**Introduction:**
**Sizes of Structural Lumber and Use**
**Text Chapter 4**

**Design Approach:**

The design of structural wood is carried on the basis of allowable stresses at service load levels. Structural calculations are based on the <u>standard net size</u> of a piece of lumber. Most structural lumber is dressed lumber.

1) <u>Dressed Lumber:</u>  Lumber that has been surfaced to the <u>standard net size</u>, which is less than the nominal size (stated)(Textbook Section 4.11 and NDS 01 Supplement).

   i.e. 8 x12 member (nominal size = 8 x 12 in.) actually is 7 ½ x 11 ½ in.  (Standard net size) NDS Table 1A Sec. 3 Supplement 2001.  Lumber is dressed on a planning machine for the purpose of obtaining smooth surfaces and uniform sizes.  Typically lumber will be S4S (surfaced on 4 sides).

**Table 1A   Nominal and Minimum Dressed Sizes of Sawn Lumber**

| Item | Thickness (inches) | | | Face Widths (inches) | | |
| | Nominal | Minimum dressed | | Nominal | Minimum dressed | |
| | | Dry | Green | | Dry | Green |
|---|---|---|---|---|---|---|
| Boards | 3/4 | 5/8 | 11/16 | 2 | 1-1/2 | 1-9/16 |
| | 1 | 3/4 | 25/32 | 3 | 2-1/2 | 2-9/16 |
| | 1-1/4 | 1 | 1-1/32 | 4 | 3-1/2 | 3-9/16 |
| | 1-1/2 | 1-1/4 | 1-9/32 | 5 | 4-1/2 | 4-5/8 |
| | | | | 6 | 5-1/2 | 5-5/8 |
| | | | | 7 | 6-1/2 | 6-5/8 |
| | | | | 8 | 7-1/4 | 7-1/2 |
| | | | | 9 | 8-1/4 | 8-1/2 |
| | | | | 10 | 9-1/4 | 9-1/2 |
| | | | | 11 | 10-1/4 | 10-1/2 |
| | | | | 12 | 11-1/4 | 11-1/2 |
| | | | | 14 | 13-1/4 | 13-1/2 |
| | | | | 16 | 15-1/4 | 15-1/2 |
| Dimension Lumber | 2 | 1-1/2 | 1-9/16 | 2 | 1-1/2 | 1-9/16 |
| | 2-1/2 | 2 | 2-1/16 | 3 | 2-1/2 | 2-9/16 |
| | 3 | 2-1/2 | 2-9/16 | 4 | 3-1/2 | 3-9/16 |
| | 3-1/2 | 3 | 3-1/16 | 5 | 4-1/2 | 4-5/8 |
| | 4 | 3-1/2 | 3-9/16 | 6 | 5-1/2 | 5-5/8 |
| | 4-1/2 | 4 | 4-1/16 | 8 | 7-1/4 | 7-1/2 |
| | | | | 10 | 9-1/4 | 9-1/2 |
| | | | | 12 | 11-1/4 | 11-1/2 |
| | | | | 14 | 13-1/4 | 13-1/2 |
| | | | | 16 | 15-1/4 | 15-1/2 |

**3**

**SECTION PROPERTIES**

2) <u>Rough Sawn:</u>  Large timbers are usually rough sawn to dimensions that are close to standard net sizes, roughly 1/8" larger than the standard dressed size.  Rough surface is usually ordered specially for architectural purposes in smaller sizes.

3) <u>Full Sawn:</u>  In this case a rough surface is obtained with actual size equal to the nominal size.

## Wood Rating

The majority of sawn lumber is graded by visual inspection, and material graded in this way (visually) is known as visually graded structural lumber.  As the lumber comes out of the mill, a person familiar with lumber grading rules examines each piece and assigns and stamps a grade. There are two broad size classifications of sawn lumber:

- Dimension Lumber: smaller (thinner) sizes of structural lumber. Dimension lumber usually ranges in the size from 2x2 through 4x16. In other words, dimension lumber is any material with a thickness (smaller dimension of a piece of wood, and width is the larger dimension) of 2 to 4 inches.
- Timbers: are the larger pieces and have a minimum nominal dimension of 5 inches. Thus, the smallest practical size timber is a 6x6 inch.

The design properties given in the NDS supplement are based on two different sets of ASTM Standards (Textbook Sections 4.3 and 4.4):

- In-grade procedures applied to Dimension lumber
- Clear wood procedures applied to timbers

The lumber grading rules which establish the allowable stresses for use in structural design have been developed over the years. The relative size of the wood was used as a guide in anticipating its use. Although most lumber is visually graded, a small % of lumber is MACHINE STRESS' RATED by subjecting each piece of wood to a non-destructive test.  This process is highly automated.  As lumber comes out of the mill, it passes through a series of rollers.  In this process, a bending load is applied about the minor axis of the cross section, and the modulus of elasticity of each piece measured.  In addition the piece is visually inspected.  The material graded using MSR is limited to a thickness of 2" or less.  MSR has less variability in mechanical properties than visually graded lumber.   Consequently, is often used to fabricate engineered wood products.

- Glulam beams
- Wood I joists and light frame

However, stress rated boards are not commonly used for structural framing because they are very thin. So we will focus on dimension lumber. It must be remarked that the allowable stress depends on the species and on the **size of the member.**

## Species (Sec. 4.5 Textbook):

A large number of species can be used to produce structural lumber. The 2001 NDS supplement (Sec. 4, Page 29) contains allowable stresses for a large number of species. The choice of species for use in design is a matter of economics typically. For a given

location only a few species groups may be available and it is prudent to check with local distributors as well as a wood products agency. The species of trees used for structural lumber are classified as hardwoods and softwoods owing not necessarily to a description of the wood properties. For example evergreens aka conifers are a large majority of the structural lumber. This will be either Douglas-Fir or Southern Pine.

## Table 4A Base Design Values for Visually Graded Dimension Lumber (2"-4" thick)[1,2]

**(All species except Southern Pine — see Table 4B)** (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

USE WITH TABLE 4A ADJUSTMENT FACTORS

| Species and commercial grade | Size classification | Bending $F_b$ | Tension parallel to grain $F_t$ | Shear parallel to grain $F_v$ | Compression perpendicular to grain $F_{c\perp}$ | Compression parallel to grain $F_c$ | Modulus of Elasticity E | Grading Rules Agency |
|---|---|---|---|---|---|---|---|---|
| **ASPEN** | | | | | | | | |
| Select Structural | | 875 | 500 | 120 | 265 | 725 | 1,100,000 | |
| No.1 | | 625 | 375 | 120 | 265 | 600 | 1,100,000 | |
| No.2 | 2" & wider | 600 | 350 | 120 | 265 | 450 | 1,000,000 | NELMA |
| No.3 | | 350 | 200 | 120 | 265 | 275 | 900,000 | NSLB |
| Stud | 2" & wider | 475 | 275 | 120 | 265 | 300 | 900,000 | WWPA |
| Construction | | 700 | 400 | 120 | 265 | 625 | 900,000 | |
| Standard | 2"-4" wide | 375 | 225 | 120 | 265 | 475 | 900,000 | |
| Utility | | 175 | 100 | 120 | 265 | 300 | 800,000 | |
| **BEECH-BIRCH-HICKORY** | | | | | | | | |
| Select Structural | | 1450 | 850 | 195 | 715 | 1200 | 1,700,000 | |
| No.1 | | 1050 | 600 | 195 | 715 | 950 | 1,600,000 | |
| No.2 | 2" & wider | 1000 | 600 | 195 | 715 | 750 | 1,500,000 | |
| No.3 | | 575 | 350 | 195 | 715 | 425 | 1,300,000 | NELMA |
| Stud | 2" & wider | 775 | 450 | 195 | 715 | 475 | 1,300,000 | |
| Construction | | 1150 | 675 | 195 | 715 | 1000 | 1,400,000 | |
| Standard | 2"-4" wide | 650 | 375 | 195 | 715 | 775 | 1,300,000 | |
| Utility | | 300 | 175 | 195 | 715 | 500 | 1,200,000 | |

**4 DESIGN VALUES**

**Allowable Stresses/Design Values (NDS Tabulated values in the NDS Supplement 01)**: Are determined by multiplying the tabulated (stresses) by the appropriate adjustment factors (Textbook, Sections 4.13-4.22, and design example in Section 4.23). Thus becoming allowable design value ($F'$). For example for tension parallel to the grain:
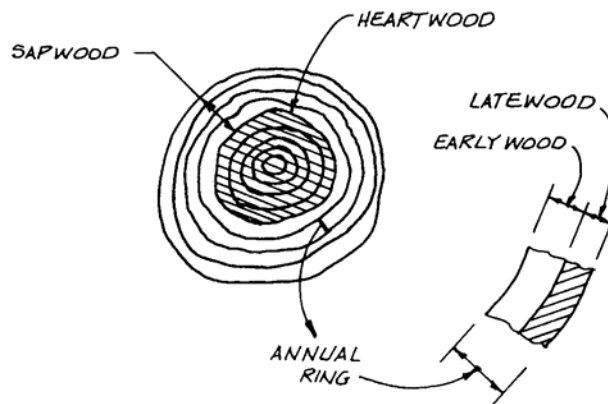


**Figure 4.4** Cross section of a log.

$$F_t' = F_t \times (\text{adjustment factors}) = \textit{Design value}$$

For an acceptable design, the axial tensile stress due to loads, $f_t$, should not exceed the allowable (adjusted) stress:

$$f_t \leq F_t^{'}$$

| Design Value (stress) | Tabulated Stress | Allowable (adjusted) Stress |
|---|---|---|
| Bending | $F_b$ | $F_b^{'}$ |
| Tension parallel to grain | $F_t$ | $F_t^{'}$ |
| Shear parallel to grain | $F_v$ | $F_v^{'}$ |
| Compression perpendicular to grain | $F_{c1}$ | $F_{cI}^{'}$ |
| Compression parallel to grain | $F_c$ | $F_c^{'}$ |
| Modulus of elasticity | $E$ | $E^{'}$ |

**Adjustment Factors**:  Some decrease other increase tabulated value (Textbook, Sections 4.13-4.22, and design example in Section 4.23, and NDS 2001 Sections 2 and 4)

Examples:

$C_D$ = load duration factor

$C_M$ = web service factor (moisture content)

$C_F$ = size factor

$C_{fu}$ = flat use factor

$C_f$ = form factor

**Stresses and adjustment factors:**

Stresses due to known loads :(NDS 2001, Section 3))

$$f_t = \frac{P}{A}, \ f_b = \frac{M}{S}, \ etc.$$

**Tabulated Values (Stresses):**  Tabulated design values listed in the NDS Supplement 2001 ED.  These values include reduction for safety (F) and are for normal load duration under the specified moisture service condition.  Modulus of elasticity (E) does not include reduction for safety and represent average values.

Dimension Lumber Page 29 NDS Supp. 2001

> Table 4A, page 30, 31:  Base design value for visually graded dimension lumber (except southern pine)
> Table 4B, page 36,37:  Base design value for visually graded southern pine
> Table 4C, page 39-42:  Design values for mechanically graded dimension lumber (MSR)

Timbers (5x5 and larger)

> Table 4D, page 43-49:  Design values for visually graded timbers (all species)

**Adjustment Factors (Sec 4.3 NDS 01 and Supplement to NDS Tables):**

A.  Wet Serviced Factor:  $C_M$

EMC = Equilibrium moisture content = the average moisture content that lumber assumes in service.

Moisture designation in grade stamp
S-Grn (surface green) MC = 19% (in service)

S-Dry (surfaced dry) MC = 15% (in service)

These values can vary depending on environmental conditions (in most buildings ranges from 7-14% EMC).  Special conditions must be analyzed individually.

Tabulated values in NDS supplement apply to members with EMC of 19% or less (regardless of S-GRN or S-Dry).  If EMC exceeds 19%for an extended period of time, table values should be multiplied by $C_M$ (see Page 30 and others for values in Table 4 NDS-Supp 01)

**Wet Service Factor, $C_M$**

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

**Wet Service Factors, $C_M$**

| $F_b$ | $F_t$ | $F_v$ | $F_{c\perp}$ | $F_c$ | E |
|-------|-------|-------|-------|-------|---|
| 0.85* | 1.0 | 0.97 | 0.67 | 0.8** | 0.9 |

\* when $(F_b)(C_F) \leq 1150$ psi, $C_M = 1.0$
\** when $(F_c)(C_F) \leq 750$ psi, $C_M = 1.0$

B.  <u>Load Duration Factor:</u>  $C_D$

Wood can handle higher stresses if loads are applied for a short period of time.  All tabulated values apply to normal duration loading (10 years) The term "duration of load" refers to the total accumulated length of time that a load is applied during the life of a structure.

Table 2.3.2 in NDS 01 provides $C_D$ to be used in the one associated with the shortest-duration of time.  Whichever combination of loads, together with the appropriate load duration factor produces the largest member size is the one that must be used in design.

**Table 2.3.2 Frequently Used Load Duration Factors, $C_D$[1]**

| Load Duration | $C_D$ | Typical Design Loads |
|---------------|-------|----------------------|
| Permanent | 0.9 | Dead Load |
| Ten years | 1.0 | Occupancy Live Load |
| Two months | 1.15 | Snow Load |
| Seven days | 1.25 | Construction Load |
| Ten minutes | 1.6 | Wind/Earthquake Load |
| Impact[2] | 2.0 | Impact Load |

1. Load duration factors shall not apply to modulus of elasticity, E, nor to compression perpendicular to grain design values, $F_{c\perp}$, based on a deformation limit.
2. Load duration factors greater than 1.6 shall not apply to structural members pressure-treated with water-borne preservatives (see Reference 30), or fire retardant chemicals.  The impact load duration factor shall not apply to connections.

## C. Size Factor: $C_F$

The size of the member has an effect on its unit stress.

- See Supplement 01 Tables: 4A, 4B, 4C, 4D and 4E

**Size Factor, $C_F$**

Tabulated bending, tension and compression parallel to grain design values for dimension lumber 2" to 4" thick shall be multiplied by the following size factors:

**Size Factors, $C_F$**

| Grades | Width (depth) | $F_b$ Thickness (breadth) 2" & 3" | $F_b$ Thickness (breadth) 4" | $F_t$ | $F_c$ |
|---|---|---|---|---|---|
| Select Structural, No.1 & Btr., No.1, No.2, No.3 | 2", 3" & 4" | 1.5 | 1.5 | 1.5 | 1.15 |
| | 5" | 1.4 | 1.4 | 1.4 | 1.1 |
| | 6" | 1.3 | 1.3 | 1.3 | 1.1 |
| | 8" | 1.2 | 1.3 | 1.2 | 1.05 |
| | 10" | 1.1 | 1.2 | 1.1 | 1.0 |
| | 12" | 1.0 | 1.1 | 1.0 | 1.0 |
| | 14" & wider | 0.9 | 1.0 | 0.9 | 0.9 |
| Stud | 2", 3" & 4" | 1.1 | 1.1 | 1.1 | 1.05 |
| | 5" & 6" | 1.0 | 1.0 | 1.0 | 1.0 |
| | 8" & wider | Use No.3 Grade tabulated design values and size factors | | | |
| Construction, Standard | 2", 3" & 4" | 1.0 | 1.0 | 1.0 | 1.0 |
| Utility | 4" | 1.0 | 1.0 | 1.0 | 1.0 |
| | 2" & 3" | 0.4 | — | 0.4 | 0.6 |

## D. Repetitive Member Factor: $C_r$ only "$F_b$"!!

The system performance of a series of small closely spaced wood members, where failure of one member is not fatal (see Supplement 01)

# Table 4A  Adjustment Factors

**Repetitive Member Factor, $C_r$**

Bending design values, $F_b$, for dimension lumber 2" to 4" thick shall be multiplied by the repetitive member factor, $C_r$ = 1.15, when such members are used as joists, truss chords, rafters, studs, planks, decking or similar members which are in contact or spaced not more than 24" on centers, are not less than 3 in number and are joined by floor, roof or other load distributing elements adequate to support the design load.

E. Flat use Factor: $C_{fu}$

Except for decking, tabulated stress for dimension lumber apply to wood members that are stressed in flexure about the strong axis – "edgewise or load applied to narrow face". If however load is a applied to the wide face – the stresses may be increased by $C_{fu}$.

**Flat Use Factor, $C_{fu}$**

Bending design values adjusted by size factors are based on edgewise use (load applied to narrow face). When dimension lumber is used flatwise (load applied to wide face), the bending design value, $F_b$, shall also be multiplied by the following flat use factors:

**Flat Use Factors, $C_{fu}$**

| Width | Thickness (breadth) | |
|---|---|---|
| (depth) | 2" & 3" | 4" |
| 2" & 3" | 1.0 | — |
| 4" | 1.1 | 1.0 |
| 5" | 1.1 | 1.05 |
| 6" | 1.15 | 1.05 |
| 8" | 1.15 | 1.05 |
| 10" & wider | 1.2 | 1.1 |

Tabulated bending stresses also for timber Beams & stringers apply for bending about x-axis. NDS does not provide $C_{fu}$ for these cases.

## 3.1.3 Definitions

NEUTRAL AXIS, in the cross section of a beam, is the line on which there is neither tension nor compression stress.

**Figure 1A Dimensions for Rectangular Cross Section**



Glued Laminated Timber          Sawn Lumber
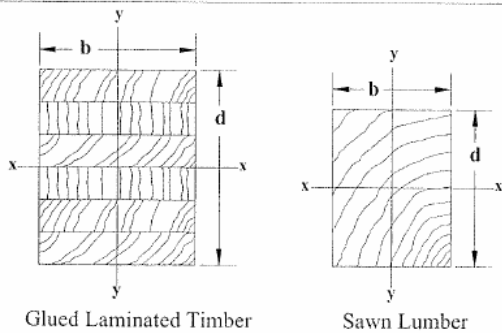
F.  Temperature Factor:  $C_t$

The strength of the wood in service is increased as the temperature cools below the normal temp in most buildings.  On the other hand, the strength decreases as temperatures are increased.  The factor $C_t$ is the multiplier that is used to reduce tabulated stresses if higher than normal temperatures are encountered in a design situation. Values of $C_t$ are given in NDS Sec. 2.3.4 for T > 100°F.  Important to note that strength will be regained when temperature returns to normal values!  Thus this factor applies for sustained conditions.

