ECE608, Fall 2016, Quiz 3

Last Name: ___________________ First Name: ____________________

I certify that I have neither given nor received unauthorized aid on this quiz.

Signed: ___________________

Use only the space provided on this page to answer the following question(s).
Do not write your answers on the other side of the page.

Evaluate the following product. Write your answer in the simplest possible form.

\[
\prod_{k=1}^{n} (3 \cdot 9^k) =
\]

The product has the form

\[
\prod_{k=1}^{n} (a \cdot a^{2^k}) = \prod_{k=1}^{n} (a^{2k+1})
\]

Using base-\(a\) logarithms,

\[
\log_a \prod_{k=1}^{n} (a^{2^k+1}) = \sum_{k=1}^{n} \log_a (a^{2^k+1}) = \sum_{k=1}^{n} 2k + 1 = ((2 \cdot 1 + 1) + (2n + 1)) \cdot n/2 =
\]

\[
(4 + 2n)n/2 = (2 + n)n.
\]

We have that

\[
\log_a \prod_{k=1}^{n} (a^{2^k+1}) = (2 + n)n
\]

Therefore,

\[
\prod_{k=1}^{n} (a^{2^k+1}) = a^{(2+n)n}.
\]

With \(a = 3\), the product evaluates to \(3^{(2+n)n}\).
Evaluate the following product. Write your answer in the simplest possible form.

\[
\prod_{k=1}^{n} (4 \cdot 16^k) =
\]

Using the solution on the previous page with \( a = 4 \), the product evaluates to \( 4^{(2+n)n} \).