Combined Bending and Compression (Sec 7.12 Text and NDS 01 Sec. 3.9)

These members are referred to as beam-columns. The basic straight line interaction for bending and axial tension (Eq. 3.9-1, NDS 01) has been modified as shown in Section 3.9.2 of the NDS 01, Eq. (3.9-3) for the case of bending about one or both principal axis and axial compression. This equation is intended to represent the following conditions:

- Column Buckling
- Lateral Torsional Buckling of Beams
- Beam-Column Interaction (P, M).

The uniaxial compressive stress, $f_c = P/A$, where *A* represents the net sectional area as per 3.6.3

3.6.3 Strength in Compression Parallel to Grain

The actual compression stress or force parallel to grain shall not exceed the allowable compression design value. Calculations of f_c shall be based on the net section area (see 3.1.2) when the reduced section occurs in the critical part of the column length that is most subject to potential buckling. When the reduced section does not occur in the critical part of the column length that is most subject to potential buckling, calculations of f_c shall be based on gross section area. In addition, f_c based on net section area shall not exceed the tabulated compression design value parallel to grain multiplied by all applicable adjustment factors except the column stability factor, $f_c \leq (F_c)(C_D)(C_M)(C_t)(C_F)(C_i)$.

3.9 Combined Bending and Axial Loading

3.9.1 Bending and Axial Tension

Members subjected to a combination of bending and axial tension (see Figure 3H) shall be so proportioned that:

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b^*} \leq 1.0 \tag{3.9-1}$$

and

$$\frac{f_b - f_t}{F_b^{**}} \leq 1.0 \tag{3.9-2}$$

where:

F_b^* = tabulated bending design value multiplied by all applicable adjustment factors except C_L

F_b^{**} = tabulated bending design value multiplied by all applicable adjustment factors except C_V

3.9.2 Bending and Axial Compression

Members subjected to a combination of bending about one or both principal axes and axial compression (see Figure 3I) shall be so proportioned that:

$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b1}}{F_{b1}'\left[1-(f_c/F_{cE1})\right]} + \frac{f_{b2}}{F_{b2}'\left[1-(f_c/F_{cE2})-(f_{b1}/F_{bE})^2\right]} \leq 1.0 \tag{3.9-3}$$

where:

$$f_c < F_{cE1} = \frac{K_{cE}\,E'}{(\ell_{e1}/d_1)^2} \text{ for either unaxial or biaxial bending}$$

and

$$f_c < F_{cE2} = \frac{K_{cE}\,E'}{(\ell_{e2}/d_2)^2} \text{ for biaxial bending}$$

and

$$f_{b1} < F_{bE} = \frac{K_{bE}\,E'}{(R_B)^2} \text{ for biaxial bending}$$

f_{b1} = actual edgewise bending stress (bending load applied to narrow face of member)

f_{b2} = actual flatwise bending stress (bending load applied to wide face of member)

d_1 = wide face dimension (see Figure 3I)

d_2 = narrow face dimension (see Figure 3I)

Effective column lengths, ℓ_{e1} and ℓ_{e2}, shall be determined in accordance with 3.7.1.2. F_c', F_{cE1} and F_{cE2} shall be determined in accordance with 2.3 and 3.7. F_{b1}', F_{b2}' and F_{bE} shall be determined in accordance with 2.3 and 3.3.3.

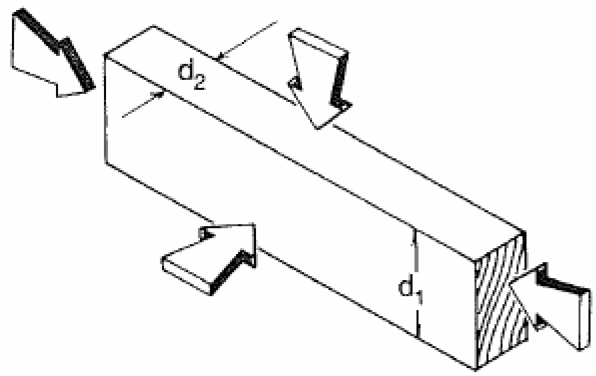
3.9.3 Eccentric Compression Loading

See 15.4 for members subjected to combined bending and axial compression due to eccentric loading, or eccentric loading in combination with other loads.

3 DESIGN PROVISIONS AND EQUATIONS

The combination of bending and axial compression is more critical due to the P-Δ effect. The bending produced by the transverse loading causes a deflection Δ. The application of the axial load, P, then results in an additional moment P*Δ; this is also know as second order effect because the added bending stress is not calculated directly. Instead, the common practice in design specifications is to include it by increasing (amplification factor) the computed bending stress in the interaction equation.

