ECE608, Fall 2016, Quiz 1

Last Name: ___________________ First Name: ____________________

I certify that I have neither given nor received unauthorized aid on this quiz.

Signed: ___________________

Use only the space provided on this page to answer the following question(s).
Do not write your answers on the other side of the page.

Answer the questions below with respect to the following problem.

Problem:
Input: Finite set $A$ and a size $s(a) \in \mathbb{Z}^+$ for each $a \in A$.
Output: A subset $A' \subseteq A$ such that $\displaystyle \sum_{a \in A'} s(a) = \sum_{a \in A \setminus A'} s(a)$.

(1) Show a set $A$ with five elements for which a set $A'$ exists.
(2) Show a set $A$ with four elements for which a set $A'$ does not exist.
(3) Explain why the problem is a combinatorial problem. Be as accurate as you can.
   For this part you should consider the given problem and not the instances that you found in parts 1 and 2.

Answer:

(1) $s(a_1) = 4$, $s(a_2) = 5$, $s(a_3) = 6$, $s(a_4) = 7$, $s(a_5) = 8$.
$A' = \{a_1, a_2, a_3\}$.

(2) $s(a_1) = 1$, $s(a_2) = 2$, $s(a_3) = 4$, $s(a_4) = 8$.
A subset that contains $a_4$ will be larger than the other subset.

(3) The set of all possible solutions consists of all the subsets of $A$. The number of subsets is $2^{|A|}$, which is finite.
Problem:
Input: Finite set $A$ and a size $s(a) \in \mathbb{Z}^+$ for each $a \in A$.
Output: A subset $A' \subseteq A$ such that $\sum_{a \in A'} s(a) = \sum_{a \in A \setminus A'} s(a)$.

(1) Show a set $A$ with four elements for which a set $A'$ exists.
(2) Show a set $A$ with five elements for which a set $A'$ does not exist.
(3) Explain why the problem is a combinatorial problem. Be as accurate as you can.
   For this part you should consider the given problem and not the instances that you found in parts 1 and 2.

Answer:

(1) $s(a_1) = 1$, $s(a_2) = 2$, $s(a_3) = 3$, $s(a_4) = 4$,
    $A' = \{a_1, a_4\}$.

(2) $s(a_1) = 1$, $s(a_2) = 2$, $s(a_3) = 4$, $s(a_4) = 8$, $s(a_5) = 16$.
    A subset that contains $a_5$ will be larger than the other subset.

(3) The set of all possible solutions consists of all the subsets of $A$. The number of subsets is $2^{|A|}$, which is finite.