Lecture 15 9/27/21

Last time: Geometric + binomial

This time: Exam review
- requires input from everyone

Seating chart will be posted before exam

Extra office hours today: 1-2 pm
3-4 pm

Any more left-handers?

Disjoint + Independent

$A \cap B = \phi$

$P(A \cap B) = P(A)P(B)$

if $P(A) \neq 0$
$P(B) \neq 0$

There must some non-ϕ intersection

if $A + B$ are independent
and $A + C$ " "
$B + C$ " "

then they may not be mutually independent

need $P(A \cap B \cap C) = P(A)P(B)P(C)$

Sets

Probability of events — Axioms

RVs

meanings of $\{1, 2\}$, $(1, 2)$

and $[1, 2]$

mutually exclusive — the intersections are ϕ

collectively exhaustive — every element is in at least one set

partition — both

$\boxed{\text{Converting English to math}}$

describing the experiment, sample space, event of interest.

set operations + combinations

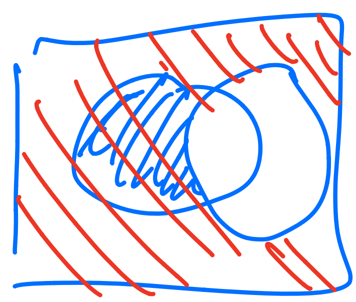
$A \cap B = B \cap A$ yes

commutativity
distributivity
associativity

De Morgan's Rules $(A \cap B)^c = A^c \cup B^c$

$(A \cup B)^c = A^c \cap B^c$

$A - B = A \cap B^c$



Probabilities

Table, Tree

Axioms + corollaries

$P(A) \geq 0$

$P(S) = 1$

$$\boxed{P(A \cup B) = P(A) + P(B) \quad \text{if } A \cap B = \phi}$$

$$P(A^c) = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A) \leq 1$$

$$P(\phi) = 0$$

generalizes

generalizes

$\subset$ ≡ a subset of

$\in$ ≡ an element of

if $A \subset B$

$$P(B) \geq P(A)$$

Conditional probability

Definition $\quad P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ if $P(B) \neq 0$

Theorem of Total probability

Bayes Rule

Concept – we know something happened (event B).

Consider what else can happen now

$\Rightarrow$ narrow the sample space

Thm total prob.

$$P(A) = \sum_{i=1}^{n} P(A / B_i)\, P(B_i)$$

if the B_i's form a partition

Bayes Rule

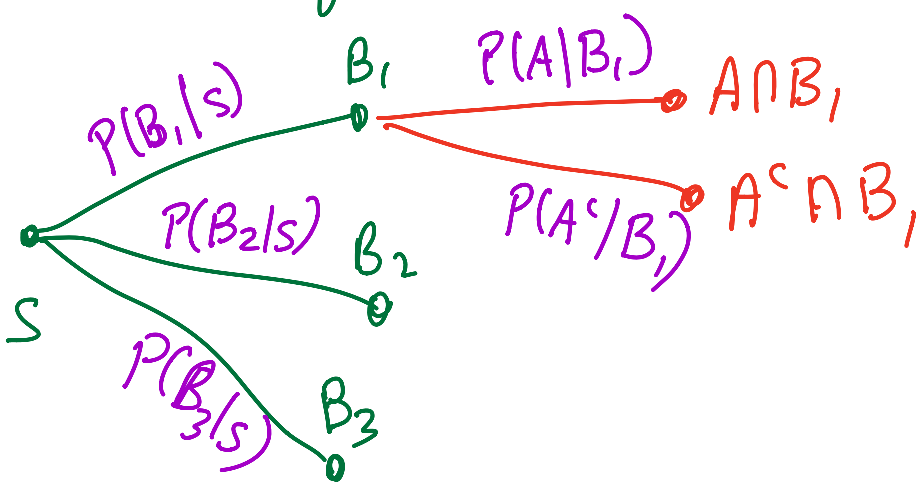
$$\boxed{P(B|A) = \frac{P(A|B)\, P(B)}{P(A)}}$$

flip the conditioning order

Table

	A	A^c	
B	$P(A\cap B)$ $P(A\mid B)P(B)$ $P(B\mid A)P(A)$	$P(A^c\cap B)$	$P(B)$
B^c	$P(A\cap B^c)$	$P(A^c\cap B^c)$	$P(B^c)$
	$P(A)$	$P(A^c)$	

