We defined a bipartite graph as follows:
A bipartite graph is an undirected graph $G = (V, E)$ in which $V$ can be partitioned into two sets, $V_1$ and $V_2$, such that $(u, v) \in E$ implies either $u \in V_1$ and $v \in V_2$, or $u \in V_2$ and $v \in V_1$.

We say that a bipartite graph $G = (V, E)$ is $d$-regular if every vertex $v \in V$ has degree $d$ exactly, and $d \geq 1$.

Answer the following questions.

1. Show a four-regular bipartite graph with the smallest possible number of vertices.

2. Specify whether or not the following statement is correct. If it is correct, prove it. If it is incorrect, show a counter example.
   Statement: In a $d$-regular bipartite graph $G = (V, E)$, where $V$ can be partitioned into $V_1$ and $V_2$, we have that $|V_1| = |V_2|$.
   Answer: Correct / Incorrect
   Correct
   Proof or Counter Example:
   Counting the number of edges in the graph by counting the edges that meet vertices in $V_1$, we have that $|E| = d|V_1|$.
   Counting the number of edges in the graph by counting the edges that meet vertices in $V_2$, we have that $|E| = d|V_2|$.
   From $|E| = d|V_1| = d|V_2|$ we have that $|V_1| = |V_2|$.
We defined a bipartite graph as follows:
A bipartite graph is an undirected graph \( G = (V, E) \) in which \( V \) can be partitioned into two sets, \( V_1 \) and \( V_2 \), such that \((u, v) \in E\) implies either \( u \in V_1 \) and \( v \in V_2 \), or \( u \in V_2 \) and \( v \in V_1 \).

We say that a bipartite graph \( G = (V, E) \) is \( d \)-regular if every vertex \( v \in V \) has degree \( d \) exactly, and \( d \geq 1 \).

Answer the following questions.

1. Show a three-regular bipartite graph with the smallest possible number of vertices.

2. Specify whether or not the following statement is correct. If it is correct, prove it. If it is incorrect, show a counter example.

Statement: In a \( d \)-regular bipartite graph \( G = (V, E) \), where \( V \) can be partitioned into \( V_1 \) and \( V_2 \), we have that \( |V_1| = |V_2| \).

Answer: Correct / Incorrect

Proof or Counter Example: