ECE 302: Probabilistic Methods in Electrical and Computer Engineering

Instructor: Prof. A. R. Reibman

PURDUE UNIVERSITY

Past Exam Questions
(Fall 2015, Spring 2016, Fall 2016, Fall 2017)
Chapters 3 and 4

Reibman

(January 2019)

SOLUTIONS

These form a collection of problems that have appeared in either Prof. Reibman's real exams or "sample exams." These can all be solved by applying the material we covered in class that appears in Chapters 3 and 4 of our textbook.

The last 3 pages of this document will be provided to you as the last pages of the exam. This will be all the formulas that will be available to you. The rest you must memorize.

The next 5 problems all refer to a discrete random variable X with the following pmf:

$$p_X(x) = \begin{cases} |x|/c & \text{for } x = -2, -1, 0, 1, 2. \\ 0 & \text{otherwise} \end{cases}$$

For these problems use the same set of answers, and clearly mark your answer next to the problem. The answers to each problem may or may not be different answers!

(a) 0

(b) 1

(c) 3

(d) 4

(e) 6

(f) None of the above

1) $\sum_x p_X(x) = 1 = \frac{1}{c}(2+1+0+1+2) = \frac{6}{c} \Rightarrow \boxed{c=6}$

2) $E(X) = \sum_x x\, p_X(x) = \frac{1}{6}(-2\cdot 2 + -1\cdot 1 + 0 + 1\cdot 1 + 2\cdot 2)$

$= 0$ (because symmetric pdf about zero)

e **Problem 1.** (5 POINTS)
Find the value of c.

a **Problem 2.** (5 POINTS)
Find $E(X)$.

3) $Var(X) = E(X^2) - E(X)^2 = E(X^2)$

$= \sum_x x^2 p_X(x) = \frac{1}{6}\cdot 2\cdot [2^2\cdot 2 + 1^2\cdot 1]$

$= 9/3 = \boxed{3}$

c **Problem 3.** (5 POINTS)
Find VAR(X).

a **Problem 4.** (5 POINTS)
Consider the random variable $Z = (X - E(X))^2$. Find $P(Z = 9)$.

b **Problem 5.** (5 POINTS)
Consider the event $A = \{X > -1.5\}$. Find $E(X|A)$.

4) there are no values of X that lead to $Z=9$, so

$\boxed{P(Z=9)=0}$

5) $E(X|A) = \sum_x x\, P_X(x|A)$

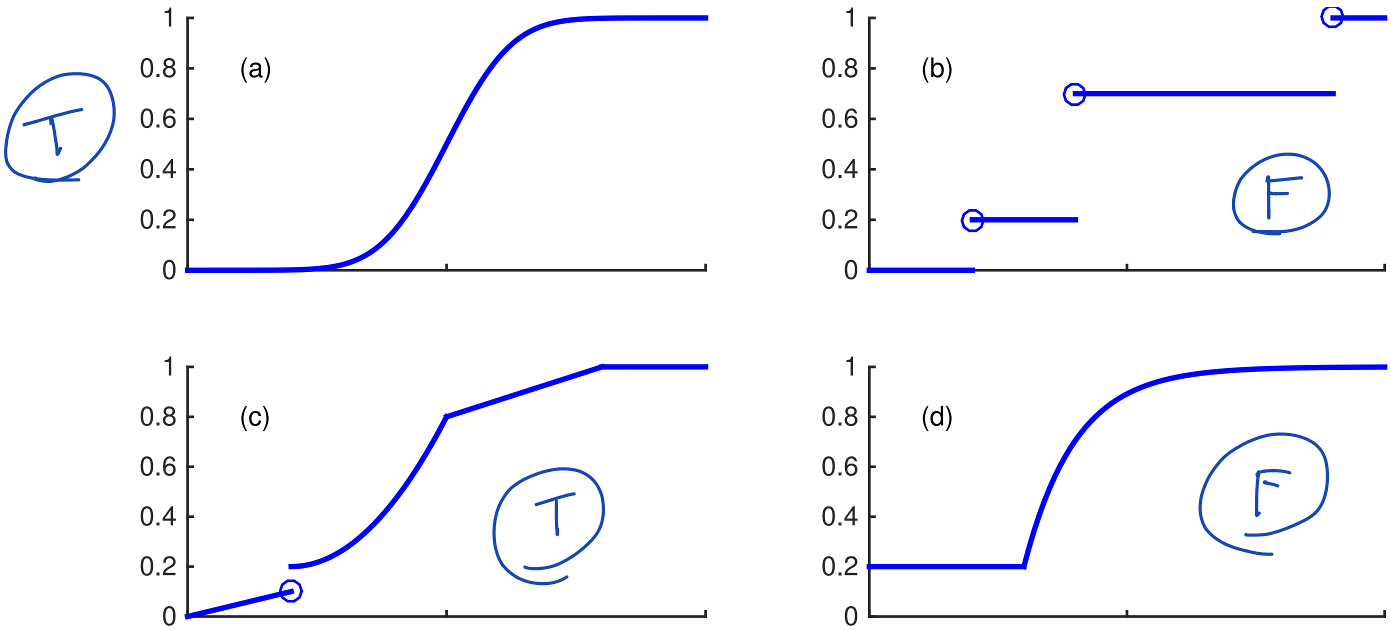
$P_X(x|A) = \frac{P(X=x \cap A)}{P(A)}$

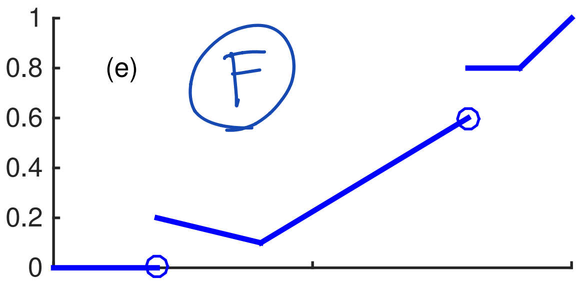
$= \begin{cases} \frac{p_X(x)}{P(A)} & x = -1, 0, 1, 2 \\ 0 & \text{else} \end{cases}$

