

Name: \_\_\_\_\_

PUID: \_\_\_\_\_

ECE 302: Probabilistic Methods in Electrical and Computer Engineering

Fall 2021

Instructor: Prof. A. R. Reibman

**PURDUE**  
UNIVERSITY

## Homework 1

Fall 2021

(Due Thursday September 2 at 11:59pm)

Homework is due on **Thursday September 2 at 11:59pm** on Gradescope. No late homework will be accepted, and no homework will be accepted without a statement. Include a brief description of all sources of information you used (including other people), not counting the text, handouts, or material posted on the web page, or state "I did not receive help on this homework". You do not need to reference any material presented in class or on the course web-site, in the textbook, nor Prof. Reibman nor TA Haoyu Chen.

**Topics:** Sample space, events, set theory; reading in Chapters 1 and Section 2.1-2.2.

**Statement:**

### Exercise 1.

Evaluate the following integration:

$$\int_0^{10} 3x^2 e^{-4x} dx$$

You may find the following equation useful:

$$\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

$$\begin{aligned} \int 3x^2 e^{-4x} dx &= 3 \cdot \left[ \frac{x^2 \cdot e^{-4x}}{-4} + \frac{1}{2} \left( e^{-4x} \cdot \frac{x}{-4} - \frac{1}{-4} \int e^{-4x} dx \right) \right] \\ &= 3 \cdot \left[ -\frac{x^2 \cdot e^{-4x}}{4} + \frac{1}{2} \left( \frac{x \cdot e^{-4x}}{-4} - \frac{1}{16} \cdot e^{-4x} \right) \right] \\ &= 3 \cdot \left( -\frac{x^2}{4} + \frac{x}{-8} - \frac{1}{32} \right) \cdot e^{-4x} \end{aligned}$$

$$\begin{aligned} \int_0^{10} 3x^2 e^{-4x} dx &= \left[ 3 \cdot \left( -\frac{x^2}{4} + \frac{x}{-8} - \frac{1}{32} \right) \cdot e^{-4x} \right]_0^{10} \\ &= 3 \cdot \left( -25 + \frac{10}{-8} - \frac{1}{32} \right) e^{-40} + \frac{3}{32} \end{aligned}$$

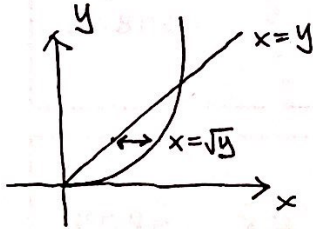
$$\int_0^{10} 3x^2 e^{-4x} dx = -\frac{2523}{32} \cdot e^{-40} + \frac{3}{32}$$

**Exercise 2.**

Given the following 2-dimensional function  $f(x,y)$ , compute  $\int_{-\infty}^{\infty} f(x,y) dx$ .

$$f(x,y) = \begin{cases} 15y & \text{for } x^2 < y < x \\ 0 & \text{otherwise} \end{cases}$$

Hint: draw a picture!



$$\begin{aligned} & \int_{-\infty}^{\infty} f(x,y) dx \\ &= \int_y^{\sqrt{y}} 15y \, dx \\ &= 15y \cdot x \Big|_y^{\sqrt{y}} \\ &= 15y^{3/2} - 15y^2, \quad y \in (0,1). \end{aligned}$$

$$\int_{-\infty}^{\infty} f(x,y) dx = \begin{cases} 15y^{3/2} - 15y^2 & , \quad y \in (0,1) . \\ 0 & , \quad \text{otherwise} . \end{cases}$$

**Exercise 3.**

The space  $S$  and three of its subsets are given by  $S = \{n \in \mathbb{Z} : 0 \leq n \leq 11\}$ ,  $A = \{1, 3, 4, 5, 9\}$ , and  $B = \{4, 7, 9, 11\}$ , and  $C = \{1, 3, 9, 11\}$ .

