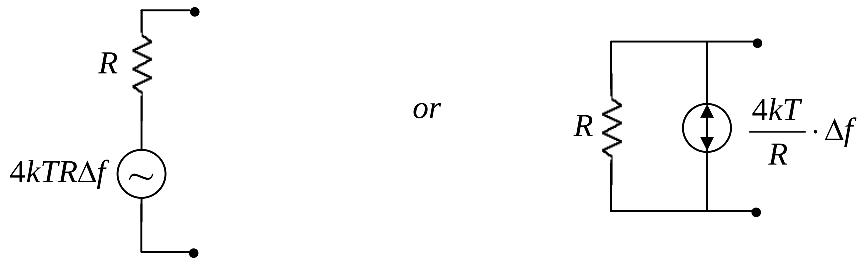
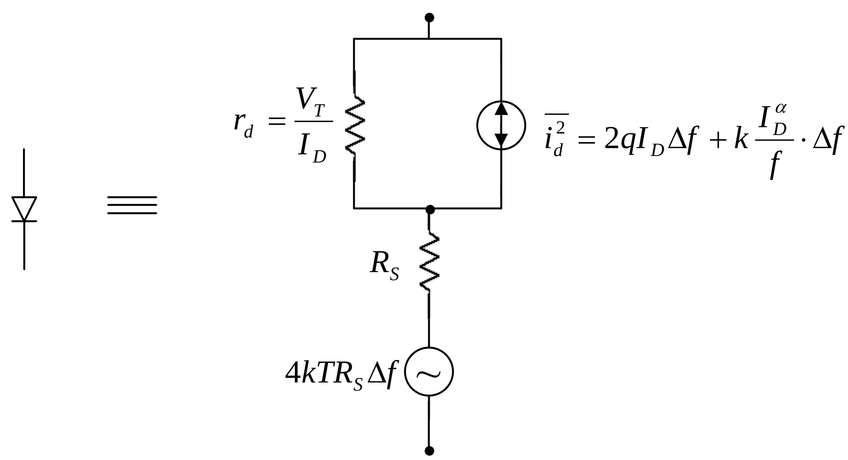
Noise in Resistors

Thermal noise (+ maybe flicker noise)



Noise in Diode

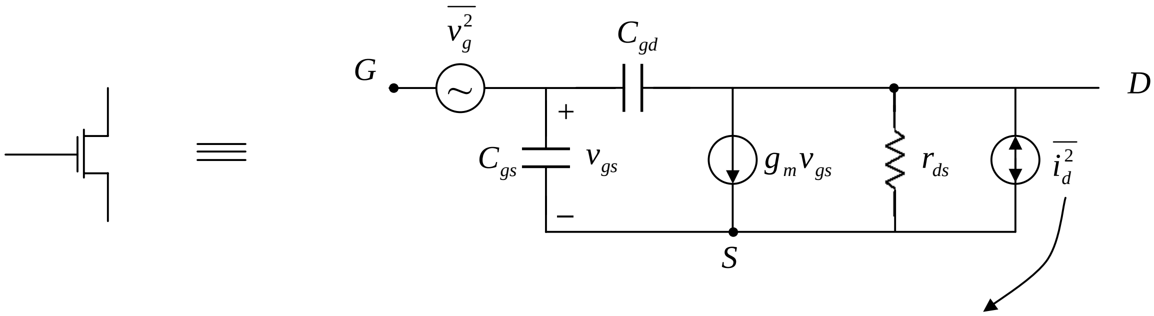
Shot noise (+ thermal noise from contact resistance + maybe flicker noise)



Noise in MOS transistor

Thermal noise (in channel) + flicker noise in gate region

Model for saturation region:



This thermal noise is not due to r_{ds} .

$$\overline{v_g^{\,2}} = \frac{k}{WLC_{ox}\,f}.\Delta f \quad \leftarrow \text{flicker noise}$$

(Experimentally they have found that flicker noise in MOS is not a function of drain current)

$$\overline{i_d^{\,2}} = 4kT\gamma g_m.\Delta f$$

$$\gamma = \frac{2}{3} \quad \rightarrow \quad long-channel\ devices$$

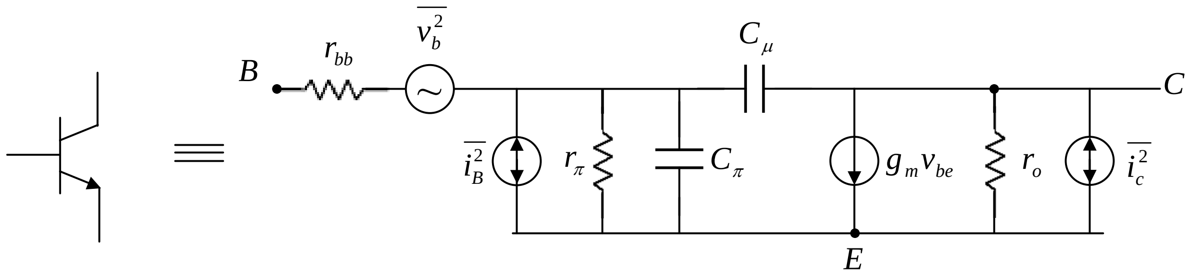
$$\gamma = 2 \sim 3 \quad \rightarrow \quad short-channel\ devices$$

Noise in Bipolar ternsistors

Thermal noise in base (contact r_{bb} and intrinsic base)

