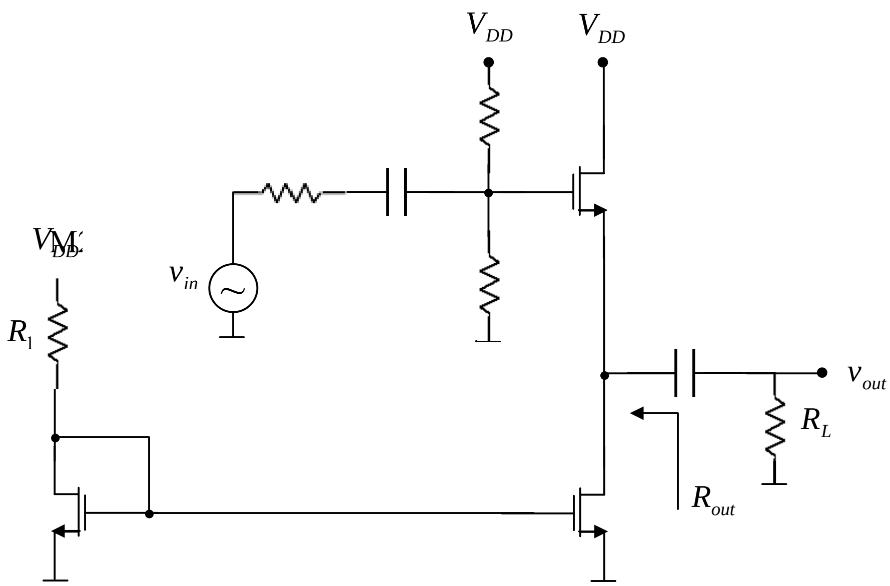
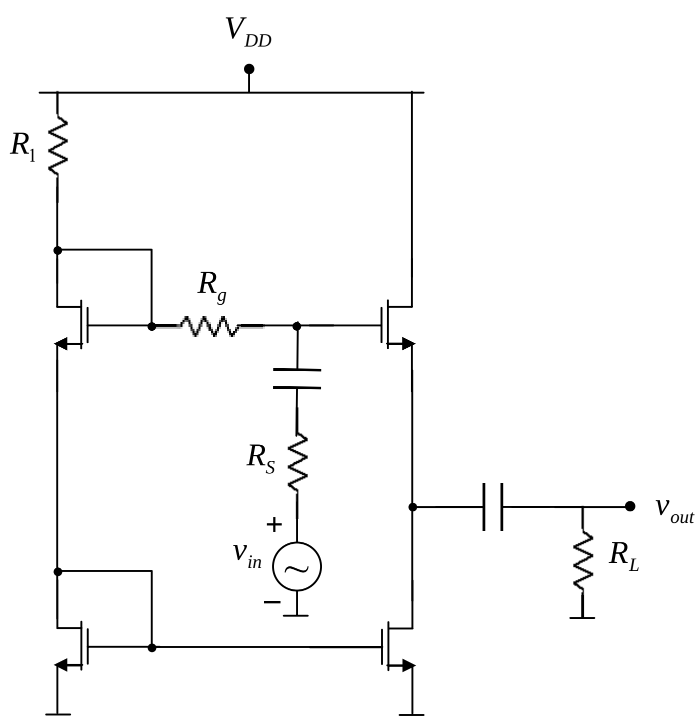
Integrated Source Follower

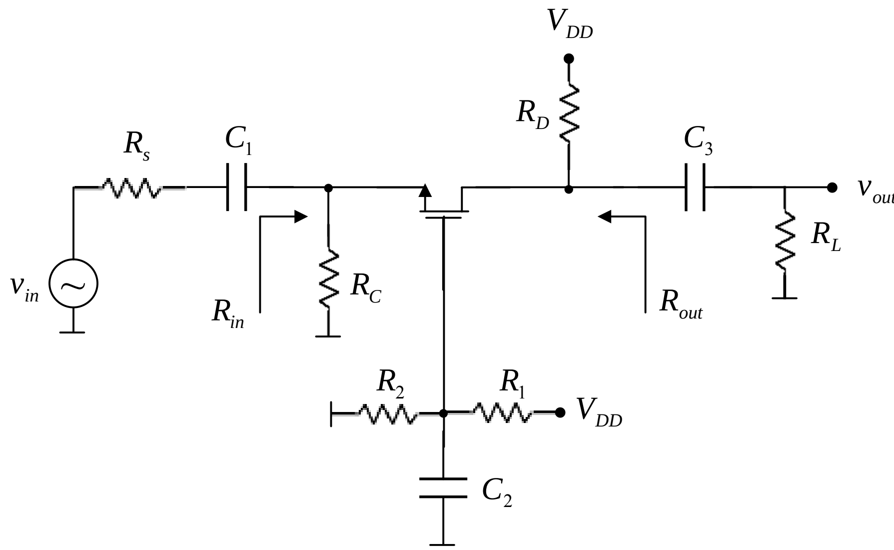


A simpler version:



Common Gate Amplifier

Non-integrated version

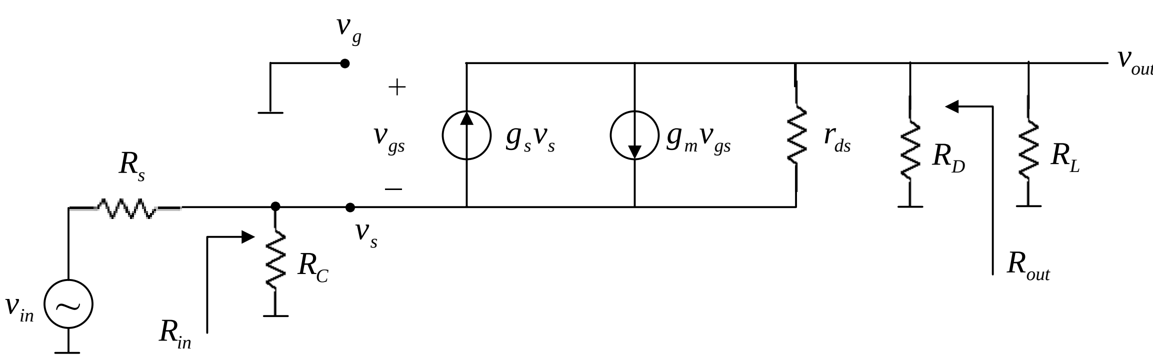


$$V_{DS} = V_{DD} - (R_D + R_C)I_D$$

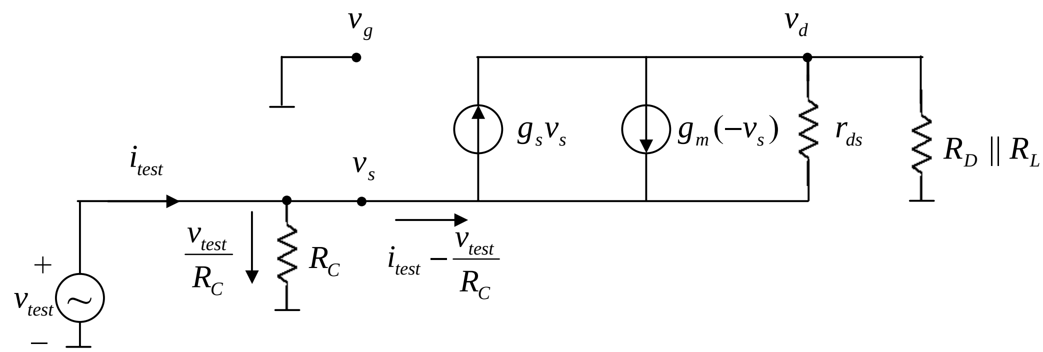
$$\left.\begin{array}{l} I_D = \dfrac{\mu_n C_{ox} W}{2L}(V_{GS} - V_{t_n})^2 \\[2ex] \left.\begin{array}{l} V_G = \dfrac{R_2}{R_1 + R_2} V_{DD} \\[2ex] V_S = R_C I_D \end{array}\right\} \rightarrow \; V_{GS} = V_G - V_S = \dfrac{R_2}{R_1 + R_2} V_{DD} - R_C I_D \end{array}\right\} \text{2 equations and 2 unknowns:} V_{GS},\ I_D .$$

from I_D you can find V_{DS} and check for ideal MOS saturation condition: $V_{DS} > V_{GS} - V_{t_n}$.

Equivalent Circuit



To calculate R_{in}



$$\left.\begin{array}{l} (i_{test} - \frac{v_{test}}{R_C}) R_D \parallel R_L = v_d \\ \\ v_s = v_{test} \end{array}\right\} \rightarrow v_{ds} = v_d - v_s$$

$$\rightarrow v_{ds} = i_{test}.R_D \parallel R_L - v_{test}\left(\frac{R_D \parallel R_L}{R_C} + 1\right)$$

Also:

$$g_s v_s + g_m v_s - \frac{v_{ds}}{r_{ds}} = i_{test} - \frac{v_{test}}{R_C}$$

$$(g_s + g_m) v_{test} + v_{test}\frac{\left(\frac{R_C + R_D \parallel R_L}{R_C}\right)}{r_{ds}} - i_{test}\frac{R_D \parallel R_L}{r_{ds}} = i_{test} - \frac{v_{test}}{R_C}$$

$$\rightarrow v_{test}\left(g_s + g_m + \frac{R_C + R_D \parallel R_L}{R_C.r_{ds}} + \frac{1}{R_C}\right) = i_{test}\left(1 + \frac{R_D \parallel R_L}{r_{ds}}\right)$$

$$\boxed{R_{in} = \frac{v_{test}}{i_{test}} = \frac{\frac{r_{ds} + R_D \parallel R_L}{r_{ds}}}{g_s + g_m + \frac{R_C + R_D \parallel R_L}{R_C.r_{ds}} + \frac{1}{R_C}}}$$

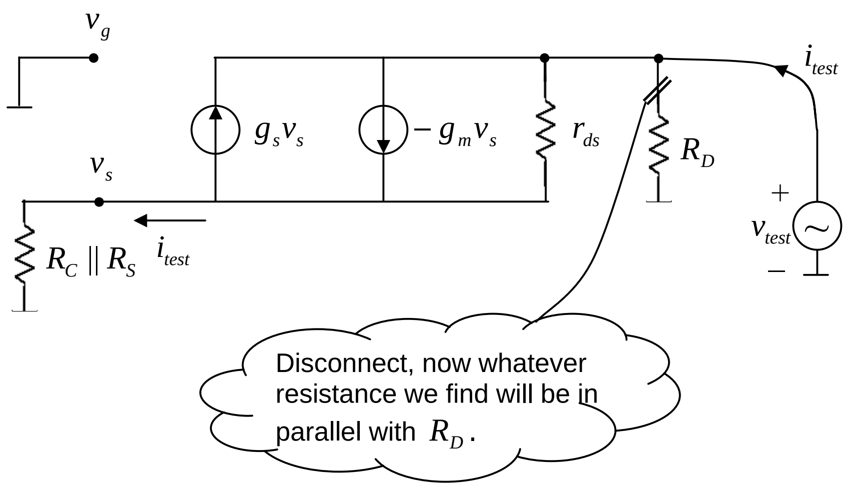
You can simplify the equation; assuming r_{ds} is large →

$$r_{ds} >> \frac{1}{g_m} + g_s$$
$$r_{ds} >> R_D \parallel R_L, \;\; R_C$$

$$\boxed{R_{in} = \frac{1}{g_m + g_s + \frac{1}{R_C}}} \quad \alpha \quad \boxed{R_{in} = \frac{1}{g_m + g_s} \parallel R_C}$$

Similar to R_{out} of source-follower

To calculate R_{out}:



$$v_s = i_{test}(R_C \parallel R_S)$$

$$i_{test} = -g_s v_s - g_m v_s + \frac{(v_{test} - v_s)}{r_{ds}}$$

$$\rightarrow \; i_{test} + (g_s + g_m + \frac{1}{r_{ds}})v_s = \frac{v_{test}}{r_{ds}}$$

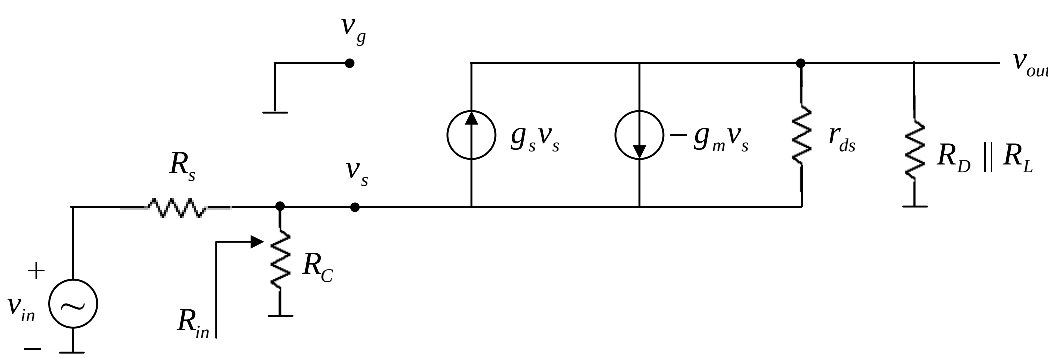
$$i_{test}\left(1+(R_C \parallel R_S)(g_s+g_m+\frac{1}{r_{ds}})\right)=\frac{v_{test}}{r_{ds}}$$

$$\rightarrow \frac{v_{test}}{i_{test}}=r_{ds}+(R_C \parallel R_S)\left[(g_s+g_m)r_{ds}+1\right]$$

$$R_{out}=R_D \parallel \frac{v_{test}}{i_{test}}=R_D \parallel \left(r_{ds}+(R_C \parallel R_S)\left[(g_s+g_m)r_{ds}+1\right]\right)$$

$$R_{out}=R_D \parallel \textit{large resistor}=R_D.$$

Voltage Gain (A_v)

