Short intermediary topic:

Building probability models

(a.k.a. assigning probabilities to events)

Putting it all together:

1. Identify random experiment (procedures and outcomes)
2. Specify the sample space S
3. Identify events of interest
4. Make a probability assignment that satisfies the axioms
5. Compute probabilities of other events via the axioms

This set of notes considers step 4 a bit more.

Many probability assignments are possible that satisfy the axioms.

A good probability assignment also reflects experimental observations or previous experience

The goal is to create the simplest model that explains all relevant aspects

Discrete sample spaces

$$S = \{ a_1, a_2, \dots, a_n \} \qquad |S| = n$$

Case 1:

"Equally likely outcomes" means $P(\{a_i\}) = \frac{1}{n}$ for $1 \leq i \leq n$

and thus $P(B) = k/n$ if B has k elements

examples: pull a ball out of an urn
draw a marble from a box
pick a card any card
roll a fair die

Case 2: Observe relative frequency of events and assign accordingly

Note: The topic of counting (chapter 2.3) is about assigning probability models in cases where all elements in a sample space are equally likely

Continuous sample spaces

In one dimension:

S is $\mathbb{R}$ (all real numbers)

or S is an interval (or union of intervals) of $\mathbb{R}$.

$\Rightarrow$ assign probabilities to intervals

Example: $S = \{x \in \mathbb{R} : x \in [0,1]\}$

A reasonable model is to say

$P(A) = b - a$, when $A = \{x \in S : x \in [a,b]\}$

when $0 \le a \le b \le 1$

$b-a$ is the length of the interval relative to the length of the sample space

In 2 dimensions:

S is $\mathbb{R}^2$ (pairs of real numbers)

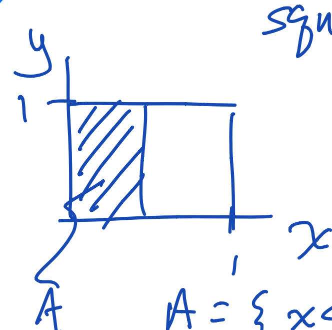
$\Rightarrow$ assign probabilities to regions

Assuming equally likely,

$$P(\text{region } R) = \frac{\text{area}(R)}{\text{area}(S)}$$

example

$S = \{\text{unit square}\}$



$A = \{x < 1/2\}$

$P(A) = 1/2$

It is not always reasonable to assume equally likely!

Be careful