Name: ____ PU ID: ________________

ECE 302: Probabilistic Methods in Electrical and Computer Engineering
Fall 2021
Instructor: Prof. A. R. Reibman

PURDUE UNIVERSITY

Homework 7

Fall 2021
(Due Thursday October 21, 11:59pm)

Homework is due on **Thursday October 21 at 11:59pm** on Gradescope. No late homework will be accepted, and no homework will be accepted without a statement. Include a brief description of all sources of information you used (including other people), not counting the text, handouts, or material posted on the web page, **or** state "I did not receive help on this homework". You do not need to reference any material presented in class or on the course web-site, in the textbook, nor Prof. Reibman nor TA Haoyu Chen.

Statement:

Topics: Common PDFs (Ch 4.4); Functions of a Random Variable (Ch 4.5)

Exercise 1. (From textbook, problem 4.63 (a-c))
Let X be a Gaussian random variable with mean 5 and variance 16.

(a) Find $P(X > 4)$, $P(X \geq 7)$, $P(2 < X < 7)$, $P(6 \leq X \leq 8)$.

(b) If $P(X < a) = 0.8869$, what is the value of a?

(c) If $P(X > b) = 0.11131$, what is the value of b?

Exercise 2.
Let X be a Gaussian variable with $E(X) = 0$ and $P(|X| \le 10) = 0.1$. What is the standard deviation of X? You may want to use the facts that $\Phi(-1.28) = 0.1$, $\Phi(-1.64) = 0.05$, and $\Phi(-1.96) = 0.025$.

Exercise 3. (From Final, Spring 2016)
A device is deployed in a remote region. The time, T, to failure, is exponentially distributed with mean 3 years. The device will not be monitored during the first 2 years, so the time before failure can be discovered is $X = max(T, 2)$. What is $E(X)$?

(Hint: Consider separately what happens when $T < 2$ and $T \geq 2$. (That is to say, consider both conditions $C = \{T < 2\}$ and C^c. This allows you to apply principles we learned in class, and only do one integration. Without this approach, you may find the following integral (without limits) helpful.)

$$\int xe^{ax}\, dx = \left(\frac{x}{a} - \frac{1}{a^2}\right) e^{ax}$$

Exercise 4.
Let random variable X have pdf $f_X(x) = 1/(2x^2)$ for $|x| \geq 1$ and $f_X(x) = 0$ for $|x| < 1$. Let $Y = \sqrt{|X|}$. Find $E(Y)$.

Exercise 5. FROM TEXTBOOK, PROBLEM 4.27 (A-C)
A voltage X is uniformly distributed in the set $\{-3, -2, \ldots, 3, 4\}$. Find the pdf and cdf of the random variables X, $Y = -2X^2 + 3$, $W = \cos(\pi X/8)$.

Exercise 6. (From textbook, problem 4.91)
Let $Y = \exp(X)$.

(a) Find the cdf of Y in terms of the cdf of X.

(b) Find the pdf of Y in terms of the pdf of X.

(c) Find the pdf of Y for the specific case when X is Gaussian. In this case, Y is a log-normal RV.

Exercise 7. (Exam 3 Fall 2016)
Let X be a voltage input to a rectifier, and let X be a continuous uniform random variable on the interval $[-1, 1]$. The rectifier output is a random variable Y, where

$$Y = g(X) = \begin{cases} 0 & X < 0 \\ X & X \geq 0 \end{cases}$$

Find and sketch the PDF of Y, $f_Y(y)$.