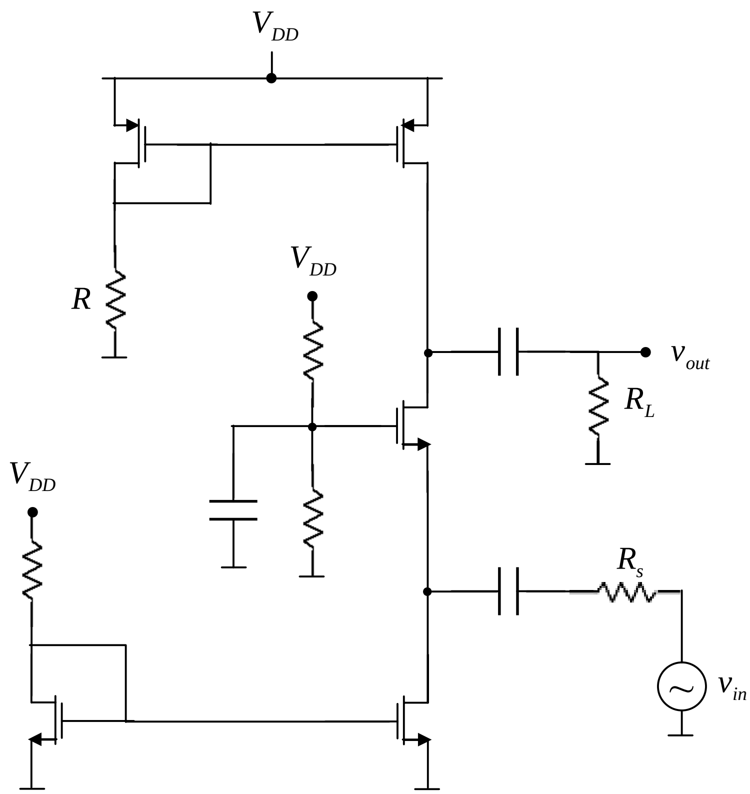


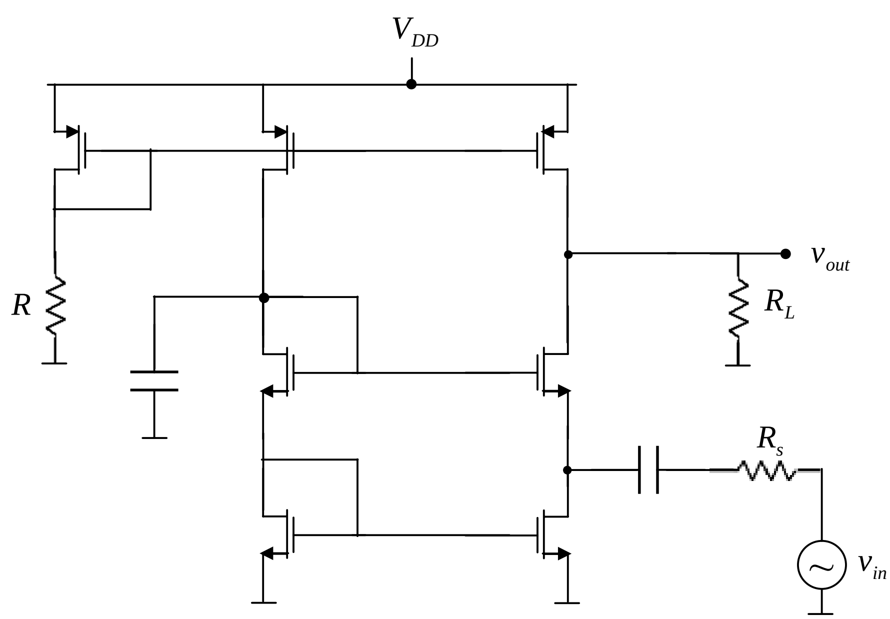
First simplify it (assume $r_{ds} >> \frac{1}{g_m+g_s}$) and ignore r_{ds}.

$$\left.\begin{aligned} v_{out} &= (g_s+g_m)v_s.R_D \parallel R_L \\ v_s &= \frac{R_{in}}{R_{in}+R_S}v_{in} \end{aligned}\right\} \Rightarrow \boxed{\frac{v_{out}}{v_{in}}=\frac{R_{in}}{R_{in}+R_S}(g_s+g_m).(.R_D \parallel R_L)}$$

$$R_{in} \cong \frac{1}{g_m+g_s+\frac{1}{R_C}}$$

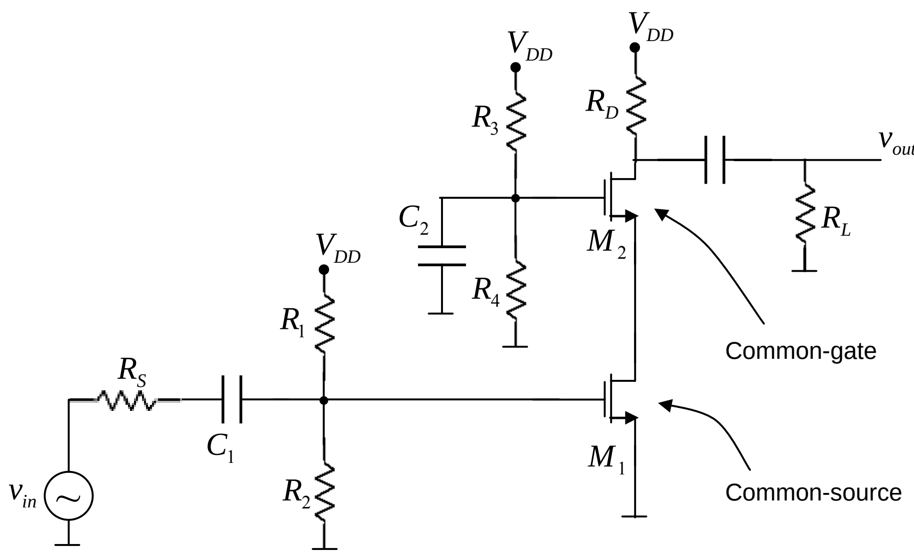
Integrated Common-gate Amplifier





<u>Cascode Amplifier</u>

- Cascode is a combination of a common source amplifier followed by a common gate.
- The configuration help reducing the Miller effect and thus much higher f_h.
- The drawback is you get gain similar to a common source amplifier but by using two transistors → which means need for higher V_{DD} and higher power consumption.



$$V_{GS} = \frac{R_2}{R_1 + R_2} V_{DD}$$

$$I_{D_1} = I_{D_2} = \frac{\mu_n C_{ox} W_1}{2L_1}(V_{GS_1} - V_{t_n})^2 = \frac{\mu_n C_{ox} W_2}{2L_2}(V_{GS_2} - V_{t_n})^2$$

