The Bellman-Ford Algorithm is the following.

Bellman-Ford\((G, w, s)\)

1. Initialize\((G, s)\)
2. for \(i \leftarrow 1\) to \(|V| - 1\) do
3. for each edge \((u, v) \in E\) do
4. Relax\((u, v, w)\)
5. for each edge \((u, v) \in E\) do
6. if \(d[v] > d[u] + w(u, v)\)
7. then return FALSE
8. return TRUE

Fill in the blanks to complete the proof of the following lemma.

**Lemma:** Let \(G = (V, E)\) be a weighted, directed graph with a source vertex \(s\) and weight function \(w: E \rightarrow R\). Assume that \(G\) contains no negative-weight cycles that are reachable from \(s\). Then the Bellman-Ford algorithm terminates with \(d[x] = \delta(s, x)\) for every vertex \(x\) that is reachable from \(s\).

**Proof:** Let \(x\) be reachable from \(s\). Let \(p = <x_0, x_1, \ldots, x_k>\), where \(x_0 = s\) and \(x_k = x\), be the shortest path from \(s\) to \(x\).

\[k \leq |V| - 1\]

The relationship between \(k\) and \(|V|\) is ________________.

Each pass of the algorithm relaxes all the edges of the graph. Among the edges relaxed

\[1, 2, \ldots, |V| - 1\]

in the \(i\)th pass, for \(i = \) ________________ is ________________.

\(\delta(s, x_i)\)

After this edge is relaxed, we have that \(d[x_i] = \) ________________.

\[d[x] = \delta(s, x)\]

Therefore, at the end of the algorithm, ________________.
The Bellman-Ford Algorithm is the following.

Bellman-Ford\((G, w, s)\)

\[
\begin{align*}
1 & \text{ Initialize}\(G, s\) \\
2 & \text{ for } i \leftarrow 1 \text{ to } |V| - 1 \text{ do} \\
3 & \quad \text{ for each edge } (u, v) \in E \text{ do} \\
4 & \quad \quad \text{ Relax}(u, v, w) \\
5 & \quad \text{ for each edge } (u, v) \in E \text{ do} \\
6 & \quad \quad \text{ if } d[v] > d[u] + w(u, v) \\
7 & \quad \quad \quad \text{ then return FALSE} \\
8 & \text{ return TRUE}
\end{align*}
\]

Proof: Let \(z\) be reachable from \(s\). Let \(p = < z_0, z_1, \ldots, z_m >\), where \(z_0 = s\) and \(z_m = z\), be

\[\ldots\]

The relationship between \(m\) and \(|V|\) is \______________\.

Each pass of the algorithm relaxes all the edges of the graph. Among the edges relaxed in the \(i\)th pass, for \(i = \_______________\) is \______________\.

After this edge is relaxed, we have that \(d[z_i] = \_______________\.

Therefore, at the end of the algorithm, \______________\.