## Table 2.3.3 Temperature Factor, $C_t$

| Design Values | In Service Moisture Conditions[1] | $C_t$ | | |
| --- | --- | --- | --- | --- |
| | | T≤100°F | 100°F<T≤125°F | 125°F<T≤150°F |
| $F_t$, E | Wet or Dry | 1.0 | 0.9 | 0.9 |
| $F_b$, $F_v$, $F_c$, and $F_{c\perp}$ | Dry | 1.0 | 0.8 | 0.7 |
| | Wet | 1.0 | 0.7 | 0.5 |

1. Wet and dry service conditions for sawn lumber, glued laminated timber, prefabricated wood I-joists, structural composite lumber, and wood structural panels are specified in 4.1.4, 5.1.5, 7.1.4, 8.1.4, and 9.3.3, respectively.

G.  Form Factor:  $C_f$

The purpose of this factor is to adjust tabulated bending stress $F_b$ for non-rectangular sections (see Section 3.3.4 in NDS 01).

## 3.3.4  Form Factor, $C_f$

Tabulated bending design values, $F_b$, for bending members with either a circular cross section or a square cross section loaded in the plane of the diagonal (diamond section) shall be multiplied by the following form factors, $C_f$:

### Table 3.3.4  Form Factors, $C_f$

| | $C_f$ |
| --- | --- |
| Round Section | 1.18 |
| Diamond Section | 1.414 |

These form factors insure that a circular or diamond shaped bending member has the same moment capacity as a square bending member having the same cross-sectional area.  If a circular member is tapered, it shall be considered a beam of variable cross section.

## Example:

Determine the tabulated and allowable design values for the following member and loading condition.

- No. 2 Hem-Fir (bending about strong axis)
- Floor beams 4x6 in @ 4' on centers.  Loads are (D+L).  High-humidity conditions exist, and moisture content may exceed 19%.

Stresses

Bending  (NDS Supp 01)

### Table 4A (Cont.) Base Design Values for Visually Graded Dimension Lumber (2"–4" thick)[1,2]

(All species except Southern Pine — see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

**USE WITH TABLE 4A ADJUSTMENT FACTORS**

| Species and commercial grade | Size classification | Design values in pounds per square inch (psi) | | | | | | Grading Rules Agency |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | | Bending $F_b$ | Tension parallel to grain $F_t$ | Shear parallel to grain $F_v$ | Compression perpendicular to grain $F_{c\perp}$ | Compression parallel to grain $F_c$ | Modulus of Elasticity E | |
| **HEM-FIR** | | | | | | | | |
| Select Structural | | 1400 | 925 | 150 | 405 | 1500 | 1,600,000 | |
| No.1 & Btr | | 1100 | 725 | 150 | 405 | 1350 | 1,500,000 | |
| No.1 | | 975 | 625 | 150 | 405 | 1350 | 1,500,000 | |
| No.2 | 2" & wider | 850 | 525 | 150 | 405 | 1300 | 1,300,000 | WCLIB |
| No.3 | | 500 | 300 | 150 | 405 | 725 | 1,200,000 | WWPA |
| Stud | 2" & wider | 675 | 400 | 150 | 405 | 800 | 1,200,000 | |
| Construction | | 975 | 600 | 150 | 405 | 1550 | 1,300,000 | |
| Standard | 2"-4" wide | 550 | 325 | 150 | 405 | 1300 | 1,200,000 | |
| Utility | | 250 | 150 | 150 | 405 | 850 | 1,100,000 | |

Tabulated value, $F_b$ = 850 psi (Tab. 4A Supp. NDS 01)

Factors (NDS 01 Sec. 4.3)

**(Table 4.3.1, NDS 01 Page 27)**

**Table 4.3.1    Applicability of Adjustment Factors for Sawn Lumber**

| | Load Duration Factor | Wet Service Factor | Temperature Factor | Beam Stability Factor | Size Factor | Flat Use Factor | Incising Factor | Repetitive Member Factor | Form Factor | Column Stability Factor | Buckling Stiffness Factor | Bearing Area Factor |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $F_b' = F_b$ × | $C_D$ | $C_M$ | $C_t$ | $C_L$ | $C_F$ | $C_{fu}$ | $C_i$ | $C_r$ | $C_f$ | - | - | - |
| $F_t' = F_t$ × | $C_D$ | $C_M$ | $C_t$ | - | $C_F$ | - | $C_i$ | - | - | - | - | - |
| $F_v' = F_v$ × | $C_D$ | $C_M$ | $C_t$ | - | - | - | $C_i$ | - | - | - | - | - |
| $F_{c\perp}' = F_{c\perp}$ × | - | $C_M$ | $C_t$ | - | - | - | $C_i$ | - | - | - | - | $C_b$ |
| $F_c' = F_c$ × | $C_D$ | $C_M$ | $C_t$ | - | $C_F$ | - | $C_i$ | - | - | $C_P$ | - | - |
| $E' = E$ × | - | $C_M$ | $C_t$ | - | - | - | $C_i$ | - | - | - | $C_T$ | - |

In many practical situations, a number of adjustment factors may have a value of 1.0.  A comprehensive summary of the modification factors for wood members is given in NDS Table 4.3.1

$C_D$ = load duration factors (Sec. 2.3.2 NDS)

$C_D$ = 1.0 (Table 2.3.2 controlled by live load)

$C_M$ = Wet service (Sec. 4.3.3 NDS 01, Supp Ch 4, Table 4A)

$C_M$ = 0.85 since $M_C > 19\%$ or 1.0 if $F_b$ $C_F \le 1150$ psi

$C_F$ = size factor (Sec. 4.3, NDS 01 and Table 4A)

$C_F = 1.3$

since $F_b x C_F = 850 \, x \, 1.3 = 1105$ psi $< 1150$ psi

$C_M = 1.0$

$C_t$ = Temperature factor (Sec. 4.3.4 NDS) $T \le 100^o F$

$C_t = 1.0$

$C_L$ = *Beam Stability Factor (Sec. 4.3.5 and 3.3.3 NSD 01)*

*(Sec. 4.4.1.2)* $d / b = \dfrac{6}{4} = 1.5 < 2.0$; *no lateral support is required*

*(Sec. 3.3.3.2)* $C_L$ = *1.0 (Also 3.3.3.3 could be invoked if needed)*

$C_{fu}$ = *1.0 (Element is not loaded on its flat side)*

$C_i$ = *Incising Factor (Sec. 4.3.8) done to increase treatment penetrations*

$C_i$ = *1.0*

$C_r$ = *Repetitive Member Factor (Sec. 4.3.9 NDS)*

$C_r$ = *1.0*

$C_f$ = *Form factor (Sec. 4.3.10 &3.3.4)*

$\therefore C_f$ = *1.0*

Finally calculate allowable stress for bending

$$F_b' = 850 \ x \ 1 \ x \ ... \ x \ 1.3 \ x \ ... = 1105 \ psi$$

Tension II to Grain

$F_T$ = Tension parallel to grain (Table 4A Supp)
$F_T = 525 \ psi$

Factors (NDS 01, Sec. 4.3 & Table 4.3.1)

$C_D = 1.0$
$C_M = 1.0$ (Table 4A, Supp Adj. Factors)
$C_t = 1.0$
$C_F = 1.3$
$C_i = 1.0$
$F_T' = 525 \ x \ 1.3 = 683 \ psi$

Shear II to Grain, $F_V$

$F_V = 150 \ psi$

Factors (NDS 4.3)

$C_D = 1.0$

$C_M = 0.97$ Table 4A Supp Adj. Factor

$C_t = 1.0$

$C_i = 1.0$

$F_V^{'} = 150\ psi\ x\ 0.97 = 146\ psi$

Compression $\perp$ to grain

$F_{C\perp} = 405\ psi$

Factors (NDS 4.3)  Table 4.3.1

(Sec. 4.3.3) and Table 4A

$C_M = 0.67$
$C_t = 1.0$
$C_i = 1.0$
$C_b$ = Bearing area factor (Sec. 4.3.13)
Assume $\ell_b \geq 6"$
$C_b = 1.0$
$F_{C_\perp} = 405\ x\ 0.67 = 271\ psi$

Compression II to grain

$F_c = 1300\ psi$
Factors  (NDS 4.3 @ Table 4.3.1)

$C_D = 1.0$  (Table 4A, Supp)
$C_M = 0.8$ or 1.0 when $(F_c)(C_F) \leq 750\ psi$
$C_F = 1.1$  (Table 4A, Supp)
$\therefore C_M = 0.8$  since $1300\ x\ 1.1 > 750$
$C_t = 1.0$
$C_i = 1.0$
$C_p = 1.0$  (This is a beam!)
$F_c^{'} = 1300\ x\ 0.8\ x\ 1.1\ x\ \ldots 1.0 = 1144\ psi$

