11/2/21 Exam review

common pmfs + pdfs

conditional prob. conditioned on events

conditional moments

PDF of $g(X) = Y$

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2 RVs joint cdf, pdf, pmf

marginal " " "

independence

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common pmfs

Bernoulli, binomial, geometric, Poisson, uniform

Bernoulli: is the next car that passes Red

$A = \{\text{red car}\}$

$$X = \begin{cases} 1 & \text{if } A) \\ 0 & \text{else} \end{cases}$$

Binomial ten cars, how many of them are red?

$$P_X(x) = \binom{10}{x} p^x (1-p)^{10-x}$$

Geometric how many cars until first red one

$p = P(\text{success})$ $S = \{0, 1, \cdots\}$

$S = \{1, \cdots\}$

write down your sample space

Poisson : cars on a busy highway, few are red.

cars that pass in a fixed time period

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common pdfs

uniform

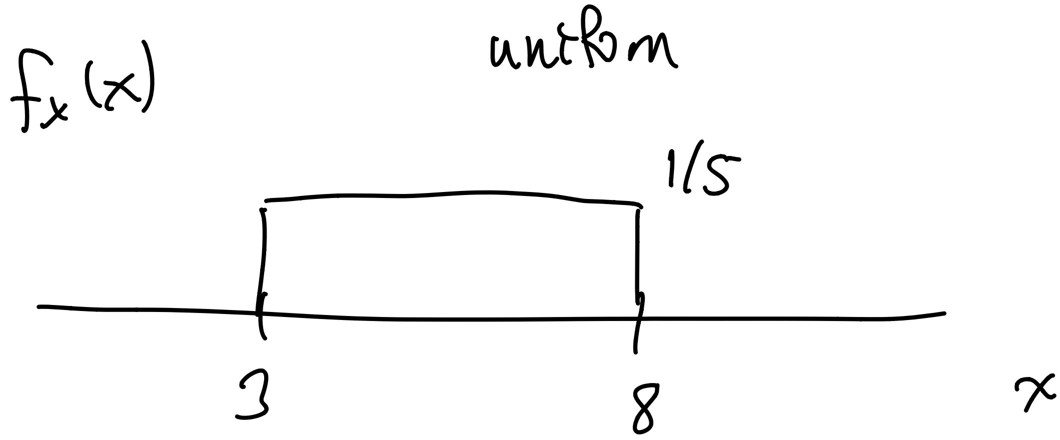
exponential

Gaussian $\rightarrow$

exponential (and geometric) RVs are memoryless

same shape regardless of where you start counting time

$$\boxed{P(X > t + h \mid X > t) = P(X > h)}$$



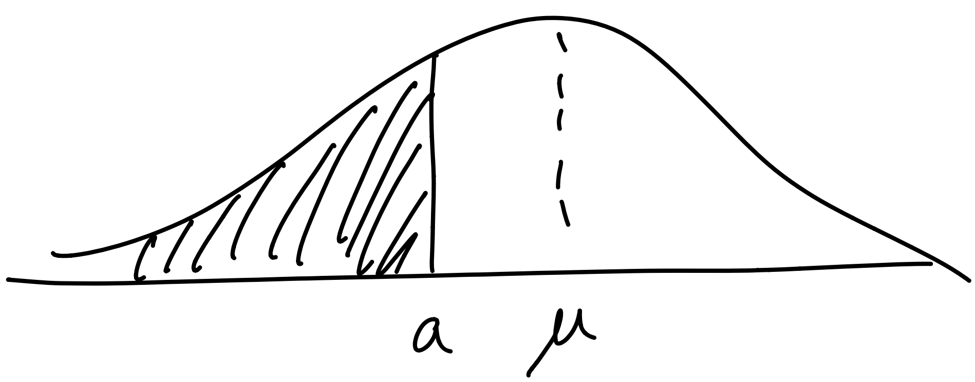
Gaussian $\quad X \sim N(\mu, \sigma^2)$

$P(X \le a)$ $\qquad Z \sim N(0,1)$

$$= P\left(\frac{X-\mu}{\sigma} \le \frac{a-\mu}{\sigma} \right)$$

$$= P\left(Z \le \frac{a-\mu}{\sigma} \right)$$

$$= \Phi\left(\frac{a-\mu}{\sigma} \right)$$



conditional pmf/pdf/cdf

- definitions
- thm of total probability
- inference (learning)