Tree — conditional prob.
sequential experiments



Random Variables

pdf
cdf
pmf

going between them

their properties

moments

discrete, mixed, continuous

definition of RV

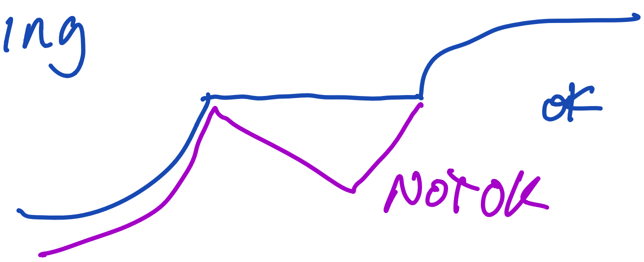
cumulative distribution function

CDF

$$P(X \le x) = F_X(x)$$

properties of CDF

non decreasing



$$\lim_{x \to -\infty} F_X(x) = 0$$

$$\lim_{x \to +\infty} F_X(x) = 1$$

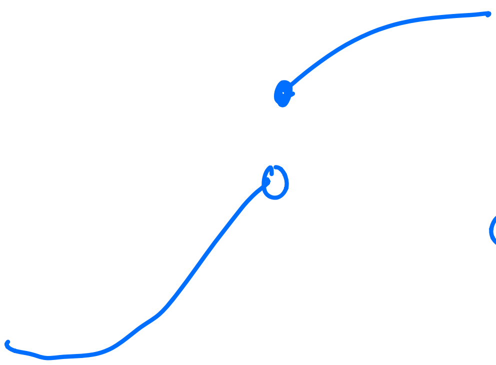
$$P(a < X \le b) = F_X(b) - F_X(a)$$

If x is in a continuous portion of $F_X(x)$

then $P(X = x) = 0$

discontinuities are ok

$$\rightarrow \quad P(X = a) = \underbrace{F_X(a) - F_X(a^-)}_{\text{jump height}}$$



continuous from right

- discontinuity is on left

cdf is a probability

pdf prob. density function

$$f_X(x) = \frac{dF_X(x)}{dx}$$

pdf is NOT a probability

$$P(a \le X \le b) = \int_a^b f_X(x)\,dx$$

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt$$

$$1 = \int_{-\infty}^{\infty} f_X(t)\,dt$$

$$E(X) = \int_{-\infty}^{\infty} t\, f_X(t)\,dt$$

$f_X(x) \geq 0 \longrightarrow$ no upper bound

Probability mass function

$$P(X = x) = p_X(x)$$

only defined discrete RVs

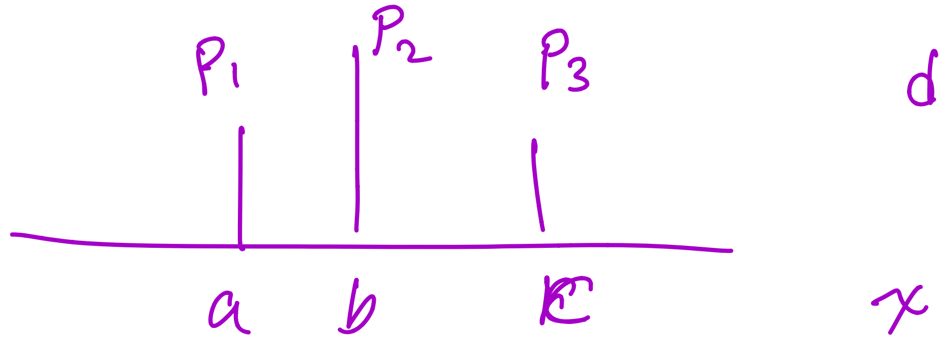
$$1 = \sum_{x \in S_X} p_X(x)$$

$$E(X) = \sum_{x \in S_X} x \, p_X(x)$$

$$E(g(X)) = \sum_{x \in S_X} g(x) \, p_X(x)$$

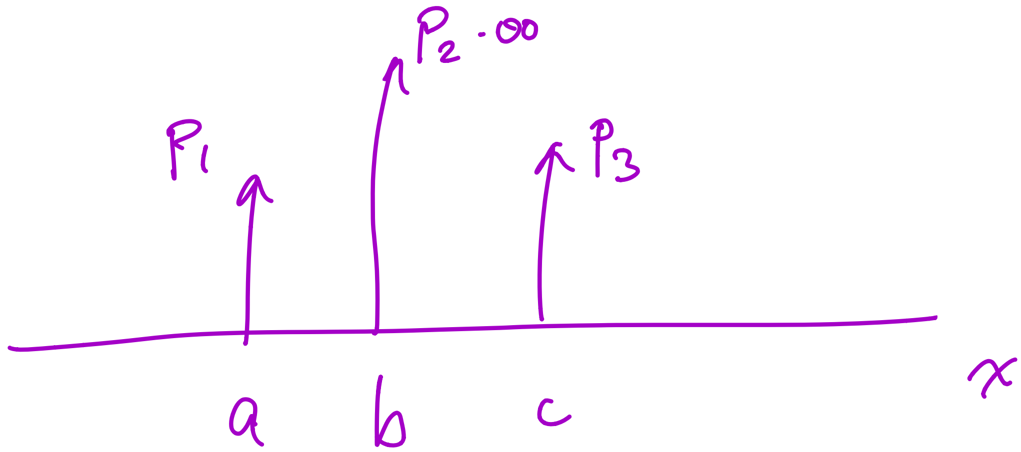
$$E(g(X)) = \int_{-\infty}^{\infty} g(t) \, f_X(t) \, dt$$