+ shot noise in base

+ shot noise in collector + flicker noise in base



$$\overline{v_g^{\;2}} = \left(4kTr_{bb} + 4kT\frac{1}{2g_m}\right)\Delta f$$

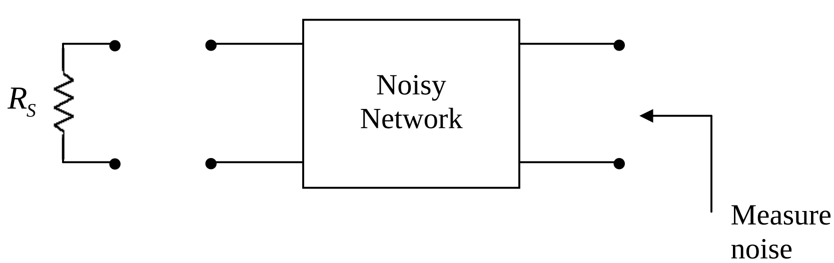
$$\overline{i_B^{\;2}} = \left(2qI_B + \frac{k'I_B}{f}\right)\Delta f$$

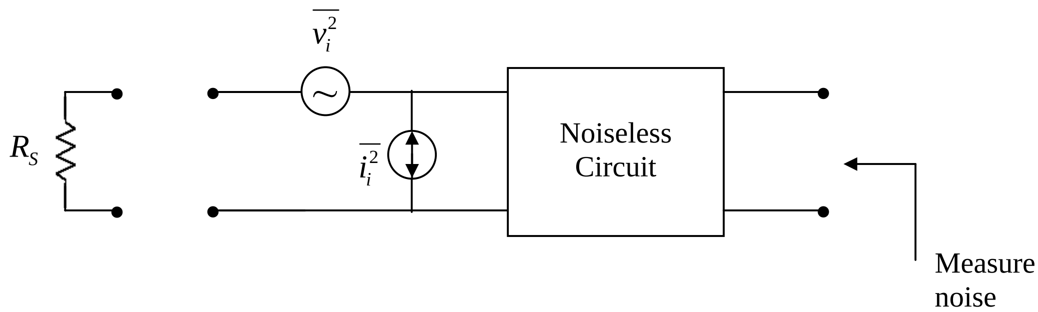
In the book you have $2q\left(I_B + \frac{kI_B}{f} + \ldots\right)$ so k and k' are related.

$$\overline{i_c^{\;2}} = 2qI_C\Delta f$$

How to calculate total Noise?

Consider the following circuits





Any noise 2-port network can be represented by a noiseless network and two input voltage and current noise sources. → (there might be some correlation between them)

Why do we need both $\overline{v_i^2}$ and $\overline{i_i^2}$?

$R_S \to 0$ → $\overline{i_i^2}$ shorts out but $\overline{v_i^2}$ still represents the noise.

$R_S \to \infty$ → $\overline{v_i^2}$ cannot generate any noise at the output but $\overline{i_i^2}$ does.

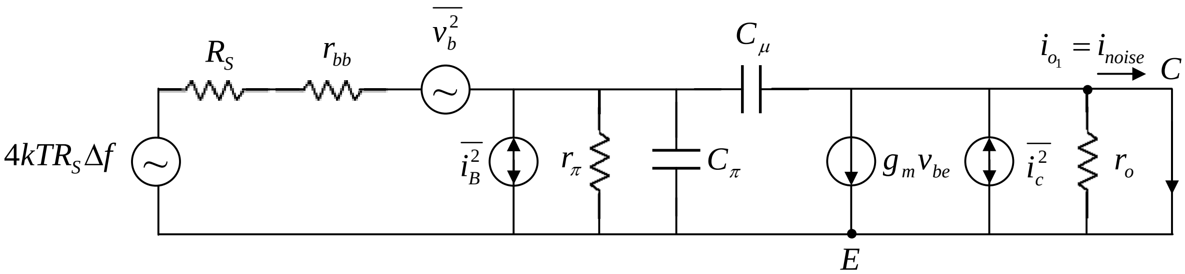
<u>One more thing</u>

To measure noise at the output, it may be easier to consider output current noise when output is short-circuited.

Example

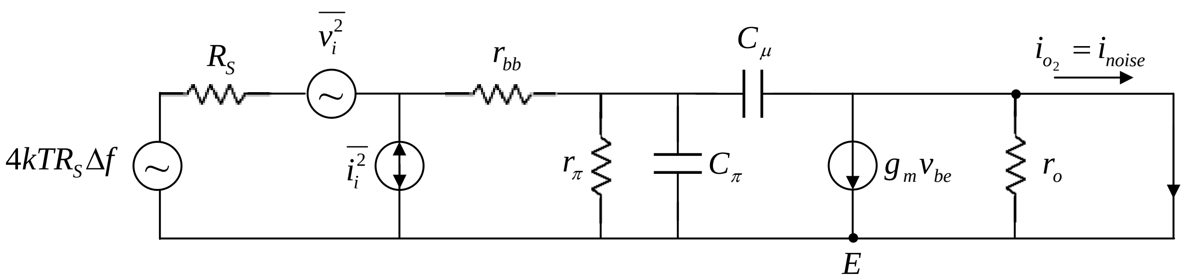
Calculate $\overline{v_i^2}$ and $\overline{i_i^2}$ for a bipolar transistor.

Noisy Network



To measure output noise we will short circuit collector.

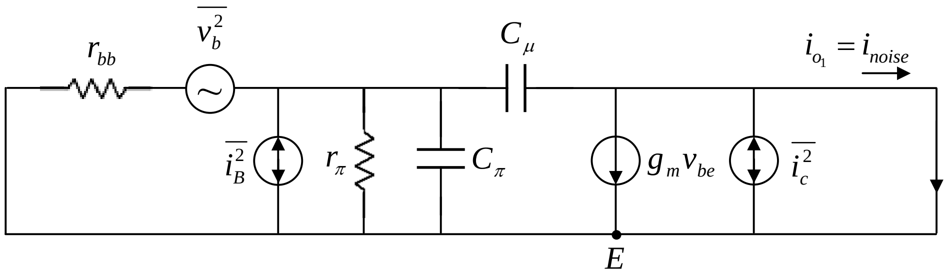
Noiseless Network



→ To find $\overline{v_i^2}$ short-circuit input ($R_S = 0$) and measure output noise in both circuits.