Figure 3I Combined Bending and Axial Compression



The most common case involves axial compression combined with bending about the strong axis of the cross section. In this case, Equation (3.9-3) reduces to:

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{b1}}{F'_{b1}\left[1-\left(\frac{f_c}{F_{cE1}}\right)\right]} \leq 1.0$$

and, the amplification factor is a number greater than 1.0 given by the expression:

$$\left(\frac{1}{1-\left(\frac{f_c}{F_{E1}}\right)}\right) = \text{Amplification factor for } \boldsymbol{f_{b1}}$$

Example of Application:

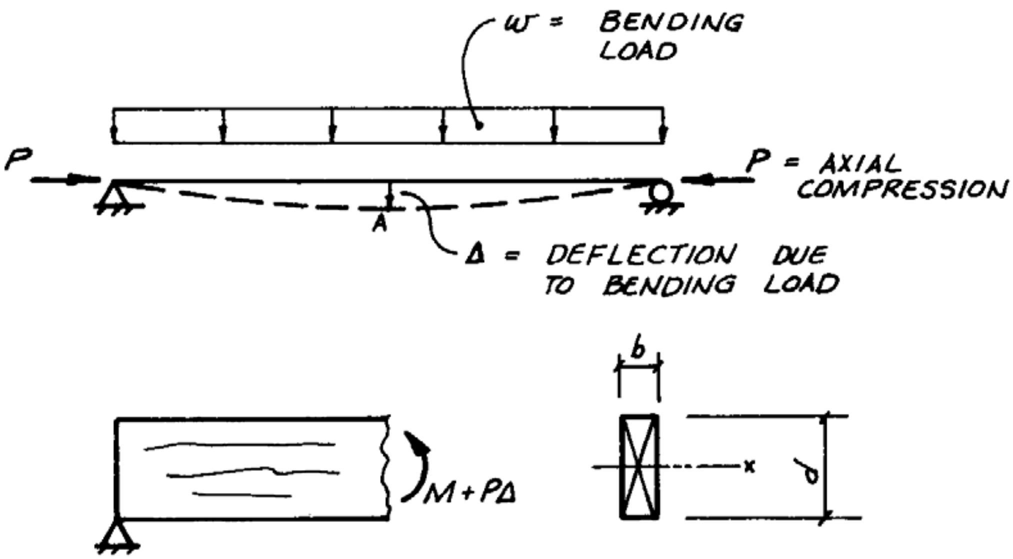


Figure 7.17a Deflected shape of beam showing P-Δ moment. The computed bending stress f_b is based on the moment M from the moment diagram. The moment diagram considers the effects of the transverse load w, but does not include the secondary moment $P \times \Delta$. The P-Δ effect is taken into account by amplifying the computed bending stress f_b.

The general interaction formula reduces to:

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{b1}}{F'_{b1}\left[1-\left(\frac{f_c}{F_{cE1}}\right)\right]} \leq 1.0$$

where:
f_c = actual compressive stress = P/A
F'_c = allowable compressive stress parallel to the grain = $F_c{*}C_D{*}C_M{*}C_t{*}C_F{*}\mathbf{C_P}{*}C_i$

Note: that F'_c includes the $\mathbf{C_P}$ *adjustment factor for stability (Sec. 3.7.1 NDS 01) to be considered in lengths of the column subject to buckling.*

C_P

3.7 Solid Columns

3.7.1 Column Stability Factor, C_P

3.7.1.1 When a compression member is supported throughout its length to prevent lateral displacement in all directions, $C_P = 1.0$.

3.7.1.2 The effective column length, ℓ_e, for a solid column shall be determined in accordance with principles of engineering mechanics. One method for determining effective column length, when end-fixity conditions are known, is to multiply actual column length by the appropriate effective length factor specified in Appendix G, $\ell_e = (K_e)(\ell)$.

3.7.1.3 For solid columns with rectangular cross section, the slenderness ratio, R_e/d, shall be taken as the larger of the ratios ℓ_{e1}/d_1 or ℓ_{e2}/d_2 (see Figure 3G) where each ratio has been adjusted by the appropriate buckling length coefficient, K_e, from Appendix G.

3.7.1.4 The slenderness ratio for solid columns, ℓ_e/d, shall not exceed 50, except that during construction ℓ_e/d shall not exceed 75.

