INTRODUCTION TO STRUCTURAL DESIGN OF REINFORCED MASONRY

In the preceding lecture on structural design of masonry, we have seen examples of unreinforced masonry bearing walls. In bearing walls, when the plan length of openings exceeds approximately one-half the total plan length of the wall, and lateral forces act on the walls whether in-plane or out of plane, flexural tensile stresses generally become so large that the walls must be designed as reinforced. Reinforcement can also be used to increase shear resistance.

Unreinforced masonry and reinforced masonry are defined from the perspective of design approach. Unreinforced masonry is designed assuming that flexural tensile stresses are resisted by masonry. Any stresses in steel are ignored. Reinforced masonry, conversely, is designed assuming that flexural tensile stresses are resisted by reinforcement alone. The flexural tensile resistance of masonry is neglected.

Using the above definition, unreinforced masonry can actually have reinforcement (for integrity or to meet prescriptive requirements).

HOW REINFORCEMENT IS USED IN MASONRY ELEMENTS

Masonry Beams

These require horizontal reinforcement placed in bond-beam units, or in fully grouted cavities between brick walls.
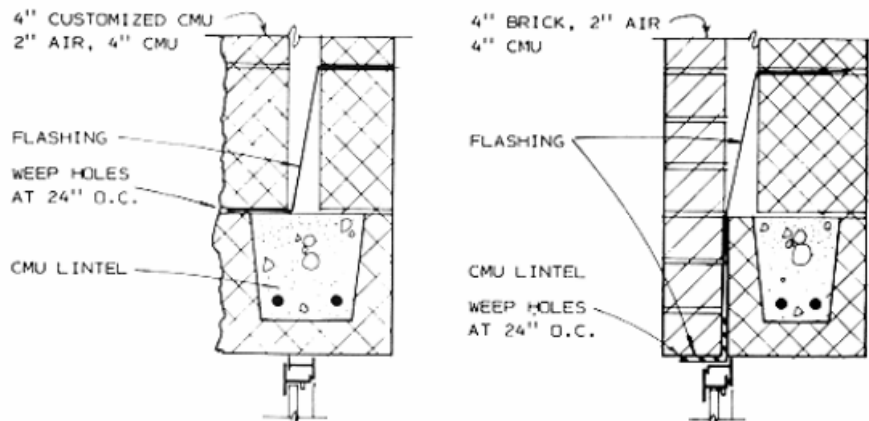
Masonry Columns and Pilasters

A column is an isolated element, meeting certain dimensional restrictions, that carries axial load and moment. A pilaster is a column that forms part of a wall, and projects out from the wall. Masonry columns can be made with solid units or hollow units. If solid units are used, they are formed to make a box. A cage of reinforcement is placed in the box, which is then filled with grout or concrete. In such applications, the solid masonry units are essentially used as stay-in-

place cover and formwork with structural function. If hollow units are used, they are laid in an overlapping pattern. Reinforcement is placed in the cells, which are then filled with grout.

Figure 1—16 in. (406 mm) Pilaster Sections
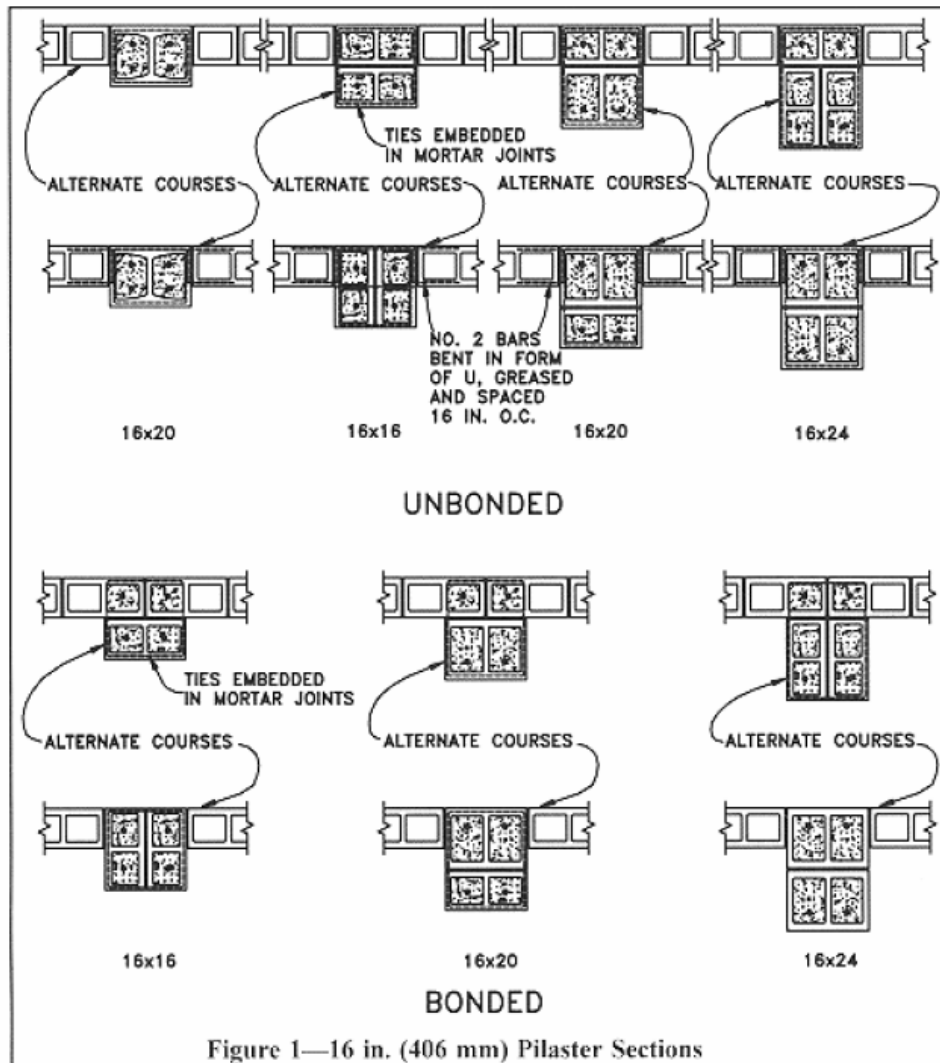
Masonry Walls