$p_X(x)$



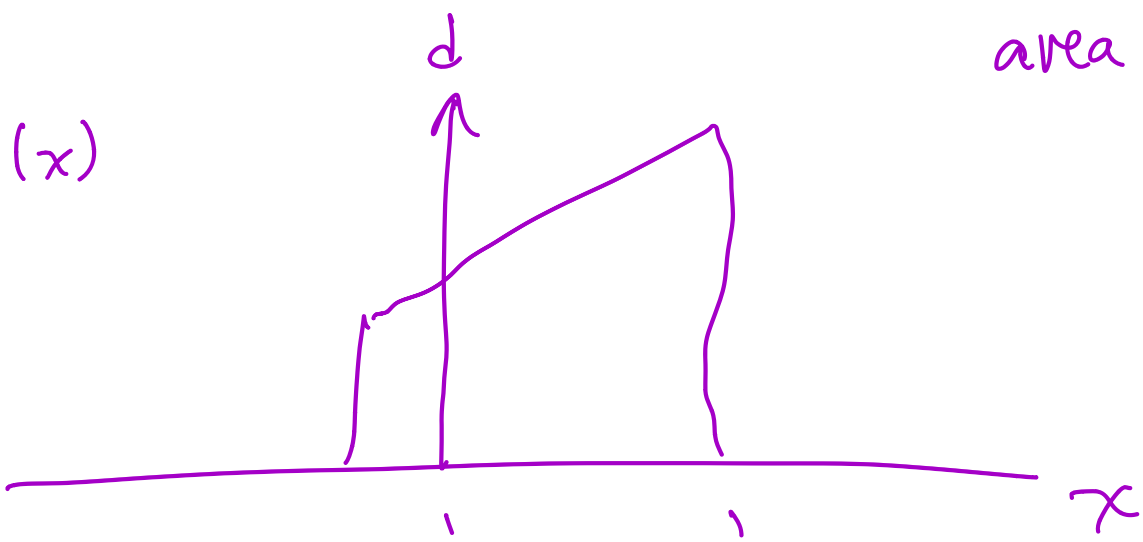
discrete RV

$f_Y(x)$



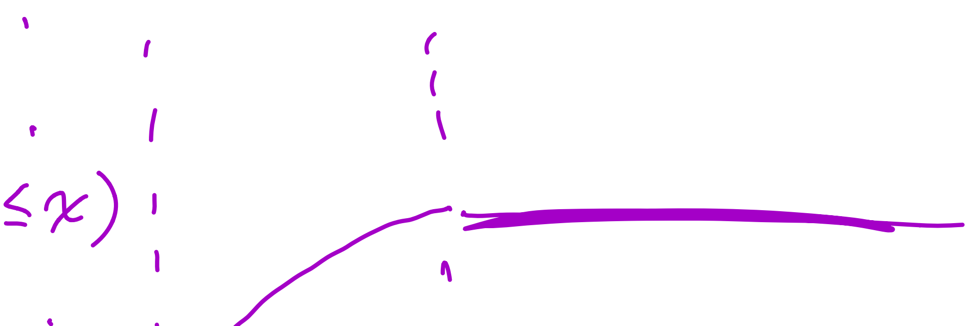
mixed RVs

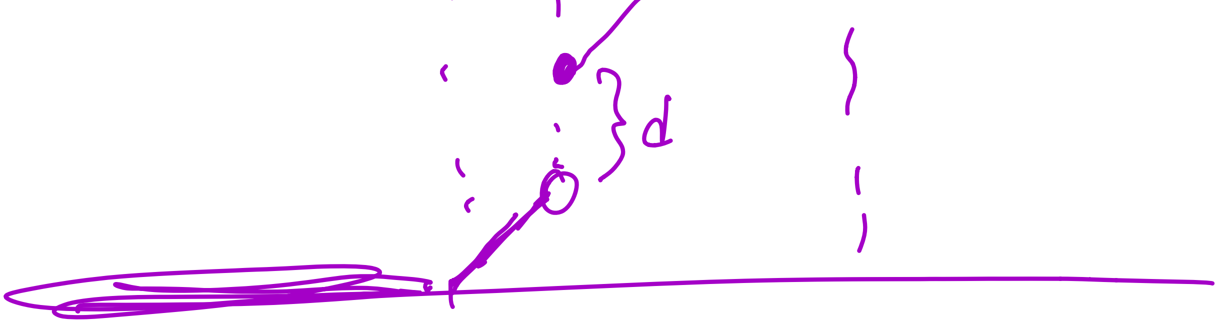
$f_X(x)$



area = 1

$F_X(x) = P(X \leq x)$

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