<u>Modulus of Elasticity, MOE</u>

From Table 4A

$E = 1,300,000\ psi$

$C_M = 0.9\ (Factors\ in\ Table\ 4A,\ Supp)$

$C_t = 1.0$

$C_i = 1.0$

$C_T = Buckling\ Stiffness\ factor\ for\ wood\ tresses\ (4.4.2)$

$N.A.$

$\therefore$

$E' = 1,300,000\ x\ 0.9 = 1,170,000\ psi$

$C_t = temperature\ factor$

$C_r = repetitive\ factor$

**Design Summary – Beams (Chapter 6 Text)**

1. <u>Determine trial beam size based on bending stress considerations</u> (long. Bending stress, II to grain – see Fig. 6.1a). For sawn lumber loaded-edgewise only are given tabulated values.

$$(S)_{reqd} = \frac{M}{F_b'}$$

Select trial member with (use Table for dressed S4S)

$$(S)_{prov} \geq (S)_{reqd}$$

recheck for appropriate size factor, $C_F$, since initially is unknown (beam size) so that

$$f_b = \frac{M}{(S_{act})} \leq F_b' \text{ (with actual } C_F)$$

2. <u>Check shear</u> (Sec. 3.4 NDS)

$$f_v = 1.5 \frac{V}{A} \leq F_v' \text{ supp. with app. factors}$$

$$f_v = \frac{V}{t_d w}$$

In this calculation a reduced shear (d- away from support face, d = overall depth) can be used V' (Sec. 3.4.31)(a)

$$f_v' = 1.5 \frac{V'}{A}$$

If this check shows the beam size selected to be inadequate, the size is revised to provide sufficient A.

Deflection Criteria (IBC 2003 Sec. 3.5 NDS 01)

Limits are established for deflections for beams, trusses, and similar members that are not to be exceeded under certain gravity loads.  Table 1604.3 in the IBC 2003 gives the necessary limits and other information necessary to ensure <u>user comfort</u> and <u>to prevent excessive cracking of plaster ceilings.</u>

### TABLE 1604.3
### DEFLECTION LIMITS[a, b, c, h, l]

| CONSTRUCTION | L | S or W[f] | D + L[d,g] |
|---|---|---|---|
| Roof members:[e] | | | |
| Supporting plaster ceiling | l/360 | l/360 | l/240 |
| Supporting nonplaster ceiling | l/240 | l/240 | l/180 |
| Not supporting ceiling | l/180 | l/180 | l/120 |
| Floor members | l/360 | — | l/240 |
| Exterior walls and interior partitions: | | | |
|   With brittle finishes | — | l/240 | — |
|   With flexible finishes | — | l/120 | — |
| Farm buildings | — | — | l/180 |
| Greenhouses | — | — | l/120 |

For SI:   1 foot = 304.8 mm.

a.  For structural roofing and siding made of formed metal sheets, the total load deflection shall not exceed l/60. For secondary roof structural members supporting formed metal roofing, the live load deflection shall not exceed l/150. For secondary wall members supporting formed metal siding, the design wind load deflection shall not exceed l/90. For roofs, this exception only applies when the metal sheets have no roof covering.

b.  Interior partitions not exceeding 6 feet in height and flexible, folding and portable partitions are not governed by the provisions of this section. The deflection criterion for interior partitions is based on the horizontal load defined in Section 1607.13.

c.  See Section 2403 for glass supports.

d.  For wood structural members having a moisture content of less than 16 percent at time of installation and used under dry conditions, the deflection resulting from $L + 0.5D$ is permitted to be substituted for the deflection resulting from $L + D$.

e.  The above deflections do not ensure against ponding. Roofs that do not have sufficient slope or camber to assure adequate drainage shall be investigated for ponding. See Section 1611 for rain and ponding requirements and Section 1503.4 for roof drainage requirements.

For Green Lumber (MC > 19%)

$$\Delta_{TOTAL} = 2.0\,(\Delta_{long\ term}) + \Delta_{short\ term} < L/180$$

$\Delta_{Live} < L/240$

For Seasoned Lumber (MC < 19%)

$$\Delta_{\text{TOTAL}} = 1.5(\Delta_{\text{Long Term}}) + \Delta_{\text{Short Term}} < L/180$$

$$\Delta_{\text{Live}} < L/240$$

where

$\Delta_{\text{Long Term}}$ = immediate deflection due to the long term portion of the design load (usually dead load)

$\Delta_{\text{Short Term}}$ = immediate deflection due to short term component of the design load (usually live load)

Bearing  - Sec. 3.10 NDS 01

### 3.10.2 Bearing Perpendicular to Grain

The actual compression stress perpendicular to grain shall be based on the net bearing area and shall not exceed the allowable compression design value perpendicular to grain, $f_{c\perp} \leq F_{c\perp}'$. When calculating bearing area at the ends of bending members, no allowance shall be made for the fact that as the member bends, pressure upon the inner edge of the bearing is greater than at the member end.

### 3.10.4 Bearing Area Factor, $C_b$

Tabulated compression design values perpendicular to grain, $F_{c\perp}$, apply to bearings of any length at the ends of a member, and to all bearings 6" or more in length at any other location. For bearings less than 6" in length and not nearer than 3" to the end of a member, the tabulated compression design value perpendicular to grain, $F_{c\perp}$, shall be permitted to be multiplied by the following bearing area factor, $C_b$:

$$C_b = \frac{\ell_b + 0.375}{\ell_b} \qquad (3.10\text{-}2)$$

where:

$\ell_b$ = bearing length measured parallel to grain, in.

This equation gives the following bearing area factors, $C_b$, for the indicated bearing length on such small areas as plates and washers:

**Table 3.10.4  Bearing Area Factors, $C_b$**

| $\ell_b$ | 0.5" | 1" | 1.5" | 2" | 3" | 4" | 6" or more |
|---|---|---|---|---|---|---|---|
| $C_b$ | 1.75 | 1.38 | 1.25 | 1.19 | 1.13 | 1.10 | 1.00 |

## Example:  Sawn Beam Design (Dimension Lumber)

- Beams are spaced 16 inches on center (roof beam)
- Buckling of the compression zone is prevented by the plywood roof sheathing
- Material is No. 1 Douglas Fir – larch
- Loads

$w_D = 19 \ lb / ft$  (Dead load per lineal ft.)

$w_L = 27 \ lb / ft$  (Live load per lineal ft.)

$\text{TOTAL} = 46 \ lb / ft$

Required load combination (Sect. 2.3.2 NDS 01 and Table 2.3.2) and Duration Factors

$D_{alone} \Rightarrow C_D = 0.9$

$D + L = C_D = 1.15$ (snow load)

Determine trial size based on bending and then check other criteria.

(Sec. 4.3.9, NDS 01 and Table 4A of the supplement, Spacing < 24" ) $C_r = 1.15$

(Table 4A Supp to NDS01) $C_F = 1.20$

(MC < 19%, normal temperature conditions, compression edge of bending member supported throughout in accordance with 4.4.1.2 and no incision)

$C_M, C_T, C_L$ and $C_i = 1.0$

$F_b = 1000 \ (1.15) \ (1.2) \ (1.15) = 1587 \ psi$

$\text{Reqd } S = \dfrac{M_{max}}{F_b^{'}} = \dfrac{12,515}{1587} = 7.9 \ in^3$

Try 2x6 S = 7.56 $in^3$ (From Table 1B Supp).

From NDS Supplement Table 4A

$C_F = 1.3 \qquad F_b^{'} = 1587 \ x \dfrac{1.3}{1.2} = 1719 \ psi$

$S_{reqd} = \dfrac{1255}{1719} = 7.32 \ in^3 < 7.56 \ \therefore o.k.$

Check for Shear (NDS Sec. 3.4)

$$f_v = \frac{3}{2}\frac{V}{b_d} \ (Rect.)$$

$$C_M, C_t = 1.0 \quad A = 8.25 \ in^2 \quad (Table \ 1B \ Supp)$$

Conservative to use $V = V_{MAX} = 46.0 \ (13.5) = 3110$

$$f_v = \frac{3}{2}\frac{(311)}{(8.25)} = 56.5 \ psi$$

$$F_V' = F_V (C_D)(C_M)(C_t)(C_i)$$
$$= 180 \ (1.15) = 207 \ psi > 56.5 \ psi \quad \therefore ok$$

Check Deflections

$$E' = E \ (C_M)(C_T)(C_i)(C_t) = E$$
$$= 1,7000 \ Ksi \ (Table \ 4A \ Supp \ NDS01)$$

$$\Delta_L = \frac{5 \ w_L L^4}{384 E' I} = \frac{5(27.0)(13.5)^4(1728)}{(384)(1,700,000)(20.8)} = 0.57''$$

$$\Delta_L = \frac{L}{240} = \frac{13.5 \ x \ 12}{240} = 0.67'' > 0.57'' \quad \therefore ok$$

Also check for long term effects by calculating dead plus live deflection does not exceed L/180 (**Do in-class**).

