Lemma: Let $G = (V,E)$ be an undirected graph that is acyclic, but if any edge is added to $E$, the resulting graph contains a cycle. Prove that $G$ is connected.

Proof: Let us consider a pair of vertices $u$ and $v$ that are not connected by an edge. Let us add the edge $(u,v)$ to the graph.

The graph now contains a cycle that includes the edge $(u,v)$ (since the __________________________________________ cycle did not exist earlier). Removing the edge $(u,v)$ from the cycle __________________________________________ leaves a path from $u$ to $v$. The path exists in the original graph.

Since this applies to every $u$ and $v$ that are not connected by an edge, the graph is connected.
Part of the proof of the following lemma is missing. Complete the proof by filling in the blanks.

Write your answer clearly and concisely, and do not use additional space.

Lemma: Let $G = (V,E)$ be an undirected graph that is connected, but if any edge is removed from $E$, the resulting graph is disconnected. Prove that $G$ is acyclic.

Proof: Assume by contradiction that $G$ contains a cycle $v_1-v_2-\cdots-v_k$, with $v_1 = v_k$. Let us remove from $G$ an edge $(v_i,v_{i+1})$ that belongs to the cycle.

Consider a path in the original graph that uses the edge $(v_i,v_{i+1})$.

The path can be updated to use the subpath $v_{i} - \cdots - v_1 = v_k - \cdots - v_{i+1}$ from $v_i$ to $v_{i+1}$ along the cycle (or part of it).

Therefore, the removal of the edge does not break the connectivity of the graph. This is a contradiction, and $G$ must be acyclic.