Find  $A \cap B \cap C$ ,  $A^c \cap B$ ,  $A - C$ , and  $(A - B) \cup B$ .

$$A \cap B \cap C = \{9\}$$

$$A^c \cap B = \{7, 11\}$$

Method 1:  $A^c = \{0, 2, 6, 7, 8, 10, 11\}$   
 $A^c \cap B = \{7, 11\}$   
Method 2:  $A^c \cap B = B - A$ : elements in  $B$  but not in  $A$   
 $= \{7, 11\}$ .

$$A - C = \{4, 5\}$$

$$(A - B) \cup B = \{1, 3, 5, 4, 7, 9, 11\}$$

$A - B = A \cap B^c = \{1, 3, 5\}$ .

**Exercise 4.**

Four marbles, numbered 1,2,3 and 4 are placed in a box. One of the marbles is drawn randomly from the box and its number,  $N_1$  is noted. (So,  $N_1=1,2,3$ , or 4.) An integer  $N_2$  is then selected at random from the values  $1, \dots, N_1$ . The outcome of this experiment is the ordered pair  $(N_1, N_2)$ , where  $N_1$  denotes the marble and  $N_2$  is just a number.

- (a) Write the sample space of the experiment.

$$S = \{ (N_1, N_2) : N_1 = 1, 2, 3, 4, N_2 = 1, \dots, N_1 \}.$$

- (b) Write the event "Marble 2 is selected".

$$\{ \text{Marble 2 is selected} \} = \{ (2, 1), (2, 2) \}$$

- (c) Write the event " $N_2 = 3$ ".

$$\{ N_2 = 3 \} = \{ (3, 3), (4, 3) \}$$

- (d) Write the event "Marble 2 is selected and  $N_2 = 3$ ".

$$\begin{aligned} \{ \text{Marble 2 is selected and } N_2 = 3 \} &= \phi & \{ \text{Marble 2 is selected} \} &= \{ (2, 1), (2, 2) \} \\ & & \{ N_2 = 3 \} &= \{ (3, 3), (4, 3) \} \\ & & \{ (2, 1), (2, 2) \} \cap \{ (3, 3), (4, 3) \} &= \phi \end{aligned}$$

Not possible!

**Exercise 5.**

An integrated circuit (IC) factory has three machines,  $X, Y$ , and  $Z$ . Test one IC from each machine, and observe if each is *acceptable* ( $a$ ) or *fails* ( $f$ ). Thus, an observation is a sequence of the three test results from each machine. For example, the observation that the circuit from  $Z$  fails while the circuits from  $X$  and  $Y$  pass is  $aaf$ .

- (a) What is the sample space?

$$S = \{aaa, aaf, afa, faa, aff, faf, ffa, fff\}$$

- (b) What are the elements of the sets

$Z_F = \{\text{circuit from } Z \text{ fails}\}$  and

$X_A = \{\text{circuit from } X \text{ is acceptable}\}$ ?

$$Z_F = \{aaf, aff, faf, fff\}$$

$$X_A = \{aaa, aaf, afa, aff\}$$

- (c) Are  $Z_F$  and  $X_A$  mutually exclusive? Are  $Z_F$  and  $X_A$  collectively exhaustive?

Mutually Exclusive?

No.  $Z_F \cap X_A = \{aaf, aff\} \neq \emptyset$

Collectively Exhaustive?

No.  $Z_F \cup X_A \neq S$ .

- (d) What are the elements of the sets

$C = \{\text{more than one circuit is acceptable}\}$  and

$D = \{\text{at least two circuits fail}\}$ ?

$$C = \{aaa, aaf, faa, afa\}$$

$$D = \{fff, faf, ffa, aff\}$$

- (e) Are  $C$  and  $D$  mutually exclusive? Are  $C$  and  $D$  collectively exhaustive?

Mutually Exclusive?

Yes.  $C \cap D = \emptyset$

Collectively Exhaustive?

Yes.  $C \cup D = S$ .

**Exercise 6. (FROM EXAM 1, FALL 2015)**

For each of the following relations, determine which is valid for arbitrary events  $A, B$ , and  $C$ . (Note: to be true "for arbitrary events", it must be true for any such event. Use a Venn diagram if it is helpful.)  
(On the exam, this was a True/False question. For this homework, show whether it is true or false.)

