Review for Topic 2: Probabilities for one RV

We do 4 things in this class

0) translate words into math
- identify experimental procedure, observation
- identify sample space, event of interest
- identify random variable and its pmf/pdf

1) Build models
- Equally likely
- Theorem of ~~total~~ probability

$$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i) \quad \text{if } B_i\text{'s form partition}$$

$$f_X(x) = \sum_{i=1}^{n} f_X(x|B_i)\,P(B_i)$$

(this may create a mixed RV)

- Independence $P(A \cap B) = P(A)P(B)$
- Any shape $f_X(x) = g(x)/c$, where $g(x)$ is piecewise continuous with $\int g(x)dx = c$
- Common pmfs (Bernoulli, binomial, geometric, uniform, Poisson, ...)
- common pdfs (Gaussian, exponential ...)
- derived RVs ($Y = g(X)$ where $g(\cdot)$ is deterministic)
(this may create a mixed RV)

2) Compute probabilities within an experiment

Axioms of probability and their corollaries

$0 \le P(A) \le 1$

$P(S) = 1 = P(A) + P(A^c)$

$P(A \cup B) = P(A) + P(B) - P(A \cup B)$ etc....

Probability mass functions, cumulative distribution functions, probability density functions

$P(a < X \le b) = F_X(b) - F_X(a)$

$P(a \le X \le b) = \int_a^b f_X(x)\,dx$

Can be for continuous, discrete, or mixed RVs

Applying Φ function

3) Learn from the experiment's outcome

Bayes Rule $\quad P(B_i | A) = \dfrac{P(A|B_i)\,P(B_i)}{P(A)}$

updating pdf after learning partial info

$$f_X(x|C) = \begin{cases} \dfrac{f_X(x)}{P(C)} & x \in C \\ 0 & \text{else} \end{cases}$$

when C is an event that includes X.

Hypothesis testing

$P_F = P(\text{decide present} \mid H_0)$

$P_D = P(\text{decide present} \mid H_1)$

4) Compute summary statistics

Mean, variance, moments

$$E(X), \quad Var(X), \quad E(X^n)$$

and their properties

$$E(aX+b) = aE(X)+b$$

$$Var(aX+b) = a^2 Var(X)$$

Mean of derived RV

$$E(X) = \int x \, f_X(x)\,dx$$

$$E(g(X)) = \int g(x) \, f_X(x)\,dx$$