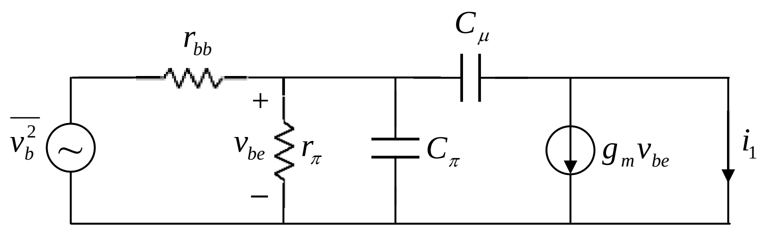
→ To find $\overline{i_i^2}$ open-circuit input ($R_S = \infty$) and measure output noise in both circuits.

Noisy network with $R_S = 0$:



Assuming noise sources are un-correlated we can use superposition.

Effect of $\overline{v_b^2}$ alone:



Ignoring the current through C_μ.

$$Z_\pi = r_\pi \parallel (C_\pi + C_\mu) = \frac{r_\pi}{1 + j\omega(C_\pi + C_\mu) r_\pi} \quad \rightarrow \left|Z_\pi\right|^2 = \frac{r_\pi^{\ 2}}{1 + \omega^2 (C_\pi + C_\mu)^2 . r_\pi^{\ 2}}$$

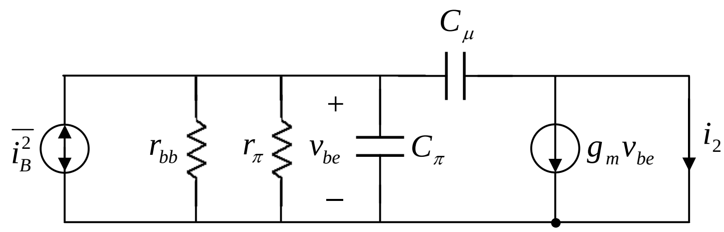
$$v_{be} = \left|\frac{Z_\pi}{Z_\pi + r_{bb}}\right| \left|\overline{v_b}\right| \quad \rightarrow \quad \overline{v_{be}^{\ 2}} = \frac{\left|Z_\pi\right|^2}{\left|r_{bb} + Z_\pi\right|^2} \overline{v_b^{\ 2}}$$

$$\left|r_{bb} + Z_\pi\right|^2 = \left|\frac{r_\pi + r_{bb} + j\omega(C_\pi + C_\mu).r_\pi.r_{bb}}{1 + j\omega(C_\pi + C_\mu).r_\pi}\right|^2 = \frac{(r_\pi + r_{bb})^2 + \omega^2 (C_\pi + C_\mu)^2 . r_\pi^{\ 2} . r_{bb}^{\ 2}}{1 + \omega^2 (C_\pi + C_\mu)^2 . r_\pi^{\ 2}}$$

$$\overline{v_{be}^{\ 2}} = \frac{r_\pi^{\ 2}}{(r_\pi + r_{bb})^2 + \omega^2 (C_\pi + C_\mu)^2 . r_\pi^{\ 2} . r_{bb}^{\ 2}} . \overline{v_b^{\ 2}}$$

$$\overline{i_1^{\ 2}} = g_m^2 \left|\overline{v_{be}^{\ 2}}\right| = g_m^2 \overline{v_{be}^{\ 2}} = \frac{g_m^{\ 2} r_\pi^{\ 2}}{(r_\pi + r_{bb})^2 + \omega^2 (C_\pi + C_\mu)^2 . r_\pi^{\ 2} . r_{bb}^{\ 2}} . \overline{v_b^{\ 2}}$$

Effect of $\overline{i_B^2}$ alone:



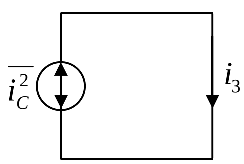
Ignore the current through C_μ.

$$Z_x = r_\pi \parallel r_{bb} \parallel (C_\pi + C_\mu) \quad \rightarrow |Z_x|^2 = \frac{(r_\pi \parallel r_{bb})^2}{1+\omega^2 (C_\pi + C_\mu)^2 . (r_\pi \parallel r_{bb})^2}$$

$$\overline{v_{be}{}^2} = \overline{i_B{}^2} . |Z_x|^2$$

$$\overline{i_2{}^2} = g_m^2 \overline{v_{be}{}^2} = \frac{g_m{}^2 (r_\pi \parallel r_{bb})^2}{1+\omega^2 (C_\pi + C_\mu)^2 . (r_\pi \parallel r_{bb})^2} . \overline{i_B{}^2} = \frac{g_m{}^2 r_\pi{}^2 r_{bb}{}^2}{(r_\pi + r_{bb})^2 + \omega^2 (C_\pi + C_\mu)^2 . r_\pi{}^2 . r_{bb}{}^2} . \overline{i_B{}^2}$$

Effect of $\overline{i_C^2}$ alone:



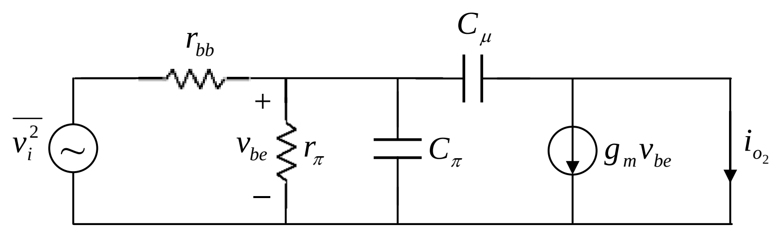
$$\overline{i_3^2} = \overline{i_C^2}$$

$$\rightarrow \quad \overline{i_{o_1}^2} = \overline{i_1{}^2} + \overline{i_2{}^2} + \overline{i_3{}^2} = \frac{g_m{}^2 r_\pi{}^2}{(r_\pi + r_{bb})^2 + \omega^2 (C_\pi + C_\mu)^2 . r_\pi{}^2 . r_{bb}{}^2} . \overline{v_b{}^2} + \frac{g_m{}^2 r_\pi{}^2 r_{bb}{}^2}{(r_\pi + r_{bb})^2 + \omega^2 (C_\pi + C_\mu)^2 . r_\pi{}^2 . r_{bb}{}^2} . \overline{i_B{}^2} + \overline{i_C{}^2}$$