When solid units are used, masonry walls are reinforced horizontally with bed joint reinforcement. Alternatively, the wall can be constructed in two wythes, and a curtain of reinforcement is placed between the wythes, and grout is then poured between the wythes. In other countries (but rarely in the US), masonry walls laid with solid units are reinforced by continuous horizontal and vertical elements of reinforced concrete. This type of masonry is sometimes referred to as "confined masonry." When masonry walls are made of hollow units, vertical reinforcement is placed in grouted cells, and horizontal reinforcement consists of bed-joint reinforcement, placed in the bed joints, or deformed horizontal reinforcement, placed in bond-beam units or units with cut-out webs.
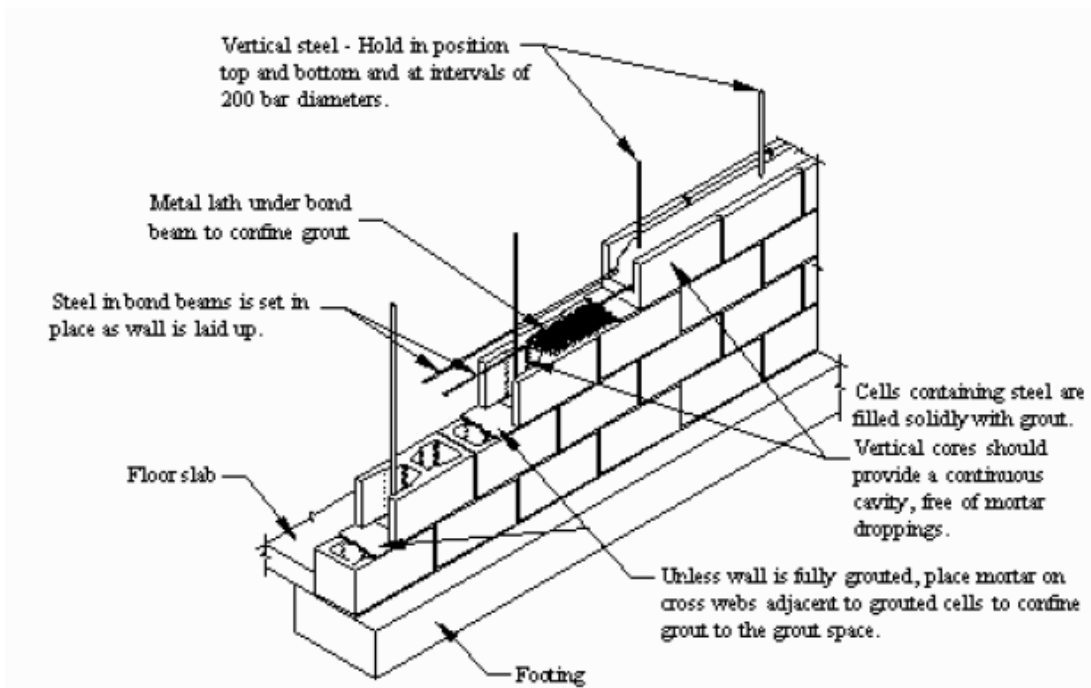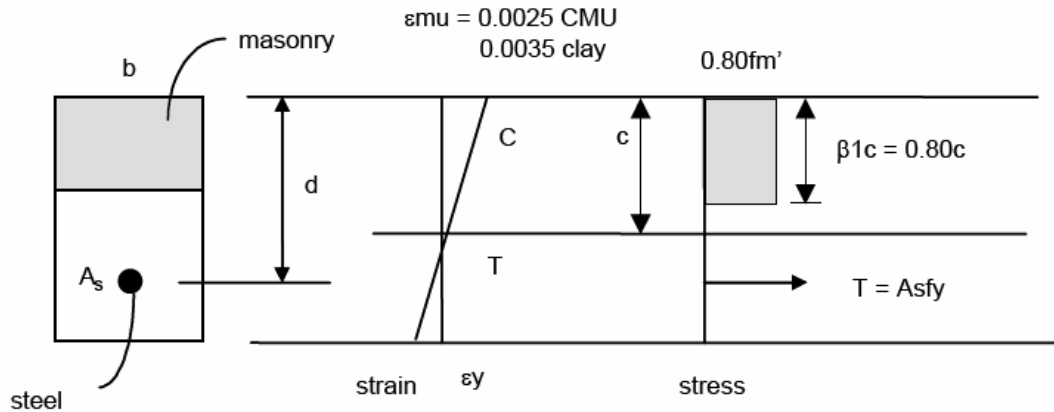
Figure 2—Reinforced Concrete Masonry Construction

