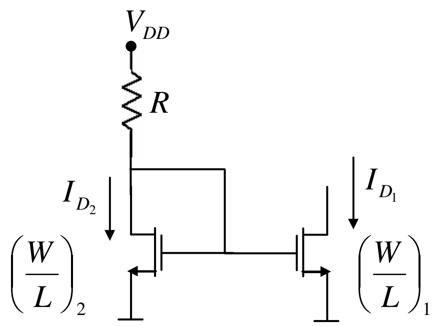
Output resistance of current mirror



$$\begin{cases} \dfrac{V_{DD} - V_{GS_2}}{R} = I_{D_2} \\ \\ I_{D_2} = \dfrac{\mu C_{ox} W_2}{2L_2}(V_{GS_2} - V_{t_n})^2 \end{cases}$$

Assuming both transistors is in ideal MOS saturation

I_{D_2} , V_{GS_2} are unknowns and you have two equations, therefore you can find them both.

(Then check for $V_{DS} > V_{GS} - V_{t_n}$)

Once you have $V_{GS\,2}$ → $V_{GS_1} = V_{GS_2}$ and $\dfrac{I_{D_1}}{I_{D_2}} = \dfrac{W_1/L_1}{W_2/L_2}$

I_{D_1} , that you will find is for the case your MOS#1 has infinite output resistance.

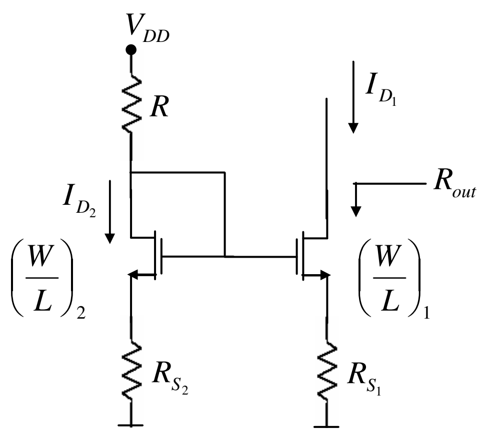
If that is not the case then:

$$I_{D_1} = \frac{W_1/L_1}{W_2/L_2} \cdot I_{D_2} \frac{(1+\lambda V_{DS1})}{(1+\lambda V_{GS})}$$

$$V_{GS1} = V_{GS2} = V_{DS2}$$

The goal is to increase R_{out} so the current I_{D1} is independent of V_{DS1}.

Source degenerate current mirrors



To find V_{GS_2}, I_{D_2}:

$$\left.\begin{array}{l} I_{D_2} = \mu_n C_{ox} \dfrac{W}{2L}(V_{GS_2} - V_{t_n})^2 \\ \\ V_{DD} - (R_{S_2} + R) I_{D_2} = V_{GS_2} \end{array}\right\} \rightarrow$$

again two unknowns, so you can find V_{GS_2}, I_{D_2}

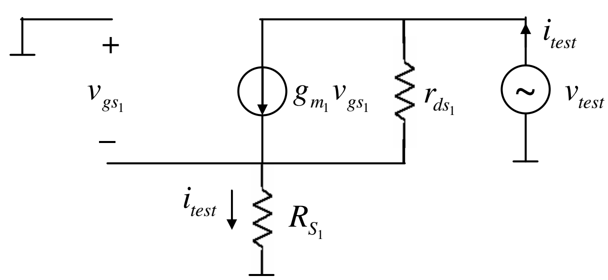
$(V_{DS_2} = V_{GS_2})$

→ check for $V_{DS} > V_{GS} - V_{t_n}$

Now, to find I_{D_1}:

$$\left.\begin{array}{l} \underset{\text{known}}{\underset{\uparrow}{V_{GS_2}}} - \underset{\text{known}}{\underset{\uparrow}{R_{S_2} I_{D_2}}} = \underset{\text{unknow}}{\underset{\uparrow}{V_{GS_1}}} + \underset{\text{unknow}}{\underset{\uparrow}{R_S I_{D_1}}} \\ \\ I_{D_1} = \dfrac{\mu_n C_{ox} W_1}{2L_1}(V_{GS} - V_{t_n})^2 \end{array}\right\} \quad \text{you can find } I_D \text{ and } V_{GS_1}$$