$$\overline{i_{o_1}^2} = \frac{g_m{}^2 r_\pi{}^2}{(r_\pi + r_{bb})^2 + \omega^2 (C_\pi + C_\mu)^2 . r_\pi{}^2 . r_{bb}{}^2} \left(\overline{v_b{}^2} + r_{bb}{}^2 \overline{i_B{}^2} \right) + \overline{i_C{}^2}$$

Now we need to calculate $\overline{i_{o_2}^2}$.

Noiseless network with $R_S = 0$



$$\overline{v_{be}^{\ 2}} = \overline{v_i^{\ 2}} \cdot \frac{|Z_\pi|^2}{|r_{bb} + Z_\pi|^2}$$

$$\overline{v_{be}^{\ 2}} = \frac{r_\pi^{\ 2}}{(r_\pi + r_{bb})^2 + \omega^2 (C_\pi + C_\mu)^2 . r_\pi^{\ 2} . r_{bb}^{\ 2}} . \overline{v_i^{\ 2}}$$

$$\overline{i_{o_2}^{\ 2}} = \frac{g_m^{\ 2} r_\pi^{\ 2}}{(r_\pi + r_{bb})^2 + \omega^2 (C_\pi + C_\mu)^2 . r_\pi^{\ 2} . r_{bb}^{\ 2}} . \overline{v_i^{\ 2}}$$

$$\overline{i_{o_1}^{\ 2}} = \overline{i_{o_2}^{\ 2}} \quad \rightarrow \quad \boxed{\overline{v_i^{\ 2}} = \overline{v_b^{\ 2}} + r_{bb}^{\ 2} \overline{i_B^{\ 2}} + \frac{(r_\pi + r_{bb})^2 + \omega^2 (C_\pi + C_\mu)^2 . r_\pi^{\ 2} . r_{bb}^{\ 2}}{g_m^{\ 2} r_\pi^{\ 2}} . \overline{i_C^{\ 2}}}$$

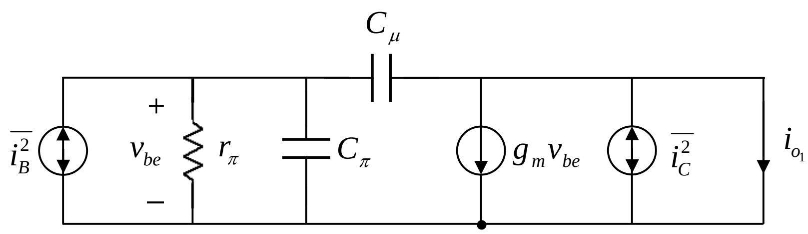
Assuming $r_\pi >> r_{bb} \quad \rightarrow \quad r_\pi + r_{bb} \approx r_\pi$.

$$\boxed{\overline{v_i^{\ 2}} = \overline{v_b^{\ 2}} + r_{bb}^{\ 2} \overline{i_B^{\ 2}} + \frac{\overline{i_C^{\ 2}}}{g_m^{\ 2}} \left(1 + \omega^2 (C_\pi + C_\mu)^2 . r_{bb}^{\ 2}\right)}$$

If $r_{bb} \rightarrow 0 \quad \Rightarrow \overline{v_i^{\ 2}} = \overline{v_b^{\ 2}} + \dfrac{\overline{i_C^{\ 2}}}{g_m^{\ 2}}$.

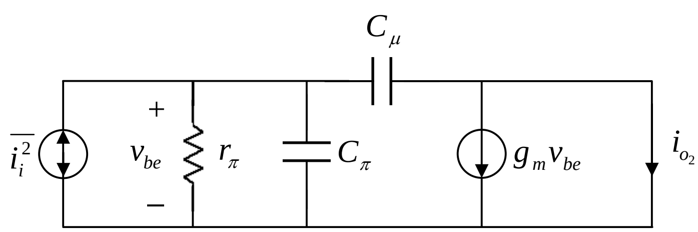
To find $\overline{i_i^2}$ assume $R_S = \infty$:

Noisy network with $R_S = \infty$



$$\overline{i_{o_1}^{\ 2}} = g_m^{\ 2}\overline{v_{be}^{\ 2}} + \overline{i_C^{\ 2}} = \frac{g_m^{\ 2} r_\pi^{\ 2}}{1+\omega^2 (C_\pi + C_\mu)^2 . r_\pi^{\ 2}} . \overline{i_B^{\ 2}} + \overline{i_C^{\ 2}}$$

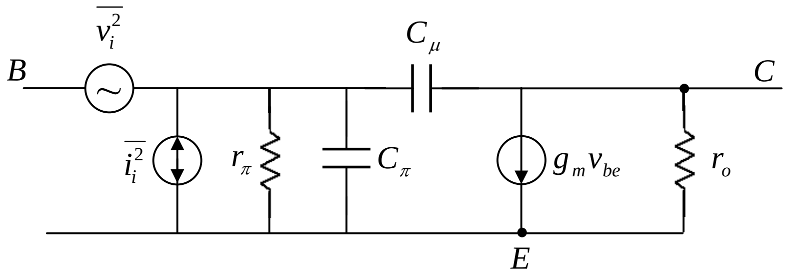
Noiseless network with $R_S = \infty$



$$\overline{i_{o_2}^{\ 2}} = \frac{g_m^{\ 2} r_\pi^{\ 2} \overline{i_i^{\ 2}}}{1+\omega^2 (C_\pi + C_\mu)^2 . r_\pi^{\ 2}} \qquad g_m r_\pi = \beta$$

$$\overline{i_{o_1}^{\ 2}} = \overline{i_{o_2}^{\ 2}} \quad \rightarrow \boxed{\overline{i_i^{\ 2}} = \overline{i_B^{\ 2}} + \frac{1+\omega^2 (C_\pi + C_\mu)^2 . r_\pi^{\ 2}}{\beta^2} \cdot \overline{i_C^{\ 2}}}$$