DESIGN OF REINFORCED MASONRY BEAMS

BACKGROUND ON STRENGTH DESIGN OF REINFORCED MASONRY BEAMS FOR FLEXURE
(Chapter 3 ACI 530-05)

Strength design of reinforced masonry beams follows the same steps used for reinforced concrete beams.

Strain in the masonry is assumed to have a maximum useful value of 0.0025 for concrete masonry and 0.0035 for clay masonry [Sec. 3.3.2 (c)]. Tension reinforcement is assumed to be somewhere on the yield plateau. Because axial load is zero, flexural capacity is equal to either the tension force or the compression force on the cross-section, multiplied by the internal lever arm (the distance between the tensile and compressive forces).
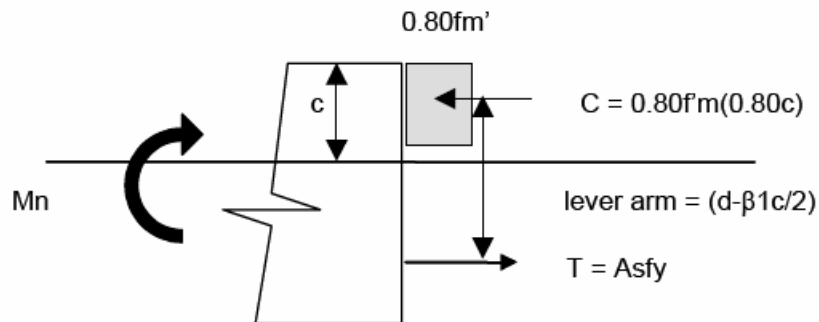
$$M_n = A_s f_y \left(d - \frac{\beta_1 c}{2} \right)$$

The depth of the compression block can be determined from equilibrium of horizontal forces.

$$T = C$$
$$A_s f_y = 0.80 f'_m \beta_1 c b$$
$$\beta_1 c = \frac{A_s f_y}{0.80 f'_m b}$$

Now define $\rho \equiv \dfrac{A_s}{bd}$ and $\omega \equiv \rho \left(\dfrac{f_y}{f'_m} \right)$. Then

$$M_n = A_s f_y \left(d - \frac{\beta_1 c}{2} \right)$$
$$M_n = A_s f_y \left(d - \frac{A_s f_y}{2 \cdot 0.80 f'_m b} \right)$$
$$M_n = \rho b d f_y \left(d - \frac{\rho b d f_y}{2 \cdot 0.80 f'_m b} \right)$$
$$M_n = \rho b d f_y \left(\frac{f'_m}{f'_m} \right) \left(d - \frac{\rho b d f_y}{2 \cdot 0.80 f'_m b} \right)$$
$$M_n = \omega b d f'_m \left(d - \frac{\omega d}{1.6} \right)$$

And finally,

$$\boxed{M_n = \omega b d^2 f'_m \left(1 - 0.63 \omega \right)}$$

This closed-form expression permits solving for the required dimensions if the steel percentage is known. The variable ω is sometimes referred to as the "tensile reinforcement index."
The steel percentage is constrained by the requirement that the steel be on the yield plateau when the masonry reaches its maximum useful strain. For this condition to be satisfied, the steel must yield before the masonry reaches its maximum useful strain. In other words, the steel percentage must be less than the balanced steel percentage, at which the steel yields just as the masonry reaches its maximum useful strain.

The balanced steel percentage for strength design can be derived based on the strains in steel and masonry:

First, locate the neutral axis under balanced conditions:

$$\frac{\varepsilon_{mu}}{\varepsilon_y} = \frac{c}{d-c}$$

$$c\varepsilon_y = \varepsilon_{mu}(d-c)$$

$$c(\varepsilon_y + \varepsilon_{mu}) = \varepsilon_{mu}d$$

$$c = d\left(\frac{\varepsilon_{mu}}{\varepsilon_y + \varepsilon_{mu}}\right)$$

Next, compute the compressive force under those conditions, and compute balanced steel area as the steel area, acting at yield, that is necessary to equilibrate that compressive force:

$$T = C$$

$$A_{sb}f_y = 0.80 f_m' \beta_1 cb$$

$$A_{sb}f_y = 0.80 f_m' \beta_1 bd\left(\frac{\varepsilon_{mu}}{\varepsilon_y + \varepsilon_{mu}}\right)$$

$$\rho_b = 0.80\left(\frac{f_m'}{f_y}\right)\beta_1\left(\frac{\varepsilon_{mu}}{\varepsilon_y + \varepsilon_{mu}}\right)$$

SUMMARY OF FLEXURAL DESIGN UNDER STRENGTH PROVISIONS

1) Estimate the steel percentage, ρ, as some portion of the strength balanced steel percentage.