Use 2x6 No. 1 DF-L MC ≤ 19%

Bearing Stress Check (Sec. 3.10 NDS01)

$$F_{C_\perp}' = F_{C_\perp} (C_M)(C_t)(C_b) = 625 \ psi$$

$$Re \ qd. \ \Delta = \frac{R}{F_{C_\perp}'} = \frac{311}{625} = 0.49 \ in^2$$

$$Re \ qd. \ \ell_b > \frac{A}{b} = \frac{0.99}{1.5} = 0.33 :< 6'' \quad \therefore ok$$

## Design of Tension Members (Sec. 3.8 NDS 01 and Chapter 7 Text)

Wood members are stressed in tension in a number of structured applications, i.e. trusses. (Tension II to grain) Table 4.3.1 NDS

$$f_t \leq F_t^{'} \quad \rightarrow \quad F_t^{'} = F_t * C_D * C_M * C_{w*} \quad \cdots$$

$$F_t = \text{Tabulated value (Supp.)}$$

$$where \ f_t = \frac{T}{A_{net}}$$

$\triangleright \ A_{net} \rightarrow (NDS \ Sec. 3.1.2)$

The cross sectional area to be used in the tension stress calculation is the net area of the member. This area is calculated by subtracting the projected area of any bolt holes from the gross-cross-sectional area of the members.



MEMBER WITH BOLT



MEMBER WITH CONNECTOR IN ONE FACE
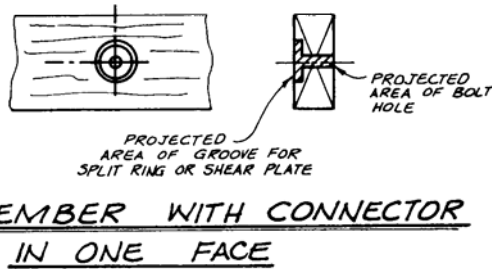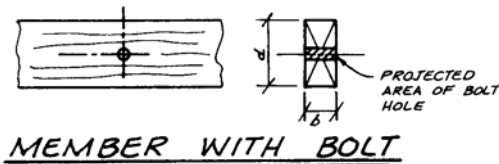
**Figure 7.2** *Net-section* through two wood members. One member is shown cut at a bolt hole. The other is at a joint with a split ring or shear plate connector in one face plus the projected area of a bolt. The bolt is required to hold the entire assembly (wood members and connectors) together. Photographs of split ring and shear plate connectors are included in Chap. 13 (Fig. 13.23a and b).

The projected areas for fasteners to be deducted from the gross area are as follows:

*Nail holes*—disregarded.

*Bolt holes*—computed as the hole diameter times the width of the wood member. The hole diameter is between $\frac{1}{32}$ and $\frac{1}{16}$ in. larger than the bolt diameter (NDS Sec. 8.1.2). In this book the bolt hole, for strength calculation purposes, is conservatively taken as the bolt diameter plus $\frac{1}{8}$ in.

*Lag bolt holes*—a function of the connection details. See NDS Appendix L for lag bolt dimensions. Drill diameters for lead holes and shank holes are given in NDS Sec. 9.1.2.

*Split ring and shear plate connectors*—a function of the connection details. See NDS Appendix K for the projected areas of split rings and shear plates.

If more than one fastener is used, the sum of the projected areas of all the fasteners *at the critical section* is subtracted from the gross area. For staggered fastener pattern, see NDS Sec. 3.1.2.

Example 7.2 Text for Spruce Pine Fir (south) No. 1

Determine the required size of the lower (tension chord) in the truss shown below. The loads are (DL + snow) and the effects of roof slope on the magnitude of snow load have already been taken into account. Joints are assumed pinned. Connections will be made with a single row of ¾"-diameter bolts. Trusses are 4 ft-0in on centers. Use No. 2 southern pine surfaced dry. MC < 0.19. Use NDS01.
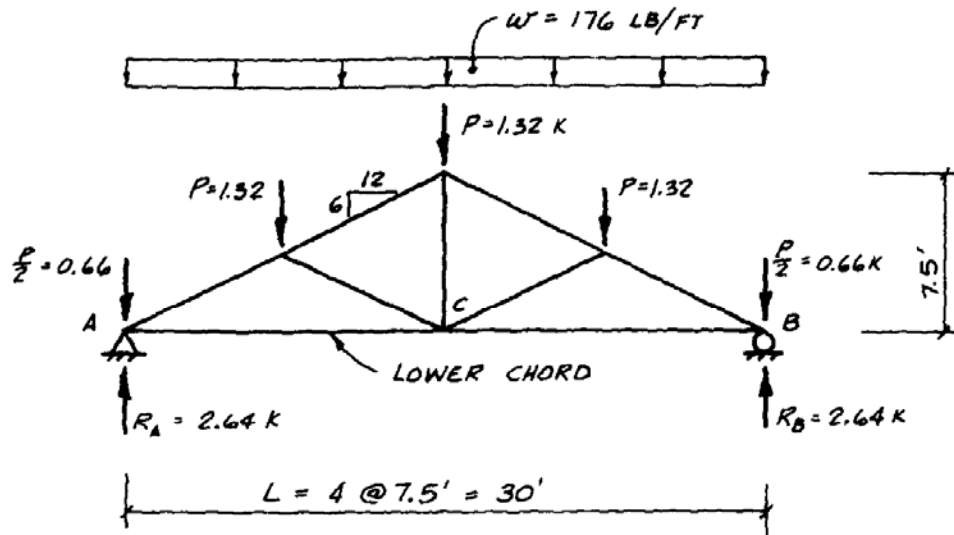


Figure 7.3a Uniform load on top chord converted to concentrated joint loads.

Total load horizontal plane $= 14(DL) + 30(SNOW) = 44\ psf$ x 4 (truss spacing) = 176 plf

Truss analysis - load to joint

*P = 0.176 x 7.5 = 1.32 kips*

Force in lower chord:

$T_{A-C}$ = (2.64- 0.66) x 2 = 3.96 Kips (service loads)

Determine required size of tension member

Assume chord will be a dimensional lumber 1-1/2" thick since M.C $\leq$ 19% $C_M = 1.0$.
Table 4.3.1 (NDS 01)

$$F_T^{'} = F_T * C_D * C_M * C_t * C_F * C_i$$

$C_D \rightarrow$ *Table 2.3.2 (NDS) Dead + Snow Load Combination*

$C_D \rightarrow$ *1.15 (snow - shorter duration)*

$C_T = 1.0$ *Sec. 2.3.4 NDS* $T \leq 100\,°F$

$$C_F = \left(\frac{12}{d}\right)^{1/9} = 1.0 \quad (see\ note\ Page\ 36\ Supp.\ NDS\ 01$$

*thickness = 1 - 1/2" , thus already incorporated in table value)*

$F_t =$ *Tension parallel to the grain for No. 2 southern pine (1.5" thick) and assume 6" wide*

   *= 725 psi (Table 4B NDS 01 Supplement)*

$F_T^{'} = (725)(1.15)(1)(1) = 834\ psi$

$$Re\,qd.\ A_{net} = \frac{P}{F_t^{'}} = \frac{3.95}{.834} = 4.74\ in^2$$

*Re qd. Gross Area = (accounting for bolt hole)*

$A_{gross} = 4.74 + (1.5)(0.75 + 1/8) = 6.053^{(4)}$

$A_{gross} = 8.25\ in^2 > 6.053$ *Table 1B Supp.*

*Use 2x6 No. 2 Southern Pine surfaced dry*

**Example of Combined Bending and Tension (Problem 7.12 Text)**
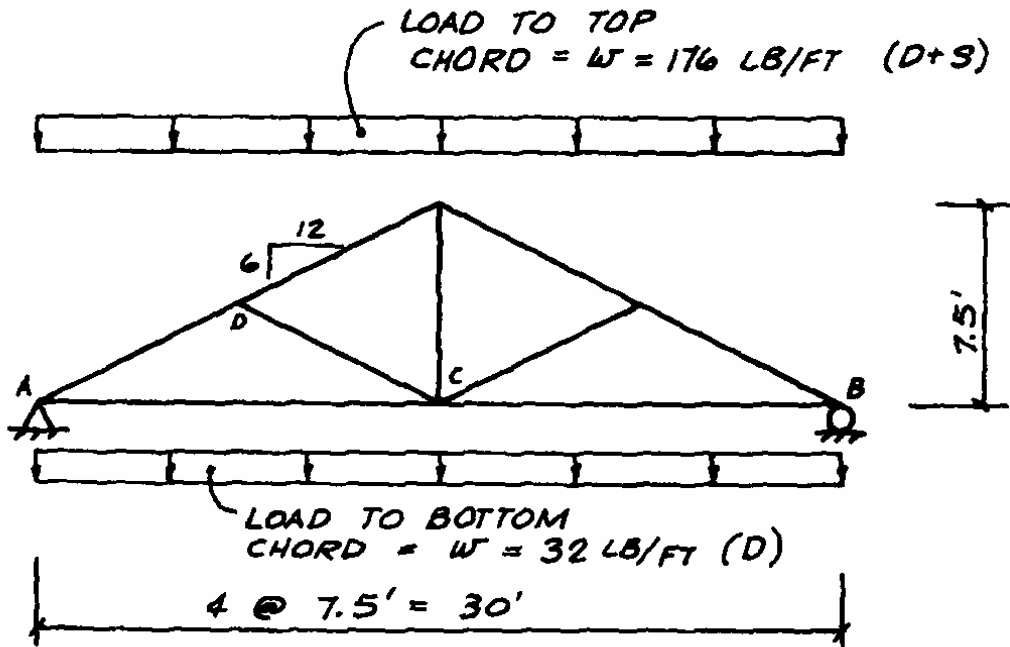


Combined Bending and Tension (Sec. 3.9 NDS 01)

Let's take the truss that we've designed the lower chord for tension only, and place an additional distributed load of 32 lb/ft (DL) applied at the lower chord. This load represents the weight of a ceiling supported by the bottom chord of the truss.