$$p_X(x \mid A) = \frac{P(X = x \cap A)}{P(A)}$$

if $P(A) \neq 0$

$$F_X(x \mid A) = \frac{P(X \leq x \cap A)}{P(A)}$$

$$f_X(x \mid A) = \frac{d}{dx} F_X(x \mid A)$$

$$f_X(x \mid A^c)$$

thm total prob

$$f_X(x) = f_X(x \mid A)\,P(A) + f_X(x \mid A^c)\,P(A^c)$$

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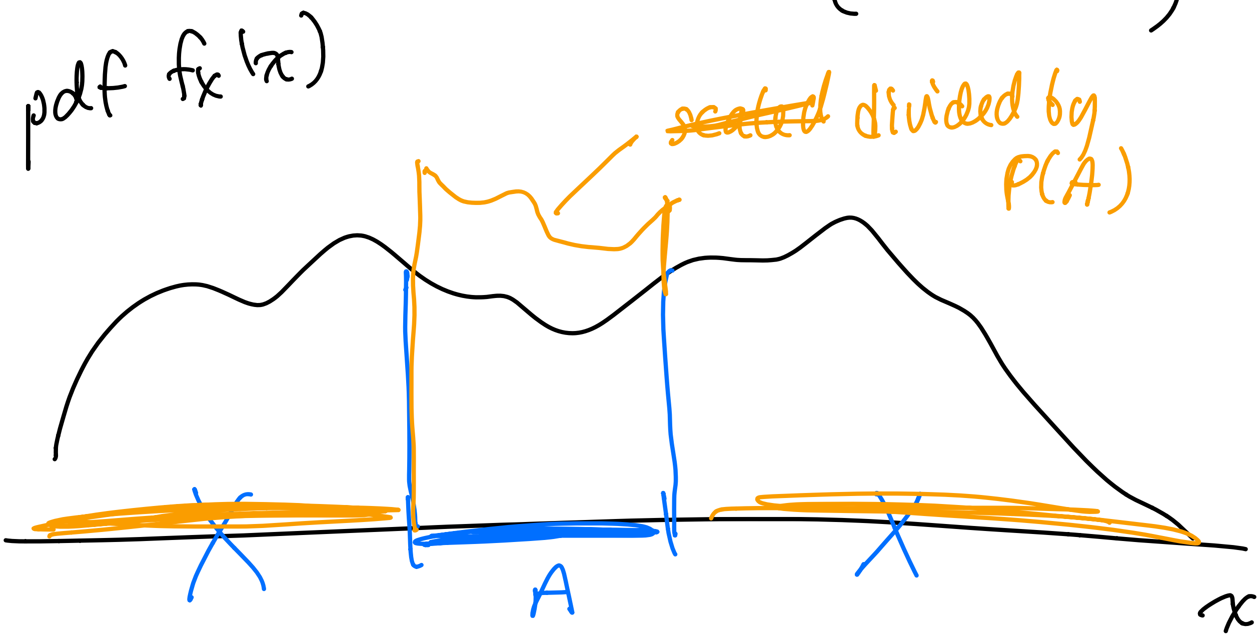
thm of total expectation

$$E(X) = E(X \mid A)\,P(A) + E(X \mid A^c)\,P(A^c)$$

conditioning on events that depend on X

$$A = \{ X \in B \}$$



$$f_X(x|A) = \begin{cases} \dfrac{f_X(x)}{P(A)} & \text{if } x \in A \\ 0 & \text{else} \end{cases}$$

$$E(X|A) = \int_{\text{all } x} x\, f_X(x|A)\, dx$$

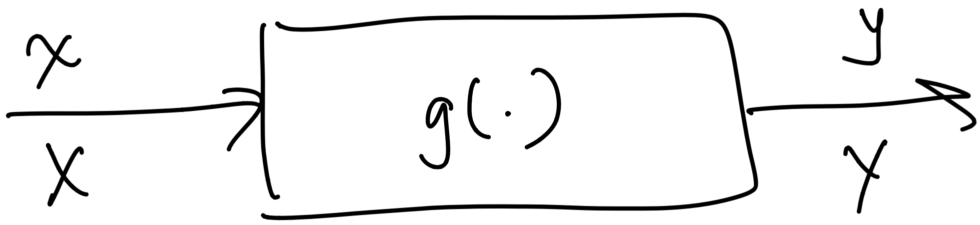
$$E(X) = \int_{\text{all } x} x\, f_X(x)\, dx$$

$Var(X|A)$ = variance of $f_X(x|A)$

$$Var(X) = E(X^2) - E(X)^2$$

$$Var(X|A) = E(X^2|A) - E(X|A)^2$$

Derived RVs



if we know $f_X(x)$, $g(x) = y$, what's $f_Y(y)$?

$$E(g(X)) = \int_{\text{all } x} g(x) f_X(x)\,dx$$

2-step process

- find cdf of Y $F_Y(y)$
- differentiate wrt y to get $f_Y(y)$
- substitute to make $f_Y(y)$ a true function of y.

check: ~~did y~~ is answer a pdf?