2) Given the width, *b,* find the corresponding required effective depth, *d,* and the corresponding required total depth, *t*.

3) Iterate as necessary.

STRUCTURAL DESIGN OF REINFORCED MASONRY BEAMS

The most common reinforced masonry beam is a lintel. Lintels are beams that support masonry over openings. Lintel design follows the same basic steps, whether allowable-stress or strength design is used:

1) Shear design: Calculate the design shear, and compare it with the corresponding resistance. Revise the lintel depth if necessary.

2) Flexural design:

a) Calculate the design moment.
b) Calculate the required flexural reinforcement. Check that it fits within minimum and maximum reinforcement limitations.

In many cases, the depth of the lintel is determined by architectural considerations. In other cases, it is necessary to determine the number of courses of masonry that will work as a beam. For example, consider the lintel in the figure below. The depth of the beam, and hence the area that is effective in resisting shear, is determined by the number of courses that we consider to comprise it. Because it is not very practical to put shear reinforcement in masonry beams, the depth of the beam may be determined by this. In other words, the beam design may start with the number of courses that are needed to that shear can be resisted by masonry alone.
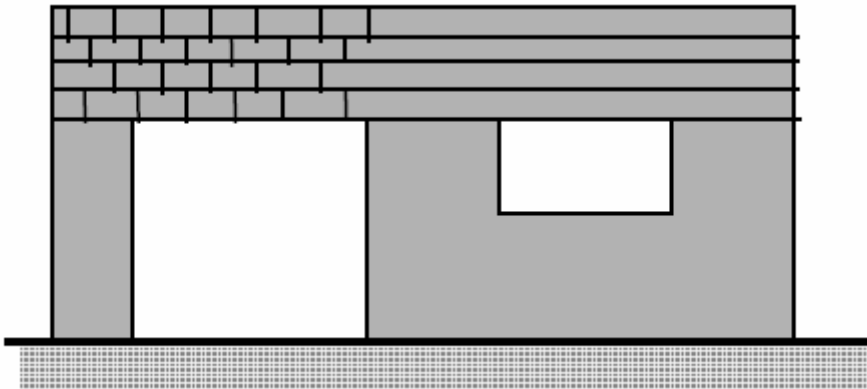
Example: Lintel Design according to Strength Provisions

Suppose that we have a uniformly distributed load of 1050 lb/ft, applied at the level of the roof of the structure shown below. Design the lintel. Assume fully grouted clay masonry with a nominal thickness of 8 in., a weight of 80 lb/ft2, and specified design strength of 1500 lb/in.2. The lintel has a span of 10 ft, and a total depth (height of parapet plus distance between the roof and the lintel) of 4 ft. These are shown in the schematic figure below. The design presumes that entire height of the lintel is grouted.
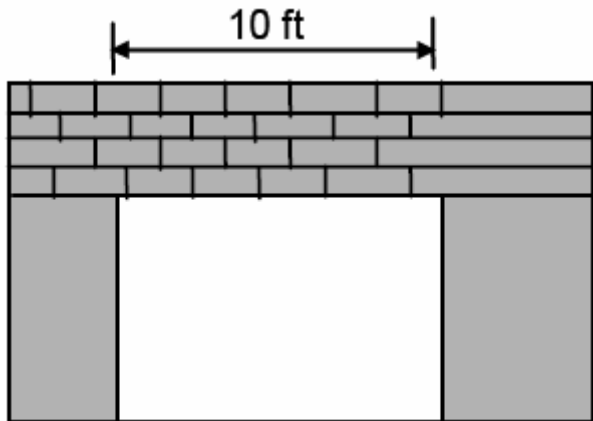
First check whether the depth of the lintel is sufficient to avoid the use of shear reinforcement. Because the opening may have a movement joint on either side, use a conservative span equal to the clear distance, plus one-half unit on each side. So the span is 10 ft plus 16 in., or 11.33 ft. Assume that 700 lb/ft of the roof load is D, and the remaining 350 lb/ft is L. The governing loading combination is 1.2D plus 1.6L.Calculate maximum bending moment and shear force:

$$M_u = \frac{w_u l^2}{8} = \frac{[(700 + 4 \cdot 80) \cdot 1.2 + 350 \cdot 1.6] \cdot 11.33^2 \, lb - ft \cdot 12 \, in./ft}{8} = 343{,}515 \, in. - lb$$

$$V_u = \frac{wl}{2} = \frac{[(700 + 4 \cdot 80) \cdot 1.2 + 350 \cdot 1.6] \cdot 11.33 \, lb}{2} = 10{,}106 \, lb$$