To calculate the output resistance:



$$V_{gs_1} = -i_{test} \cdot R_{S_1}$$

$$i_{test} R_{S_1} + r_{ds_1}(i_{test} - g_{m_1} V_{gs_1}) = v_{test}$$

$$i_{test} R_{S_1} + r_{ds_1} i_{test} + g_{m_1} i_{test} \cdot R_{S_1} \cdot r_{ds_1} = v_{test}$$

$$i_{test}(r_{ds_1} + R_{S_1} + g_{m_1} R_{S_1} r_{ds_1}) = v_{test}$$

$$\frac{v_{test}}{i_{test}} = R_{out} = r_{ds_1}(1 + g_{m_1} R_{S_1}) + R_{S_1} \approx r_{ds_1}(1 + g_{m_1} R_{S_1})$$

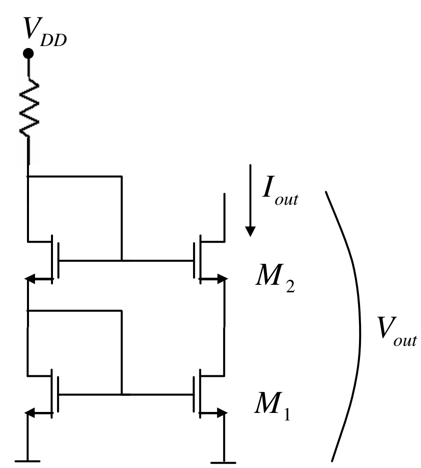
Since R_{S1} is typically much smaller than r_{ds1}

Therefore the output resistance has been increased by $(1+g_m R_{S_1})$.

The penalty is now you have voltage drop across R_{S_1} (and also R_{S_2}), so it is harder to keep M1 in saturation.

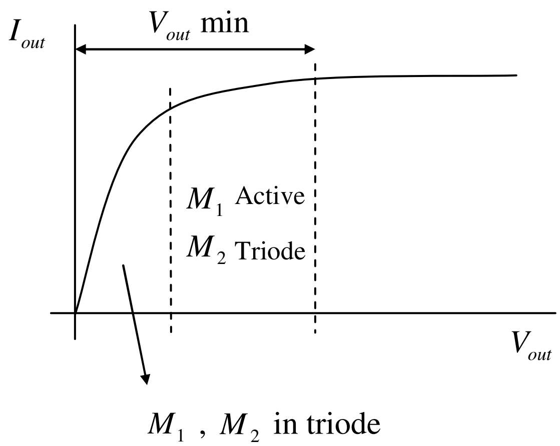
You can do the same thing by using a transistor instead of a resistor.

Cascode Current Source

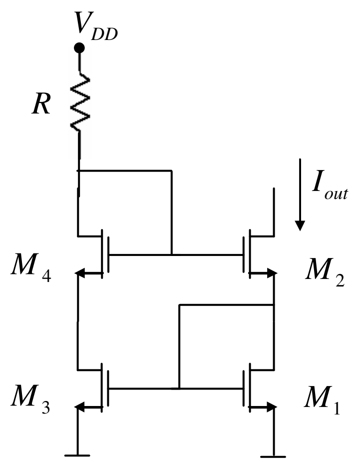


Same analogy with resistors → $R_{out} = r_{ds_2}(1+g_m r_{ds_1}) + r_{ds_1}$

But now you need to keep both transistors in saturation.

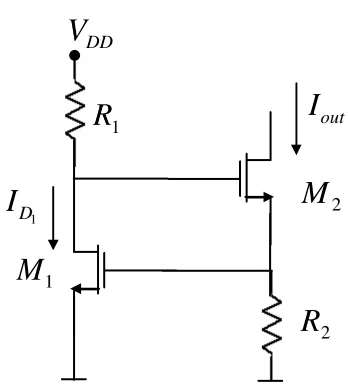


Wilson Current Source



$$R_{out} \approx (1 + g_{m_2} r_{ds_3}) r_{ds_2}$$

Threshold Reference Current Source



$$I_{out} = \frac{V_{GS_1}}{R_2} = \frac{V_{th} + \sqrt{\dfrac{2I_D}{\mu C_{ox} \dfrac{W_1}{L_1}}}}{R_2}$$

$$\left.\begin{array}{l}\text{Make } I_{D_1} \text{ small} \\ \dfrac{W_1}{L_1} \text{ large}\end{array}\right\}$$