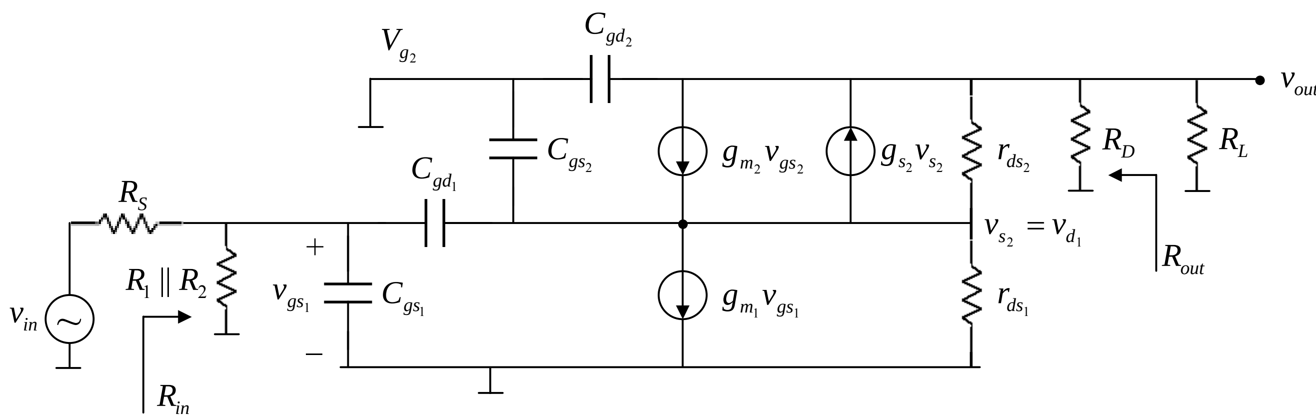
has body effect

find $I_{D_1} = I_{D_2}$ and V_{GS_2}

$$V_{G_2} = \frac{R_4}{R_3 + R_4} V_{DD} \quad , \quad V_{GS_2} = V_{G_2} - V_{S_2} \Rightarrow \text{ find } V_{S_2} = V_{D_1} = V_{DS_1} \text{ check } V_{DS_1} > V_{GS_1} - V_{t_n} \,.$$

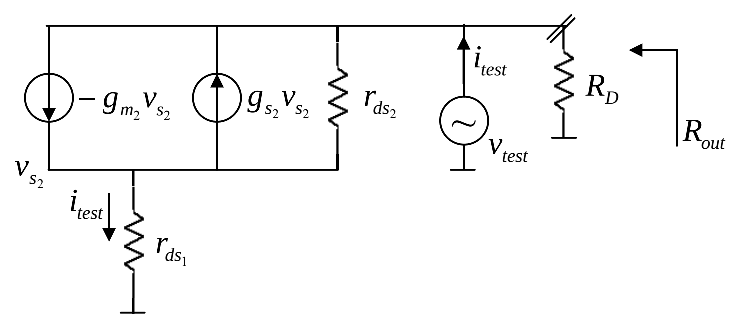
$$V_{D_2} = V_{DD} - R_D I_{D_2} \Rightarrow V_{DS_2} = V_{D_2} - V_{S_2} \text{ check } V_{DS_2} > V_{GS_2} - V_{t_n} \,.$$

Small-signal equivalent circuit:



$$R_{in} = R_1 \parallel R_2$$

To check R_{out}:



$$v_{s_2} = i_{test} . r_{ds_1}$$

$$i_{test} = -g_{m_2} v_{s_2} - g_{s_2} v_{s_2} + \frac{v_{test} - v_{s_2}}{r_{ds_2}}$$

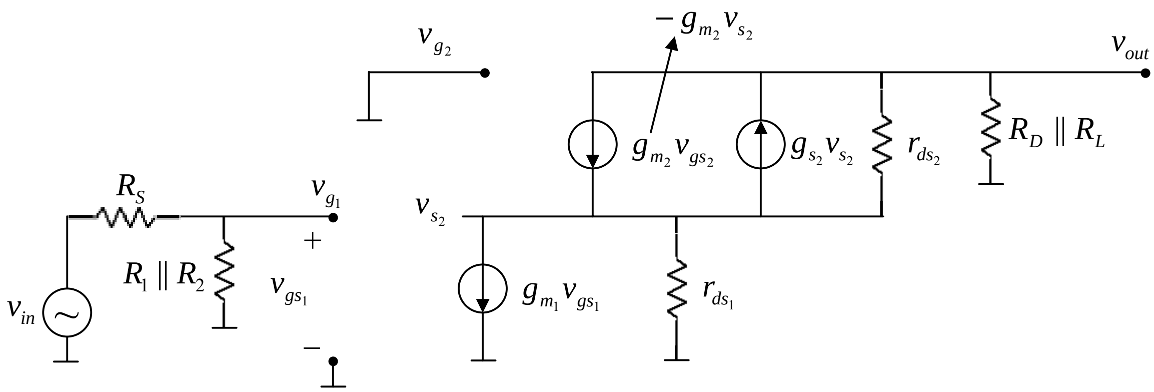
$$i_{test} \left(1 + g_{m_2} r_{ds_1} + g_{s_2} r_{ds_1} + \frac{r_{ds_1}}{r_{ds_2}} \right) = \frac{v_{test}}{r_{ds_2}}$$

$$\frac{v_{test}}{i_{test}} = r_{ds_2}\left(1 + r_{ds_1}\left(g_{m_2} + g_{s_2} + \frac{1}{r_{ds_2}}\right)\right)$$

$$R_{out} = R_D \parallel \underbrace{r_{ds_2}\left(1 + r_{ds_1}\left(g_{m_2} + g_{s_2} + \frac{1}{r_{ds_2}}\right)\right)}_{\text{much larger than } r_{ds_2} \text{ itself}}$$

$\Rightarrow R_{out} = R_D$.