Because this is a reinforced element, shear capacity is calculated using Section 3.3.4.1.2.1 of the 2005 MSJC Code:

$$V_m = \left[4.0 - 1.75 \left(\frac{M}{Vd_v} \right) \right] A_n \sqrt{f'_m} + 0.25P$$

As (M/Vd_v) increases, V_m decreases. Because (M/Vd_v) need not be taken greater than 1.0 (2005 MSJC Code Section 3.3.4.1.2.1), the most conservative (lowest) value of V_m is obtained with (M/Vd_v) equal to 1.0. Also, axial load, P, is zero:

$$V_m = \left[4.0 - 1.75(1.0)\right] A_n \sqrt{f'_m}$$

$$V_m = 2.25 A_n \sqrt{f'_m}$$

$$V_u = 10,106 \ lb \le \phi V_n = 0.8 \cdot 2.25 \cdot 7.63 \ in. \cdot 46 \ in. \sqrt{1500} = 24,468 \ lb$$

Also according to Equation (3-20)

$$V_n \le 4 \sqrt{f_m^1}$$

This does not govern, and the shear design is acceptable. Now check the required flexural reinforcement:

$$M_n = A_s f \ (lever \ arm)$$

$$M_n \approx A_s f_y \cdot 0.9d$$

In our case,

$$M_n^{required} = \frac{M_u}{\phi} = \frac{M_u}{0.9} = \frac{343,515 \ lb-in.}{0.9} = 381,684 \ lb-in.$$

$$A_s^{required} \approx \frac{M_n^{required}}{0.9 d f_y} = \frac{381,684 \ lb-in.}{0.9 \cdot 44 \ in. \cdot 60,000 \ lb/in.^2} = 0.16 \ in.^2$$

Because of the depth of the beam, this can easily be satisfied with a #4 bar. Also include 2-#4 bars at the level of the roof (bond beam reinforcement). The flexural design is quite simple.

The 2005 MSJC Code has no minimum reinforcement requirements for flexural members. This will probably be addressed in future editions of the Code.
Section 3.3.4.2.2.2 of the 2005 MSJC Code does require that the nominal flexural strength of a beam not be less than 1.3 times the nominal cracking capacity, calculated using the modulus of rupture from 3.1.8.2 and Table 3.1.8.2.1. In our case, the nominal cracking moment for the 4-ft deep section is

$$M_{cr} = S \, f_r = \frac{bt^2}{6} f_r = \frac{7.63 \cdot 48^2 \, in.^3}{6} \cdot 250 \, lb/in.^2 = 732{,}480 \, in. - lb$$

This value, multiplied by 1.3, is 952,224 in.-lb, which exceeds the nominal capacity of this lintel with the provided #4 bar. Flexural reinforcement must be increased to

$$A_s \approx 0.16 \, in.^2 \left(\frac{952{,}224}{381{,}684} \right) = 0.40 \, in.$$

Use a #6 bar.

Finally, Section 3.3.3.5 of the 2005 MSJC imposes maximum flexural reinforcement limitations that are based on a series of critical strain gradients. As explained in the Commentary, the limitations can in some cases be more severe than those used in the past. They generally do not govern for members with little or no axial load, like this lintel. They may govern for members with significant axial load, such as tall shear walls.

## DESIGN OF REINFORCED MASONRY BEAM-COLUMNS

Introduction

Reinforced masonry beam-columns are reinforced masonry elements subjected to combinations of axial force and flexure. Reinforced masonry beam-columns, like those of reinforced concrete, are designed using moment-axial force interaction diagrams. Combinations of axial force and moment lying inside the diagram represent permitted designs; combinations lying outside, prohibited ones. Unlike reinforced concrete, however, reinforced masonry beam-columns rarely take the form of isolated rectangular elements with four longitudinal bars and transverse ties. The most common form for a reinforced masonry beam-column is a wall, loaded out-of-plane by eccentric gravity load, alone or in combination with wind.

<u>**BACKGROUND ON MOMENT-AXIAL FORCE INTERACTION DIAGRAMS BY THE STRENGTH APPROACH**</u>

Using the strength approach, we seek to construct interaction diagrams that represent combinations of axial and flexural capacity. This can be done completely by hand, or with the help of a spreadsheet.