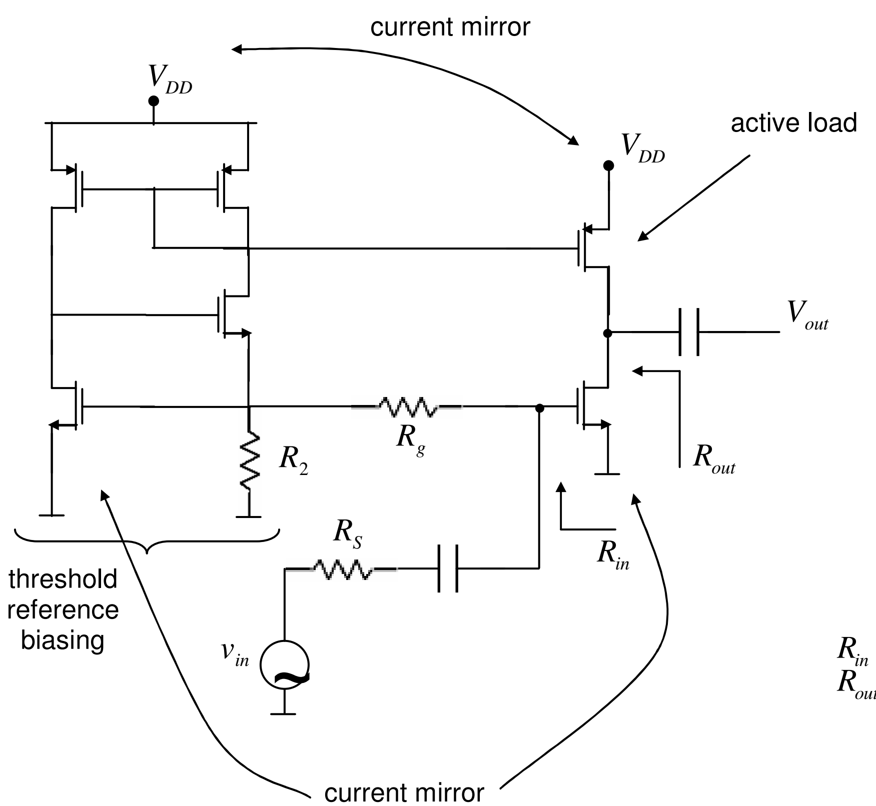
$$\Rightarrow \quad \sqrt{\frac{2I_D}{\frac{\mu C_{ox} W_1}{2L_1}}} << V_{t_n}$$

$$\Rightarrow \quad I_{out} \approx \frac{V_{t_n}}{R_2}$$

Integrated Common Source Amplifier

Components:

* Active load for gain
* Current source for biasing

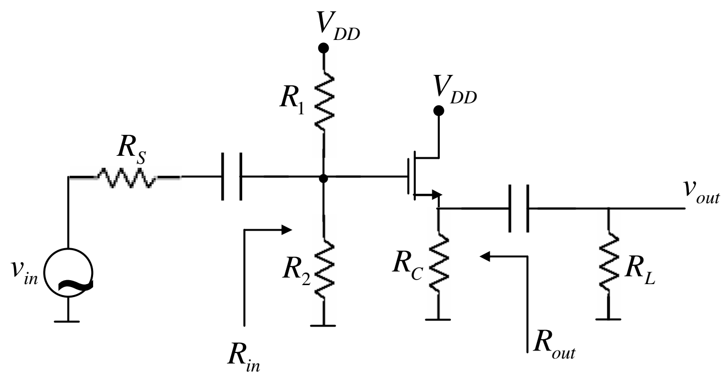


$R_{in} \sim R_G$
$R_{out} = r_{ds(nmos)} \parallel r_{ds(pmos)}$

$$A_v = \frac{-g_m (r_{ds_{nmos}} \parallel r_{ds_{pmos}} \parallel R_L) R_g}{R_S + R_g}$$

Common Drain Amplifier

Source-follower



$$V_{DS} = V_{DD} - V_S = V_{DD} - I_D R_C$$

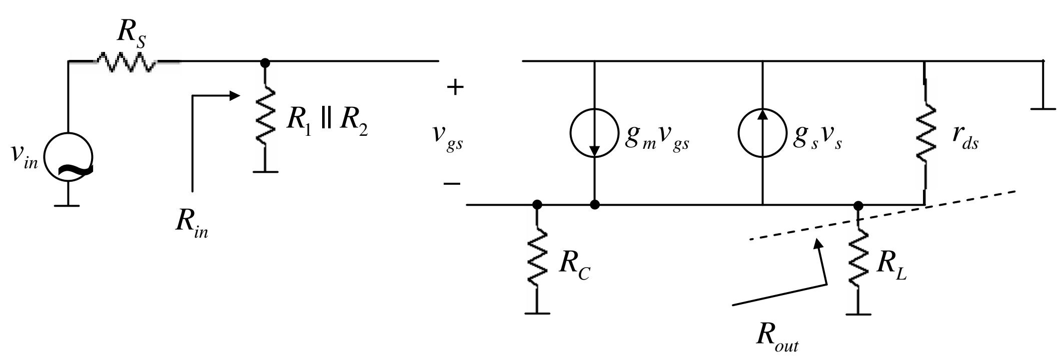
$$\left.\begin{aligned} V_G &= \frac{R_1}{R_1 + R_2} V_{DD} \\ V_S &= I_D R_C \end{aligned}\right\} \rightarrow$$

$$\left.\begin{aligned} V_{GS} &= \frac{R_1}{R_1 + R_2} V_{DD} - I_D R_C \\ I_D &= \frac{\mu_n C_{ox} W}{2L} (V_G - V_{t_n})^2 \end{aligned}\right\}$$