False

(a)  $(A \cup B \cup C)^c = A^c \cup B^c \cup C^c$

True

(b)  $(A \cap B) \cup (A \cap B^c) \cup (A^c \cap B) = (A^c \cap B^c)^c$

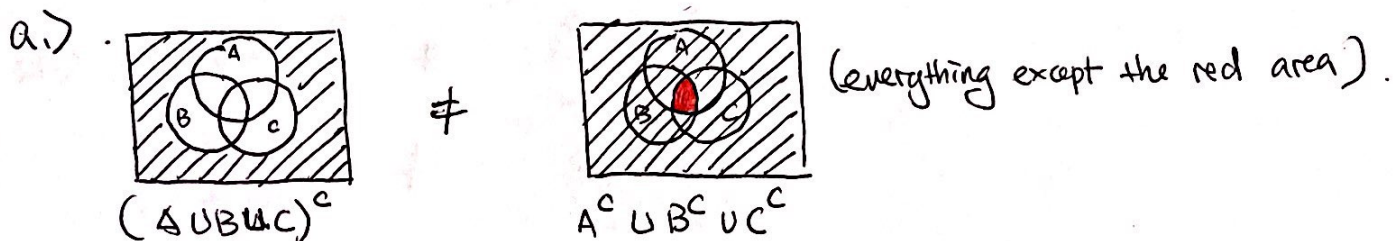
False

(c)  $(A - B) - C = A - (B - C)$ .

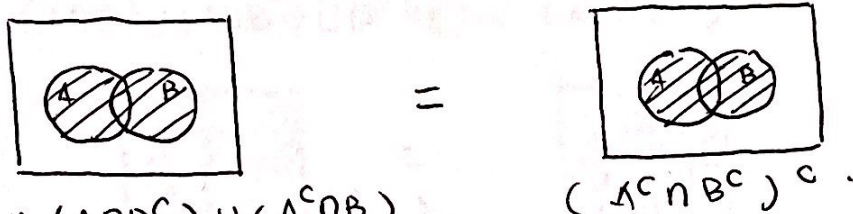
True

(d)  $(A \cup B) \cap (A^c \cup B^c) = (A \cap B^c) \cup (A^c \cap B) \cup (A^c \cap B \cap C^c)$

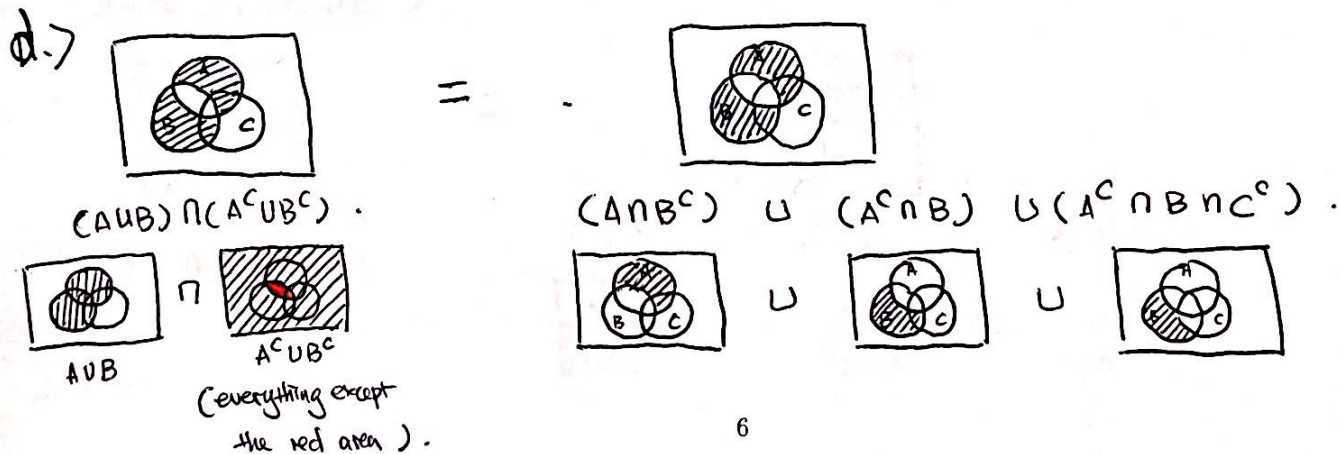
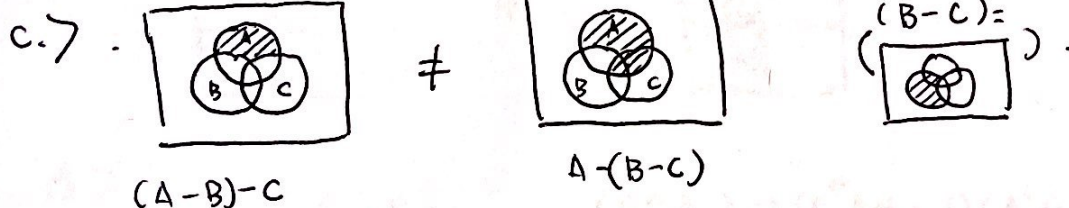
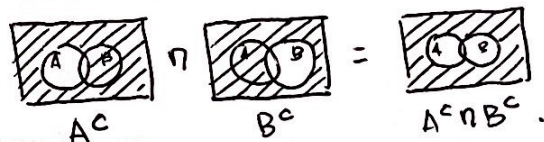
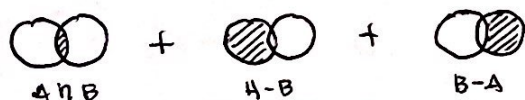
Hint: be careful, item (d) is tricky.