Strength Interaction Diagrams Hand-Calculations

By hand, we can compute three points (pure compression, pure flexure, and the balance point). We then draw a straight line between pure compression and the balance point, and either a straight line or an appropriate curve between the balance point and pure flexure. This approach is commonly applied to reinforced concrete columns.
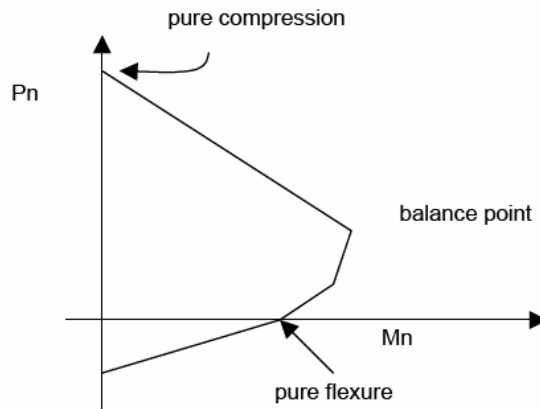
Pure Compression

$$P_0 = 0.80 \cdot 0.80 \, f_m'(A_c - A_{st}) + A_{st}f_y$$

As in ACI 318-05, the leading factor of 0.80 in effect imposes a minimum design eccentricity.

Pure Flexure

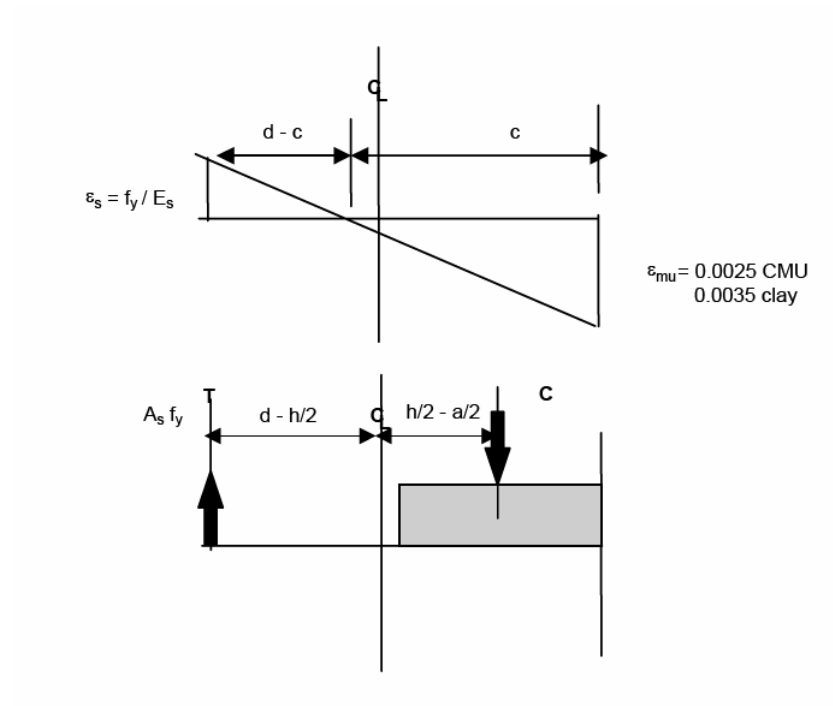As before, the possible contribution of compressive reinforcement is small, and can be neglected.

$$\rho = \frac{A_s}{bd}$$

$$\omega = \rho\left(\frac{f_y}{f_m'}\right)$$

$$M_n = \omega b d^2 f_m'(1 - 0.63\omega)$$

Balance Point

$$\frac{c}{d-c} = \frac{\varepsilon_{mu}}{\varepsilon_y}$$

$$c = d\left(\frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y}\right)$$

Next calculate the corresponding tensile and compressive forces:

$$T = A_s f_y$$
$$C = 0.80 f_m'(\beta_1 c)b$$
$$P_n = C - T$$
$$M_n = T\left(d - \frac{h}{2}\right) + C\left(\frac{h}{2} - \frac{\beta_1 c}{2}\right)$$

Example: Moment-Axial Force Interaction Diagram by the Strength Approach (hand-calculation)

Construct the moment-axial force interaction diagram by the strength approach for a nominal 8-in. CMU wall, fully grouted, with $f_m' = 1500$ lb/in.2 and reinforcement consisting of #5 bars at 48 in., placed in the center of the wall. Compute the interaction diagram per foot of wall length.

For the case of a wall with reinforcement at mid-depth, the reference axis for moment is located at the plastic centroid (geometric centroid) of the cross section, which is also at mid-depth. This leads to results which appear considerably different from what we are used to for a symmetrically reinforced column. For example, using the geometric centroid (the level of the reinforcement) as the reference axis, the contribution of the reinforcement to the moment is always zero. In what is apparently even stranger, the balanced-point axial force does not coincide with the maximum moment capacity. As a result, hand calculations are useful for some reinforced masonry beam-columns, but not all.

