Independence of events

In general, is $P(A|B) > P(A)$?

$P(A|B) < P(A)$?

$P(A|B) = P(A)$?

<u>It depends.</u>

Example 1:

	A		
B	$1/3$	$1/6$	$1/2$
	$1/6$	$1/3$	$1/2$
	$1/2$	$1/2$	

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/3}{1/2}$$

$$= 2/3 > P(A)$$

Example 2:

	A		
B	$1/6$	$1/3$	$1/2$
	$1/3$	$1/6$	$1/2$
	$1/2$	$1/2$	

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{1/2}$$

$$= 1/3 < P(A)$$

Example 3:

	A		
B	$1/4$	$1/4$	$1/2$
	$1/4$	$1/4$	$1/2$
	$1/2$	$1/2$	

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/2}$$

$$= 1/2 = P(A)$$

Section 2.5 Independence of events

Two events A and B are independent if and only if $P(A \cap B) = P(A)P(B)$

This implies $P(A|B) = P(A)$
and $P(B|A) = P(B)$

In words: if we learn that A happened, we learn nothing more about the likelihood of B happening

Independence is often a natural consequence of the world.

Examples:
- outcomes of coin flips
- outcomes of successive die tosses

Independence may also be stated as an assumption in the problem definition.

DO NOT assume independence when it is not stated or when it is not warranted by the physical situation

There are many events that are NOT independent.
Ex: {study for exam} and {get a good grade}.

When asked to prove independence, you must rely on the math. Show $P(A \cap B) = P(A)P(B)$.

Let's start w/ a simple example.

Suppose A and B are disjoint.

Are they also independent?

NO if we learned A happened, then we know B could not have happened, so A and B cannot have happened.

Disjoint events have $P(A \cap B) = 0$,

but $P(A)P(B) = 0$ only happens if either $P(A) = 0$ or $P(B) = 0$.

So independent events can only be disjoint if $P(A) = 0$ or $P(B) = 0$.

Reminder and contrast

Disjoint A and B:	$P(A \cup B) = P(A) + P(B)$
Independent A and B:	$P(A \cap B) = P(A)P(B)$

Independence of more than 2 events, ex: A, B, C

Need 1) mutual (pairwise) independence

$$P(A \cap B) = P(A)\,P(B)$$
$$P(A \cap C) = P(A)\,P(C)$$
$$P(B \cap C) = P(B)\,P(C)$$

<u>and</u> 2) joint independence

$$P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$$

More generally, for n events, need all possible combinations of 2 or more events to be independent

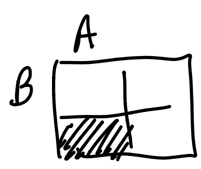
$$P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}) = P(A_{i_1})\,P(A_{i_2}) \cdots P(A_{i_k})$$

for $1 \leq i_1 \leq i_2 \leq \cdots \leq i_k \leq n$

and for $k = 2, 3, \ldots, n$

<u>NOTE</u>: If A and B and C are independent, then unions, intersections, complements of A, B, and/or C are also independent.

Ex: $P(A \cap B^c) = P(A) - P(A \cap B)$



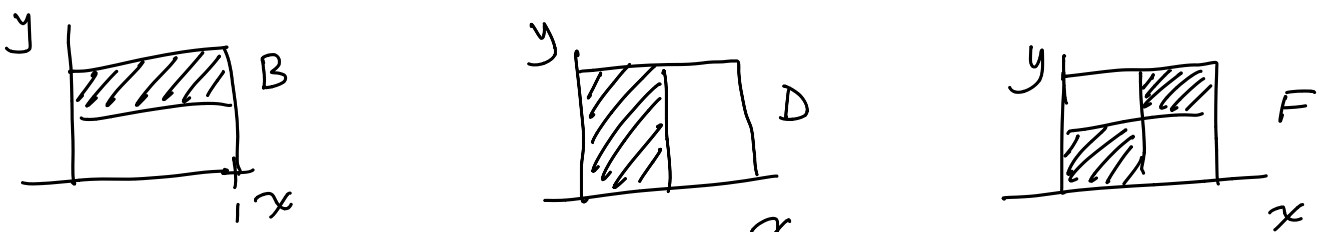
$$= P(A) - P(A \cap B) = P(A) - P(A)P(B)$$
$$= P(A)[1 - P(B)] = P(A)\,P(B^c)$$

<u>Example</u> $S = \{(x,y) \in \mathbb{R}^2 : 0 \le x \le 1, 0 \le y \le 1\}$
equally likely

Let $B = \{y < 1/2\}$

$D = \{x < 1/2\}$ and

$F = \{x < 1/2 \text{ and } y < 1/2\} \cup \{x > 1/2 \text{ and } y > 1/2\}$



$P(B) = 1/2$ $P(D) = 1/2$ $P(F) = 1/2$

$$\left.\begin{aligned} P(B \cap D) &= 1/4 = P(B)P(D) \\ P(B \cap F) &= 1/4 = P(B)P(F) \\ P(D \cap F) &= 1/4 = P(D)P(F) \end{aligned}\right\}$$ pairwise independent

$P(B \cap D \cap F) = P(\emptyset) = 0 \neq P(B)P(D)P(F)$

NOT jointly idependent