3.7.1.5 The column stability factor shall be calculated as follows:

$$C_P = \frac{1+(F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1+(F_{cE}/F_c^*)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}} \qquad (3.7\text{-}1)$$

where:

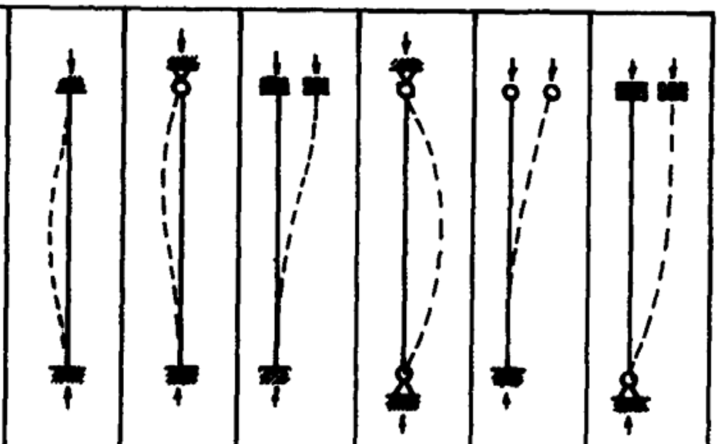
F_c^* = tabulated compression design value multiplied by all applicable adjustment factors except C_P (see 2.3)

$F_{cE} = \dfrac{K_{cE}\,E'}{(\ell_e/d)^2}$

K_{cE} = 0.510 - 0.839(COV_E)
= 0.3 for visually graded lumber
= 0.384 for machine evaluated lumber (MEL)
= 0.418 for products with $COV_E \le 0.11$ (see Appendix F.2)

c = 0.8 for sawn lumber
c = 0.85 for round timber poles and piles
c = 0.9 for glued laminated timber or structural composite lumber

The actual unsupported column length multiplied by the appropriate length factor in Appendix G of the NDS 01 yields the effective column length, l_e.



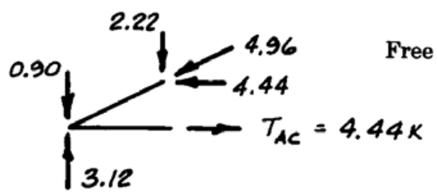
Buckling modes						
Theoretical K_e value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design K_e when ideal conditions approximated	0.65	0.80	1.2	1.0	2.10	2.4

End condition code

Rotation fixed, translation fixed
Rotation free, translation fixed
Rotation fixed, translation free
Rotation free, translation free

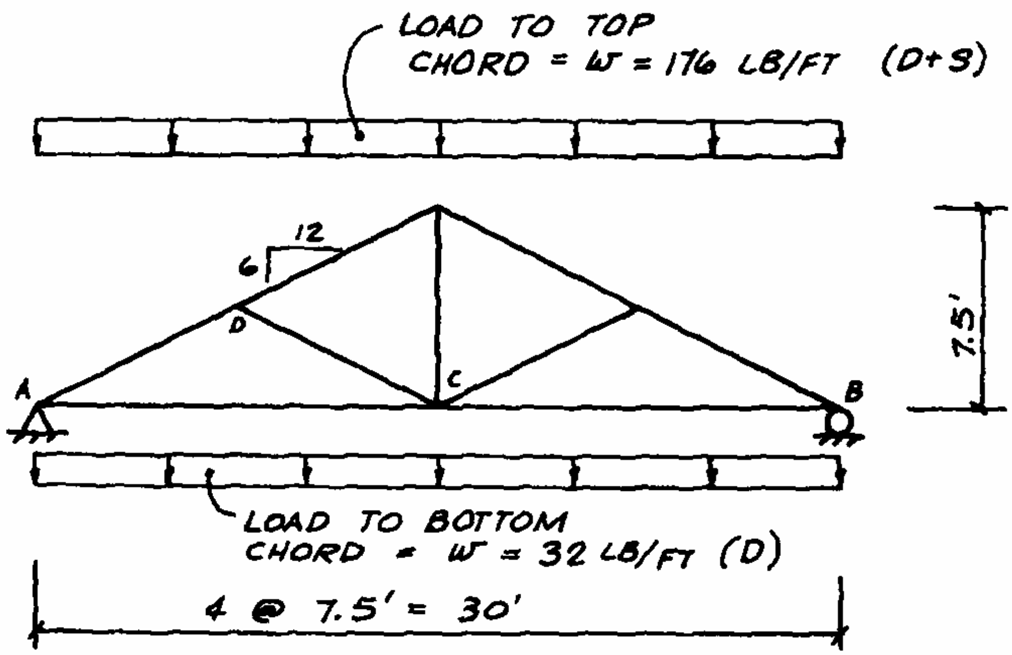
Design Problem (Sec. 7.13 Text)

The top chord of the truss analyzed in the case of tension and bending of the lower chord is considered for the case of combined compression and bending. The bending of the top chord is due to dead plus snow being applied along the length of the member. The truss analysis provides the forces in the member from A-D.