$P(A) = 4/6.$

$E(X|A) = \frac{6}{4}\cdot\frac{1}{6}(-1+0+1+2) = 1$

Problem 6. (10 POINTS (2 POINTS EACH))
For each of the following 5 graphs, clearly indicate T (true) if the graph indicates a **valid** cumulative distribution function or F (false) otherwise.





b) is not valid because it is not right-continuous

c) is not valid because $\lim_{x \to -\infty} F_X(x) \neq 0$

e) is not valid because it is not non-decreasing

Problem 7. (5 POINTS)
Which of the following is NOT a property that the cumulative distribution function $F_X(x)$ of a random variable X must satisfy?

(a) $0 \leq F_X(x) \leq 1$ for every x. ✓

(b) $\lim_{x \to \infty} F_X(x) = 1$ ✓

(c) $\lim_{x \to -\infty} F_X(x) = 0$ ✓

(d) $F_X(x)$ is continuous at every x X it need not be continuous

(e) $P(X > x) = 1 - F_X(x)$ ✓

(f) $F_X(x)$ is an increasing function of x X it must just be non-decreasing

(g) (f) and (d)

(h) None of the above.

(i) All of the above.

Problem 8. (5 POINTS)
Which of the following statements are NOT NECESSARILY true about the probability density function $f_X(x)$ of a random variable X?

(a) $f_X(x) \leq 1$ for every x. NOT necessarily true

(b) $\int_{-\infty}^{\infty} f_X(x)dx = 1$ always true

(c) $\int_a^b f_X(x)dx = P(a < X \leq b)$ if $a < b$ NOT necessarily true (see equation 4.11)

(d) $f_X(x) \geq 0$ for every x. always true

(e) answers (a) and (c)

(f) None of the above.

(g) All of the above.

Problem 9. (5 POINTS)
Which of the following statements is a correct way to compute $P(X > 2)$ for a continuous random variable X?

(a) $f_X(2)$.

(b) $\int_2^\infty f_X(x)dx$

(c) $\int_2^\infty f_X(X)dX$

(d) $F_X(2)$

(e) $1 - F_X(2)$.

(f) answers (b) and (e)

(g) answers (c) and (d)

(h) None of the above.

probabilities can be computed using either the cdf $F_X(x)$ or the pdf $f_X(x)$ (since this is a continuous RV)

Problem 10. (6 POINTS)
Which is the correct mathematical expression for the following probability:
The probability that X varies from its mean μ by no more than 2 is more than 95%.
(You may find it useful to draw a sketch.)

"varies from its mean by no more than 2" means $|x-\mu| \le 2$

(a) $P(|X - \mu| \le 2) > 0.95$

(b) $P(|X - \mu| < 2) > 0.95$

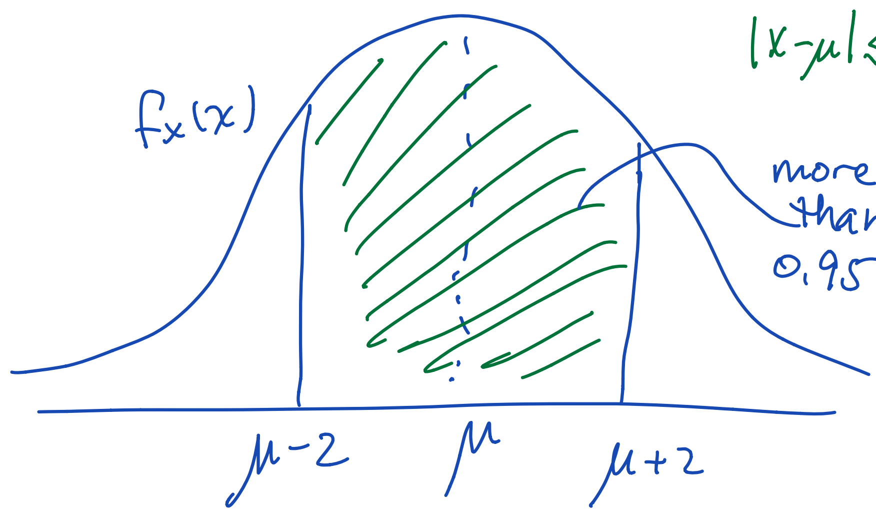
(c) $P(|X - \mu| > 2) > 0.95$

(d) $P(|X - \mu| > 2) \le 0.05$

(e) $P(|X - \mu| \ge 2) \le 0.05$

(f) answers (a) and (d)

(g) None of the above.



Is (d) also true?

No! Suppose we say $a = P(|X-\mu| \le 2)$, and that $a > 0.95$.

then $1-a = P(|X-\mu| > 2)$, but $1-a < 0.5$

strictly less than

Problem 11. (20 POINTS)
A discrete random variable X has the following probability mass function (PMF):

$$p_X(x) = \begin{cases} cx & \text{for } x = 1, 2, 3, 4. \\ 1/2 & \text{for } x = 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of the constant c.

(b) Find $E(X)$.

(c) Find $Var(X)$.