Therefore the noise equivalent circuit of a bipolar transistor, ignoring the effect of r_{bb} is:



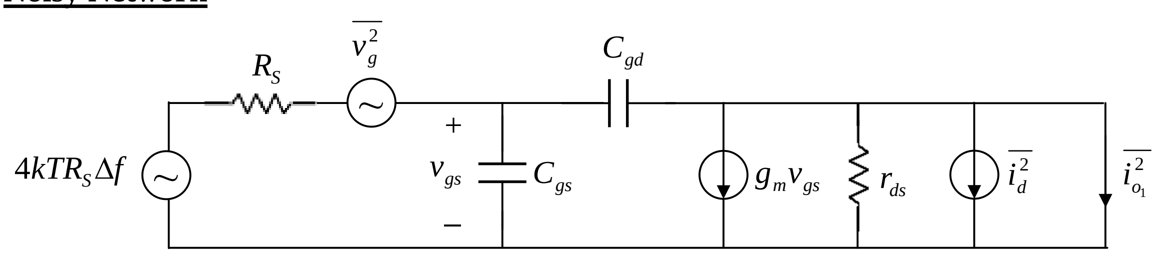
$$\boxed{\overline{v_i^2} = \overline{v_b^2} + \frac{\overline{i_C^2}}{g_m^2} = 4kT\frac{1}{2g_m}.\Delta f + \frac{2qI_C\Delta f}{g_m^2}}$$

$$\overline{i_i^2} = \overline{i_B^2} + \frac{1+\omega^2(C_\pi + C_\mu)^2.r_\pi^2}{\beta^2}.\overline{i_C^2}$$

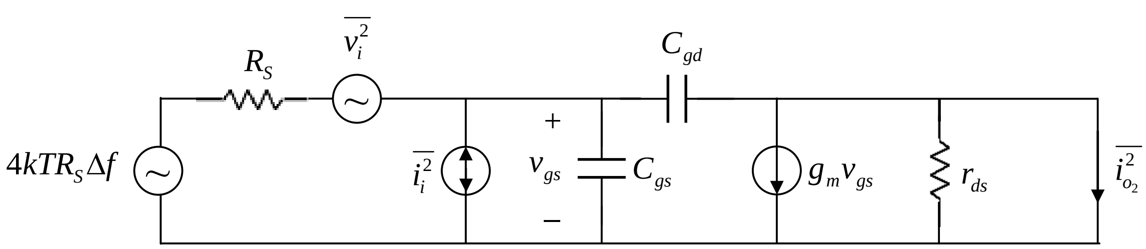
$$\boxed{\overline{i_i^2} = \left(2qI_B + \frac{k'I_B}{f}\right)\Delta f + \frac{1+\omega^2(C_\pi + C_\mu)^2.r_\pi^2}{\beta^2} \times 2qI_C\Delta f}$$

Noise equivalent circuit of MOS transistor with noise sources at the input:

<u>Noisy Network</u>

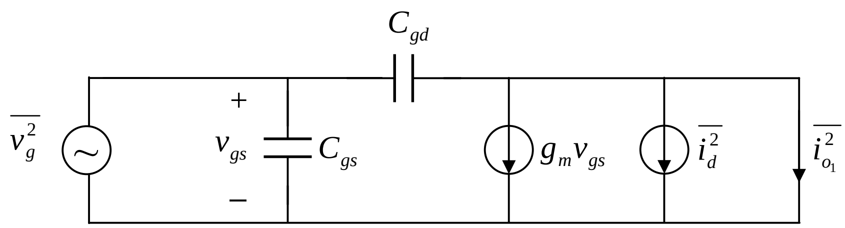


Noiseless network with noise sources at the input:



To find $\overline{v_i^2}$ assume $R_S = 0$.

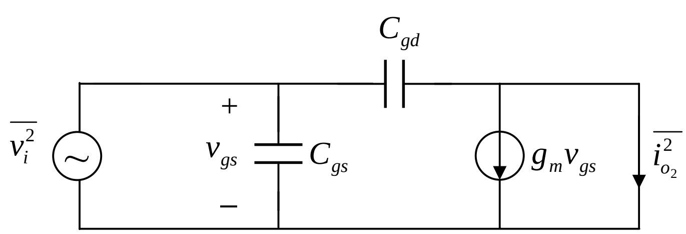
Noisy network with $R_S = 0$



$\overline{v_g^2} = \overline{v_{gs}^2}$

Ignoring current through C_{gd} $\rightarrow$ $\overline{i_{o_1}^2} = \overline{i_d^2} + g_m^2 \overline{v_g^2}$.

