For the following claim, prove that it is correct or show a counter example.

Claim: If $f(n)$ is $O(g(n))$ and $f(n)$ is $\Omega(g(n))$ then $g(n)$ is $\Theta(f(n))$.

Answer: The claim is correct.

Proof/counter example:
If $f(n)$ is $O(g(n))$, then there exist $n_1 > 0$ and $c_1 > 0$ such that $f(n) \leq c_1 g(n)$ for all $n \geq n_1$. This implies that $g(n) \geq \frac{1}{c_1} f(n)$ for all $n \geq n_1$.

If $f(n)$ is $\Omega(g(n))$, then there exist $n_2 > 0$ and $c_2 > 0$ such that $f(n) \geq c_2 g(n)$ for all $n \geq n_2$. This implies that $g(n) \leq \frac{1}{c_2} f(n)$ for all $n \geq n_2$.

Using $n_0 = \max \{n_1, n_2\}$, we have that $\frac{1}{c_1} f(n) \leq g(n) \leq \frac{1}{c_2} f(n)$ for all $n \geq n_0$. This implies that $g(n)$ is $\Theta(f(n))$. 