Random Experiments   Section 2.1

Experiment
- a procedure to follow and an observation of an outcome

Random Experiment
- an experiment where the outcome is uncertain

Outcome, denoted ω, ζ, is the basic result

Sample Space S (a set)
- the collection of all possible outcomes

Event $A, B, C, \ldots$ (also a set)
- the collection of (some) possible outcomes

Examples

1. Flip a coin twice, observe sequence of H and T
2. Flip a coin twice, observe how many H

$S_1 = \{HH, HT, TH, TT\}$ ← each outcome equally likely

$S_2 = \{0, 1, 2\}$ ← different probabilities for each outcome

Applications and their random experiments

Fall 2016
(August 23, 2016)

WHAT IS THE SAMPLE SPACE? WHAT IS THE EVENT OF INTEREST?

Problem 1. BOOK
Flip a coin 3 times. What is the probability you get the sequence Head,Tail,Head?

$S_1 = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$A_1 = \{HTH\}$

Note: you must observe the sequence to be able to express the event A.

Problem 2. BOOK
Flip a coin 3 times. What is the probability you get 2 heads and a tail?

There are several correct answers. Some are more useful.

$S_2 = \{0, 1, 2, 3\}$ where this indicates the number of heads

$B_2 = \{2\}$

Can also have same sample space S_1 above, with

$B_1 = \{HHT, HTH, THH\}$.

Note $P(B_1) = P(B_2)$.

Another option:

$S_3 = \{0, 1, 2, 3\}$ where this indicates # of Tails

$B_3 = \{1\}$

And $S_4 = \{2H, \text{not-}2H\}$ and $B_4 = \{2H\}$.

Time is almost always continuous

WHAT IS THE SAMPLE SPACE? WHAT IS THE EVENT OF INTEREST?

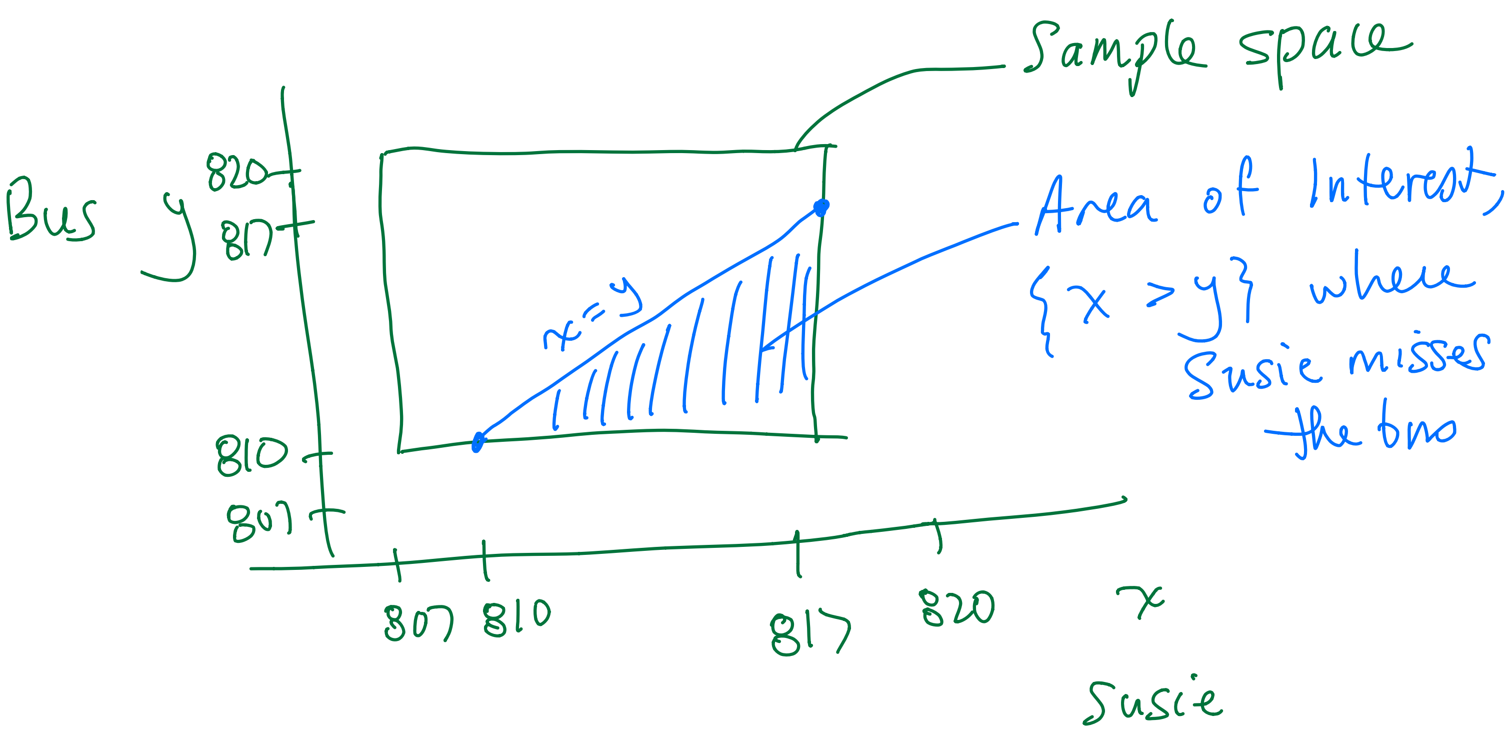
Problem 3. (From final exam Fall 2015)
Susie wants to take the 8:15 bus in the morning. Let X be the time she arrives at the bus-stop, which is a uniform random variable between 8:07 and 8:17. Let Y be the time the bus arrives at the bus-stop, which is a uniform random variable between 8:10 and 8:20. Assume that X and Y are independent.