Pure Compression

Because the compressive reinforcement in the wall is not supported laterally, it is not counted in calculated capacity.

$$P_0 = 0.80 \cdot 0.80\, f_m'\left(A_c - A_{st}\right) + A_{st} f_y$$
$$P_n = 0.80 \cdot 0.80 \cdot 1500\ lb/in.^2 \cdot (7.63 \cdot 48 - 0.31) + 0.31 \cdot 0\ lb/in.^2$$
$$P_n = 351{,}293\ lb$$

Per foot of wall length, the design capacity will be the above value, divided by 4 (the length of the wall in feet), and multiplied by the strength reduction factor of 0.90:

$\varphi P_n = 79{,}041$ lbs

Pure Flexure

As before, neglect the influence of compressive reinforcement:

$$d = \left(\frac{1}{2}\right) 7.63 \; in. = 3.81 \; in.$$

$$\rho = \frac{A_s}{bd} = \frac{0.31 \; in.^2}{48 \cdot 3.81 \; in.^2} = 1.70 \cdot 10^{-3}$$

$$\omega = \rho\left(\frac{f_y}{f_m'}\right) = (1.70 \cdot 10^{-3}) \cdot \left(\frac{60000}{1500}\right) = 0.0678$$

$$M_n = \omega bd^2 f_m'(1 - 0.63\omega)$$

$$M_n = 0.0678 \cdot 48 \, in. \cdot 3.81^2 \, in.^2 \cdot 1500 \, lb/in.^2 \cdot (1 - 0.63 \cdot 0.0678)$$

$$M_n = 67,838 \; lb - in.$$

Per foot of wall length, the design capacity is the above value, divided by 4 and multiplied by the strength reduction factor of 0.90:

$$\phi M_n = 15,264 \; lb - in.$$

Balance Point

First, locate the neutral axis:

$$c = d\left(\frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y}\right)$$

$$c = d\left(\frac{0.0025}{0.0025 + 0.0207}\right)$$

$$c = 0.55d$$

$$c = 0.55 \cdot 3.81 \; in. = 2.08 \; in.$$

The value of β_1 is prescribed in Section 3.3.2 (g) of the 2005 MSJC as 0.80:

$$T = A_s f_y = 0.31 in.^2 \cdot 60000\,lb/in.^2 = 18,600 \ lb$$

$$C = 0.80\, f_m^{'}(\beta_1 c)b = 0.80\, f_m^{'}(0.80 \cdot 0.55 \ d)\ b$$

$$C = 0.80 \cdot 1500\,lb/in^2\,(0.80 \cdot 0.55 \cdot 3.81 in.) \cdot 48\ in. = 96,561 \ lb$$

$$P_n = C - T = 77,961\,lb$$

$$M_n = T\left(d - \frac{h}{2}\right) + C\left(\frac{h}{2} - \frac{\beta_1 c}{2}\right)$$

$$M_n = 18,600\,lb\,(3.81 - 3.81)\,in. + 96,561\,lb\left(\frac{7.63}{2} - \frac{0.80 \cdot 0.55 \cdot 3.81}{2}\right)in.$$

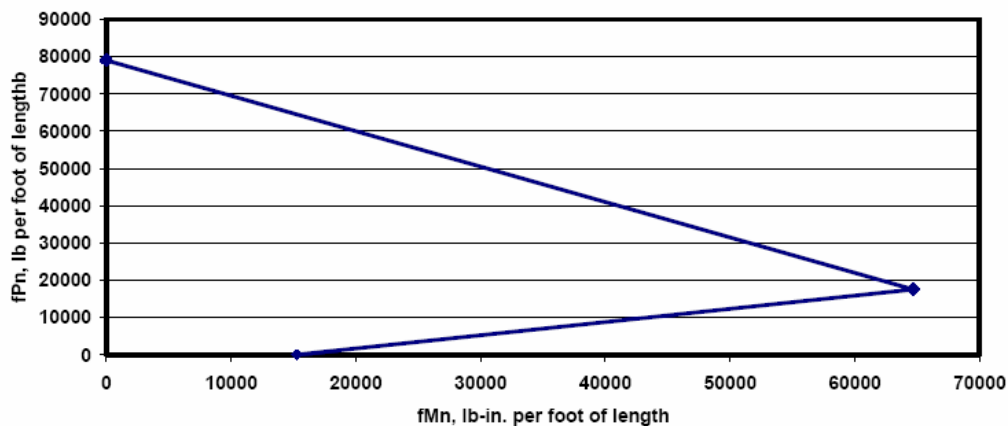$$M_n = 0\,lb - in. + 287,442\,lb - in. = 287,442 \ lb - in.$$

Per foot of wall length, the design capacities are the above values, divided by 4 and multiplied by the strength reduction factor of 0.90:

$$\phi P_n = 17,541 \ lb$$

$$\phi M_n = 64,674 \ lb - in.$$

Plot of Strength Interaction Diagram by Hand

Strength Interaction Diagram by Hand
8-in. solid CMU wall, f'm=1500 psi, #5 bars @ 48 in.

As we shall shortly see, the points that we have calculated are correct. The form of the diagram is misleading, however, because the balance point is actually not the point of maximum moment. It is incorrect to draw the diagram with a straight line from the balance point to the pure compression point. The balance point becomes the point of maximum moment as the reinforcement is placed farther apart than about 70% of the thickness of the wall.