**Figure 3H  Combined Bending and Axial Tension**
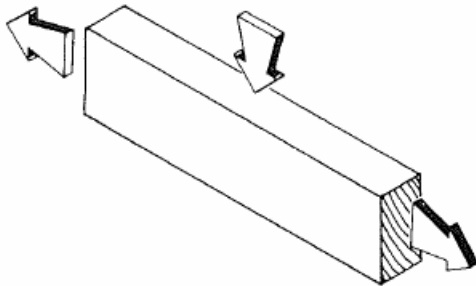
Design Example:

Determine the size of the lower chord of the truss. Use No. 2 Southern pine surface dry (MC ≤ 19%). Connections will be made with a single row of ¾" diameter bolts also. Connections are then assumed to be pinned. Lateral buckling is prevented by ceiling. Trusses are 4' on center.

i)    Determine Force in lower chord

Resolve distributed load into joint loads



Free body diagram of joint A.
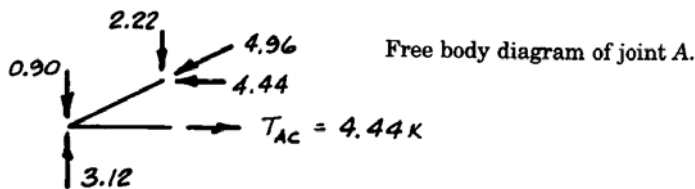
Estimate trial size of member: from previous example a 2x6 was needed with the additional load in the bottom chord try 2x8

Calculate force in member:

Load diagram for lower truss chord: taking advantage of symmetry
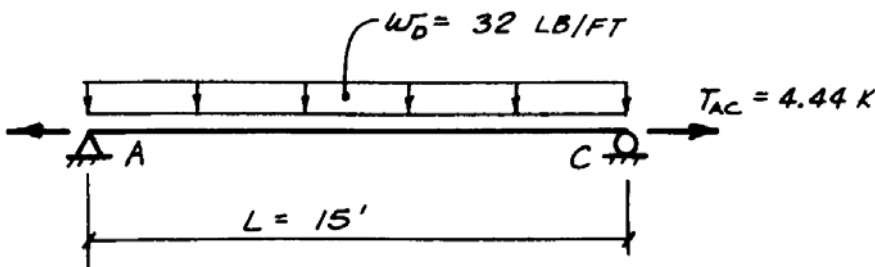


Note that due to load application chord will be subjected to combined bending and tension!!

CE 479
Wood Design Lecture Notes
JAR

ii)    Member Design

Try 2x8" from NDS 01 supp. For No. 1 – Southern Pine surface dry

Table 4B

$F_b = 1500\ psi$

$F_T = 825\ psi$

$S_{xx} = 13.14\ in^3$,  thickness $= 1.5"$, wide $= 7\,1/4"$

<u>1. Axial tension</u>:  first check tension at net section (midspan-bolt location).  Because of bolted connection M=0 at this section.
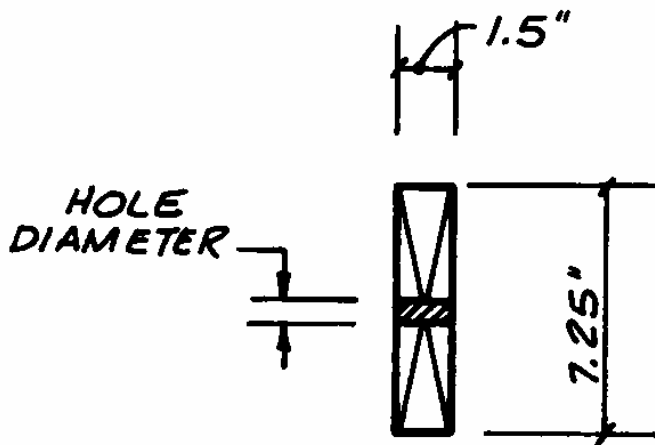


$A_{net} = (7.25)\,(1.5) - (1.5)\,(1/8 + 0.75) = 9.56\ in^2$

$f_t = \dfrac{4.44}{9.56} = 0.464\ ksi = 464\ psi\ (reqd)$

$F_{t_1}' = F_{t_{11}} * C_D * C_M * C_t * C_F * C_i\ \ (allowable)$

The duration load factor used for the independent tension check is the $C_D$ of (DL + snow)

$C_D = 1.15$  (Use that of shortest-duration load in the combination- Snow)

$C_M = 1.0$  (MC $\leq$ 19%)

$C_t = 1.0 \quad (\text{T} \leq 100^\text{o}\text{F})$

$C_F = 1.0$ (Table 4B-Adjust. Factors NDS 01 Supp.)

$\therefore F'_{t_{11}} = (825)\,(1.15) = 949 \; psi$

$949 > 464 \; psi$

2. Axial Tension + Bending Tension

The combined tensile stresses are analyzed using a straight-line interaction formula. (NDS, Sec. 3.9.1 page 21) – tension/tension Eq. 3.9-1

### 3.9.1 Bending and Axial Tension

Members subjected to a combination of bending and axial tension (see Figure 3H) shall be so proportioned that:

$$\frac{f_t}{F'_t} + \frac{f_b}{F^*_b} \leq 1.0 \qquad (3.9\text{-}1)$$

and

$$\frac{f_b - f_t}{F^{**}_b} \leq 1.0 \qquad (3.9\text{-}2)$$

where:

$F^*_b$ = tabulated bending design value multiplied by all applicable adjustment factors except $C_L$

$F^{**}_b$ = tabulated bending design value multiplied by all applicable adjustment factors except $C_V$

$F^*_b$ = tabulated design value (bending) multiplied by all applicable adjustment factors except $C_L$ (This is because buckling is not an issue in tension).

$C_L$ = beam stability factor = 1.0

$\therefore Thus : Tension : \ f_t = \dfrac{4440}{7.25 x1.5} \ 408 \ psi \ \ (at \ point \ of \ max. bending \ stress \ no \ hole)$

$F_t^{'} = 949 \ psi \, (same \ as \ pure \ tension)$

$Bending : \ f_b = \dfrac{M}{S} = \dfrac{(\,32\,)(\,15\,)^2 \, x12}{(\,8\,)(\,13.14\,)} = 822 \ psi$

$F_b^* = 1500 * C_D * C_r$

$C_D = 1.15 \ (Dead \ load \ included \ bending)$

$C_r = 1.0 \ (Table \ 4B$ - $Adj. \ fact \ S = 48'' > 24'' \, Page \ 36$

$F_b^* = (\,1500\,)(\,1.15\,) = 1725 \ psi$

$\dfrac{f_t}{F_t^{'}} + \dfrac{f_b}{F_b^*} \le 1.0$

$\dfrac{408}{908} + \dfrac{822}{1725} = 0.93 < 1.0 \ \ \therefore \ ok$

<u>Axial tension plus compression due to bending (net bending compression)</u>

Check bending compression stress without axial tension (remember axial tension in the truss chord is caused by snow). Also self-weight of truss would reduce compression, thus do not include. Hence the combination of least axial tension with compression due to dead load on the chord causing bending is not critical.

$F_b^{'} = 1500 x0.9 = 1350 \ psi$

$1350 > 822 \, psi(compression)$

(Lateral buckling is prevented)

$\therefore$ Use 2 x 8 No. 1 southern pine (check for shear and deflection due to chord bending must be carried out as in the beam example)

---

**Thus, no need for** $\dfrac{f_b - f_t}{F_b^{**}}$ **check (Eq. 3.9-2)**

---

**Combined Bending and Compression (Sec 7.12 Text and NDS 01 Sec. 3.9)**

These members are referred to as beam-columns. The basic straight line interaction for bending and axial tension (Eq. 3.9-1, NDS 01) has been modified as shown in Section 3.9.2 of the NDS 01, Eq. (3.9-3) for the case of bending about one or both principal axis and axial compression. This equation is intended to represent the following conditions:

- Column Buckling
- Lateral Torsional Buckling of Beams
- Beam-Column Interaction (P, M).