Susie will miss the bus if it arrives before she does. What is the probability that Susie will miss the bus?
(Hint: It may be helpful to draw a diagram indicating the relationship between X, Y, and the event that Susie misses the bus.)

$S = \{ (x,y) : 807 \le x \le 817 \text{ and } 810 \le y \le 820 \}$

x continuous, $x \in \mathbb{R}$
y continuous, $y \in \mathbb{R}$

$(x,y) \in \mathbb{R}^2$



$A = \{ (x,y) : x > y \}$

"equally likely" $\Rightarrow$

$P(A) = \dfrac{|A|}{|S|}$ — area of A / area of S

WHAT IS THE SAMPLE SPACE? WHAT IS THE EVENT OF INTEREST?

Problem 4. Book
A block of information is transmitted repeatedly over a noisy channel until an error-free block arrives at the receiver. Each block is in error with probability 0.1. What is the probability that more than 4 retransmissions are required?

(a) observe how many transmissions are necessary before an error free block arrives

OR (b) observe # of retransmissions

$S_b = \{0, 1, 2, 3, \ldots\}$ ← retransmissions start from 0

$A_b = \{5, 6, 7, \ldots\}$ ← "more than 4" does not include 4

OR

$S_a = \{1, 2, 3, 4, \ldots\}$ ← transmissions do NOT start from 0

$A_a = \{6, 7, 8, \ldots\}$ ← total # transmissions is one more than # retransmissions

WHAT IS THE SAMPLE SPACE? WHAT IS THE EVENT OF INTEREST?

Problem 6.
A company has a machine that makes 1-kilo-ohm resistors. What is the probability the resistor's resistance is within ±5% of the desired value?

$S = \{ r : \quad 0 \leq r < \infty \}$ continuous outcomes

(technically, the resistance can be anything greater than 0)

$A = \{ r \in S : \quad 950 \leq r \leq 1050 \}$

WHAT IS THE SAMPLE SPACE? WHAT IS THE EVENT OF INTEREST?

Problem 8. BOOK

You are designing a packet voice transmission system. A speech signal is segmented into 10 ms chunks, and sent in packets. However, to save bandwidth, the signal is only packetized when the speaker is NOT silent.

If you have a group of N speakers in *different* conversations, chances are not all of them will be speaking at once.

You design your system to handle up to 8 simultaneous speakers. How many calls can you admit into your system, and ensure that you exceed capacity no more than 1% of the time? (here, $=0.01$ is ok)

$S = \{0, 1, 2, 3, \ldots\}$ # speakers at a time

$A = \{9, 10, 11, \ldots\}$ exceed capacity with more than 8 speakers

want $P(A) \leq 0.01$

Note: This provisioning problem is equivalent, mathematically, to many others. How many PCs should you have in the computer lab to have smaller than 1% chance there's no computer available when a student arrives at the lab? (here, 0.01 is not ok)

How many processors should you have in your multicore machine to ensure there is less than a 1% chance of 2 programs having to share a processor?

$B = \{0, 1, 2, \ldots, 8\}$ event you don't exceed capacity (Note: $B = A^c$)

WHAT IS THE SAMPLE SPACE? WHAT IS THE EVENT OF INTEREST?

Problem 9. Book
You are designing a system to go into space (or, into a hot humid environment, or a cold dry environment, ..). You want to model the lifetime of the system, using the lifetime of the components. What is the probability the system will function correctly for more than 1 year?

Experiment: deploy system
observe when it fails, T

Sample Space $S = \{T \in \mathbb{R} : T \geq 0\}$

Event of Interest $A = \{T \in S : T > 1 \text{ year}\}$

WHAT IS THE SAMPLE SPACE? WHAT IS THE EVENT OF INTEREST?

