Let $G = (V,E)$ be an undirected graph to which BFS was applied. Let $G_\Pi = (V_\Pi,E_\Pi)$ be the BFS tree, where $V_\Pi = \{ v \in V : \Pi[v] \neq NULL \} \cup \{ s \}$, and $E_\Pi = \{ (\Pi[v],v) \in E : v \in V_\Pi \setminus \{ s \} \}$.

Let $u_1$, $u_2$ and $u_3$ be three vertices in $V$ such that $u_1$ is an ancestor of $u_2$ in the BFS tree, and $u_2$ is an ancestor of $u_3$ in the BFS tree.

(1) Is it possible that the edge $(u_1,u_3)$ exists in $G_\Pi$?

(2) Is it possible that the edge $(u_1,u_3)$ exists in $G$?

Answers:

(1) Yes/No. No. Explanation:
From the fact that $u_1$ is an ancestor of $u_2$ we have that $d[u_2] \geq d[u_1]+1$.
From the fact that $u_2$ is an ancestor of $u_3$ we have that $d[u_3] \geq d[u_2]+1$.
From these two inequalities we have that $d[u_3] \geq d[u_1]+2$.
If the edge $(u_1,u_3)$ exists in $G_\Pi$ then $d[u_3] = d[u_1]+1$.
It is not possible to have $d[u_3] \geq d[u_1]+2$ and $d[u_3] = d[u_1]+1$. Therefore, the edge $(u_1,u_3)$ cannot exist.

(2) Yes/No. No. Explanation:
From the fact that $u_1$ is an ancestor of $u_2$ and $u_2$ is an ancestor of $u_3$ we have that the vertices were entered in the order $u_1-u_2-u_3$.
Suppose that the edge $(u_1,u_3)$ exists.
After $u_1$ was entered, its adjacent vertices, including $u_3$, should have been labeled.
This yields $d[u_3] = d[u_1]+1$.
From the same arguments as above, this leads to a contradiction.
Let $G = (V,E)$ be an undirected graph to which BFS was applied. Let $G_{\Pi} = (V_{\Pi}, E_{\Pi})$ be the BFS tree, where $V_{\Pi} = \{v \in V: \Pi[v] \neq NULL \} \cup \{s\}$, and $E_{\Pi} = \{(\Pi[v], v) \in E : v \in V_{\Pi}\setminus \{s\}\}$.

Let $u_1$, $u_2$ and $u_3$ be three vertices in $V$ such that $u_1$ is a descendant of $u_2$ in the BFS tree, and $u_2$ is a descendant of $u_3$ in the BFS tree.

(1) Is it possible that the edge $(u_1, u_3)$ exists in $G_{\Pi}$?

(2) Is it possible that the edge $(u_1, u_3)$ exists in $G$?

Answers:

(1) Yes/No. No. Explanation:
From the fact that $u_1$ is a descendant of $u_2$ we have that $d[u_1] \geq d[u_2]+1$.
From the fact that $u_2$ is a descendant of $u_3$ we have that $d[u_2] \geq d[u_3]+1$.
From these two inequalities we have that $d[u_1] \geq d[u_3]+2$.
If the edge $(u_1, u_3)$ exists in $G_{\Pi}$ then $d[u_3] = d[u_1]+1$, or $d[u_1] = d[u_3]-1$.
It is not possible to have $d[u_1] \geq d[u_3]+2$ and $d[u_1] = d[u_3]-1$. Therefore, the edge $(u_1, u_3)$ cannot exist.

(2) Yes/No. No. Explanation:
From the fact that $u_1$ is a descendant of $u_2$ and $u_2$ is a descendant of $u_3$ we have that the vertices were entered in the order $u_3-u_2-u_1$.
Suppose that the edge $(u_1, u_3)$ exists.
After $u_3$ was entered, its adjacent vertices, including $u_1$, should have been labeled.
This yields $d[u_1] = d[u_3]+1$.
From the same arguments as above, this leads to a contradiction.