(d) What is the largest value of x for which $P(X > x) > 0.75$? (Hint, a sketch of the PMF can be helpful.)

a) $\sum_{\text{all } x} p_X(x) = 1$ so $1 = c[1+2+3+4] + \frac{1}{2}$

$= 10c + \frac{1}{2} \Rightarrow \boxed{c = \frac{1}{20}}$

b) $E(X) = \sum_{\text{all } x} x p_X(x) = c[1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3 + 4 \cdot 4] + \frac{1}{2} \cdot 5$

$= c(30) + \frac{5}{2} = \frac{30}{20} + \frac{5}{2} = \boxed{4}$

c) $Var(X) = E(X^2) - E(X)^2$

$E(X^2) = \sum_{\text{all } x} x^2 p_X(x) = c[1 \cdot 1^2 + 2 \cdot 2^2 + 3 \cdot 3^2 + 4 \cdot 4^2] + \frac{5^2}{2}$

$= c(1 + 8 + 27 + 64) + \frac{25}{2}$

$= c(100) + \frac{25}{2} = 5 + 12.5 = 17.5 = \frac{35}{2}$

So $Var(X) = \frac{35}{2} - (4)^2 = \frac{35}{2} - \frac{32}{2} = \boxed{\frac{3}{2}}$

$\boxed{\text{d on next page}}$

Problem 11. (20 POINTS)
A discrete random variable X has the following probability mass function (PMF):

$$p_X(x) = \begin{cases} cx & \text{for } x = 1, 2, 3, 4. \\ 1/2 & \text{for } x = 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of the constant c.

$c = \frac{1}{20}$

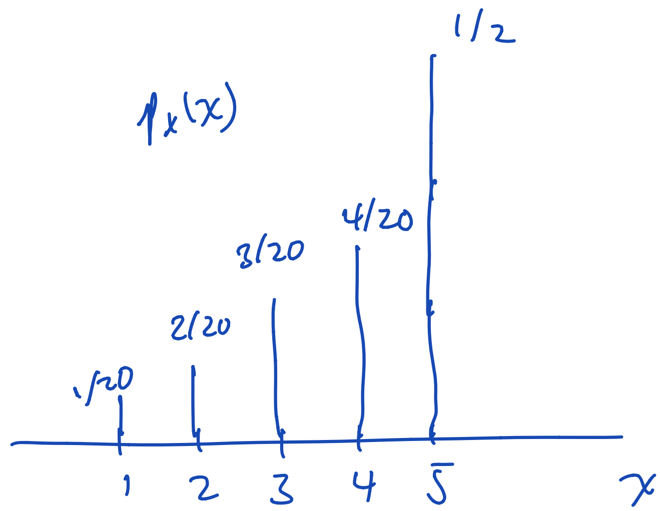
(b) Find $E(X)$.

(c) Find $Var(X)$.

(d) What is the largest value of x for which $P(X > x) > 0.75$? (Hint, a sketch of the PMF can be helpful.)

d) The problem is small enough we can make a table

x	$P(X > x)$
$x \geq 5$	0
$4 \leq x < 5$	$1/2$
$3 \leq x < 4$	$14/20 = 0.70$
$2 \leq x < 3$	$17/20 = 0.85$
$1 \leq x < 2$	$19/20$
$x < 1$	1



$\Rightarrow$ largest value x satisfying the constraint is technically $2.99\bar{9}$

but the largest value of $x \in S_X$ satisfying the constraint is $\boxed{x = 2.}$

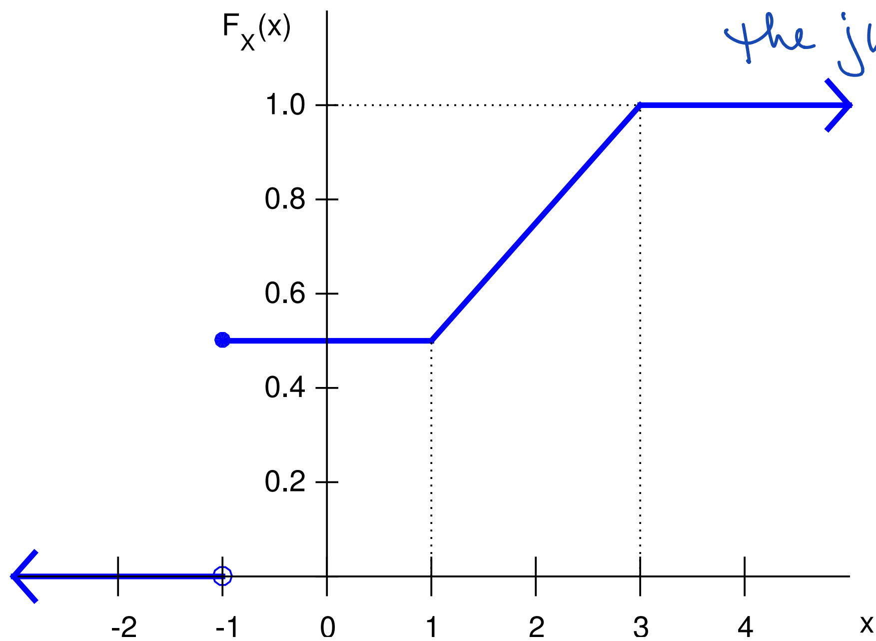
Problem 12. (15 POINTS)
The figure below shows the cumulative distribution function of a random variable X.

(a) What is $P(X < -1)$?

(b) Find and sketch the PDF of X. Label relevant values.

(c) Compute $E(X)$.

a) $P(X<-1)=0$ because this does not include the jump at $x=-1$



b) $f_X(x)$ is the derivative of $F_X(x)$ and can be constructed piece by piece from the figure.

