First Name: ___________________ Last Name: ____________________

I certify that I have neither given nor received unauthorized aid on this quiz.
Signed: ___________________

Use only the space provided on this page to answer the following questions.

For each one of the following claims, indicate whether it is true or false. If it is true, prove it. If it is false, show a counter example.

(1) If \( f(n) \) is \( \Theta(g(n)) \) then \( f(n) \) is \( O(g(n)) \).
(2) If \( f(n) \) is \( O(g(n)) \) then \( g(n) \) is \( \Omega(f(n)) \).

Answers:

(1) True. If \( f(n) \) is \( \Theta(g(n)) \) then there exist positive constants \( c_1, c_2 \) and \( n_0 \) such that
\[ 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \]
for all \( n \geq n_0 \).

From \( f(n) \leq c_2 g(n) \) for all \( n \geq n_0 \) we have that \( f(n) \) is \( O(g(n)) \).

(2) True. If \( f(n) \) is \( O(g(n)) \) then there exist positive constants \( c \) and \( n_0 \) such that
\[ f(n) \leq cg(n) \]
for all \( n \geq n_0 \).

From \( f(n) \leq cg(n) \) we have that \( g(n) \geq \frac{1}{c} f(n) \).

Using \( c' = \frac{1}{c} \) and \( n_0 \) we have that \( g(n) = \Omega(f(n)) \).
For each one of the following claims, indicate whether it is true or false. If it is true, prove it. If it is false, show a counter example.

1) If $f(n)$ is $\Theta(g(n))$ then $f(n)$ is $\Omega(g(n))$.

2) If $f(n)$ is $\Omega(g(n))$ then $g(n)$ is $O(g(n))$.

Answers:

1) True. If $f(n)$ is $\Theta(g(n))$ then there exist positive constants $c_1,c_2$ and $n_0$ such that

$0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0$.

From $c_1 g(n) \leq f(n)$ for all $n \geq n_0$ we have that $f(n)$ is $\Omega(g(n))$.

2) True. If $f(n)$ is $\Omega(g(n))$ then there exist positive constants $c$ and $n_0$ such that

$0 \leq cg(n) \leq f(n)$ for all $n \geq n_0$.

From $cg(n) \leq f(n)$ we have that $g(n) \leq \frac{1}{c}f(n)$.

Using $\tilde{c} = \frac{1}{c}$ and $n_0$ we have that $g(n) = O(f(n))$. 