Set theory (Section 2.1.3)

A set is a collection of objects

$A, B, C \ldots$ a set (capital roman letters)

Set notation:

what we'll call a member of the set

$$S = \{ x \in \mathbb{Z} : \underline{x > 5} \}$$

the set

set delineation

"a member of"

the set of all integers

"such that"

a restriction on what integers are actually in the set

$\mathbb{R} \leftarrow$ the set of all real numbers

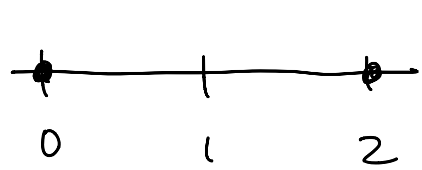
$\in$ a member of, or an element of

$\notin$ not an element of

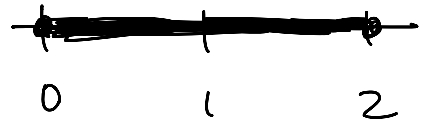
$\subset$ a subset of

S the sample space, containing all possible elements

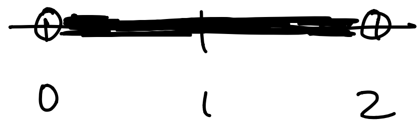
Discrete set of integers $S = \{0, 2\}$



Set of real numbers $S = [0, 2]$



Set of real numbers $S = (0, 2)$



These 3 are all different sets!

Additional examples of sets

$$B = \{ t \in \mathbb{R} : \quad 0 \leq t < \infty \}$$

$$A = \{ t \in \mathbb{R} : \quad 1 \leq t \leq 2 \}$$

and

$$B = \{ \text{Purdue ECE students} \}$$

$$A = \{ \text{ECE 302 section 2 students} \}$$

For these, $A \subset B$, or A is a subset of B if every element in A is also an element in B

(for every $w \in A$, then $w \in B$)

More definitions

$A = B$ ← sets are equal
if and only if (iff)
they contain the same elements

otherwise $A \neq B$, the sets are not equal.

Note: $A = B$ iff $A \subset B$ and $B \subset A$.
(This is a common way to prove that one set is equal to another.)

The set with no elements is the null set
or the empty set
Denoted ϕ or $\{\}$

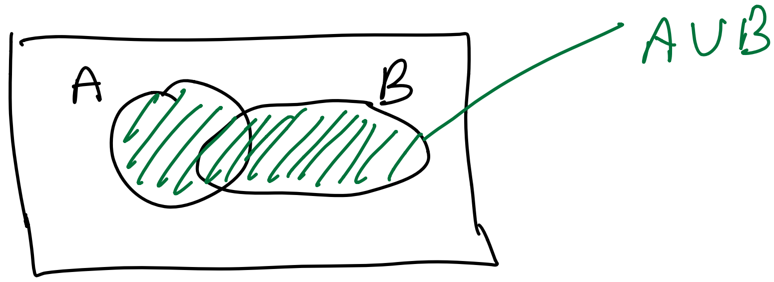
Questions: Is $\phi \subset A$? Is $A \subset A$?

Is $\phi \in A$? Is $A \in A$?

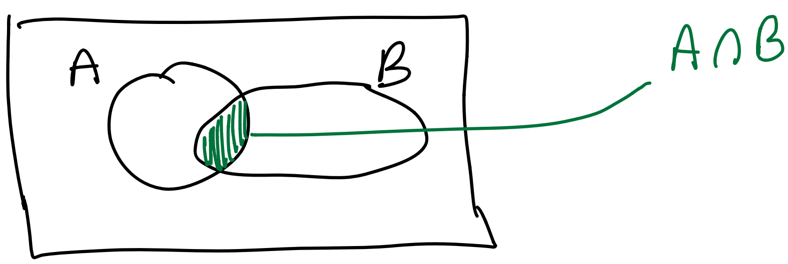
Three basic set operations:
Union, Intersection, Complement

Union $A \cup B = \{x : x \in A \text{ or } x \in B\}$

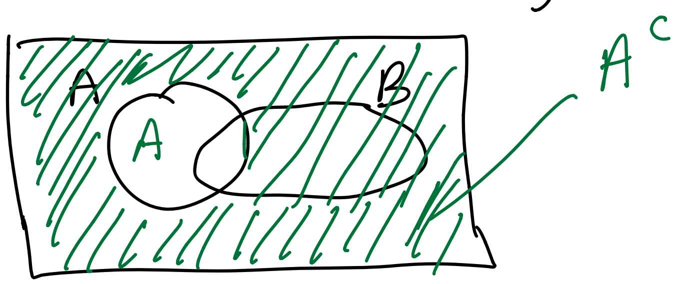
(also $A+B$)



Intersection $A \cap B = \{x : x \in A \text{ and } x \in B\}$
(also AB)



Complement $A^c = \{x : x \notin A\}$
(also $\bar{A}$)



Sets A and B are disjoint or mutually exclusive if $A \cap B = \phi$.