Regions where $F_X(x)$ are constant have $f_X(x)=0$.

These regions are $x<-1$, $-1<x<1$, and $x>3$.

Exactly at $x=-1$, there's a jump of $1/2$, so this contributes $\frac{1}{2}\delta(x+1)$

When $1<x<3$, the slope of $F_X(x) = \frac{1-0.5}{3-1} = \frac{1}{4}$

$f_X(x) = \frac{1}{2}\delta(x+1) + \frac{1}{4}[u(x-1)-u(x-3)]$

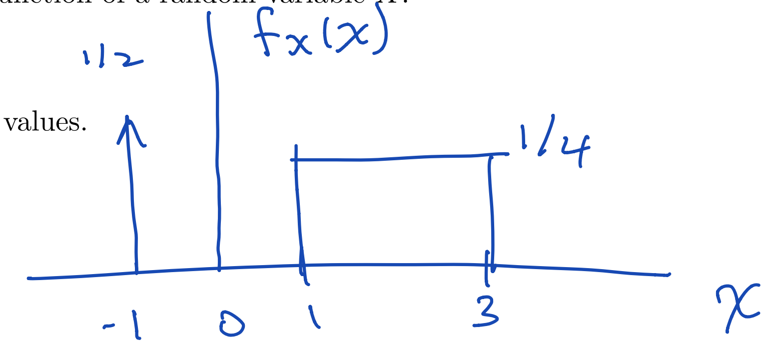
(continued on next page)

Problem 12. (15 POINTS)
The figure below shows the cumulative distribution function of a random variable X.

(a) What is $P(X < -1)$?

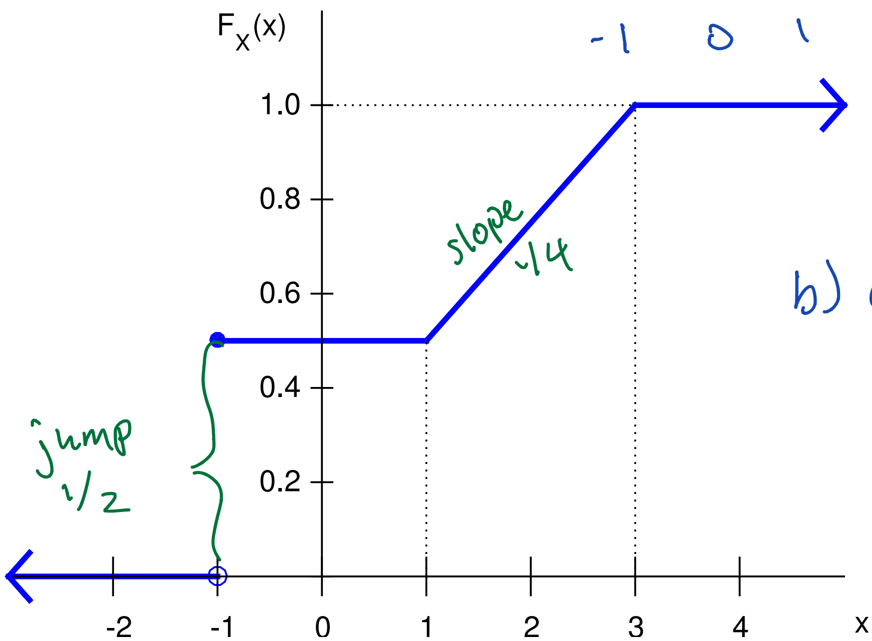
(b) Find and sketch the PDF of X. Label relevant values.

(c) Compute $E(X)$.



b) continued

sketch of $f_X(x)$



c) $$E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx = \underbrace{\int_{-\infty}^{\infty} \frac{1}{2} x\,\delta(x+1)\,dx}_{\text{apply the sifting property}} + \frac{1}{4}\int_{1}^{3} x\,dx$$

$$= \frac{1}{2}(-1) + \frac{1}{4}\,\frac{x^2}{2}\Big|_1^3$$

$$= -\frac{1}{2} + \frac{1}{8}(9-1) = \boxed{\frac{1}{2}}$$

Note that this is between min of -1 and max of 3.

If you forgot the effect of the δ function you might have gotten an answer that would not be between the min and the max

Problem 13. (10 POINTS)
The bandwidth of a video being delivered on the internet has a continuous distribution on the interval $(0, 4)$, with a PDF $f_X(x) = a(1+x)^{-2}$ on the interval $(0, 4)$.

(a) Find the constant a.

(b) What is the probability that the bandwidth of a video is less than 0.6.

a) Know $1 = \int_{-\infty}^{\infty} f_X(x)\,dx$ so

$$\int_0^4 \frac{a}{(1+x)^2}\,dx = 1 = \int_0^4 a\,u(x)^{-2}\,dx$$

$$= \frac{a u(x)^{-1}}{(-1)}\Big|_0^4 = -\frac{a}{(1+x)}\Big|_0^4 = a\left(-\frac{1}{5}+1\right) = a\frac{4}{5}$$

$$\Rightarrow \boxed{a = 5/4}$$

b) $$P(X < 0.6) = \int_0^{0.6} \frac{a}{(1+x)^2}\,dx = \frac{-a}{1+x}\Big|_0^{0.6}$$

$$= a\left(-\frac{1}{1.6}+1\right) = \frac{5}{4}\left(-\frac{10}{16}+\frac{16}{16}\right) = \frac{5}{4}\cdot\frac{6}{16}$$