Two equations and two unknowns

You can find both V_{GS} and I_D $\Rightarrow$ test $V_{DS} > V_G - V_{t_n}$ for ideal MOS saturation.

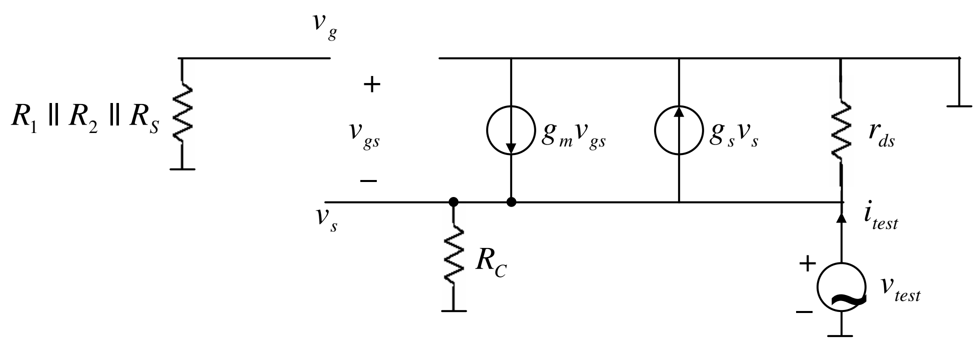
Equivalent Circuit:



$R_{in} = R_1 \parallel R_2$

$R_{out} = ?$

$$g_s = \frac{\gamma g_m}{2\sqrt{V_{SB} + |2\Phi_f|}}$$



$$\left.\begin{aligned} v_g &= 0 \\ v_s &= v_{test} \end{aligned}\right\} \rightarrow \quad \frac{v_{test}}{R_C \parallel r_{ds}} + g_s v_s - g_m v_{gs} = i_{test}$$

$$\frac{v_{test}}{R_C \parallel r_{ds}} + g_s v_{test} + g_m v_{test} = i_{test}$$

$$\frac{i_{test}}{v_{test}} = \frac{1}{R_C \parallel r_{ds}} + g_s + g_m \quad \rightarrow \quad R_{out} = \frac{1}{g_m} \parallel \frac{1}{g_s} \parallel R_C \parallel r_{ds}$$

$$\Rightarrow \boxed{R_{out} = \frac{1}{g_m + g_s} \parallel R_C \parallel r_{ds}} \quad \rightarrow \quad R_{out} \approx \frac{1}{g_m + g_s}$$

Small value at high currents, since $g_m \uparrow$ as $I_D \uparrow$.

To find $A_v = \dfrac{v_{out}}{v_{in}}$

$$v_g = \frac{R_1 \parallel R_2}{R_S + R_1 \parallel R_2} \cdot v_{in}$$

$$v_{out} = (g_m v_{gs} - g_s v_s)(r_{ds} \parallel R_C \parallel R_L)$$

$$v_s = v_{out}\ , \quad v_{gs} = v_g - v_s = v_g - v_{out}$$

$$\rightarrow \quad v_{out} = (g_m v_g - (g_m + g_s)v_{out})(r_{ds} \parallel R_C \parallel R_L)$$

$$v_{out} + (g_m + g_s)(r_{ds} \parallel R_C \parallel R_L)v_{out} = g_m (r_{ds} \parallel R_C \parallel R_L)v_{gs}$$

$$v_{out}(1 + (g_m + g_s)(r_{ds} \parallel R_C \parallel R_L)) = \frac{g_m (r_{ds} \parallel R_C \parallel R_L) \cdot R_1 \parallel R_2}{R_S + R_1 \parallel R_2} v_{in}$$

$$A_v = \frac{v_{out}}{v_{in}} = \underbrace{\frac{g_m (r_{ds} \parallel R_C \parallel R_L)}{1 + (g_m + g_s)(r_{ds} \parallel R_C \parallel R_L)}}_{<1} \cdot \underbrace{\frac{R_1 \parallel R_2}{R_S + R_1 \parallel R_2}}_{<1}$$

At best you can get $A_v \sim \dfrac{g_m}{g_m + g_s}$.