Example: Design of Masonry Beam-Columns by the Strength Approach

Once we have developed the moment-axial force interaction diagram by the strength approach, the actual design simply consists of verifying that the combination of factored design axial force and moment lies within the diagram of nominal axial and flexural capacity, reduced by strength reduction factors.

Consider the bearing wall designed as unreinforced. It has an eccentric axial load plus out of plane wind load of 25 psf.
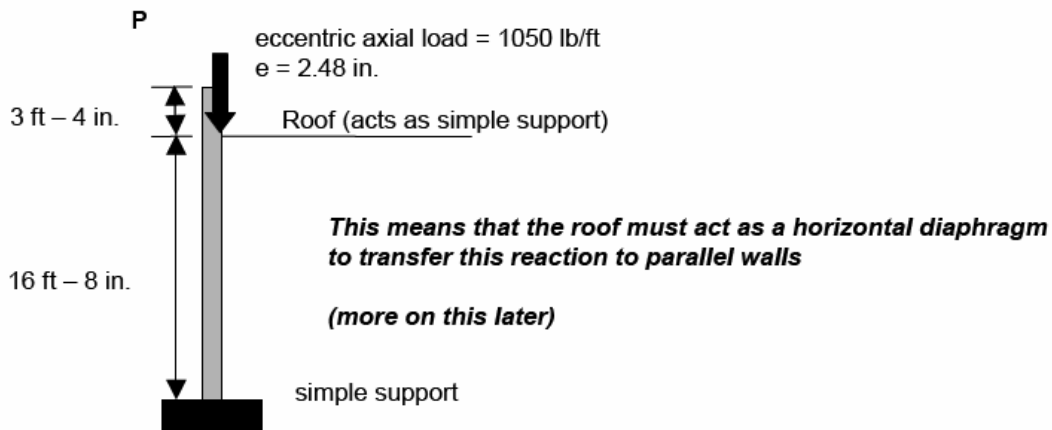
At each horizontal plane through the wall, the following condition must be met:

- *combinations of factored axial load and moment must lie within the moment- axial force interaction diagram, reduced by strength-reduction factors.*

Because flexural capacity increases with increasing axial load, the most critical loading combination is probably 0.9D + 1.6W.

From the unreinforced example, we know that the critical point on the wall is at or near the midspan of the wall. Due to wind only, the unfactored moment at the base of the parapet (roof level) is:

$$M = \frac{qL^2}{2} = \frac{25 \ lb/\ ft \cdot 3.33^2 \ ft^2}{2} \cdot 12 \ in./\ ft = 1663 \ lb - in.$$

The maximum moment is close to that occurring at mid-height. The moment from wind load is the superposition of approximately one-half moment at the upper support due to wind load on the parapet only, plus the midspan moment in a simply supported beam with that same wind load:

$$M_{midspan} = -\frac{1663}{2} + \frac{qL^2}{8} = -\frac{1663}{2} + \frac{25\ lb/ft \cdot 16.67^2\ ft^2}{8} \cdot 12\ in./ft = 9589\ lb - in.$$

Unfactored moment diagrams due to eccentric axial load and wind are as shown below:

M = Pe = 2604 lb-in.   1663 lb-in.

1302 lb-in.   9589 lb-in.
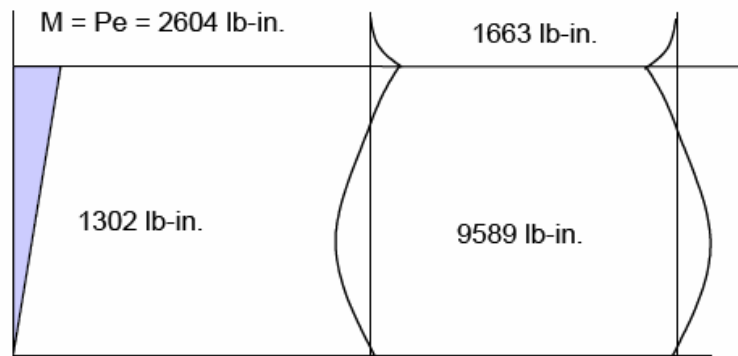
Check the adequacy of the wall with 8-in. nominal units, a specified compressive strength, f_m', of 1500 lb/in.2, and #5 bars spaced at 48 in. All design actions are calculated per foot of width of the wall. At the mid-height of the wall, the axial force due to 0.9D is:

$$P_u = 0.9(700\ lb) + 0.9(3.33\ ft + 8.33\ ft) \cdot 48\ lb/ft = 1134\ lb$$

At the mid-height of the wall, the factored design moment, M_u, is given by:

$$M_u = P_u \frac{e}{2} + M_{u\ wind} = \left(\frac{1}{2}\right) 0.9 \cdot 700\ lb \cdot 2.48\ in. + 1.6 \cdot 9589\ lb - in. = 16{,}124\ lb - in.$$

In each foot of wall, the design actions are P_u = 1134 lb, and M_u = 16,124 lb-in. That combination lies within the interaction diagram of design capacities, and the design is satisfactory.