$$= \boxed{\frac{15}{32}}$$

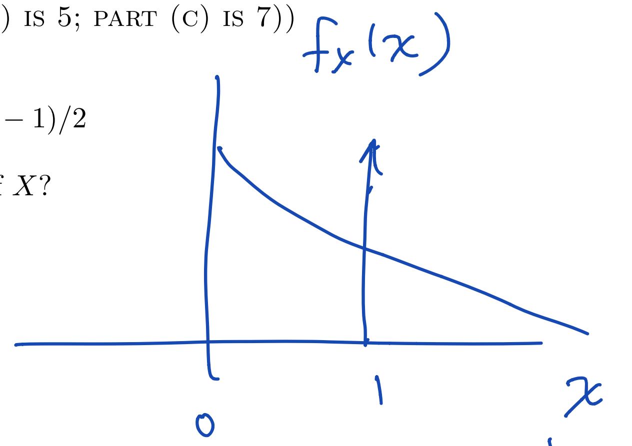
Problem 14. (20 POINTS (PART (A) IS 8 POINTS; PART (B) IS 5; PART (C) IS 7))
The probability density function of a random variable X is

$$f_X(x) = (1/4)e^{-x/2}u(x) + \delta(x-1)/2$$

(a) What is $F_X(x)$, the cumulative distribution function of X?

(b) What is $P(2 \le X \le 4)$?

(c) What is $E(X)$, the expected value of X?



(a mixed RV)

a) $F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt$

3 regions to consider:
when $x < 0$, $0 \le x < 1$, and $x \le 1$.

when $x<0$, $F_X(x) = 0$

when $0 \le x < 1$, $F_X(x) = \int_0^x \frac{1}{4} e^{-t/2}\,dt = \frac{1}{2}\left(1 - e^{-x/2}\right)$

when $1 \le x$, $F_X(x) = \frac{1}{2} + \int_0^x \frac{1}{4} e^{-t/2}\,dt$

this term is from integrating the $\frac{1}{2}\delta(x-1)$

$= \frac{1}{2} + \frac{1}{2}\left(1 - e^{-x/2}\right)$

Combining, $\boxed{F_X(x) = \frac{1}{2}\left(1 - e^{-x/2}\right)u(x) + \frac{1}{2}u(x-1)}$

see sketch on next page

b) $P(2 \le X \le 4) = F_X(4) - F_X(2)$

note: no jumps in $F_X(x)$ at $x=4$, so

$P(2 \le X \le 4) = P(2 \le X < 4)$

$= \left(1 - \frac{1}{2}e^{-4/2}\right) - \left(1 - \frac{1}{2}e^{-2/2}\right)$

$= \boxed{\frac{1}{2}\left(e^{-1} - e^{-2}\right)}$

part c on next page

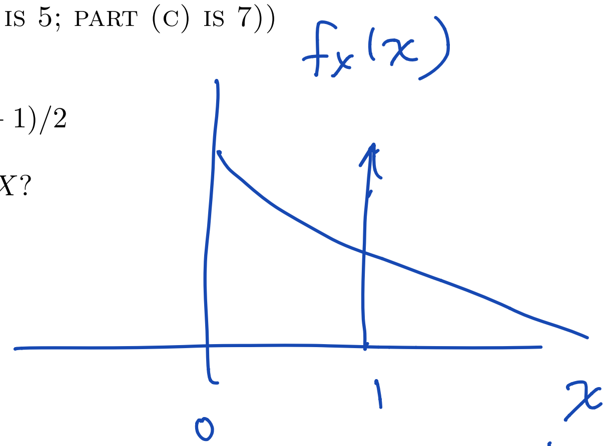
Problem 14. (20 POINTS (PART (A) IS 8 POINTS; PART (B) IS 5; PART (C) IS 7))
The probability density function of a random variable X is

$$f_X(x) = (1/4)e^{-x/2}u(x) + \delta(x-1)/2$$

(a) What is $F_X(x)$, the cumulative distribution function of X?

(b) What is $P(2 \le X \le 4)$?

(c) What is $E(X)$, the expected value of X?



(a mixed RV)

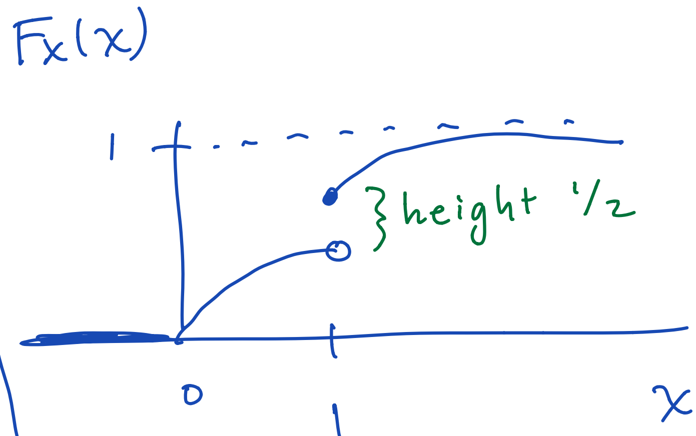
c) $E(X) = \int_0^\infty x f_X(x)\,dx$

$$= 1 \cdot \frac{1}{2} + \frac{1}{4}\int_0^\infty x e^{-x/2}\,dx$$

$$= \frac{1}{2} + 1 = \boxed{\frac{3}{2}}$$

from the δ function.