Free body diagram of joint *A*.

The length of the member from A-D is 8.39 feet. Although two load combinations, D-only, and D + S must be considered, it has been determined that the D + S combination controls the design and only those calculations are included herein.



Let's try a 2 X 8 Southern Pine No. 1 (Table 4B NDS Supplement 01)

F_B = *1500* psi
F_C = 1650 psi (parallel to the grain)
E = 1,700,000 psi

The tabulated values in 4B are size specific thus C_F (comp. II to the grain) and C_F (bending) are equal to 1.0.

Section properties for 2x8 are A = 10.88 in^2 and S = 13.14 in^3.

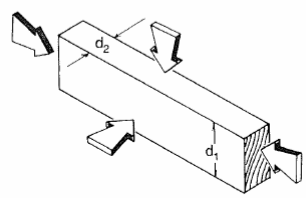
Axial check:

1. Stability: Column buckling can occur away from the truss joints. Use gross are to calculate f_c:

   $f_c = P/A = 4960/10.88 = 456$ *psi*

   lateral support is provided by the roof diaphragm, thus as the member is used edgewise, buckling is prevented about the weak axis (d_2 direction)

Figure 3I Combined Bending and Axial Compression



$$\left(\frac{l_e}{d}\right)_1 = \frac{8.39*12}{7.25} = 13.9\,ft$$

$E' = E\,(C_M)(C_t)$ = 1,700,000 (1)(1) = 1,700,000 psi

For visually graded sawn lumber (Section 3.7.1.5)

$K_{cE} = 0.3$

$c = 0.8$

$F_{cE} = (K_{cE}*E')/(l_e/d_1)^2$ = (0.3*1,7000,000)/$(13.9)^2$ = 2645 psi

$F_c^{**} = F_c(C_D)(C_M)(C_t)(C_F)(C_i)$ = 1650(1.15)(1.0)(1.0)(1.0)(1.0) = 1898 psi

Substituting in Eq. (3.7-1) results in C_P = 0.792

$F'_c = F^*_c\,(C_P)$= 1898 * 0.792 = 1502 psi > 456 psi OK!

Note that at the connections the reduced area should be used to check compression, but there is no possibility of buckling (braced location). Assuming a bolt diameter plus 1/8" for the opening diameter of 0.875", the net area is 1.5*(7.25-.875) = 9.56 in^2. Thus the acting axial stress is 4960/9.56 = 518 psi. The design stress is $F'_c = F^*_c$ = 1898 > 518 , OK!

Bending check:

Assume pinned ends for the chord member and take load and span on the horizontal plane:

$$M = \frac{wL^2}{8} = \frac{176(7.5)^2 * 12}{8 * 1000} = 14.9 \text{ in-k}$$

$f_b = M/S$ = 14.9* 1000/13.14 = 1130 psi

The truss chord acting as a beam has full lateral support and the lateral stability factor C_l = 1.0 and $F'_b = 1500 * 1.15 = 1725$ *psi* > *1130 psi* OK!

Combined Stress check:

There is no bending about the weak axis, and the axial load is concentric. Thus, Equation (3.9-3) reduces to:

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{b1}}{F'_{b1}\left[1 - \left(\frac{f_c}{F_{cE1}}\right)\right]} \leq 1.0$$

The load duration factor that controls is snow, C_D = 1.15. Therefore, the previously determined values of F'_C and F'_b are still valid for use in the interaction formula. The elastic buckling factor depends on the slenderness ratio about the strong axis of 13.92. It must be noted that the buckling stress, F_{cE}, for the axial check is based on the $(l_e/d)_{max}$ whereas for the P-Δ effect is based on the axis about which the bending moment occurs (strong axis based on d_1). In this example, it is a coincidence that the two values of the buckling stress, F_{cE}, are the same.

F_{cE} = 2645 psi

$$\left(\frac{456}{1502}\right)^2 + \left(\frac{1}{1 - \left(\frac{456}{2645}\right)}\right)\frac{1130}{1725} = 0.884 < 1.0 \quad \text{OK! Use 2 x 8 No. 1 Southern Pine}$$

It is appropriate in an actual design situation to check the D-load only case with a load duration factor, C_D, of 0.9.