Ce 479
Reinforced Masonry
Fall 2004

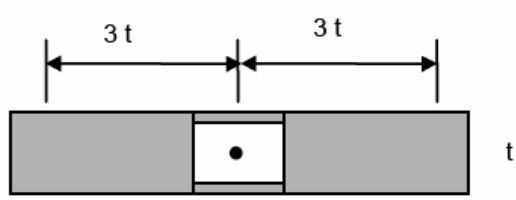
INTRODUCTION TO STRUCTURAL DESIGN OF REINFORCED MASONRY

DESIGN OF REINFORCED MASONRY BEAM-COLUMNS

Introduction

Reinforced masonry beam-columns are reinforced masonry elements subjected to combinations of axial force and flexure. Reinforced masonry beam-columns, like those of reinforced concrete, are designed using moment-axial force interaction diagrams. Combinations of axial force and moment lying inside the diagram represent permitted designs; combinations lying outside, prohibited ones. Unlike reinforced concrete, however, reinforced masonry beam-columns rarely take the form of isolated rectangular elements with four longitudinal bars and transverse ties. The most common form for a reinforced masonry beam-column is a wall, loaded out-of-plane by eccentric gravity load, alone or in combination with wind.

For example, the figure on the next page shows a portion of a wall, with a total effective width of 6*t* prescribed by the allowable-stress provisions of the 2002 MSJC Code (Section 2.3.3.3.1). Although this effective width is not prescribed by the strength provisions of the 2002 MSJC Code, the same value will be used.



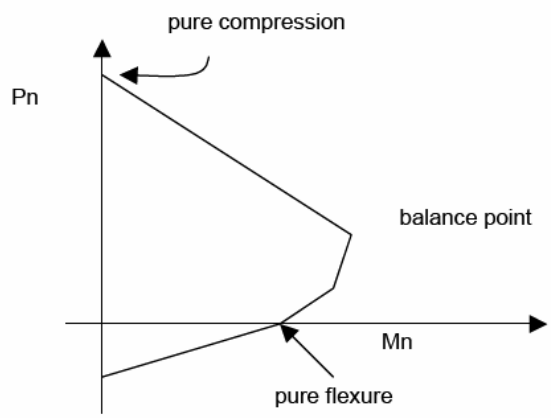
effective width on each side of bar

BACKGROUND ON MOMENT-AXIAL FORCE INTERACTION DIAGRAMS BY THE STRENGTH APPROACH

Using the strength approach, we seek to construct interaction diagrams that represent combinations of axial and flexural capacity. This can be done completely by hand, or with the help of a spreadsheet.

Strength Interaction Diagrams Hand-Calculations

By hand, we can compute three points (pure compression, pure flexure, and the balance point). We then draw a straight line between pure compression and the balance point, and either a straight line or an appropriate curve between the balance point and pure flexure. This approach is commonly applied to reinforced concrete columns.



Pure Compression

$$P_0 = 0.80 \cdot 0.80 f_m^{'}(A_c - A_{st}) + A_{st} f_y$$

<u>As in ACI 318-02, the leading factor of 0.80 in effect imposes a minimum design eccentricity.</u>

Pure Flexure

As before, the possible contribution of compressive reinforcement is small, and can be neglected.

$$\rho = \frac{A_s}{bd}$$

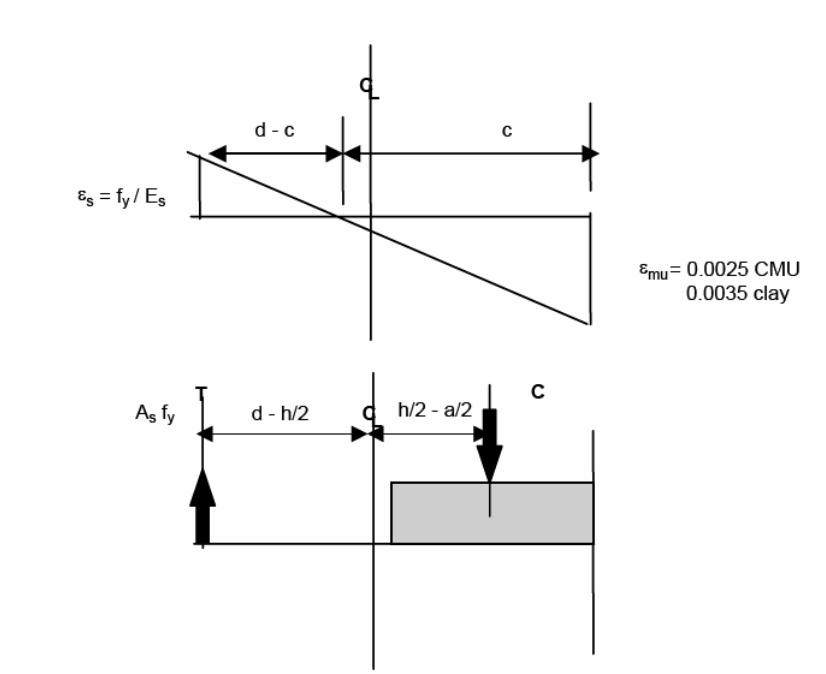
$$\omega = \rho\left(\frac{f_y}{f_m^{'}}\right)$$