Combinations of set operations

We use parentheses to denote order of operations.

Commutative

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

Associative

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

Distributive

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Identities $A \cup \emptyset = A$ $\quad A \cap S = A$

Complements $(A^c)^c = A$

$$A \cup A^c = S \qquad A \cap A^c = \emptyset$$
$$A \cup S = S \qquad A \cap \emptyset = \emptyset$$

Set difference (aka relative complement)

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

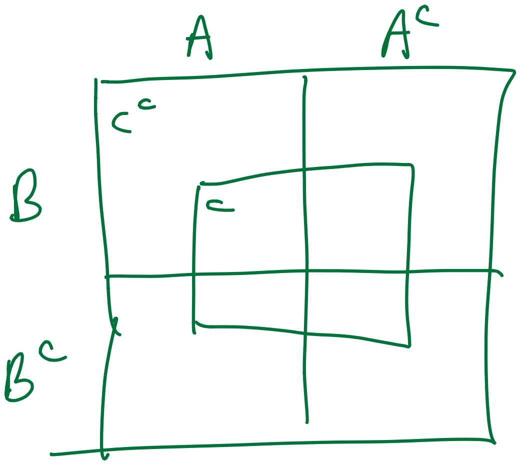
those elements in A that are not elements of B.

$$A - B = A \cap B^c$$

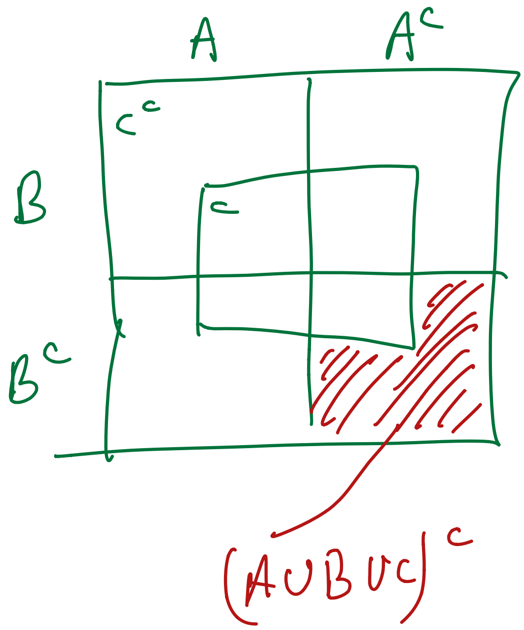
Note: do not subtract the values of the elements; remove elements

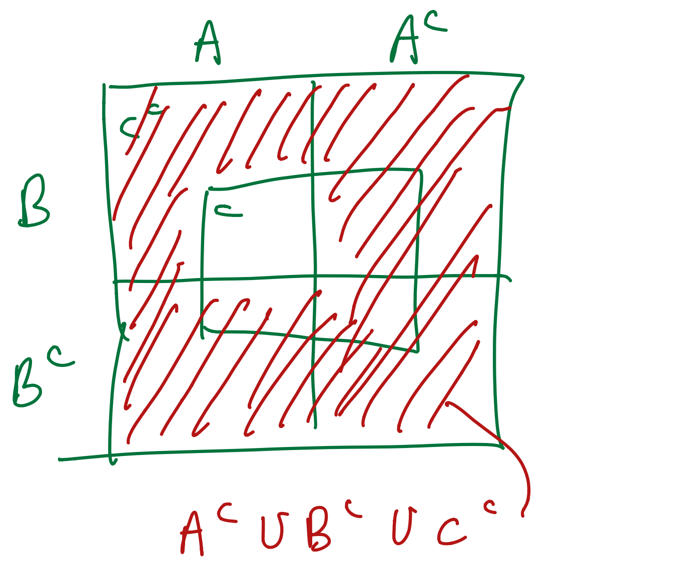
Example from HW

a) $(A \cup B \cup C)^c = A^c \cup B^c \cup C^c$



A is everything on Left
B is everything on top
C is inside the inner box





$\boxed{\text{FALSE}}$

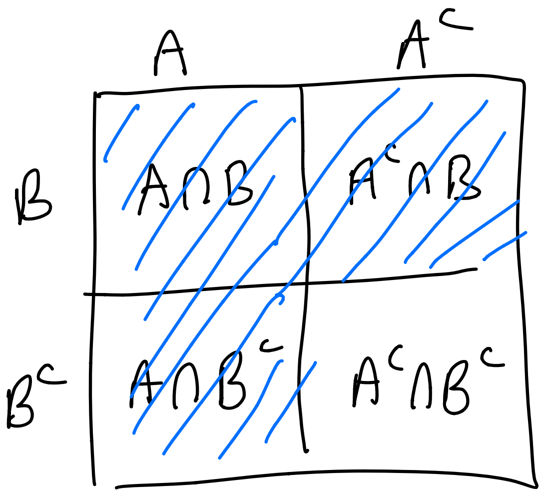
DeMorgan's Laws: combining complements, intersections, and unions

1st

$$(A \cup B)^c = A^c \cap B^c$$

$A \cup B$ is shaded

$(A \cup B)^c$ is unshaded.



