Dynamic programming was used for finding the shortest paths in a directed graph with set of vertices \( \{A, B, C, D, E\} \), and resulted in the matrices below.

The matrix on the left provides the weights of the shortest paths.
The matrix on the right provides the intermediate vertices through which the shortest paths were obtained.

\[
\begin{array}{cccccc}
A & B & C & D & E \\
A & 0 & 1 & -3 & 2 & -4 \\
B & 3 & 0 & -4 & 1 & -1 \\
C & 7 & 4 & 0 & 5 & 3 \\
D & 2 & -1 & -5 & 0 & -2 \\
E & 8 & 5 & 1 & 6 & 0 \\
\end{array}
\quad
\begin{array}{cccccc}
A & B & C & D & E \\
A & - & C & D & E & A \\
B & D & - & D & B & A \\
C & D & C & - & B & A \\
D & D & C & D & - & A \\
E & D & C & D & E & - \\
\end{array}
\]

1. What is the weight of the shortest path from \( A \) to \( B \)?
2. What is the shortest path from \( A \) to \( B \)?

Answers:

1. Weight of the shortest path from \( A \) to \( B \):
   From the \((A, B)\) entry of the matrix on the left, it is one.

2. Shortest path from \( A \) to \( B \):
   Using the matrix on the right:
   \((A, B)\) goes through \( C \) to yield \(< A, C, B >\).
   \((A, C)\) goes through \( D \) to yield \(< A, D, C, B >\).
   \((A, D)\) goes through \( E \) to yield \(< A, E, D, C, B >\).
   \((A, E)\) goes through \( A \), no change.
   \((E, D)\) goes through \( E \), no change.
   \((D, C)\) goes through \( D \), no change.
   \((C, B)\) goes through \( C \), no change.

Note that it is necessary to use the matrix on the right.
"Guessing" by using entries from the matrix on the left may leave subpaths (instead of edges) in the shortest path.
Dynamic programming was used for finding the shortest paths in a directed graph with set of vertices \( \{ A, B, C, D, E \} \), and resulted in the matrices below. The matrix on the left provides the weights of the shortest paths. The matrix on the right provides the intermediate vertices through which the shortest paths were obtained.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>1</td>
<td>-3</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>0</td>
<td>-4</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>7</td>
<td>4</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>-1</td>
<td>-5</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>A</th>
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<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>C</td>
<td>D</td>
<td>E</td>
</tr>
<tr>
<td>B</td>
<td>D</td>
<td>-</td>
<td>D</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
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<td>C</td>
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<td>D</td>
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<td>D</td>
<td>E</td>
</tr>
</tbody>
</table>

1. What is the weight of the shortest path from \( C \) to \( E \)?
2. What is the shortest path from \( C \) to \( E \)?

Answers:

1. Weight of the shortest path from \( C \) to \( E \):
   \[ 3 \]
2. Shortest path from \( C \) to \( E \):
   \[ < C, B, D, A, E > \]
   See solution on the previous page.