- the "1" is from the value of x at the δ function
- the "1/2" is from the multiplier in front of the δ function



Problem 15. (15 POINTS)
Let X be a continuous RV with PDF

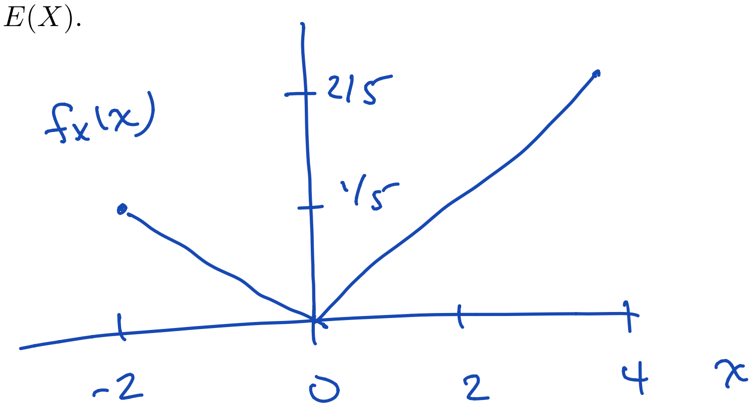
$$f_X(x) = \begin{cases} |x|/10 & \text{for } -2 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch $f_X(x)$

(b) What is $P(X < 0)$?

(c) Find $E(X)$.

a)



b) $P(X<0) = \int_{-2}^{0} f_X(x)\,dx = \int_{-2}^{0} \frac{-x}{10}\,dx$

$= \left.\frac{-x^2}{20}\right|_{-2}^{0} = 0 - -\frac{4}{20} = \boxed{\frac{1}{5}}$

c) $E(X) = \int_{-\infty}^{\infty} x\, f_X(x)\,dx = \int_{-2}^{0} x\left(-\frac{x}{10}\right)dx + \int_{0}^{4} x\left(\frac{x}{10}\right)dx$

$= \left.-\frac{x^3}{30}\right|_{-2}^{0} + \left.\frac{x^3}{30}\right|_{0}^{4}$

$= -\frac{8}{30} + \frac{64}{30} = \frac{56}{30} = \boxed{\frac{28}{15}}$

It makes sense that this is positive, and that it is greater than 1.

The next 4 problems all refer to a random variable X with the following cdf:

$$F_X(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ x^2 & \text{for } 0 < x < 1 \\ 1 & x \geq 1 \end{cases}$$

For these problems use the same set of answers, and clearly mark your answer next to the problem. The answers to each problem may or may not be different answers!

(a) 0

(b) 0.1

(c) 0.2

(d) 1/3

(e) 0.6

(f) 2/3

(g) 1

(h) None of the above

a **Problem 16.** (5 POINTS)
Find $P(X = 0.5)$.

continuous RV, so $P(X = \text{anything}) = 0$.

e **Problem 17.** (5 POINTS)
Find $P(0.2 \leq X \leq 0.8)$.

$F_X(0.8) - F_X(0.2) = \left(\frac{8}{10}\right)^2 - \left(\frac{2}{10}\right)^2 = 0.6$

c **Problem 18.** (5 POINTS)
Find $f_X(0.1)$.

$f_X(x) = \frac{d}{dx} F_X(x) = 2x$ when $0 \leq x \leq 1$

so $f_X(0.1) = 0.2$

f **Problem 19.** (5 POINTS)
Find $E(X)$

$$E(X) = \int_0^1 x\,(2x)\,dx = \frac{2x^3}{3}\Big|_0^1 = 2/3$$

Problem 20. (20 POINTS)
A random variable X has the cumulative distribution function

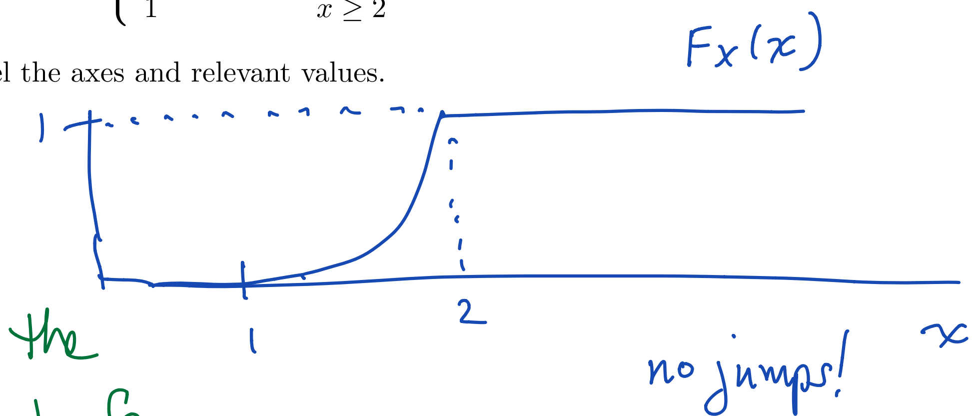
$$F_X(x) = \begin{cases} 0 & x < 1 \\ x^2 - 2x + 1 & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

(a) Sketch the CDF, and label the axes and relevant values.

(b) Find the PDF, $f_X(x)$.

