$\qquad$ PU ID: $\qquad$

ECE 302: Probabilistic Methods in Electrical and Computer Engineering
Fall 2021
PURDUE
Instructor: Prof. A. R. Reibman

## Homework 3

Fall 2021
(Due Thursday September 16 at 11:59pm)
Homework is due on Thursday September 16 at 11:59pm on Gradescope. No late homework will be accepted, and no homework will be accepted without a statement. Include a brief description of all sources of information you used (including other people), not counting the text, handouts, or material posted on the web page, or state "I did not receive help on this homework". You do not need to reference any material presented in class or on the course web-site, in the textbook, nor Prof. Reibman nor TA Haoyu Chen.

## Statement:

Topics: Independence (Section 2.5); Random variables, PMF, PDF, and CDF (Section 3.1-3.2, Sections 4.1-4.2, except Section 4.2.2)).

## Exercise 1.

An experiment consists of picking one of two urns at random and then selecting a ball from the urn and noting its color (black or white). Let A be the event "urn 1 is selected" and B the event "a black ball is observed." Under what conditions are A and B independent?
(Hint: recall that independence is a mathematical definition, so determine what mathematical conditions are necessary, and how that translates into the world.)

## Exercise 2.

Let the sample space $S=\left\{\omega_{1}, \omega_{2}, \omega_{3}\right\}, P\left(\omega_{i}\right)=1 / 3$ for $i=1,2,3$, and let random variables $X, Y$, and $Z$ be:

$$
\begin{array}{lll}
X\left(\omega_{1}\right)=1, & X\left(\omega_{2}\right)=2 & X\left(\omega_{3}\right)=3 \\
Y\left(\omega_{1}\right)=2, & Y\left(\omega_{2}\right)=3 & Y\left(\omega_{3}\right)=1 \\
Z\left(\omega_{1}\right)=3, & Z\left(\omega_{2}\right)=1 & Z\left(\omega_{3}\right)=2
\end{array}
$$

(a) $X, Y$, and $Z$ have the same probability mass function. Find the PMF of $X$.
(a)
(b) Find the PMF of $X+Y-Z$
$\square$
(c) Find the PMF of $Z /|X-Y|$.
(c)

Exercise 3. (Textbook 3.20)
Two dice are tossed; let $X$ be the absolute value of the difference in the number of dots facing up.
(a) Find and sketch the pmf of $X$.
$\square$
(b) Find the probability that $|X| \leq k$ for all $k$.
(b)

Exercise 4. (From textbook, problems 4.13, 4.23, plus)
A random variable $X$ has cdf:

$$
F_{X}(x)= \begin{cases}0 & \text { for } x<0 \\ 1-\frac{1}{4} e^{-2 x} & \text { for } x \geq 0\end{cases}
$$

(a) Plot the cdf. Is this a discrete, continuous, or mixed RV?
$\square$
(b) Find and plot the pdf.
$\square$
(c) Use the cdf to find $P(X \leq 2), P(X=0), P(X<0), P(2<X<6)$.
$\square$
$P(X \leq 2)=$

$$
P(X=0)=
$$

$$
P(X<0)=
$$

$$
P(2<X<6)=
$$

Exercise 5. (From textbook, problems 4.17, 4.39)
A random variable $X$ has pdf:

$$
f_{X}(x)= \begin{cases}c\left(1-x^{2}\right) & -1 \leq x \leq 1 \\ 0 & \text { otherwise }\end{cases}
$$

(a) Find $c$ and sketch the pdf.
(a) $c=$
(a) Sketch:
(b) Find and sketch the cdf of $X$.
(b) CDF:
(b) Sketch:
(c) Use the cdf to find $P(X=0), P(0<X<0.5), P(|X-0.5|<0.25)$.

$$
P(X=0)=
$$

$$
P(0<X<0.5)=
$$

$$
P(|X-0.5|<0.25)=
$$