<u>Voltage Gain (A_v)</u>



$$v_{g_1} = \frac{R_1 \parallel R_2}{R_S + R_1 \parallel R_2} v_{in} = v_{gs_1}$$

If r_{ds_1} , r_{ds_2} can be ignored, then: $g_{m_1}v_{gs_1} = -g_{m_2}v_{s_2} - g_{s_2}v_{s_2} \rightarrow v_{s_2} = -\dfrac{g_{m_1}}{g_{m_2} + g_{s_2}}.v_{gs_1}$

$$v_{out} = (-g_{m_2}v_{gs_2} + g_{s_2}v_{s_2})R_D \parallel R_L$$

$$v_{out} = (g_{m_2} + g_{s_2})v_{s_2}.R_D \parallel R_L = \cancel{(g_{m_2} + g_{s_2})}.R_D \parallel R_L.\frac{-g_{m_1}}{\cancel{(g_{m_2} + g_{s_2})}}.v_{gs_1}$$

$$v_{out} = -g_{m_1}.R_D \parallel R_L.\frac{R_1 \parallel R_2}{R_S + R_1 \parallel R_2}.v_{in}$$

$$\boxed{A_v = -g_{m_1}.R_D \parallel R_L.\frac{R_1 \parallel R_2}{R_S + R_1 \parallel R_2}} \rightarrow \text{ exactly like C.S.}$$

General Case

When r_{ds_1} , r_{ds_2} cannot be ignored:

$$v_{g_1} = v_{gs_1} = \frac{R_1 \parallel R_2}{R_S + R_1 \parallel R_2} v_{in}$$

$$\underbrace{g_{m_1} v_{gs_1} + \frac{v_{s_2}}{r_{ds_1}}}_{\text{current in M1}} = \underbrace{\frac{-v_{out}}{R_D \parallel R_L}}_{\text{current in load}}$$

$$\underbrace{g_{m_2} v_{s_2} + g_{s_2} v_{s_2} - \frac{v_{out} - v_{s_2}}{r_{ds_2}}}_{\text{current in M2}} = \underbrace{\frac{v_{out}}{R_D \parallel R_L}}_{\text{current in load}} \rightarrow \text{ find } v_{s_2} \text{ as a function of } v_{out}.$$

$$\Rightarrow v_{s_2}\left(g_{m_2} + g_{s_2} + \frac{1}{r_{ds_2}}\right) = v_{out}\left(\frac{1}{r_{ds_2}} + \frac{1}{R_D} + \frac{1}{R_L}\right)$$

$$v_{s_2} = v_{out} \frac{\left(\frac{1}{r_{ds_2}} + \frac{1}{R_D} + \frac{1}{R_L}\right)}{\left(g_{m_2} + g_{s_2} + \frac{1}{r_{ds_2}}\right)}$$

$$g_{m_1} \cdot \frac{R_1 \parallel R_2}{R_S + R_1 \parallel R_2} v_{in} + \frac{v_{s_2}}{r_{ds_1}} = \frac{-v_{out}}{R_D \parallel R_L}$$

$$\Rightarrow \quad g_{m_1} \cdot \frac{R_1 \parallel R_2}{R_S + R_1 \parallel R_2} v_{in} = -v_{out}\left[\frac{\left(\frac{1}{r_{ds_2}} + \frac{1}{R_D} + \frac{1}{R_L}\right)}{\left(g_{m_2} + g_{s_2} + \frac{1}{r_{ds_2}}\right)} \cdot \frac{1}{r_{ds_1}} + \frac{1}{R_D} + \frac{1}{R_L}\right]$$

$$\boxed{A_v = \frac{v_{out}}{v_{in}} = -g_{m_1} \cdot \frac{R_1 \parallel R_2}{R_S + R_1 \parallel R_2} \cdot \frac{1}{\left(\frac{\frac{1}{r_{ds_2}} + \frac{1}{R_D} + \frac{1}{R_L}}{g_{m_2} + g_{s_2} + \frac{1}{r_{ds_2}}}\right) \cdot \frac{1}{r_{ds_1}} + \frac{1}{R_D} + \frac{1}{R_L}}}$$

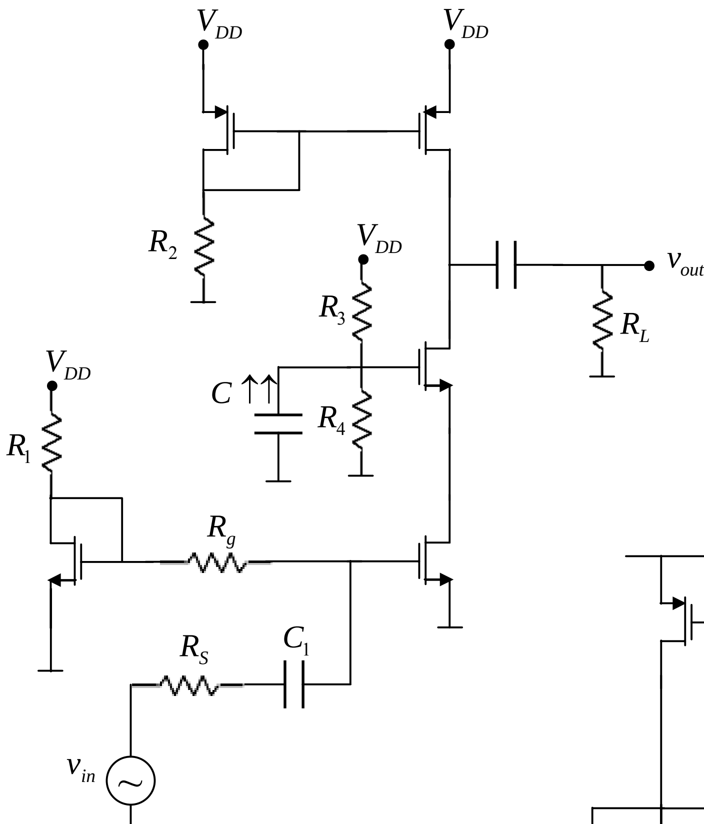
Bandwidth

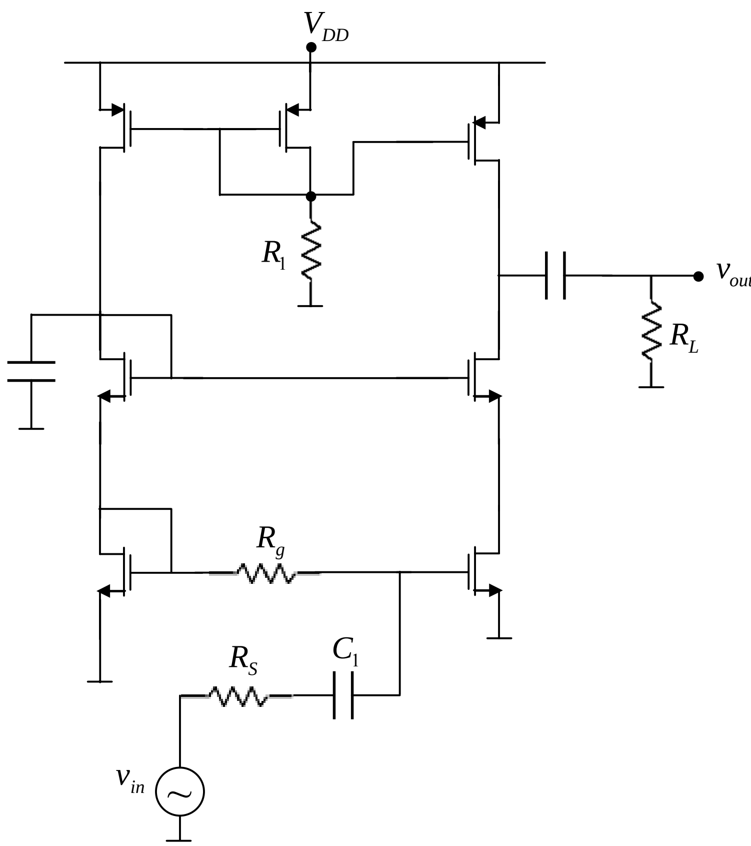
Miller effect of C_{gd} does not exist, because the gain of first stage $A_v = \frac{-(g_{m_2} + g_{s_2})}{g_{m_1}} \approx -1$

→ bandwidth is expected to be much higher.

