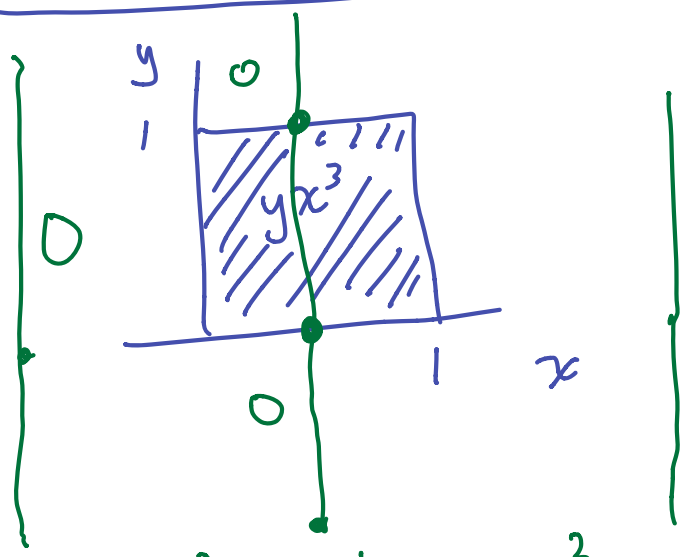
Integration of 2D functions

case 1: indefinite integration

$$\int y x^3 \, dy = x^3 \frac{y^2}{2} + c$$

case 2: $f(x,y) = \begin{cases} y x^3 & 0<x<1 \text{ and } 0<y<1 \\ 0 & \text{otherwise} \end{cases}$

$$\int f(x,y)\,dy$$



if $x<0$ then

$$\int \dots = 0$$

if $x>1$, $\int \dots = 0$

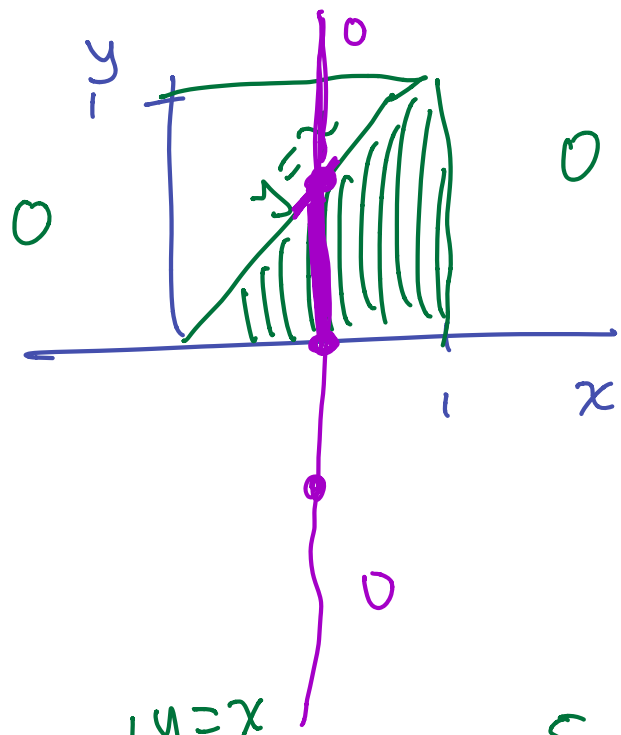
if $0<x<1$

$$\int \dots = \int_0^1 y x^3 \, dy = \frac{x^3 y^2}{2}\Big|_0^1 = \frac{x^3}{2}$$

$$\int f(x,y)\,dy = \begin{cases} \frac{x^3}{2} & \text{if } 0<x<1 \\ 0 & \text{else} \end{cases}$$

case 3:

$$f(x,y) = \begin{cases} yx^3 & 0<x<1 \text{ and } 0<y<x \\ 0 & \text{otherwise} \end{cases}$$



$\int f(x,y)\,dy$

if $x<0$ or $x>1$

then $\int \cdots = 0$

if $0<x<1$

then $\displaystyle\int_0^x yx^3\,dy = \frac{y^2x^3}{2}\Big|_{y=0}^{y=x} = \frac{x^5}{2}$

$$\int \cdots = \begin{cases} x^5/2 & \text{if } 0<x<1 \\ 0 & \text{else} \end{cases}$$

case 4:

$$f(x,y) = \begin{cases} yx^3 & 0<x<1 \text{ and } 0<y<x \\ 0 & \text{otherwise} \end{cases}$$

$$\iint f(x,y)\,dx\,dy = \iint f(x,y)\,dy\,dx$$

$$= \int_0^1 \underbrace{\int_0^x y\, x^3\, dy}\, dx$$

$$= \int_0^1 \frac{x^5}{2}\, dx = \frac{x^6}{12}\Big|_0^1$$

$$= \frac{1}{12}$$