ECE608, Fall 2015, Quiz 2

Last Name: ___________________ First Name: ____________________

I certify that I have neither given nor received unauthorized aid on this quiz.

Signed: ___________________

Use only the space provided on this page to answer the following question(s).
Do not write your answers on the other side of the page.

For the following claim, indicate whether it is true or false. If it is true, prove it. If it is false, show a counter example.

If $f_1(n)$ is $\Theta(g_1(n))$ and $f_2(n)$ is $\Theta(g_2(n))$, then $(f_1 + f_2)(n)$ is $\Theta(\max \{ g_1(n), g_2(n) \})$.

True.

Since $f_1(n)$ is $\Theta(g_1(n))$, there exist positive constants $c_{11}, c_{12}$ and $n_{10}$ such that $0 \leq c_{11} g_1(n) \leq f_1(n) \leq c_{12} g_1(n)$ for all $n \geq n_{10}$.

Since $f_2(n)$ is $\Theta(g_2(n))$, there exist positive constants $c_{21}, c_{22}$ and $n_{20}$ such that $0 \leq c_{21} g_2(n) \leq f_2(n) \leq c_{22} g_2(n)$ for all $n \geq n_{20}$.

Therefore, $(f_1 + f_2)(n) \leq 2 \max \{ f_1(n), f_2(n) \} \leq 2 \max \{ c_{12} g_1(n), c_{22} g_2(n) \} \leq 2 \max \{ c_{12}, c_{22} \} \max \{ g_1(n), g_2(n) \}$ for $n \geq n_0 = \max \{ n_{10}, n_{20} \}$.

In addition, $(f_1 + f_2)(n) \geq \max \{ f_1(n), f_2(n) \} \geq \max \{ c_{11} g_1(n), c_{21} g_2(n) \} \geq \min \{ c_{11}, c_{21} \} \max \{ g_1(n), g_2(n) \}$ for $n \geq n_0 = \max \{ n_{10}, n_{20} \}$.

With $c_1 = \min \{ c_{11}, c_{21} \}$, $c_2 = 2 \max \{ c_{12}, c_{22} \}$ and $n_0 = \max \{ n_{10}, n_{20} \}$, we have that $0 \leq c_1 \max \{ g_1(n), g_2(n) \} \leq (f_1 + f_2)(n) \leq c_2 \max \{ g_1(n), g_2(n) \}$ for all $n \geq n_0$. 
For the following claim, indicate whether it is true or false. If it is true, prove it. If it is false, show a counter example.

If $f_1(n)$ is $\Theta(g_1(n))$ and $f_2(n)$ is $\Theta(g_2(n))$, then $(f_1 \cdot f_2)(n)$ is $\Theta(g_1(n) \cdot g_2(n))$.

True.

Since $f_1(n)$ is $\Theta(g_1(n))$, there exist positive constants $c_{11}, c_{12}$ and $n_{10}$ such that $0 \leq c_{11}g_1(n) \leq f_1(n) \leq c_{12}g_1(n)$ for all $n \geq n_{10}$.

Since $f_2(n)$ is $\Theta(g_2(n))$, there exist positive constants $c_{21}, c_{22}$ and $n_{20}$ such that $0 \leq c_{21}g_2(n) \leq f_2(n) \leq c_{22}g_2(n)$ for all $n \geq n_{20}$.

Therefore, $(f_1 \cdot f_2)(n) \leq c_{12}c_{22}g_1(n)g_2(n)$ for $n \geq n_0 = \max \{n_{10}, n_{20}\}$.

In addition, $(f_1 \cdot f_2)(n) \geq c_{11}g_1(n)c_{21}g_2(n) = c_{11}c_{21}g_1(n)g_2(n)$ for $n \geq n_0 = \max \{n_{10}, n_{20}\}$.

With $c_1 = c_{11}c_{21}$, $c_2 = c_{12}c_{22}$ and $n_0 = \max \{n_{10}, n_{20}\}$, we have that $0 \leq c_1g_1(n)g_2(n) \leq (f_1 \cdot f_2)(n) \leq c_2g_1(n)g_2(n)$ for all $n \geq n_0$. 