HW: find BW of cascode using OCτ method.

Integrated Cascode

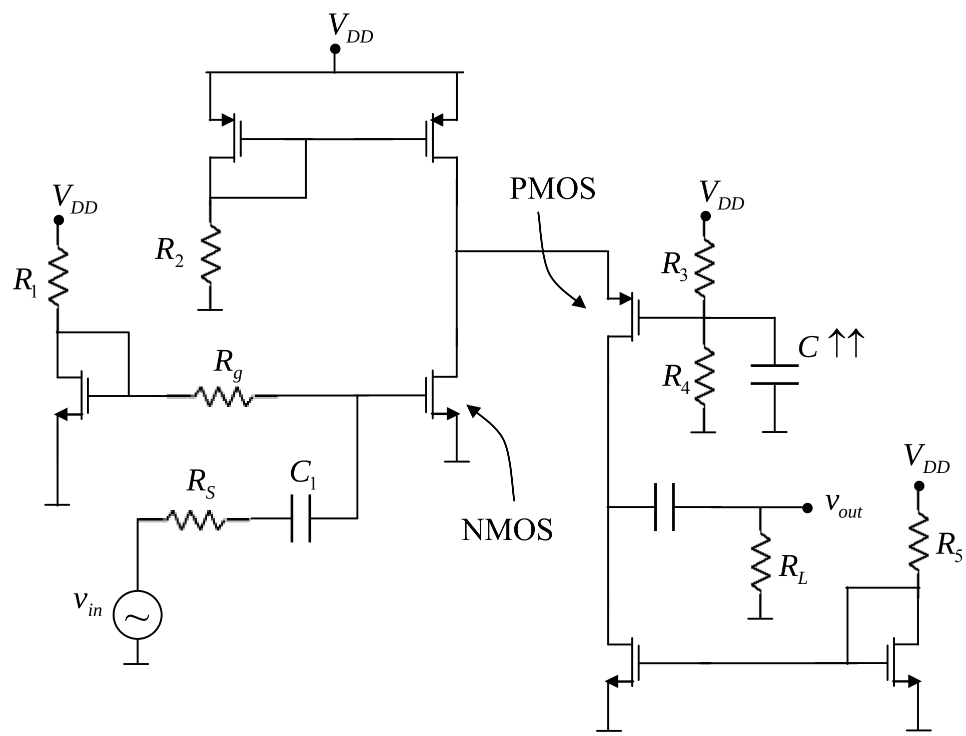




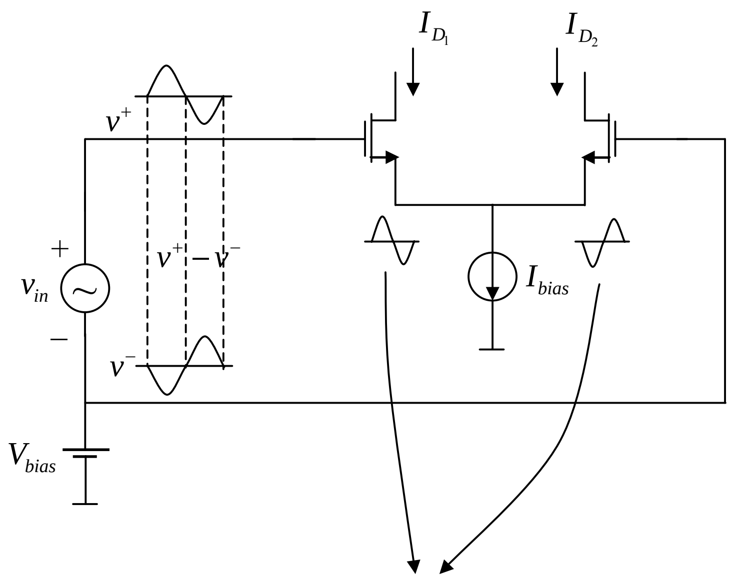
Folded Cascode

- Advantages → only 2 transistors are stacked → low V_{DD} .

- Disadvantages → use PMOS as gain stage.



MOS differential pair



Without v_{in}, both transistors are biased at exactly same V_{GS}. With identical W/L ratio $I_{D_1} = I_{D_2} = \frac{I_{bias}}{2}$.

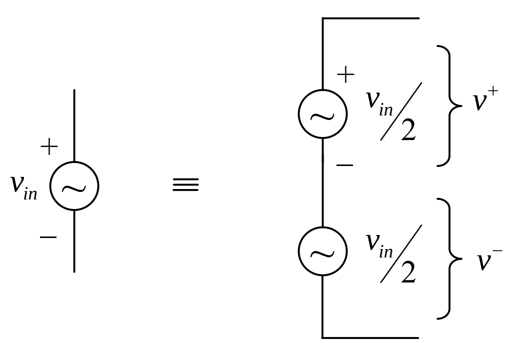
* Identical waveforms only 180 different in phase

→ The sum of two waveforms is 0v.

→ Source of the two transistors although at high impedance in DC is AC ground.

$$\Rightarrow \; i_{d_1} = g_{m_1}.v^+ = g_m \frac{v_{in}}{2} \quad \longleftarrow \quad \text{ignoring transistor output resistance}$$

Since:

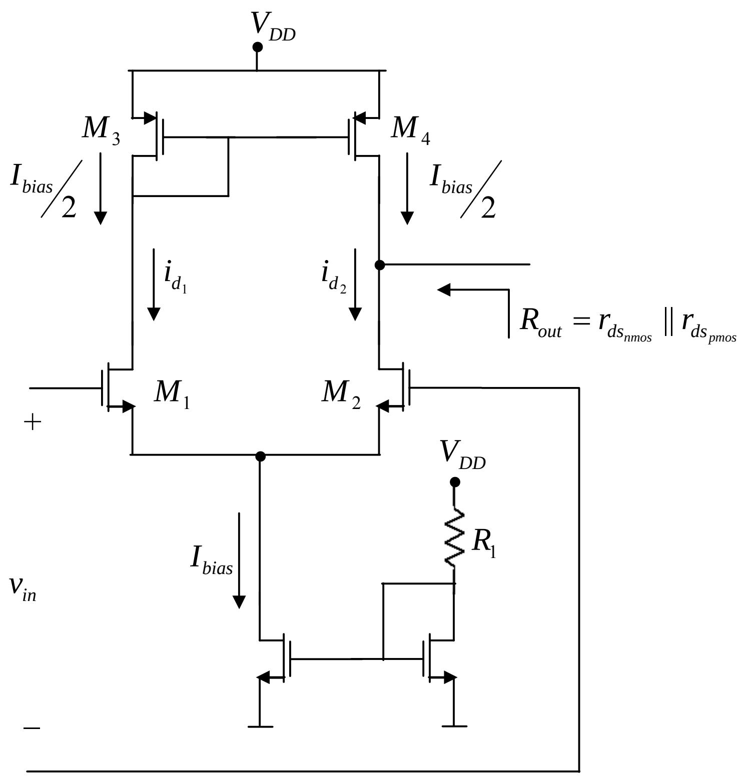


$$i_{d_2} = +g_{m_2}.v^- = -g_m \frac{v_{in}}{2}$$

$$i_{out} = i_{d_2} - i_{d_1} = g_m v_{in}$$

If the load is an active load (current mirror), then:

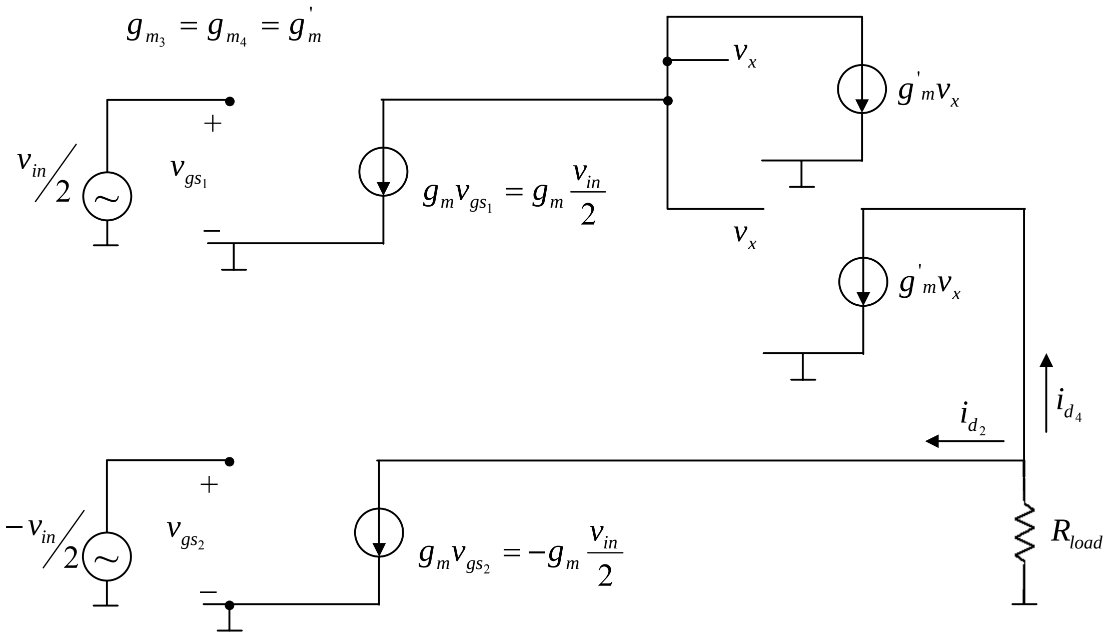
Differential Input → single-ended output



To calculate A_v let's ignore r_{ds} of all transistors.

$$g_{m_1} = g_{m_2} = g_m$$

$$g_{m_3} = g_{m_4} = g'_m$$



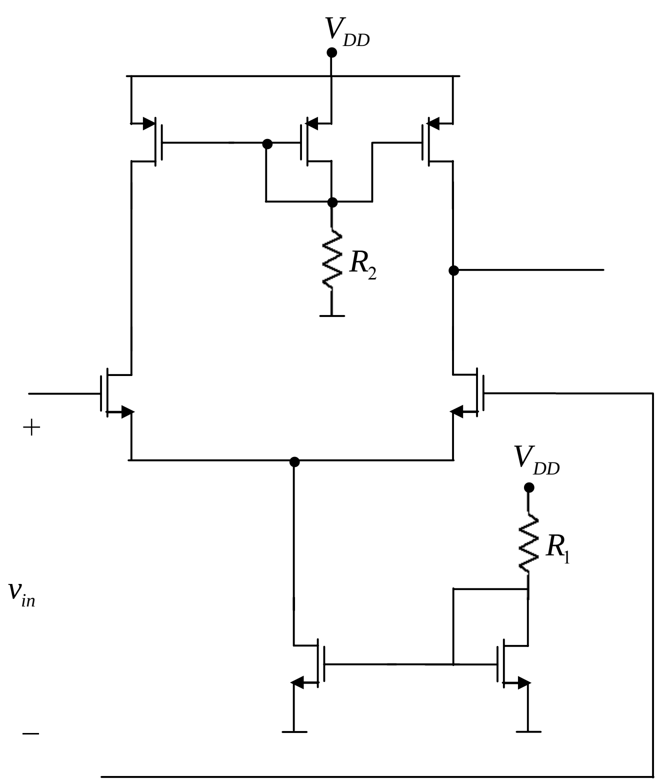
$$g'_m v_x = -g_m \frac{v_{in}}{2} \quad \rightarrow \quad v_x = \frac{-g_m}{g'_m} \frac{v_{in}}{2}$$

$$i_{d_2} = \frac{-g_m v_{in}}{2}$$

$$i_{d_1} = g'_m v_x = g'_m \cdot \frac{-g_m}{g'_m} \frac{v_{in}}{2} = \frac{-g_m v_{in}}{2}$$

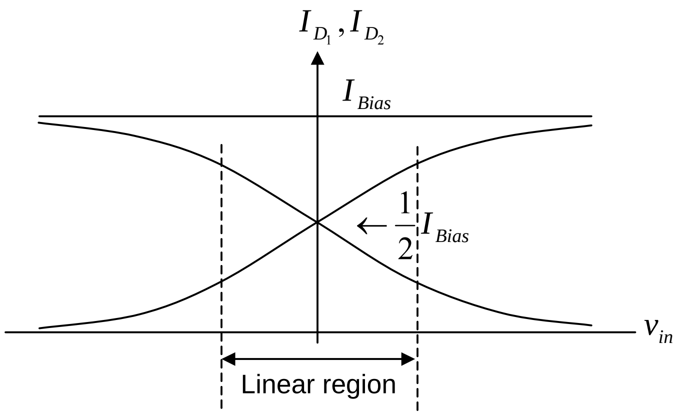
$$\boxed{\begin{aligned} v_{out} &= R_{load} \cdot (-i_{d_2} - i_{d_4}) = R_{load} \cdot g_m v_{in} \\ R_{load} &= R_L \parallel R_{out} = R_L \parallel r_{ds_{nmos}} \parallel r_{ds_{pmos}} \end{aligned}}$$

Note that in this case the active load is actually in the AC path. Now consider the following circuit:



$$v_{out} = R_{load} g_m \cdot \frac{v_{in}}{2}$$

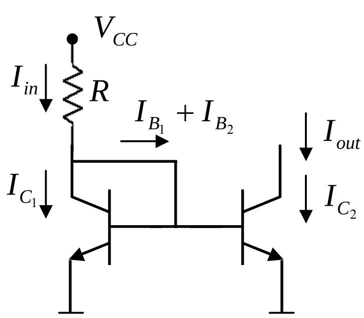
because you don't have amplification through PMOS transistors.