(c) Find $E(X)$

(d) Find the variance of X.



a) when sketching the CDF, always look for potential jumps at the edges of the cases!

when $x=1$, $F_X(1) = 1^2 - 2\cdot 1 + 1 = 0$ (no jump)

when x just less than 2, $F_X(2^-) = 2^2 - 2(2) + 1 = 1$ (no jump)

b) $f_X(x) = \frac{d}{dx} F_X(x) = \begin{cases} 0 & \text{else} \\ 2x-2 & 1 \le x < 2 \end{cases}$

c) $E(X) = \int_1^2 x(2x-2)\,dx = 2\int_1^2 (x^2 - x)\,dx = \frac{2x^3}{3}\Big|_1^2 - \frac{2x^2}{2}\Big|_1^2$

$= \frac{16}{3} - \frac{2}{3} - (4-1) = \boxed{5/3}$

Note: this is between 1 and 2 $\Rightarrow$ makes sense

d) $E(X^2) = \int_1^2 x^2(2x-2)\,dx = 17/6$

So $Var(X) = E(X^2) - E(X)^2 = \frac{17}{6} - \left(\frac{5}{3}\right)^2$

$= \frac{51-50}{18} = \boxed{\frac{1}{18}}$

Problem 21. (10 POINTS)
Given the Cumulative Distribution Function

$$F_X(x) = \begin{cases} 0 & \text{for } x < 1 \\ (x^2 - 2x + 2)/2 & \text{for } 1 \le x \le 2 \\ 1 & \text{for } x > 2 \end{cases}$$

(a) Find and sketch the PDF $f_X(x)$.

(b) Find $E(X)$

a) when given a CDF, always look for potential jumps at the edges of the cases!

when $x=1$, $F_X(1) = \frac{1^2 - 2\cdot 1 + 2}{2} = \frac{1}{2}$ a jump!

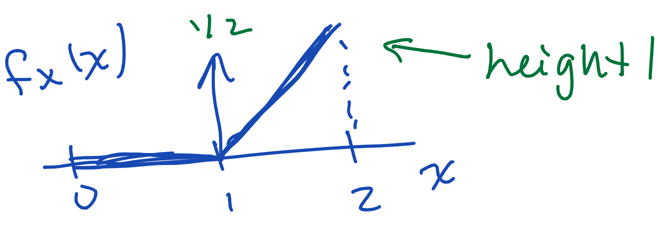
when $x=2^-$, $F_X(2^-) = \frac{2^2 - 2\cdot 2 + 2}{2} = 1$ no jump

The jump @ $x=1$ means that $f_X(x)$ has a delta function with area $1/2$ at $x=1$.

For the rest of $f_X(x)$ compute $f_X(x) = \frac{dF_X(x)}{dx}$

when $1 < x < 2$, $f_X(x) = \frac{2x-2}{2} = x-1$

so $f_X(x) = \frac{1}{2}\delta(x-1) + (x-1)[u(x-1) - u(x-2)]$



b) $E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_{-\infty}^{\infty} \frac{x\,\delta(x-1)}{2}\,dx + \int_1^2 x(x-1)\,dx$

$= \frac{1}{2}\cdot 1 + \left(\frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_1^2 = \frac{1}{2} + \left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - \frac{1}{2}\right) = \boxed{4/3}$

Note: this makes sense since it's between 1 and 2.

Problem 22. (28 POINTS (4 POINTS FOR (B), 8 POINTS EACH FOR (A,C,D))
The probability density function of a random variable X is

$$f_X(x) = \begin{cases} 0.1 + Cx & 0 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

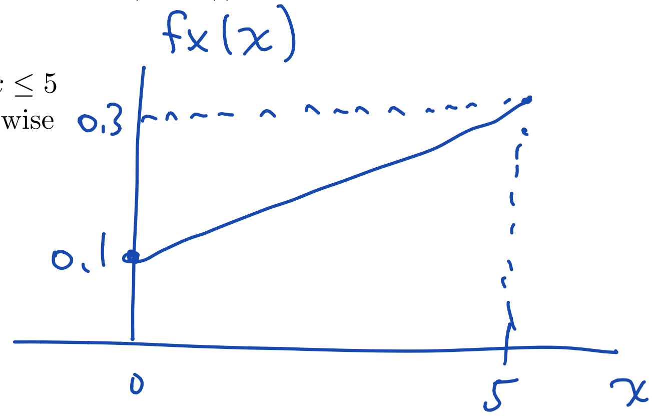
(a) Find the value of C that makes $f_X(x)$ a valid PDF.

(b) Sketch the PDF. Label axes and relevant values.

(c) What is $P(2 \le X \le 4)$?

(d) What is $E(X)$, the expected value of X?

b)



a) Know $1 = \int_{-\infty}^{\infty} f_X(x)\,dx = \int_0^5 (0.1 + Cx)\,dx$

$$= 0.1x\Big|_0^5 + C\frac{x^2}{2}\Big|_0^5 = 0.1(5) - 0 + C\frac{25}{2} - 0 = 1$$

$$\Rightarrow \frac{25C}{2} = \frac{1}{2} \Rightarrow C = \boxed{1/25}$$

c) $P(2 \le X \le 4) = \int_2^4 f_X(x)\,dx = \left(0.1x + \frac{Cx^2}{2}\right|_2^4$

$$= 0.2 + 6C = \boxed{\frac{11}{25}}$$

d) $E(X) = \int_0^5 x f_X(x)\,dx = \int_0^5 (0.1x + Cx^2)\,dx$

$$= \left(\frac{0.1x^2}{2} + \frac{Cx^3}{3}\right|_0^5 = \frac{2.5}{2} + \frac{C5^3}{3}$$

$$= \frac{5}{4} + \frac{5}{3} = \boxed{\frac{35}{12}}$$

Problem 23. (MULTIPLE CHOICE: 5 POINTS)

Which expression is correct for computing the expected value of a continuous random variable X?

(a)
$$E(X) = \int_{-\infty}^{x} t f_X(t) dt$$

(b)
$$E(X) = \int_{-\infty}^{\infty} t f_X(t) dt$$

(c)
$$E(X) = \int_{-\infty}^{\infty} f_X(t) dt$$

(d)
$$E(X) = \int_{-\infty}^{\infty} F_X(t) dt$$

(e) None of the above.