The uniaxial compressive stress, $f_c = P/A$, where $A$ represents the net sectional area as per 3.6.3

## 3.6.3 Strength in Compression Parallel to Grain

The actual compression stress or force parallel to grain shall not exceed the allowable compression design value. Calculations of $f_c$ shall be based on the net section area (see 3.1.2) when the reduced section occurs in the critical part of the column length that is most subject to potential buckling. When the reduced section does not occur in the critical part of the column length that is most subject to potential buckling, calculations of $f_c$ shall be based on gross section area. In addition, $f_c$ based on net section area shall not exceed the tabulated compression design value parallel to grain multiplied by all applicable adjustment factors except the column stability factor, $f_c \leq (F_c)(C_D)(C_M)(C_t)(C_F)(C_i)$.

# 3.9 Combined Bending and Axial Loading

## 3.9.1 Bending and Axial Tension

Members subjected to a combination of bending and axial tension (see Figure 3H) shall be so proportioned that:

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b^*} \leq 1.0 \tag{3.9-1}$$

and

$$\frac{f_b - f_t}{F_b^{**}} \leq 1.0 \tag{3.9-2}$$

where:

$F_b^*$ = tabulated bending design value multiplied by all applicable adjustment factors except $C_L$

$F_b^{**}$ = tabulated bending design value multiplied by all applicable adjustment factors except $C_V$

## 3.9.2 Bending and Axial Compression

Members subjected to a combination of bending about one or both principal axes and axial compression (see Figure 3I) shall be so proportioned that:

$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b1}}{F_{b1}'\left[1 - (f_c/F_{cE1})\right]}$$
$$+ \frac{f_{b2}}{F_{b2}'\left[1 - (f_c/F_{cE2}) - (f_{b1}/F_{bE})^2\right]} \leq 1.0 \tag{3.9-3}$$

where:

$$f_c < F_{cE1} = \frac{K_{cE}E'}{(\ell_{e1}/d_1)^2} \quad \text{for either uniaxial or biaxial bending}$$

and

$$f_c < F_{cE2} = \frac{K_{cE}E'}{(\ell_{e2}/d_2)^2} \quad \text{for biaxial bending}$$

and

$$f_{b1} < F_{bE} = \frac{K_{bE}E'}{(R_B)^2} \quad \text{for biaxial bending}$$

$f_{b1}$ = actual edgewise bending stress (bending load applied to narrow face of member)

$f_{b2}$ = actual flatwise bending stress (bending load applied to wide face of member)

$d_1$ = wide face dimension (see Figure 3I)

$d_2$ = narrow face dimension (see Figure 3I)

Effective column lengths, $\ell_{e1}$ and $\ell_{e2}$, shall be determined in accordance with 3.7.1.2. $F_c'$, $F_{cE1}$ and $F_{cE2}$ shall be determined in accordance with 2.3 and 3.7. $F_{b1}'$, $F_{b2}'$ and $F_{bE}$ shall be determined in accordance with 2.3 and 3.3.3.

## 3.9.3 Eccentric Compression Loading

See 15.4 for members subjected to combined bending and axial compression due to eccentric loading, or eccentric loading in combination with other loads.

The combination of bending and axial compression is more critical due to the P-△ effect. The bending produced by the transverse loading causes a deflection △. The application of the axial load, P, then results in an additional moment P*△; this is also know as second order effect because the added bending stress is not calculated directly. Instead, the common practice in design specifications is to include it by increasing (amplification factor) the computed bending stress in the interaction equation.

**Figure 3I   Combined Bending and Axial Compression**
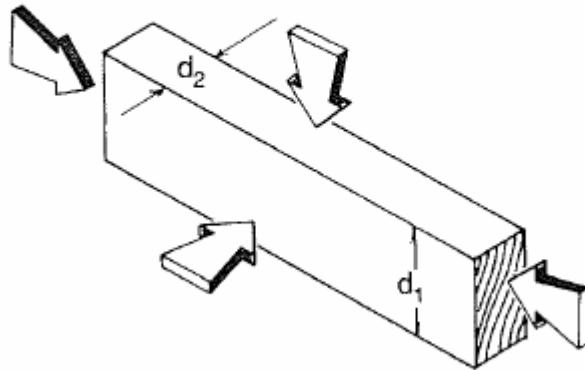


The most common case involves axial compression combined with bending about the strong axis of the cross section. In this case, Equation (3.9-3) reduces to:

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{b1}}{F'_{b1}\left[1-\left(\frac{f_c}{F_{cE1}}\right)\right]} \leq 1.0$$

and, the amplification factor is a number greater than 1.0 given by the expression:

$$\left(\frac{1}{1-\left(\frac{f_c}{F_{E1}}\right)}\right) = \text{Amplification factor for } f_{b1}$$

**Example of Application:**
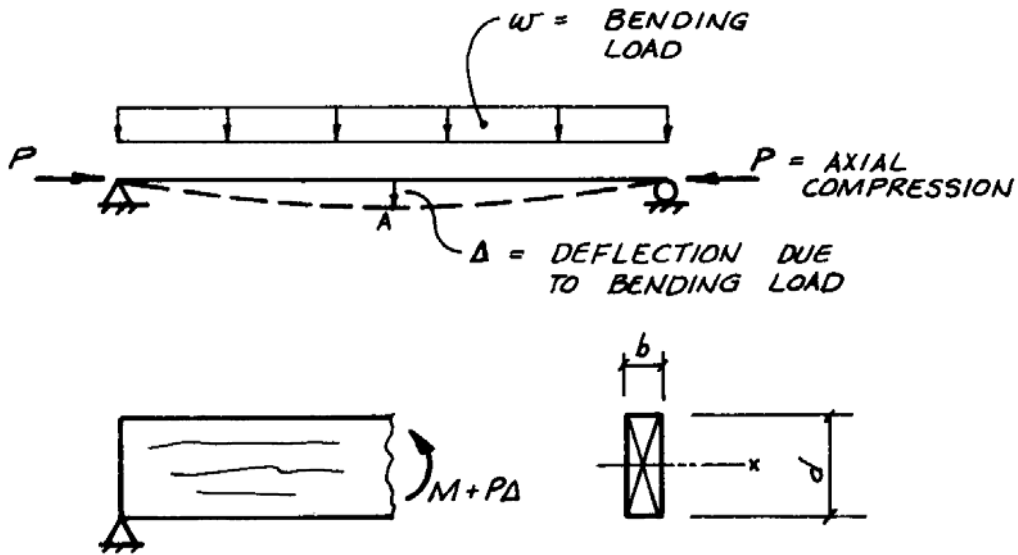


**Figure 7.17a** Deflected shape of beam showing $P$-$\Delta$ moment. The computed bending stress $f_b$ is based on the moment $M$ from the moment diagram. The moment diagram considers the effects of the transverse load $w$, but does not include the secondary moment $P \times \Delta$. The $P$-$\Delta$ effect is taken into account by amplifying the computed bending stress $f_b$.

The general interaction formula reduces to:

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{b1}}{F'_{b1}\left[1-\left(\frac{f_c}{F_{cE1}}\right)\right]} \leq 1.0$$

where:
$f_c$ = actual compressive stress = $P/A$
$F'_c$ = allowable compressive stress parallel to the grain = $F_c*C_D*C_M*C_t*C_F*\mathbf{C_P}*C_i$

*Note: that $F'_c$ includes the $\mathbf{C_P}$ adjustment factor for stability (Sec. 3.7.1 NDS 01) to be considered in lengths of the column subject to buckling.*

## $C_P$

# 3.7  Solid Columns

### 3.7.1  Column Stability Factor, $C_p$

3.7.1.1 When a compression member is supported throughout its length to prevent lateral displacement in all directions, $C_p = 1.0$.

3.7.1.2 The effective column length, $\ell_e$, for a solid column shall be determined in accordance with principles of engineering mechanics. One method for determining effective column length, when end-fixity conditions are known, is to multiply actual column length by the appropriate effective length factor specified in Appendix G, $\ell_e = (K_e)(\ell)$.

3.7.1.3 For solid columns with rectangular cross section, the slenderness ratio, $R_e/d$, shall be taken as the larger of the ratios $\ell_{e1}/d_1$ or $\ell_{e2}/d_2$ (see Figure 3G) where each ratio has been adjusted by the appropriate buckling length coefficient, $K_e$, from Appendix G.

3.7.1.4 The slenderness ratio for solid columns, $\ell_e/d$, shall not exceed 50, except that during construction $\ell_e/d$ shall not exceed 75.