Turns out that the linearity is improved compared to single stage device.

(Constant gain as you increase the input signal.)

Bipolar transistor current Mirrors



$$V_{BE_1} = V_{BE_2}$$

$$I_{C_1} = A_{E_1}.I_S.e^{V_{BE_1}/nV_T}$$

$$I_{C_2} = A_{E_2}.I_S.e^{V_{BE_2}/nV_T}$$

$\dfrac{I_{C_1}}{I_{C_2}} = \dfrac{A_{E_2}}{A_{E_1}} \leftarrow$ ratio of emitter areas determine the current.

Assuming both transistors are identical except for emitter area → same I_S
→ same β
→ same n

$$I_{in} = I_{C_1} + I_{B_1} + I_{B_2} = \frac{V_{CC} - V_{BE_{on}}}{R} = \frac{V_{CC} - 0.7}{R}$$

$$I_{B_2} = \frac{A_{E_2}}{A_{E_1}}.I_{B_1} \quad , \quad I_{C_1} = \beta I_{B_1}$$

$$\Rightarrow \beta I_{B_1} + I_{B_1} + \frac{A_{E_2}}{A_{E_1}}.I_{B_1} = \frac{V_{CC} - 0.7}{R}$$

$$I_{B_1} = \frac{V_{CC} - 0.7}{R\left(1 + \beta + \frac{A_{E_2}}{A_{E_1}}\right)}$$

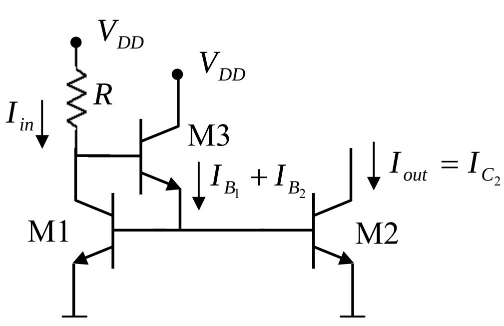
$$I_{C_2} = I_{out} = \beta I_{B_2} = \beta.\frac{A_{E_2}}{A_{E_1}}.I_{B_1}$$

$$\boxed{I_{out} = \frac{(V_{CC} - 0.7)\beta.\frac{A_{E_2}}{A_{E_1}}}{R\left(1 + \beta + \frac{A_{E_2}}{A_{E_1}}\right)} = \frac{\beta.\frac{A_{E_2}}{A_{E_1}}}{1 + \beta + \frac{A_{E_2}}{A_{E_1}}}.I_{in}}$$

if $A_{E_1} = A_{E_2} \Rightarrow \boxed{I_{out} = \frac{\beta}{2 + \beta}.I_{in} = \frac{1}{1 + 2/\beta}.I_{in}}$

$$\boxed{R_{out} = r_{o_2}}$$

At the cost of higher power consumption, you can do the following:

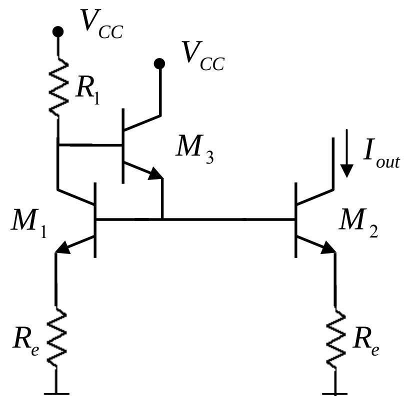


$$I_{in} \approx I_{C_1} = \frac{V_{DD} - 2V_{BE_{on}}}{R} = \frac{V_{DD} - 1.4}{R}$$

$$I_{out} = I_{C_2} = \frac{A_{E_2}}{A_{E_1}}.I_{C_1} = \frac{A_{E_2}}{A_{E_1}}.I_{in} = \frac{A_{E_2}}{A_{E_1}}.\frac{V_{DD} - 1.4}{R}$$

$$R_{out} = r_{o_2}$$

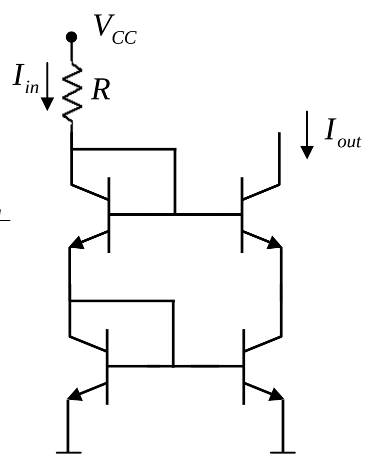
In order to increase the output resistance:



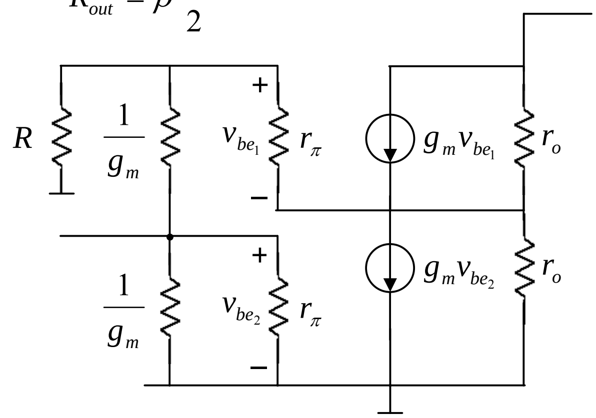
$$R_{out} = r_{o_2}(1 + g_{m_2} R_e)$$

Cascode Current Source

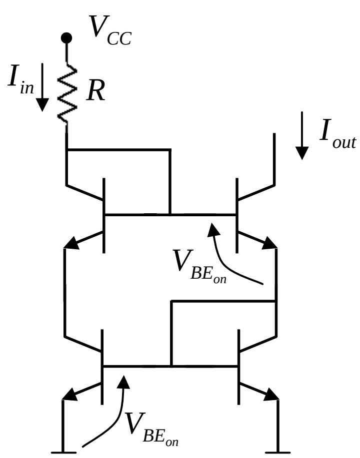
$$I_{in} = \frac{V_{CC} - 2V_{BE_{on}}}{R}$$



$$R_{out} = \beta \frac{r_o}{2}$$



Wilson Current Source



$$I_{in} = \frac{V_{CC} - 2V_{BE_{on}}}{R}$$

$$R_{out} = \beta \, {}^{r_o}\!/_{2}$$