Noiseless network with $R_S = 0$



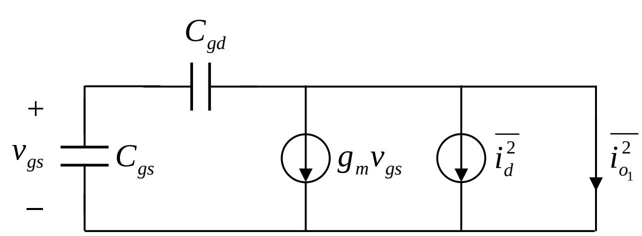
$\overline{v_{gs}^2} = \overline{v_i^2}$

$\rightarrow$ $\overline{i_{o_2}^2} = g_m^2 \overline{v_i^2}$

$\rightarrow$ $\overline{i_{o_1}^2} = \overline{i_{o_2}^2} \rightarrow \boxed{\overline{v_i^2} = \overline{v_g^2} + \frac{1}{g_m^2} \cdot \overline{i_d^2}}$

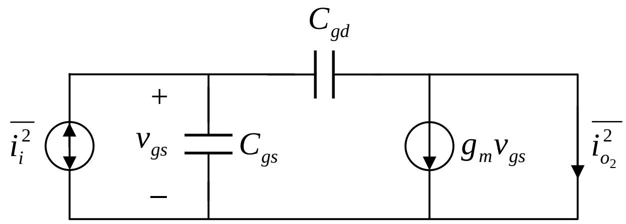
To find $\overline{i_i^2}$ assume $R_S = \infty$.

Noisy network with $R_S = \infty$:



Ignoring current through C_{gd} $\rightarrow$ $\overline{i_{o_1}^2} = \overline{i_d^2}$.

Noiseless network with $R_S = \infty$

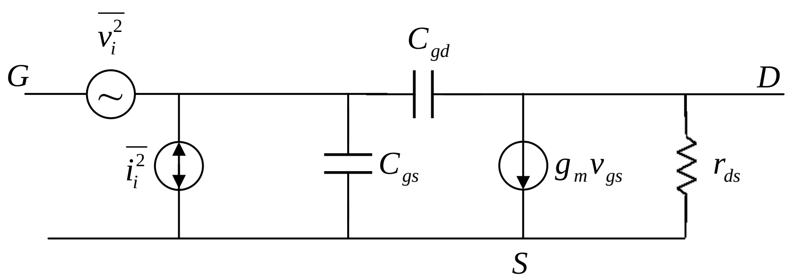


Ignoring C_{gd} current → $\overline{i_{o_2}^2} = g_m^2 \overline{v_{gs}^2} = g_m^2 \times Z^2 \cdot \overline{i_i^2}$.

$$\overline{i_{o_2}^2} = \frac{1}{\omega^2 (C_{gs} + C_{gd})^2} \cdot \overline{i_i^2} \cdot g_m^2$$

$$\overline{i_{o_1}^2} = \overline{i_{o_2}^2} \quad \rightarrow \quad \boxed{\overline{i_i^2} = \frac{\omega^2 (C_{gs} + C_{gd})^2}{g_m^2} \cdot \overline{i_d^2}}$$

Therefore the noise equivalent circuit for a MOS transistor is:

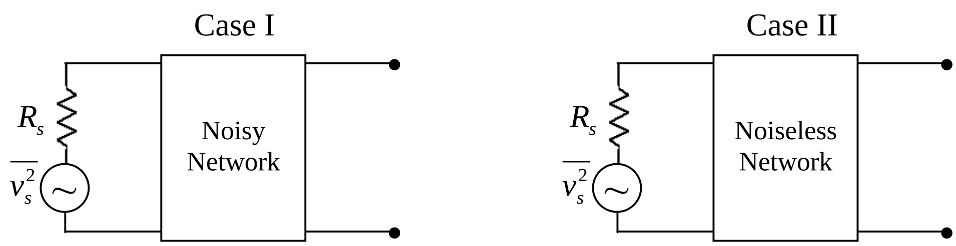


$$\overline{v_i^2} = \overline{v_g^2} + \frac{\overline{i_d^2}}{g_m^2} = \frac{k}{WLC_{ox} f} . \Delta f + \frac{4kT\gamma\Delta f}{g_m}$$

$$\overline{i_i^2} = \frac{\omega^2 (C_{gs} + C_{gd})^2}{g_m^2} \cdot \overline{i_d^2} = \frac{\omega^2 (C_{gs} + C_{gd})^2}{g_m^2} \times 4kT\gamma g_m \Delta f$$

Noise Figure and Noise Temperature

Noise Figure → shows how noisy a device/amplifier is.

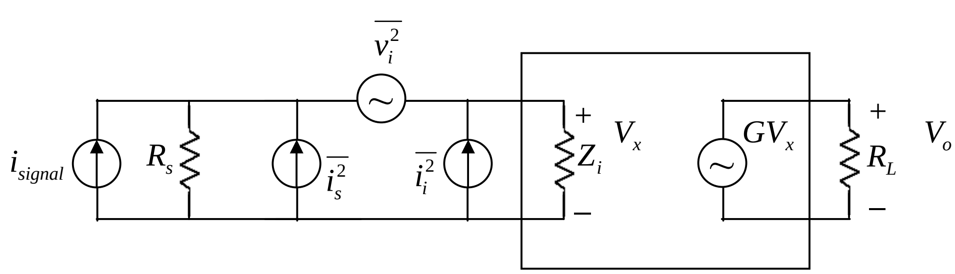


$$\text{Noise figure} = \frac{\text{Available noise power at the output in case I}}{\text{Available noise power at the output in case II}}$$

You can also show noise figure in terms of S/N ratio.

$$\text{Noise figure} = \text{F} = \frac{\text{input S/N ratio}}{\text{output S/N ratio}}$$

We can calculate the noise figure for a noisy network with equivalent input noise current and voltage.



$v_x = V_i \dfrac{Z_i}{Z_i + R_S} + i_i \dfrac{R_S Z_i}{Z_i + R_S} + i_s \dfrac{R_S Z_i}{Z_i + R_S}$, assuming uncorrelated current/voltage noise source.