$$M_n = \omega b d^2 f_m^{'}(1 - 0.63\omega)$$

Balance Point

$$\frac{c}{d-c} = \frac{\varepsilon_{mu}}{\varepsilon_y}$$

$$c = d\left(\frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y}\right)$$



Next calculate the corresponding tensile and compressive forces:

$$T = A_s f_y$$
$$C = 0.80 f_m^{'}(\beta_1 c)b$$
$$P_n = C - T$$
$$M_n = T\left(d - \frac{h}{2}\right) + C\left(\frac{h}{2} - \frac{\beta_1 c}{2}\right)$$

Example: Moment-Axial Force Interaction Diagram by the Strength Approach (hand-calculation)

Construct the moment-axial force interaction diagram by the strength approach for a nominal 8-in. CMU wall, fully grouted, with $f_m{}'$ = 1500 lb/in.^{2} and reinforcement consisting of #5 bars at 48 in., placed in the center of the wall. Compute the interaction diagram per foot of wall length.

For the case of a wall with reinforcement at mid-depth, the reference axis for moment is located at the plastic centroid (geometric centroid) of the cross section, which is also at mid-depth. This leads to results which appear considerably different from what we are used to for a symmetrically reinforced column. For example, using the geometric centroid (the level of the reinforcement) as the reference axis, the contribution of the reinforcement to the moment is always zero. In what is apparently even stranger, the balanced-point axial force does not coincide with the maximum moment capacity. As a result, hand calculations are useful for some reinforced masonry beam-columns, but not all.

Pure Compression

Because the compressive reinforcement in the wall is not supported laterally, it is not counted in calculated capacity.

$$P_0 = 0.80 \cdot 0.80 f_m^{'}(A_c - A_{st}) + A_{st} f_y$$
$$P_n = 0.80 \cdot 0.80 \cdot 1500 \ lb/in.^2 \cdot (7.63 \cdot 48 - 0.31) + 0.31 \cdot 0 \ lb/in.^2$$
$$P_n = 351{,}293 \ lb$$

Per foot of wall length, the design capacity will be the above value, divided by 4 (the length of the wall in feet), and multiplied by the strength reduction factor of 0.90:

φP_n *= 79,041 lbs*

Pure Flexure

As before, neglect the influence of compressive reinforcement:

$$d = \left(\frac{1}{2}\right)7.63\ in. = 3.81\ in.$$

$$\rho = \frac{A_s}{bd} = \frac{0.31\ in.^2}{48 \cdot 3.81\ in.^2} = 1.70 \cdot 10^{-3}$$

$$\omega = \rho\left(\frac{f_y}{f_m^{'}}\right) = (1.70 \cdot 10^{-3}) \cdot \left(\frac{60000}{1500}\right) = 0.0678$$

$$M_n = \omega b d^2 f_m^{'}(1 - 0.63\omega)$$

$$M_n = 0.0678 \cdot 48 in. \cdot 3.81^2\ in.^2 \cdot 1500\, lb / in.^2 \cdot (1 - 0.63 \cdot 0.0678)$$

$$M_n = 67{,}838\ lb - in.$$

Per foot of wall length, the design capacity is the above value, divided by 4 and multiplied by the strength reduction factor of 0.90:

$$\phi M_n = 15{,}264\ lb - in.$$

Balance Point

First, locate the neutral axis:

$$c = d\left(\frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y}\right)$$

$$c = d\left(\frac{0.0025}{0.0025 + 0.0207}\right)$$

$$c = 0.55d$$

$$c = 0.55 \cdot 3.81\ in. = 2.08\ in.$$

The value of β_1 is prescribed in Section 3.2.2 (g) of the 2002 MSJC as 0.80:

$$T = A_s f_y = 0.31 in.^2 \cdot 60000 lb/in.^2 = 18{,}600\ lb$$
$$C = 0.80 f_m^{'}(\beta_1 c)b = 0.80 f_m^{'}(0.80 \cdot 0.55\ d)\ b$$
$$C = 0.80 \cdot 1500 lb/in^2 (0.80 \cdot 0.55 \cdot 3.81 in.) \cdot 48\ in. = 96{,}561\ lb$$

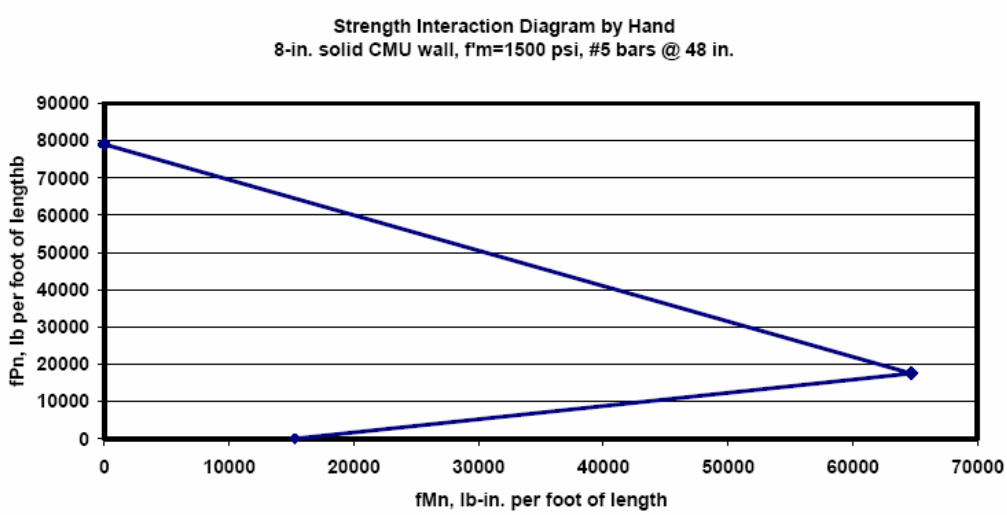
$$P_n = C - T = 77{,}961 lb$$
$$M_n = T\left(d - \frac{h}{2}\right) + C\left(\frac{h}{2} - \frac{\beta_1 c}{2}\right)$$

$$M_n = 18{,}600 lb\ (3.81 - 3.81)\ in. + 96{,}561 lb\left(\frac{7.63}{2} - \frac{0.80 \cdot 0.55 \cdot 3.81}{2}\right) in.$$
$$M_n = 0 lb - in. + 287{,}442 lb - in. = 287{,}442\ lb - in.$$

Per foot of wall length, the design capacities are the above values, divided by 4 and multiplied by the strength reduction factor of 0.90:

$$\phi P_n = 17{,}541\ lb$$
$$\phi M_n = 64{,}674\ lb - in.$$

Plot of Strength Interaction Diagram by Hand

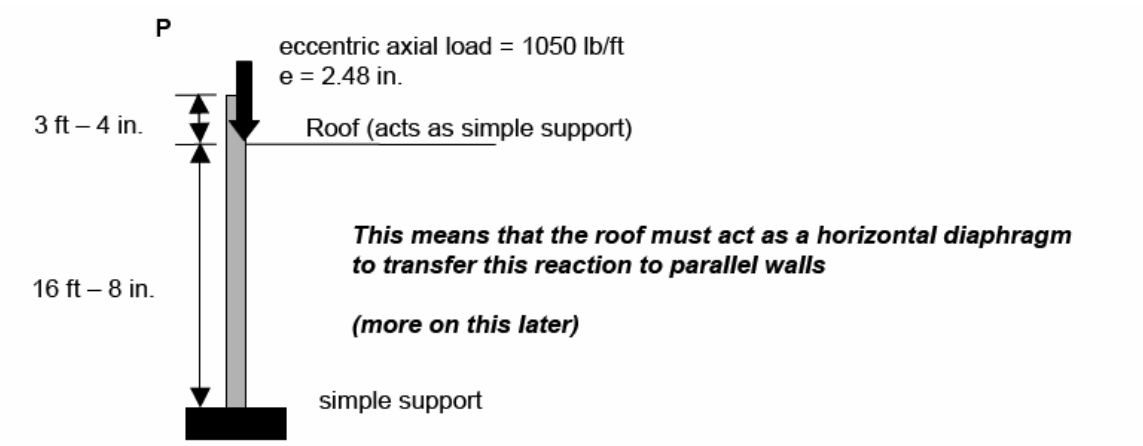


As we shall shortly see, the points that we have calculated are correct. The form of the diagram is misleading, however, because the balance point is actually not the point of maximum moment. It is incorrect to draw the diagram with a straight line from the balance point to the pure compression point. The balance point becomes the point of maximum moment as the reinforcement is placed farther apart than about 70% of the thickness of the wall.

Example: Design of Masonry Beam-Columns by the Strength Approach

Once we have developed the moment-axial force interaction diagram by the strength approach, the actual design simply consists of verifying that the combination of factored design axial force and moment lies within the diagram of nominal axial and flexural capacity, reduced by strength reduction factors.

Consider the bearing wall designed as unreinforced. It has an eccentric axial load plus out of plane wind load of 25 psf.



At each horizontal plane through the wall, the following condition must be met:

- *combinations of factored axial load and moment must lie within the moment- axial force interaction diagram, reduced by strength-reduction factors.*

Because flexural capacity increases with increasing axial load, the most critical loading combination is probably 0.9D + 1.6W .

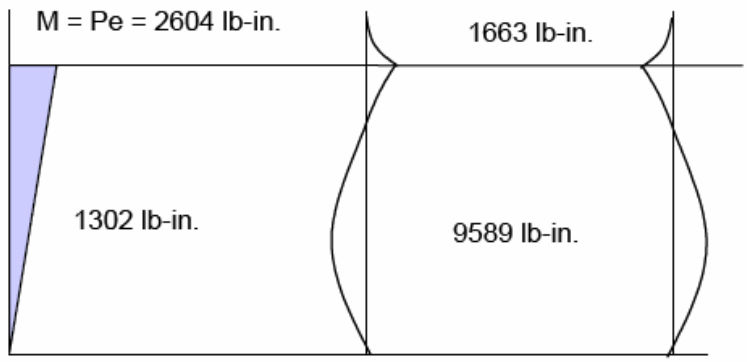
From the unreinforced example, we know that the critical point on the wall is at or near the midspan of the wall. Due to wind only, the unfactored moment at the base of the parapet (roof level) is:

$$M = \frac{qL^2}{2} = \frac{25\ lb/ft \cdot 3.33^2\ ft^2}{2} \cdot 12\ in./ft = 1663\ lb - in.$$

The maximum moment is close to that occurring at mid-height. The moment from wind load is the superposition of approximately one-half moment at the upper support due to wind load on the parapet only, plus the midspan moment in a simply supported beam with that same wind load:

$$M_{midspan} = -\frac{1663}{2} + \frac{qL^2}{8} = -\frac{1663}{2} + \frac{25\ lb/ft \cdot 16.67^2\ ft^2}{8} \cdot 12\ in./ft = 9589\ lb - in.$$

Unfactored moment diagrams due to eccentric axial load and wind are as shown below:



Check the adequacy of the wall with 8-in. nominal units, a specified compressive strength, f_m', of 1500 lb/in.^{2}, and #5 bars spaced at 48 in. All design actions are calculated per foot of width of the wall. At the mid-height of the wall, the axial force due to 0.9D is:

$$P_u = 0.9(700\ lb) + 0.9(3.33\ ft + 8.33\ ft) \cdot 48\ lb/ft = 1134\ lb$$

At the mid-height of the wall, the factored design moment, M_u, is given by:

$$M_u = P_u \frac{e}{2} + M_{u\ wind} = \left(\frac{1}{2}\right) 0.9 \cdot 700\ lb \cdot 2.48\ in. + 1.6 \cdot 9589\ lb - in. = 16{,}124\ lb - in.$$

In each foot of wall, the design actions are P_u = 1134 lb, and M_u = 16,124 lb-in. That combination lies within the interaction diagram of design capacities, and the design is satisfactory.