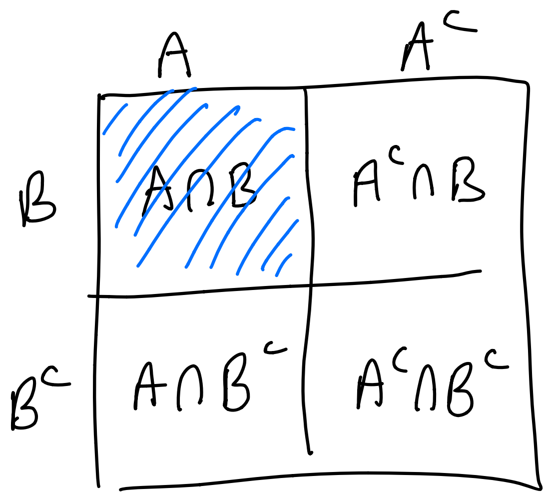
2nd

$$(A \cap B)^c = A^c \cup B^c$$

$(A \cap B)$ is shaded

$(A \cap B)^c$ is unshaded

as is $A^c \cup B^c$



Unions and Intersections can be repeated for an arbitrary number of sets

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup A_3 \cdots \cup A_n$$

or even an infinite number of sets

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup \cdots$$

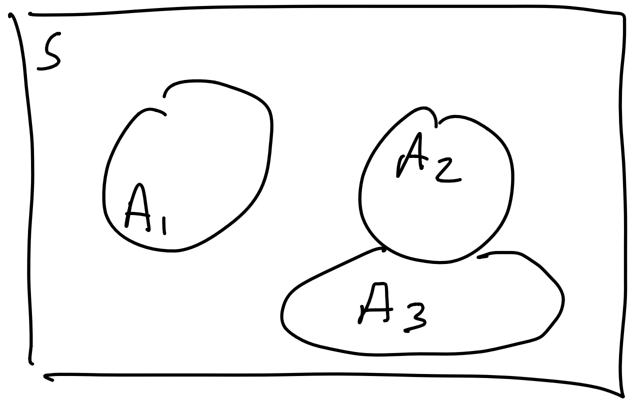
Definitions:

mutually exclusive: $A_1, A_2, \ldots, A_n$ are mutually exclusive if for every $i \neq j$ $A_i \cap A_j = \emptyset$, i.e., if A_i and A_j are disjoint

Collectively exhaustive: $A_1, A_2, \ldots, A_n$ are collectively exhaustive if $\bigcup_{i=1}^{n} A_i = S$

partition: $A_1, A_2, \ldots, A_n$ form a partition if they are both mutually exclusive and collectively exhaustive

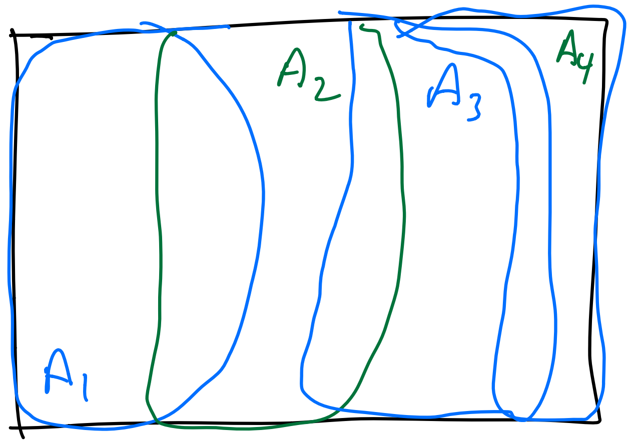
Visually:



Mutually exclusive

$A_i \cap A_j = \phi$ for all i, j

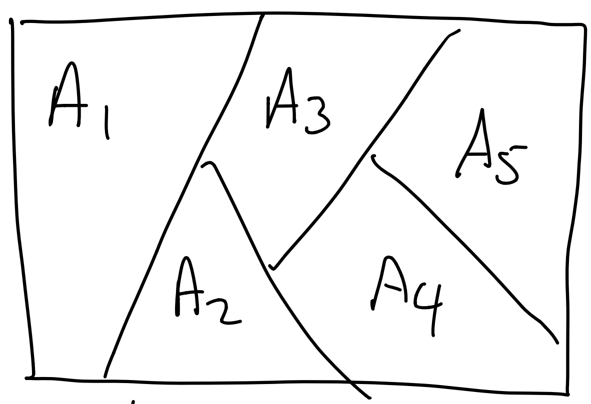
Collectively exhaustive



$$\bigcup_{i=1}^{n} A_i = S$$

Every element in S is in at least one event A_i

Partition



Both mutually exclusive and collectively exhaustive

$A_1, A_2, \ldots, A_5$ form a partition