Noise power in R_L :

$$N_{out\ \ noisy} = \frac{|G|^2}{R_L}\overline{v_x^2} = \frac{|G|^2}{R_L}\left(\overline{v_i^2}\,\frac{|Z_i|^2}{|Z_i+R_S|^2} + \left(\overline{i_i^{\,2}} + \overline{i_s^{\,2}}\right)\frac{|R_S Z_i|^2}{|Z_i+R_S|^2}\right)$$

$$N_{out\ \ noiseless\ \ network} = \frac{|G|^2}{R_L}.\overline{i_s^{\,2}}.\frac{|R_S Z_i|^2}{|Z_i+R_S|^2} \quad , \quad \overline{i_s^{\,2}} = \frac{4kT\Delta f}{R_s}$$

noise figure = F = $\dfrac{N_{out\ \ noisy}}{N_{out\ \ noiseless\ \ network}}$

$$F = 1 + \frac{\overline{v_i^{\,2}}}{4kTR_S\,\Delta f} + \frac{\overline{i_i^{\,2}}}{4kT\,\dfrac{1}{R_s}\,\Delta f}$$

$\rightarrow$ noise figure depends on R_S and input current/voltage noise sources.

There is an optimum R_S that results in a minimum F:

$$\frac{\partial F}{\partial R_S} = 0 \quad \rightarrow \quad \frac{\overline{v_i^{\,2}}}{4kT\Delta f}\left(\frac{-1}{R_{S_{opt}}^2}\right) + \frac{\overline{i_i^{\,2}}}{4kT\Delta f} = 0$$

$$R_{S_{opt}}^2 = \frac{\overline{v_i^{\,2}}}{\overline{i_i^{\,2}}}$$ optimum source impedance for minimum noise

Noise Figure is at its minimum when $R_S = R_{S_{opt}}$, therefore:

$$F_{\min} = 1 + \frac{\overline{v_i^{\,2}}}{4kTR_{S_{opt}}\Delta f} + \frac{\overline{i_i^{\,2}}}{4kT\dfrac{1}{R_{S_{opt}}}\Delta f}$$

$$\boxed{R_{S_{opt}}^2 = \frac{\overline{v_i^{\,2}}}{\overline{i_i^{\,2}}}}$$

$$F_{\min} = 1 + \frac{\overline{i_i^{\,2}}.R_{S_{opt}}^2}{4kTR_{S_{opt}}\Delta f} + \frac{\overline{i_i^{\,2}}}{4kT\dfrac{1}{R_{S_{opt}}}\Delta f} = 1 + \overline{i_i^{\,2}}\left(\frac{R_{S_{opt}}}{4kT\Delta f} + \frac{R_{S_{opt}}}{4kT\Delta f}\right)$$

$$\boxed{F_{\min} = 1 + \frac{2\overline{i_i^{\,2}}.R_{S_{opt}}}{4kT\Delta f} = 1 + \frac{2\overline{v_i^{\,2}}}{4kTR_{S_{opt}}\Delta f}}$$

For MOS transistor: at frequencies higher than noise corner frequency (a few MHz), one can ignore the flicker noise. Therefore:

$$\left.\begin{aligned} \overline{v_i^{\,2}} &= \frac{4kT\gamma\Delta f}{g_m} \quad \rightarrow no \quad \frac{1}{f} \quad component \\ \overline{i_i^{\,2}} &= \frac{\omega^2(C_{gs}+C_{gd})^2}{g_m^2} \times 4kT\gamma g_m\Delta f \end{aligned}\right\}$$

$$R_{S_{opt}}^2 = \frac{\overline{v_i^{\,2}}}{\overline{i_i^{\,2}}}$$

$$R_{S_{opt}}^2 = \frac{\dfrac{4kT\gamma\Delta f}{g_m}}{\dfrac{4kT\gamma\Delta f}{g_m} \times \omega^2(C_{gs}+C_{gd})^2}$$

$$R_{S_{opt}}^2 = \frac{1}{\omega^2 (C_{gs} + C_{gd})^2}$$

$$R_{S_{opt}} = \frac{\pm 1}{\omega(C_{gs} + C_{gd})} \qquad F_{\min} > 1 \rightarrow \boxed{R_{S_{opt}} = \frac{1}{\omega(C_{gs} + C_{gd})}}$$

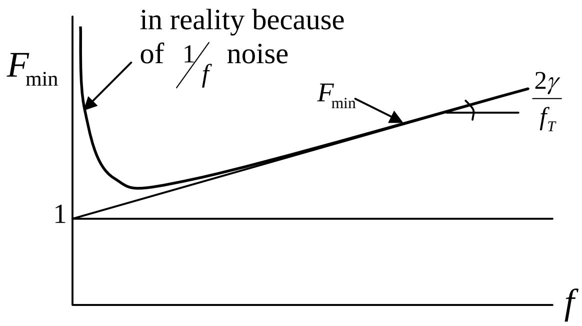
$$F_{\min} = 1 + \frac{2\overline{v_i^2}}{4kTR_{S_{opt}}\Delta f} = 1 + \frac{2 \times \dfrac{4kT\gamma\Delta f}{g_m}}{4kT\Delta f \cdot \dfrac{1}{\omega(C_{gs} + C_{gd})}}$$

$$\boxed{F_{\min} = 1 + \frac{2\gamma\omega(C_{gs} + C_{gd})}{g_m}}$$

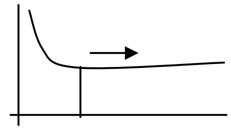
minimum noise figure for MOS transistor

$$\omega_T = \frac{g_m}{(C_{gs} + C_{gd})} \rightarrow cut-off \quad frequency \Rightarrow F_{\min} = 1 + \frac{2\gamma\omega}{\omega_T}$$

$$\boxed{F_{\min} = 1 + \frac{2\gamma f}{f_T}}$$



For bipolar transistors: again ignoring flicker noise for frequencies above noise corner frequency:



$$\overline{v_i^2} = \frac{4kT}{2g_m}\Delta f + \frac{2qI_C\Delta f}{g_m^2}$$

$$\overline{i_i^2} = 2qI_B\Delta f + \frac{1+\omega^2(C_\pi + C_\mu)^2 r_\pi^2}{\beta^2}\times 2qI_C\Delta f$$

assuming large β and small parasitic:

$$\overline{i_i^2} \approx 2qI_B\Delta f$$

$$\rightarrow R_{S_{opt}}^2 = \frac{\overline{v_i^2}}{\overline{i_i^2}} = \frac{\frac{4kT}{2g_m}\Delta f + \frac{2qI_C\Delta f}{g_m^2}}{2qI_B\Delta f} = \frac{kT}{q}\times\frac{1}{g_m.I_B} + \frac{I_C/I_B}{g_m^2}$$