b.)



$(A \cap B) \cup (A \cap B^c) \cup (A^c \cap B)$



# Alternative

## Exercise 6. (FROM EXAM 1, FALL 2015)

For each of the following relations, determine which is valid for arbitrary events  $A, B$ , and  $C$ . (Note: to be true "for arbitrary events", it must be true for any such event. Use a Venn diagram if it is helpful.)  
(On the exam, this was a True/False question. For this homework, show whether it is true or false.)

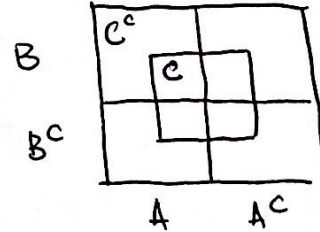
F (a)  $(A \cup B \cup C)^c = A^c \cup B^c \cup C^c$

T (b)  $(A \cap B) \cup (A \cap B^c) \cup (A^c \cap B) = (A^c \cap B^c)^c$

F (c)  $(A - B) - C = A - (B - C)$

T (d)  $(A \cup B) \cap (A^c \cup B^c) = (A \cap B^c) \cup (A^c \cap B) \cup (A^c \cap B \cap C^c)$

Hint: be careful, item (d) is tricky.



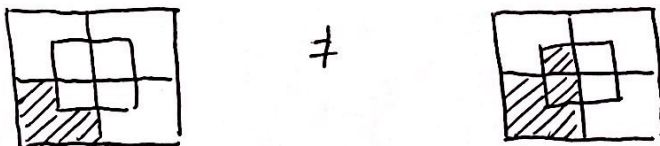
a.)  $(A \cup B \cup C)^c \neq A^c \cup B^c \cup C^c$



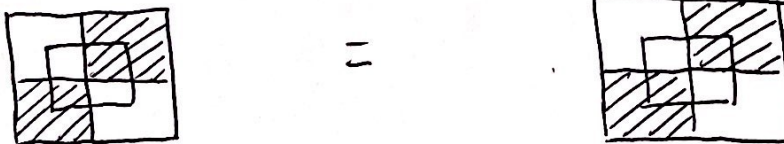
b.)  $(A \cap B) \cup (A \cap B^c) \cup (A^c \cap B) = (A^c \cap B^c)^c$



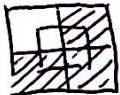
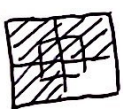
c.)  $(A - B) - C \neq A - (B - C)$



d.)  $(A \cup B) \cap (A^c \cup B^c) = (A \cap B^c) \cup (A^c \cap B) \cup (A^c \cap B \cap C^c)$



$(A \cup B) \cap (A^c \cup B^c)$



$A \cap B^c$



$A^c \cap B$



$A^c \cap B \cap C^c$