$$F_Y(y) = P(Y \le y) = P(g(X) \le y)$$

manipulate this until see something like

$$P(X \le h(y)) = F_X(h(y))$$

or $P(h_1(y) < X < h_2(y)) = F_X(h_2(y)) - F_X(h_1(y))$

or $P(X > h(y)) = 1 - F_X(h(y))$

tranlate that into something

w/ $F_X(\ldots)$

→ expression for $F_Y(y)$ ~~of~~

differentiate

$$f_Y(y) = \frac{d F_Y(y)}{dy}$$

use chain rule.

ex: $F_Y(y) = F_X(h(y))$

then $f_Y(y) = f_X(h(y)) \frac{d h(y)}{dy}$

→ expression for $f_Y(y)$

may still have $f_X(\cdot)$

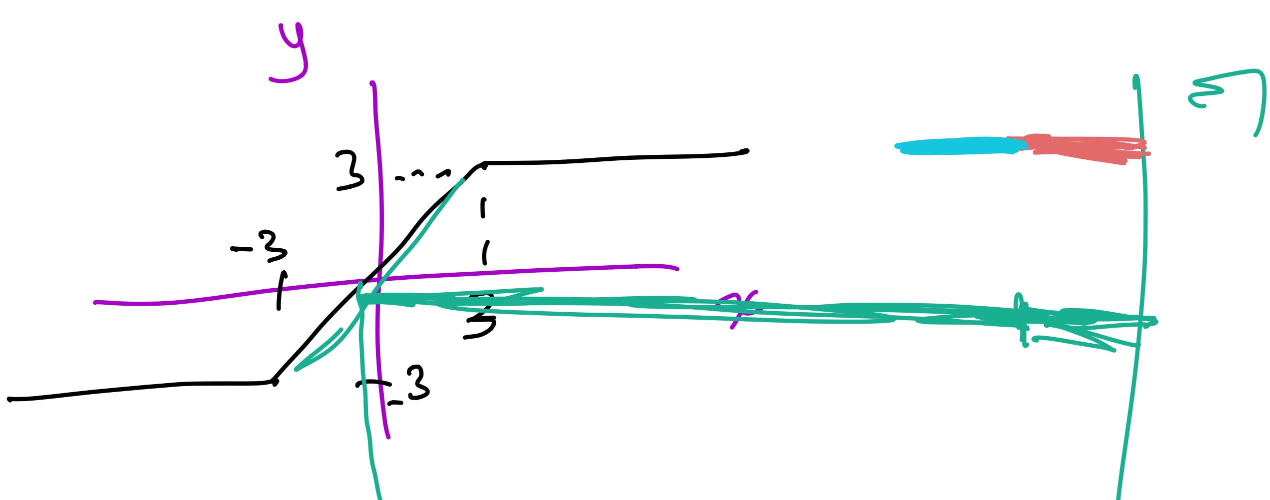
ex: $f_Y(y) = 3y\, f_X(3y)$

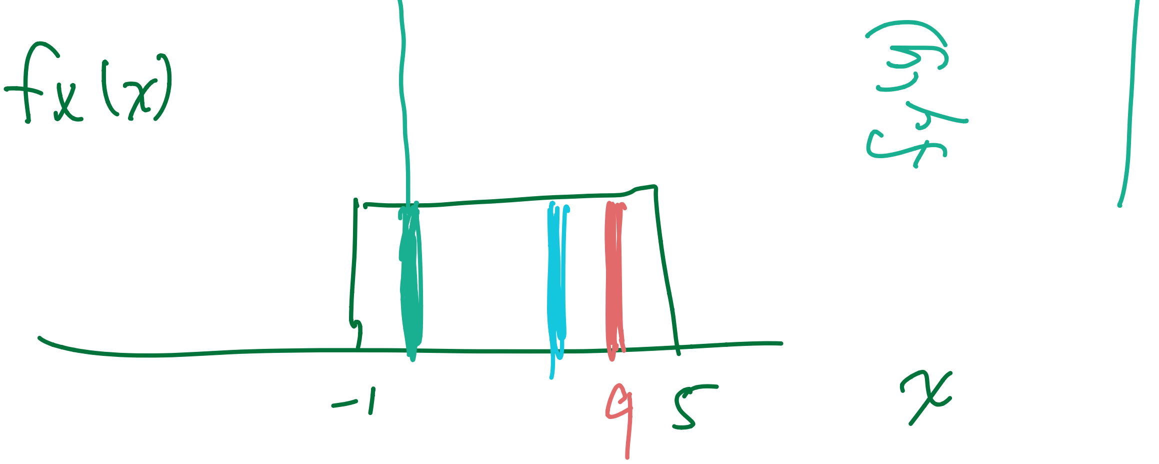
substitute $x = 3y$ everywhere in $f_X(x)$ expression

$\Rightarrow$ $f_Y(y)$ in terms of y

- bounds in terms of y
- will be a pdf

$$g(x) = \begin{cases} x & -3 < x < 3 \\ -3 & x < -3 \\ 3 & x > 3 \end{cases}$$





$$F_Y(y) = P(Y \le y)$$