Problem 24. (MULTIPLE CHOICE: 5 POINTS)
The variance of a random variable X with mean μ can be computed using

(a) $E((X-\mu)^2)$

(b) $E(X)^2 - E(X^2)$

(c) $E(X^2) + E(X)^2$

(d) $E(X^2) - E(X)^2$

(e) both (a) and (c)

(f) both (a) and (d)

(g) None of the above

Problem 25. (10 POINTS)
Let X be a random variable with mean μ and variance σ^2. Define two new random variables, $Y = 2 - X$ and $Z = 3X + 1$. Express $Var(X + Y + Z)$ in terms of μ and σ^2.

$Y = 2 - X$

$Z = 3X + 1$

$X + Y + Z = X + 2 - X + 3X + 1$
$= 3X + 3$

$Var(3X+3)$
$= 9\,Var(X)$
$= \boxed{9\sigma^2}$

Note! in general, $Var(X+Y) \neq Var(X) + Var(Y)$

Problem 26. (10 POINTS)
Let X be a random variable with mean μ and variance σ^2, and let $Y = 2X + 3$ and $Z = -4X + 5$. Express $Var(Y + Z)$ in terms of μ and σ.

$Var(Y+Z) = Var(2X + 3 - 4X + 5)$
$= Var(-2X + 8) = 4\,Var(X)$
$= \boxed{4\sigma^2}$

Problem 27. (10 POINTS)
Let X be a random variable with mean μ and variance σ^2, and let $Y = 3X + 4X^2$. Express $E(Y)$ in terms of μ and σ.

$E(Y) = E(3X + 4X^2) = 3E(X) + 4E(X^2)$

we know $\sigma^2 = E(X^2) - E(X)^2 = E(X^2) - \mu^2$

so $E(X^2) = \sigma^2 + \mu^2$

Then $E(Y) = 3\mu + 4(\sigma^2 + \mu^2)$

Problem 48. (True/False: 5 points each, total 20 points)
Label each statement T or F to the left of the problem number.

F (a) $E(g(X)) = g(E(X))$ Not always. Example: $g(x) = X^2$

F (b) Let X be a random variable and let a be a constant. Then $P(X \geq a) = 1 - F_X(a)$.

T (c) A deck of 52 cards is fairly dealt to 2 hands, each with 26 cards. The probability that both hands get 2 aces is $\binom{4}{2}\binom{48}{24}/\binom{52}{26}$

T (d) Let X be a random variable and let $Y = aX + b$, where a, b are constants.
Then VAR(Y)=a^2VAR(X).

b) $P(X > a) = 1 - F_X(a)$ There could be a jump at $x = a$

Discrete Random Variables

- Bernoulli Random Variable, parameter p
  $S = \{0, 1\}$
  $p_0 = 1 - p, p_1 = p; 0 \leq p \leq 1$
  $E(X) = p; \mathrm{VAR}(X) = p(1-p)$

- Binomial Random Variable, parameters (n, p)
  $S = \{0, 1, \ldots, n\}$
  $p_k = \binom{n}{k} p^k (1-p)^{n-k}; k = 0, 1, \ldots, n; 0 \leq p \leq 1$
  $E(X) = np; \mathrm{VAR}(X) = np(1-p)$

- Geometric Random Variable, parameter p
  $S = \{0, 1, \ldots\}$
  $p_k = p(1-p)^k; k = 0, 1, \ldots, ; 0 \leq p \leq 1$
  $E(X) = (1-p)/p; \mathrm{VAR}(X) = (1-p)/p^2$

- Poisson Random Variable, parameter α
  $S = \{0, 1, \ldots\}$
  $p_k = \alpha^k e^{-\alpha}/k! \quad k = 0, 1, \ldots,$
  $E(X) = \alpha; \mathrm{VAR}(X) = \alpha$

- Uniform Random Variable
  $S = \{0, 1, \ldots, L\}$
  $p_k = 1/L \quad k = 0, 1, \ldots, L$
  $E(X) = (L+1)/2; \mathrm{VAR}(X) = (L^2 - 1)/12$

Continuous Random Variables

- Uniform Random Variable
  Equally likely outcomes
  $S = [a, b]$
  $f_X(x) = 1/(b-a), \quad a \leq x \leq b$
  $E(X) = (a+b)/2; \quad \mathrm{VAR}(X) = (b-a)^2/12$

- Exponential Random Variable, parameter λ
  $S = [0, \infty)$
  $f_X(x) = \lambda \exp(-\lambda x), \quad x \geq 0, \ \lambda > 0$
  $E(X) = 1/\lambda; \quad \mathrm{VAR}(X) = 1/\lambda^2$

- One Gaussian Random Variable, parameters μ, σ^2
  $S = (-\infty, \infty)$
  $f_X(x) = \exp(-(x-\mu)^2/(2\sigma^2))/\sqrt{2\pi\sigma^2}$
  $E(X) = \mu; \quad \mathrm{VAR}(X) = \sigma^2$

Other useful formulas

$$\sum_{k=0}^{n} r^k = \frac{1-r^{n+1}}{1-r}$$

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r} \qquad \text{if } |r| < 1$$

$$\sum_{k=1}^{\infty} k r^{k-1} = \frac{1}{(1-r)^2} \qquad \text{if } |r| < 1$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$$

$$\sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} = (a+b)^n$$

$$\int x e^{ax}\, dx = \left(\frac{x}{a} - \frac{1}{a^2}\right) e^{ax}$$

$$\int x^2 e^{ax}\, dx = e^{ax}\left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)$$