = \frac{\left(\frac{W}{L}\right)_7 (V_{GS_7} - V_{tn})^2}{\left(\frac{W}{L}\right)_{13} (V_{GS_{13}} - V_{tn})^2} \qquad \text{(I)}$$

On the other hand $\left.\begin{matrix} I_{D_7} = I_{D_6} \\ I_{D_{13}} = I_{D_{11}} \end{matrix}\right\}$ current mirror

$$\frac{I_{D_6}}{I_{D_{11}}} = \frac{\left(\frac{W}{L}\right)_6}{\left(\frac{W}{L}\right)_{11}} = \frac{I_{D_7}}{I_{D_{13}}} \qquad \text{(II)}$$

From the PMOS current mirrors

$$\rightarrow \text{(I), (II)} \rightarrow \frac{(V_{GS_7} - V_{tn})^2}{(V_{GS_{13}} - V_{tn})^2} = \frac{\left(\frac{W}{L}\right)_{13}}{\left(\frac{W}{L}\right)_7} \times \frac{\left(\frac{W}{L}\right)_6}{\left(\frac{W}{L}\right)_{11}}$$

$$\rightarrow \frac{(V_{GS_7} - V_{tn})}{(V_{GS_{13}} - V_{tn})} = \sqrt{\frac{\left(\frac{W}{L}\right)_{13}}{\left(\frac{W}{L}\right)_7} \times \frac{\left(\frac{W}{L}\right)_6}{\left(\frac{W}{L}\right)_{11}}} = 1$$

Since $\left\{\begin{matrix} W_{11} = W_{13} = 25 \\ W_6 = W_7 = 300 \end{matrix}\right.$

$$\rightarrow \boxed{V_{GS_7} - V_{tn} = V_{GS_{13}} - V_{tn}} \rightarrow \boxed{V_{GS_7} = V_{GS_{13}}}$$

$$(V_a = V_b)$$

On the other hand:

$$\begin{cases} V_{G_{16}} = V_{D_{12}} = V_{G_{12}} \quad \Rightarrow \quad connected\ together \\ \\ V_{S_{16}} = V_b = V_a = V_{S_{12}} \end{cases}$$

$$V_{GS_{16}} = V_{GS_{12}}$$

So we can write:

$$\frac{(V_{GS_7} - V_{tn})}{(V_{GS_{16}} - V_{tn})} = \frac{(V_{GS_{13}} - V_{tn})}{(V_{GS_{12}} - V_{tn})} = \sqrt{\frac{\dfrac{2I_{D_{13}}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_{13}}}{\dfrac{2I_{D_{12}}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_{12}}}}$$

Since $I_{D_{12}} = I_{D_{13}} \quad \Rightarrow$

$$\frac{(V_{GS_7} - V_{tn})}{(V_{GS_{16}} - V_{tn})} = \sqrt{\frac{\left(\frac{W}{L}\right)_{12}}{\left(\frac{W}{L}\right)_{13}}}$$

$$\Rightarrow \quad R_C g_{m_7} = \frac{\left(\frac{W}{L}\right)_7}{\left(\frac{W}{L}\right)_{16}} \times \frac{(V_{GS_7} - V_{tn})}{(V_{GS_{16}} - V_{tn})} = \frac{\left(\frac{W}{L}\right)_7}{\left(\frac{W}{L}\right)_{16}} \sqrt{\frac{\left(\frac{W}{L}\right)_{12}}{\left(\frac{W}{L}\right)_{13}}} = \frac{300}{W_{16}}$$

Our analysis is not entirely correct, since Q_{16} is not in ideal MOS saturation!

Since I's are independent of Bias/μ's, etc. g_m's are also independent of Bias/temperature/process.