3.7.1.5 The column stability factor shall be calculated as follows:

$$C_p = \frac{1+\left(F_{cE}/F_c^*\right)}{2c} - \sqrt{\left[\frac{1+\left(F_{cE}/F_c^*\right)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}} \qquad (3.7\text{-}1)$$

where:

$F_c^*$ = tabulated compression design value multiplied by all applicable adjustment factors except $C_p$ (see 2.3)

$F_{cE} = \dfrac{K_{cE}\,E'}{\left(\ell_e/d\right)^2}$

$K_{cE}$ = $0.510 \cdot 0.839(COV_E)$

= 0.3 for visually graded lumber
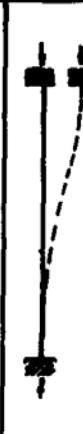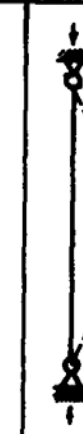
= 0.384 for machine evaluated lumber (MEL)
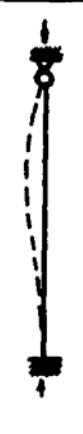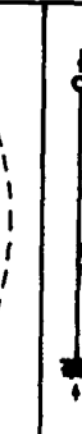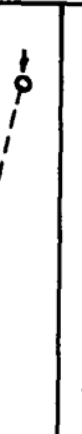
= 0.418 for products with $COV_E \le 0.11$ (see Appendix F.2)

c = 0.8 for sawn lumber

c = 0.85 for round timber poles and piles

c = 0.9 for glued laminated timber or structural composite lumber

32

The actual unsupported column length multiplied by the appropriate length factor in Appendix G of the NDS 01 yields the effective column length, $l_e$.
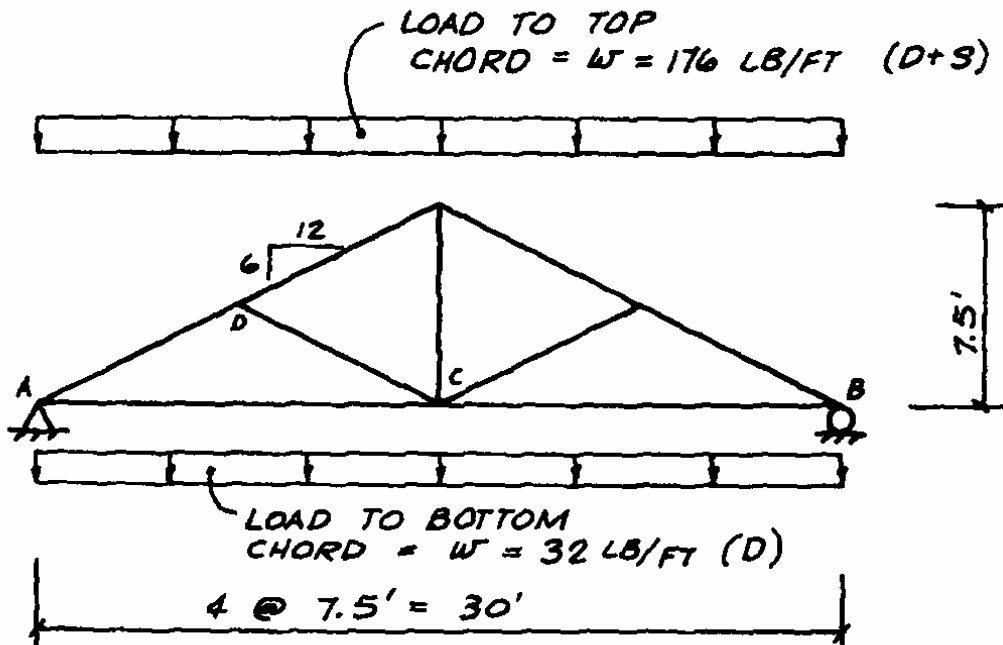
| Buckling modes | | | | | | |
|---|---|---|---|---|---|---|
| Theoretical $K_e$ value | 0.5 | 0.7 | 1.0 | 1.0 | 2.0 | 2.0 |
| Recommended design $K_e$ when ideal conditions approximated | 0.65 | 0.80 | 1.2 | 1.0 | 2.10 | 2.4 |
| End condition code | | Rotation fixed, translation fixed | | | | |
| | | Rotation free, translation fixed | | | | |
| | | Rotation fixed, translation free | | | | |
| | | Rotation free, translation free | | | | |

**Design Problem (Sec. 7.13 Text)**

The top chord of the truss analyzed in the case of tension and bending of the lower chord is considered for the case of combined compression and bending. The bending of the top chord is due to dead plus snow being applied along the length of the member. The truss analysis provides the forces in the member from A-D.

2.22

0.90

4.96

4.44

$T_{AC} = 4.44K$

3.12

Free body diagram of joint A.
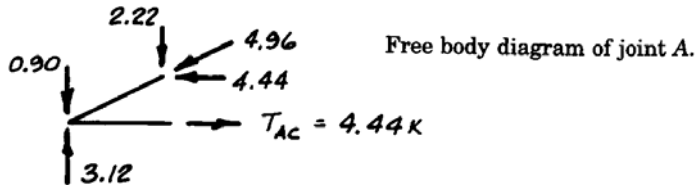
The length of the member from A-D is 8.39 feet. Although two load combinations, D-only, and D + S must be considered, it has been determined that the D + S combination controls the design and only those calculations are included herein.

LOAD TO TOP
CHORD = W = 176 LB/FT  (D+S)

12
6

D

C

A

B

7.5'

LOAD TO BOTTOM
CHORD = W = 32 LB/FT (D)

4 @ 7.5' = 30'

Let's try a 2 X 8 Southern Pine No. 1 (Table 4B NDS Supplement 01)
        $F_B = 1500$ psi
        $F_C = 1650$ psi (parallel to the grain)
        $E = 1,700,000$ psi

The tabulated values in 4B are size specific thus $C_F$ (comp. II to the grain) and $C_F$ (bending) are equal to 1.0.

Section properties for 2x8 are A = 10.88 in$^2$ and S = 13.14 in$^3$.

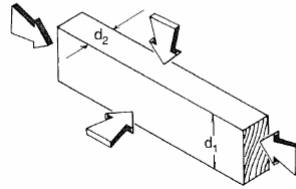## Axial check:

1. Stability: Column buckling can occur away from the truss joints. Use gross are to calculate $f_c$:

$f_c = P/A = 4960/10.88 = 456$ psi

lateral support is provided by the roof diaphragm, thus as the member is used edgewise, buckling is prevented about the weak axis ($d_2$ direction)



**Figure 3I   Combined Bending and Axial Compression**

$$\left(\frac{l_e}{d}\right)_1 = \frac{8.39*12}{7.25} = 13.9\,ft$$

$E' = E\,(C_M)(C_t) = 1,700,000\,(1)(1) = 1,700,000$ psi

For visually graded sawn lumber (Section 3.7.1.5)

$K_{cE} = 0.3$

$c = 0.8$

$F_{cE} = (K_{cE}*E')/(l_e/d_1)^2 = (0.3*1,7000,000)/(13.9)^2 = 2645$ psi

$F_c^{**} = F_c(C_D)(C_M)(C_t)(C_F)(C_i) = 1650(1.15)(1.0)(1.0)(1.0)(1.0) = 1898$ psi

Substituting in Eq. (3.7-1) results in $C_P = 0.792$

$F'_c = F^*_c\,(C_P)= 1898 * 0.792 = 1502$ psi $> 456$ psi  OK!

Note that at the connections the reduced area should be used to check compression, but there is no possibility of buckling (braced location). Assuming a bolt diameter plus 1/8" for the opening diameter of 0.875", the net area is $1.5*(7.25-.875) = 9.56$ in$^2$. Thus the acting axial stress is $4960/9.56 = 518$ psi. The design stress is $F'_c = F^*_c = 1898 > 518$ , OK!

### Bending check:

Assume pinned ends for the chord member and take load and span on the horizontal plane:

$$M = \frac{wL^2}{8} = \frac{176(7.5)^2 * 12}{8*1000} = 14.9 \text{ in-k}$$

$f_b = M/S = 14.9* 1000/13.14 = 1130$ psi

The truss chord acting as a beam has full lateral support and the lateral stability factor $C_l = 1.0$ and $F'_b = 1500 * 1.15 = 1725$ psi $> 1130$ psi OK!

### Combined Stress check:

There is no bending about the weak axis, and the axial load is concentric. Thus, Equation (3.9-3) reduces to:

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{b1}}{F'_{b1}\left[1 - \left(\frac{f_c}{F_{cE1}}\right)\right]} \le 1.0$$

The load duration factor that controls is snow, $C_D = 1.15$. Therefore, the previously determined values of $F'_C$ and $F'_b$ are still valid for use in the interaction formula. The elastic buckling factor depends on the slenderness ratio about the strong axis of 13.92. It must be noted that the buckling stress, $F_{cE}$, for the axial check is based on the $(l_e/d)_{max}$ whereas for the P-$\Delta$ effect is based on the axis about which the bending moment occurs (strong axis based on $d_1$). In this example, it is a coincidence that the two values of the buckling stress, $F_{cE}$, are the same.

$F_{cE} = 2645$ psi

$$\left(\frac{456}{1502}\right)^2 + \left(\frac{1}{1 - \left(\frac{456}{2645}\right)}\right)\frac{1130}{1725} = 0.884 < 1.0 \quad \text{OK! Use 2 x 8 No. 1 Southern Pine}$$

It is appropriate in an actual design situation to check the D-load only case with a load duration factor, $C_D$, of 0.9.

### Connections

Chapter 11, 12, 13 and 14 in the textbook:

- Nailed
- Bolted
- Screws
- Split Ring and Metal Plate Connectors