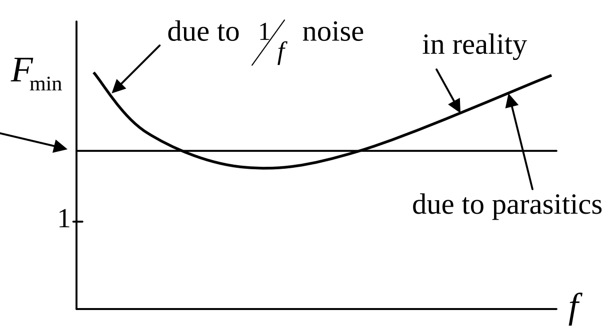
$$\rightarrow R_{S_{opt}}^2 = \frac{V_T/I_B}{g_m} + \frac{\beta}{g_m^2} = \frac{r_\pi}{g_m} + \frac{\beta}{g_m^2} = 2\frac{\beta}{g_m^2}$$

$$R_{S_{opt}} = \frac{\pm\sqrt{2\beta}}{g_m} \rightarrow \boxed{R_{S_{opt}} = \frac{\sqrt{2\beta}}{g_m}}$$

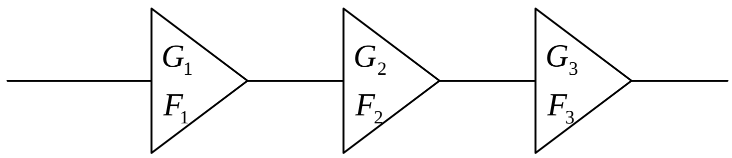
$$F_{\min} = 1 + \frac{2\overline{i_i^2}R_{S_{opt}}}{4kT\Delta f} \rightarrow F_{\min} = 1 + \frac{2\times 2qI_B\Delta f\times\frac{\sqrt{2\beta}}{g_m}}{4kT\Delta f}$$

$$\overline{i_i^2} = 2qI_B\Delta f$$

$$\boxed{F_{\min} = 1 + \sqrt{\frac{2}{\beta}}}$$



Noise figure of cascaded stages:



F → noise figure

G → power gain = (voltage gain x current gain)

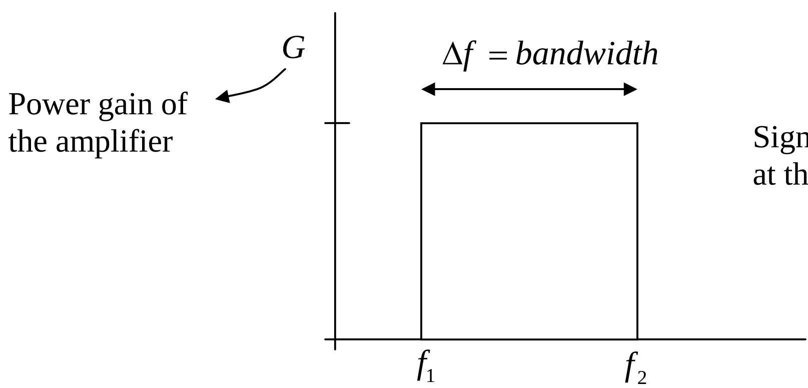
$$F_{overall} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots$$

<u>Minimum Detectable Signal (MDS)</u>

Also known as the <u>noise floor</u> of the system

<u>Important observation</u>: → an input signal is detectable only if its output is above noise level.

To calculate MDS, you assume your amplifier has an ideal box shape transfer function:



Signal power at the output

$* S_{out} = G.S_{in}$

Signal power at the input

Noise power at the output

Noise figure, since amplifier adds noise

$* N_{out} = F.G.N_{in}$

Noise power at the input

$$\frac{S_{out}}{N_{out}} = \frac{1}{F}\left(\frac{S_{in}}{N_{in}}\right)$$

$$\rightarrow \left(\frac{S}{N}\right)_{out} = \frac{1}{F}\left(\frac{S}{N}\right)_{in}$$

Signal to noise ratio at output

Signal to noise ratio at input

Assume that you need $\left(\frac{S}{N}\right)_{out} = 3dB$ to be able to detect the output signal → ($3dB = \times 2$)

→ $\left(\frac{S}{N}\right)_{out} = 2 \quad \Rightarrow \quad \left(\frac{S}{N}\right)_{in} = 2F$

$$N_{in} = \frac{1}{2}\left(\frac{\overline{U_n^2}}{2R_S}\right) = \frac{1}{2}\times\frac{4kTR_S\Delta f}{2R_S} = kT\Delta f$$

Because only ½ of the noise goes to the amplifier. The other half is wasted in R_S.

$$S_{in}\big|_{\min} = 2FN_{in} = 2FkT\Delta f$$

$$MDS = V_{in}\big|_{\min} = \sqrt{4R_S \times S_{in}\big|_{\min}}$$

Minimum limit of detectable signal

$$MDS = V_{in}\big|_{\min} = \sqrt{4R_S \times 2FkT\Delta f}$$

$10\log F = 8dB$

$\rightarrow F = 6.3$

Example: Determine MDS of an amplifier with 8dB Noise figure and bandwidth of 2.1KHz ($R_s = 50\Omega$)

$$MDS = \sqrt{4\times 50\times 2\times 6.3\times 2.1\times 10^{3}\times 4.16\times 10^{-21}}$$

$$MSD = 0.14\mu v$$