Problem 10. (FROM SAMPLE EXAM 1 FALL 2015)
Zeros and ones are sent over a noisy communication channel, where the transmission of each bit can be considered to be independent sequential experiements. The probability that each 0 is correctly sent is 0.9, while the probability that each 1 is correctly sent is 0.85. The digit 0 is sent with probability 0.6.

(a) Find the probability that an error occurs, for each bit sent.

(b) Given that you detect a 1, what is the probability that a 1 had been sent.

(c) If the string 0010 is sent, what is the probability the string is correctly received.

a) observe the pair of the sent bit and the received bit

$S = \{(0,0), (0,1), (1,0), (1,1)\}$

$A_{err} = \{(0,1), (1,0)\}$

b) If you detect a 1, observe what bit was sent

$S_{R_1} = \{0, 1\}$ ← indicates all possible bits that could have been sent given that R_1 (i.e., detect/receive a 1)

$A = \{1\}$

c) observe possible sequence of bits received given that 0010 was sent

$S = \{0000, 0001, 0010, 0011, \ldots, 1110, 1111\}$
(S has 16 elements)

$A = \{0010\}$ ← correctly received

WHAT IS THE SAMPLE SPACE? WHAT IS THE EVENT OF INTEREST?

Problem 11. (Problem setup is from exam 2, fall 2015)
Five cars start out on a cross-country race. The probability that a car breaks down and drops out of the race is 0.2. Cars break down independently of each other.

(a) What is the probability that exactly two cars finish the race?

(b) What is the probability that at most two cars finish the race?

(c) What is the probability that at least three cars finish the race?

Experiment 1 run race and obsenve how many cars finish

Experiment 2 run race and obsenve how many cars drop out

Sample Space $S_1 = \{0,1,2,3,4,5\}$

$S_2 = \{0,1,2,3,4,5\}$

Event of Interest

a) $A_1 = \{2\}$ $\quad$ $A_2 = \{3\}$

b) $B_1 = \{0,1,2\}$ $\quad$ $B_2 = \{3,4,5\}$

c) $C_1 = \{3,4,5\} = B_1^c$ $\quad$ $C_2 = \{0,1,2\} = B_2^c$

WHAT IS THE SAMPLE SPACE? WHAT IS THE EVENT OF INTEREST?

Problem 14. FROM QUIZ 1 SPRING 2016
Select a ball from a container that contains balls numbered from 1 to 10. Observe the number on the ball. You want to compute the probability that the observed number is even.
What is the sample space S? What is the event of interest?

$$S = \{1, 2, 3, \ldots, 10\}$$

$$A = \{2, 4, 6, 8, 10\}$$

WHAT IS THE SAMPLE SPACE? WHAT IS THE EVENT OF INTEREST?

Problem 16. (FROM FINAL SPRING 2016)
A super-computer has three cooling components that operate independently. Each fails with probability 1/10. The super-computer will overheat if any two (or three) cooling components fail. What is the probability the super-computer overheats?

Observe # operating (not failed) cooling components

$S = \{0, 1, 2, 3\}$

$A = \{0, 1\}$

observe # failed cooling components

$S = \{0, 1, 2, 3\}$

$A = \{2, 3\}$

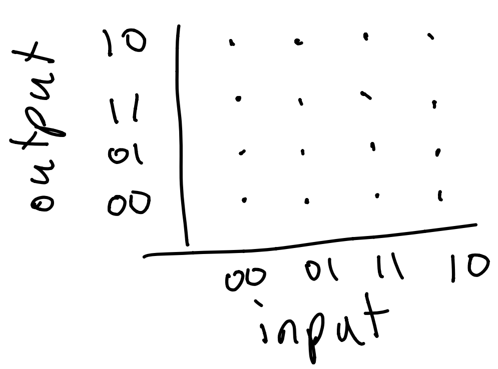
Example: QPSK

Quadrature Phase Shift Keying (QPSK) is a digital modulation scheme used in cellular communications, wireless LANs, and satellite and cable TV.

Its goal is to communicate 2 bits of information in one symbol each time period. To do this, each symbol must have 4 possible distinct values.

On any communication channel, there is a chance that the message sent is not exactly the message received. We call this a "noisy channel".

Send $N=2$ bits in one symbol;
receive 2 bits that may be different.
4 inputs, 4 outputs $\Rightarrow$ 16 possible outcomes



Ex: send 01, receive 10

$\omega = (01, 10)$

What is $A_1 = \{\text{no errors}\}$

$A_2 = \{\text{one bit in error}\}$

$A_3 = \{\text{two bits in error}\}$

$A_i \cap A_j = \phi$ and $A_1 \cup A_2 \cup A_3 = S$

$\Rightarrow A_1, A_2, A_3$ form a partition